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Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Fast-rolling relaxion

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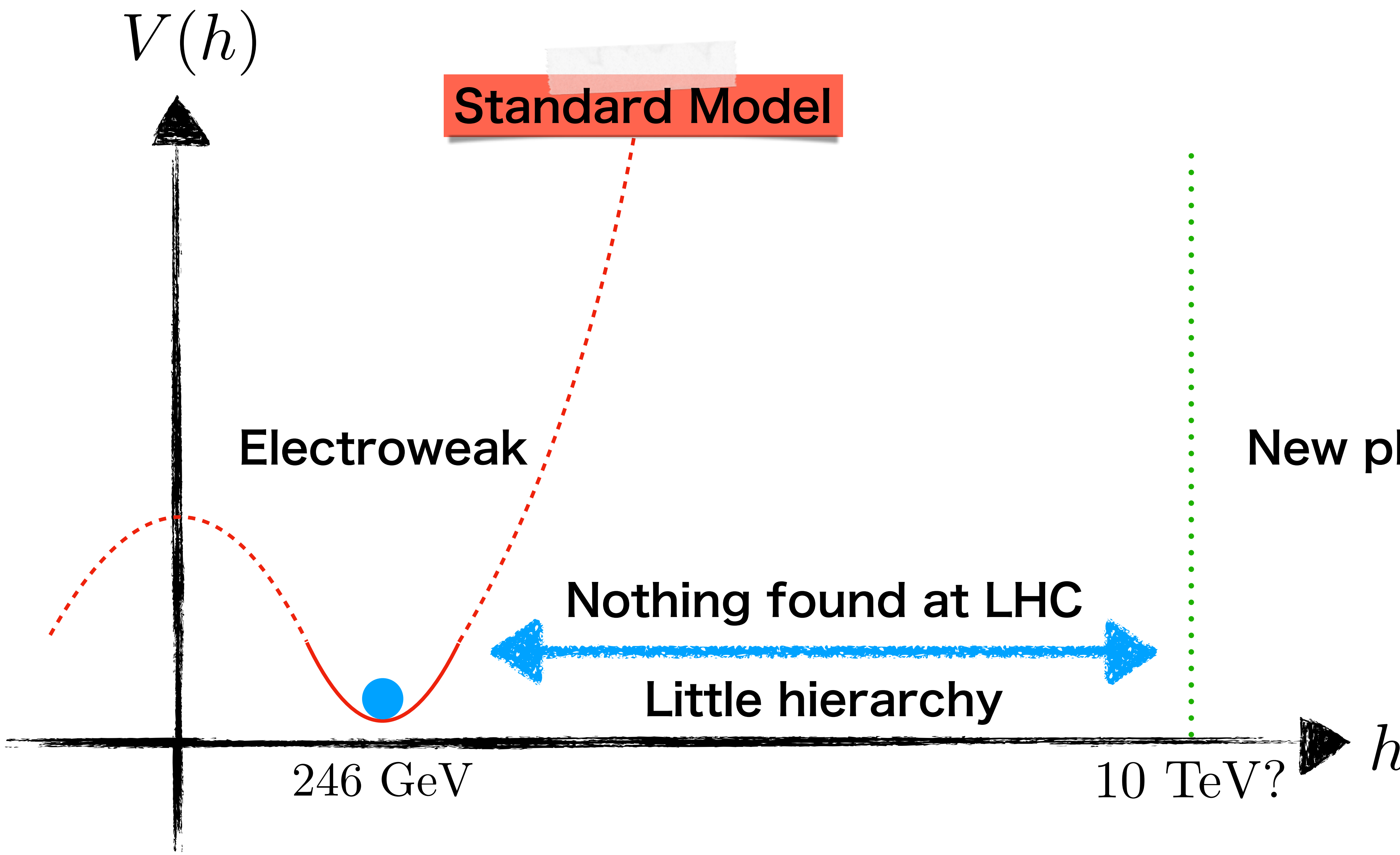
1904.02545/hep-ph

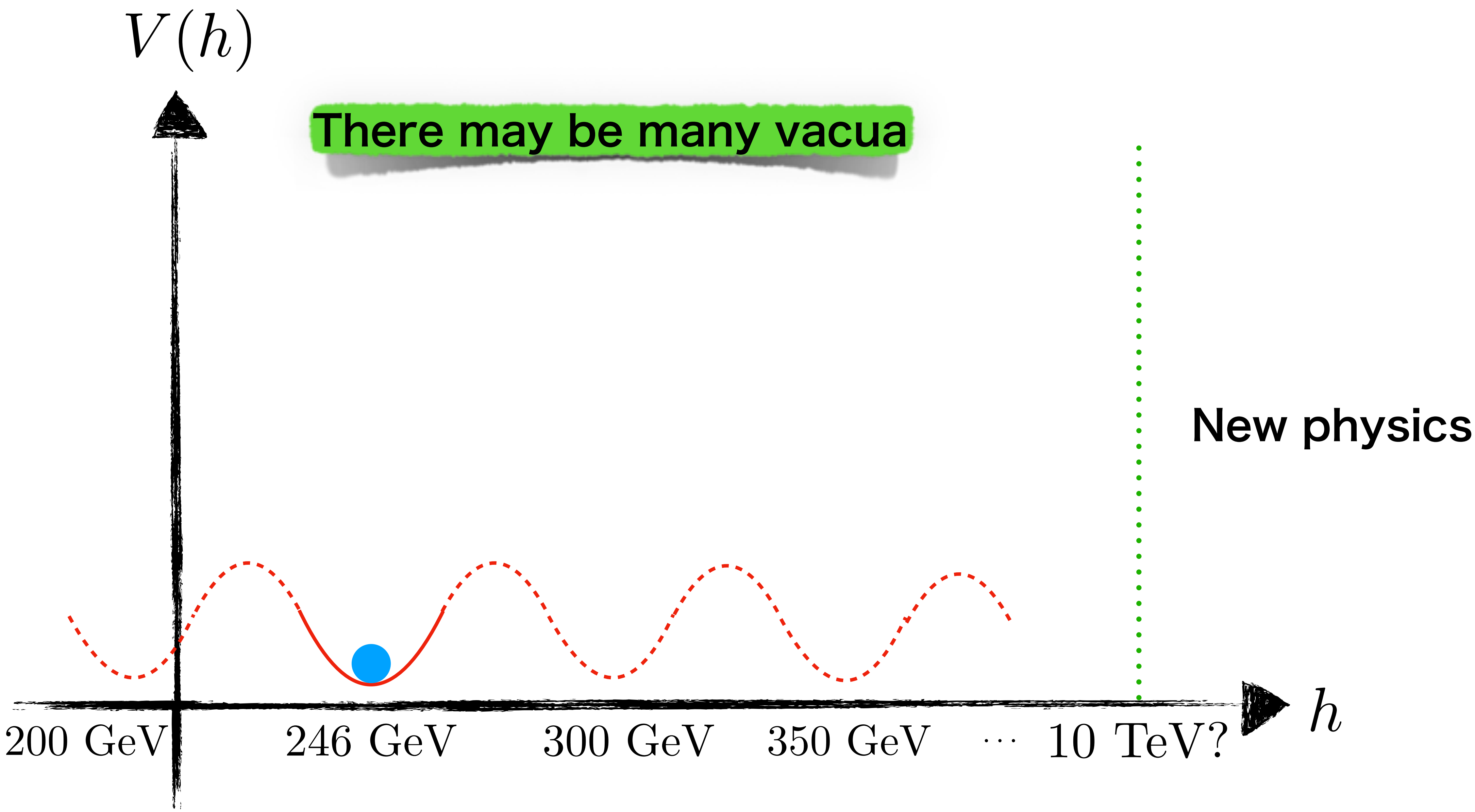
In collaboration with M. Ibe (ICRR, IPMU) and M. Suzuki (TDLI)

Introduction

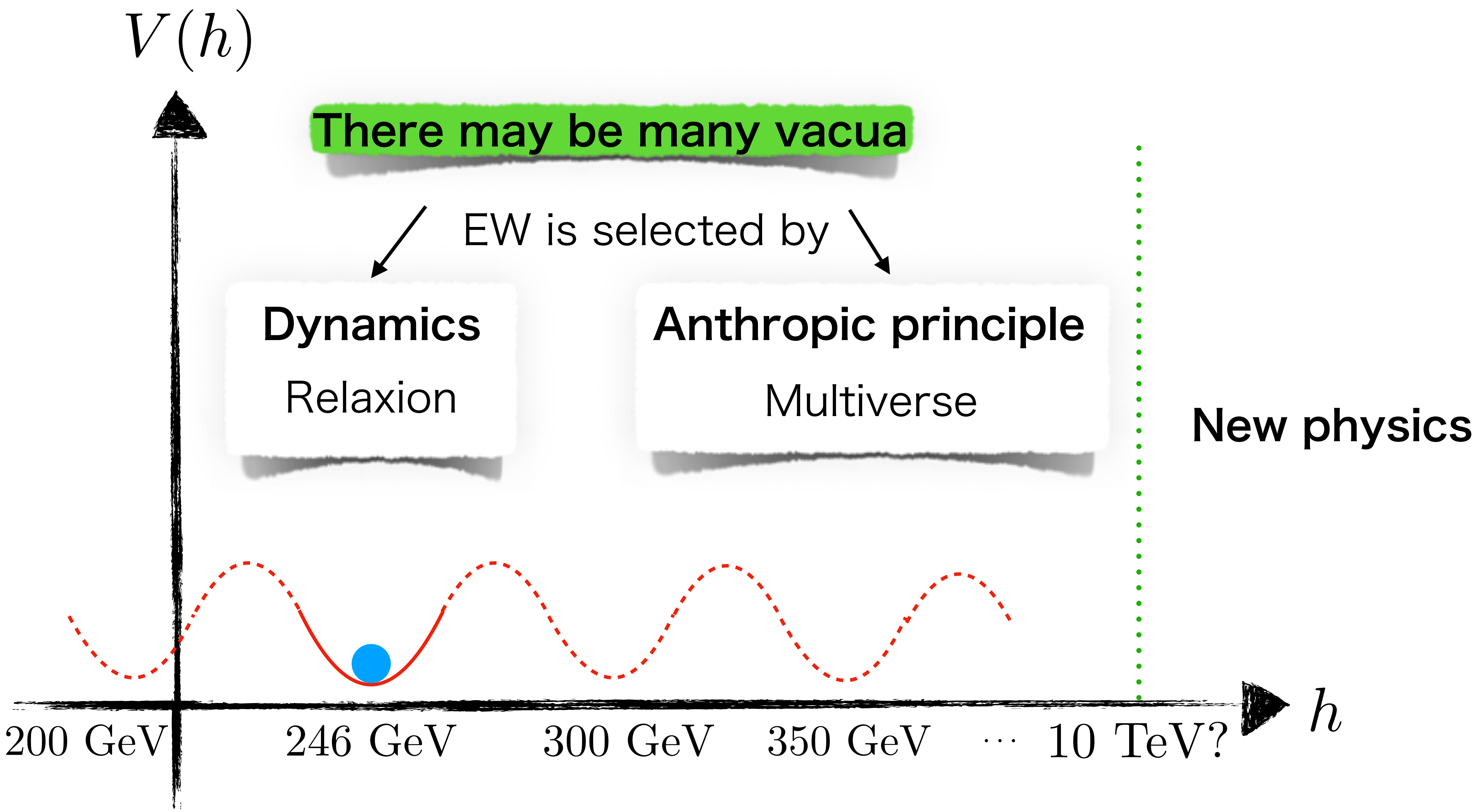
Little hierarchy



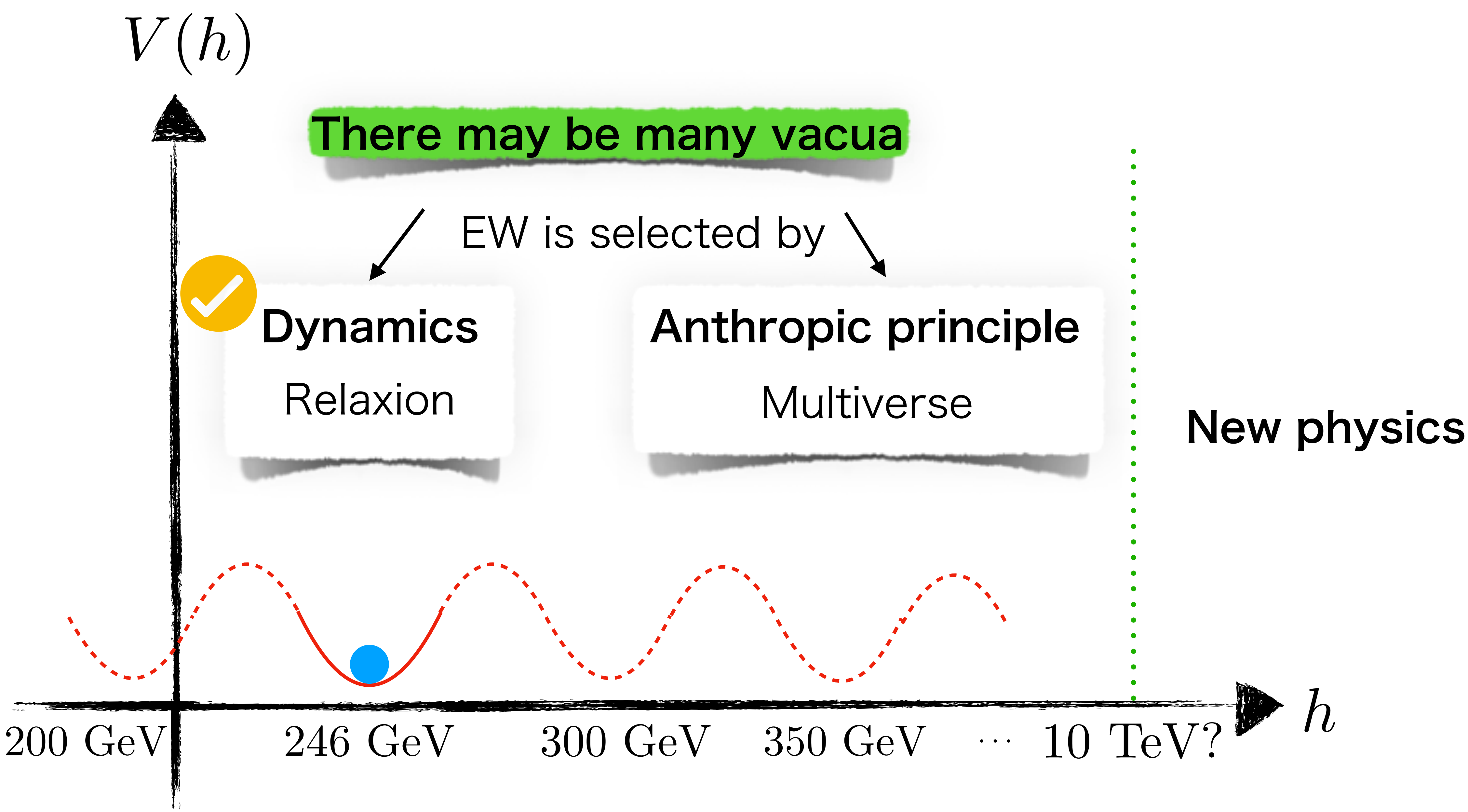
Many EW vacua



Many EW vacua



Many EW vacua



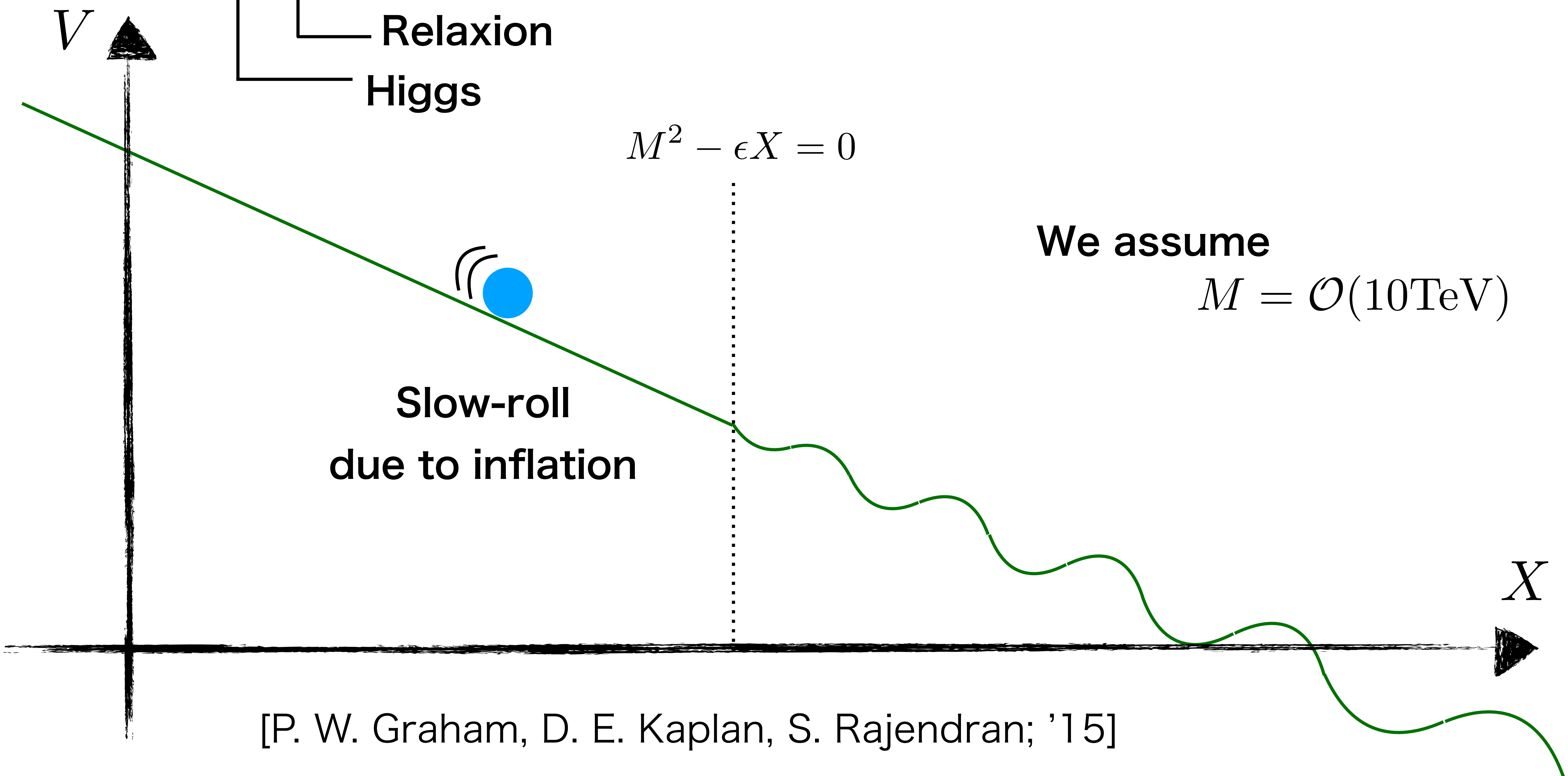
Relaxion mechanism

Higgs mass Slope Back reaction

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

Relaxion

Higgs



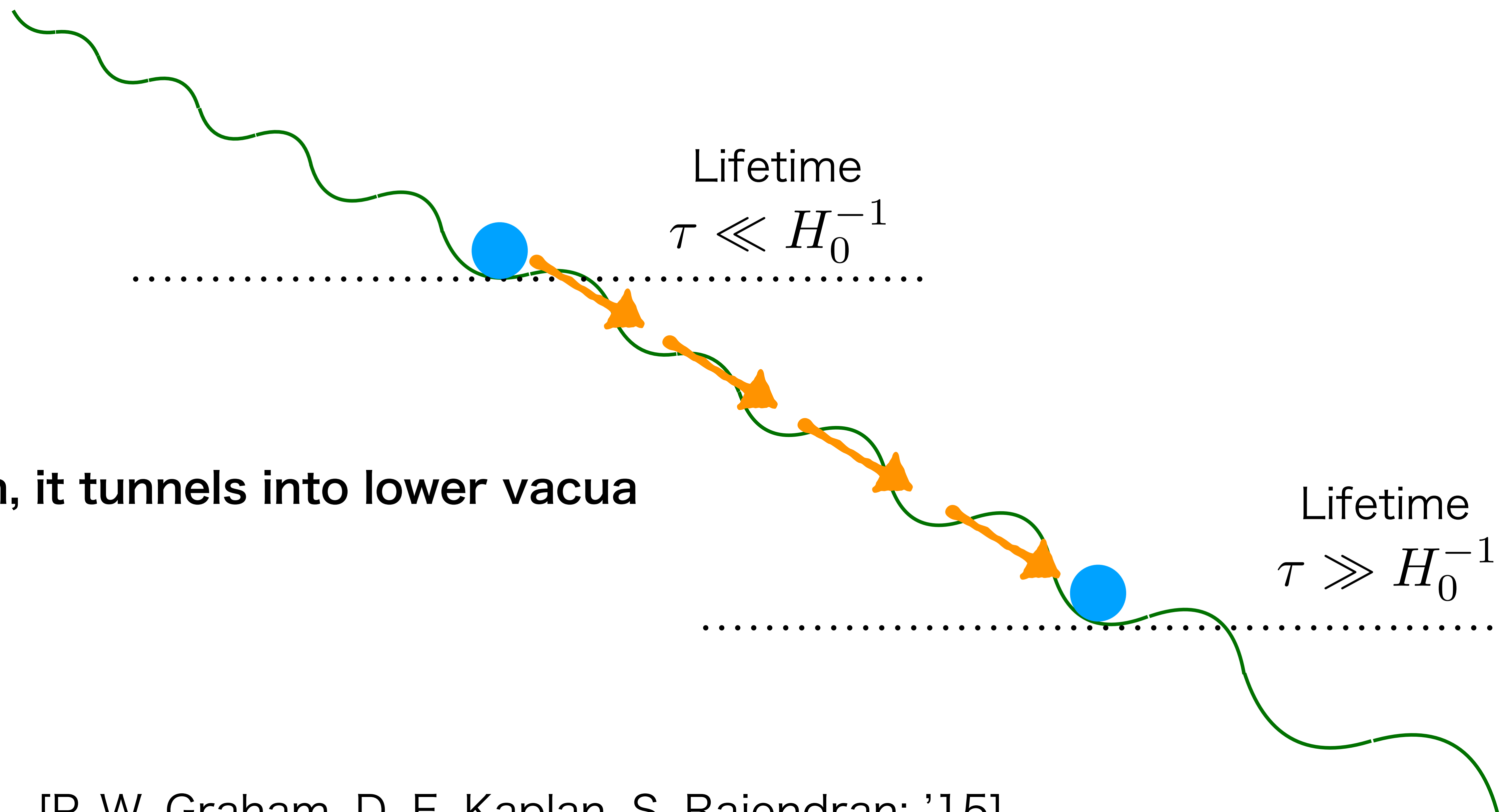
We assume

$M = \mathcal{O}(10\text{TeV})$

[P. W. Graham, D. E. Kaplan, S. Rajendran; '15]

Quantum tunneling

The relaxion stops classically at $\dfrac{dV}{dX} = 0$



Then, it tunnels into lower vacua

[P. W. Graham, D. E. Kaplan, S. Rajendran; '15]

Timescale of tunneling

At the last tunneling, $\gamma \ll H_0^4$

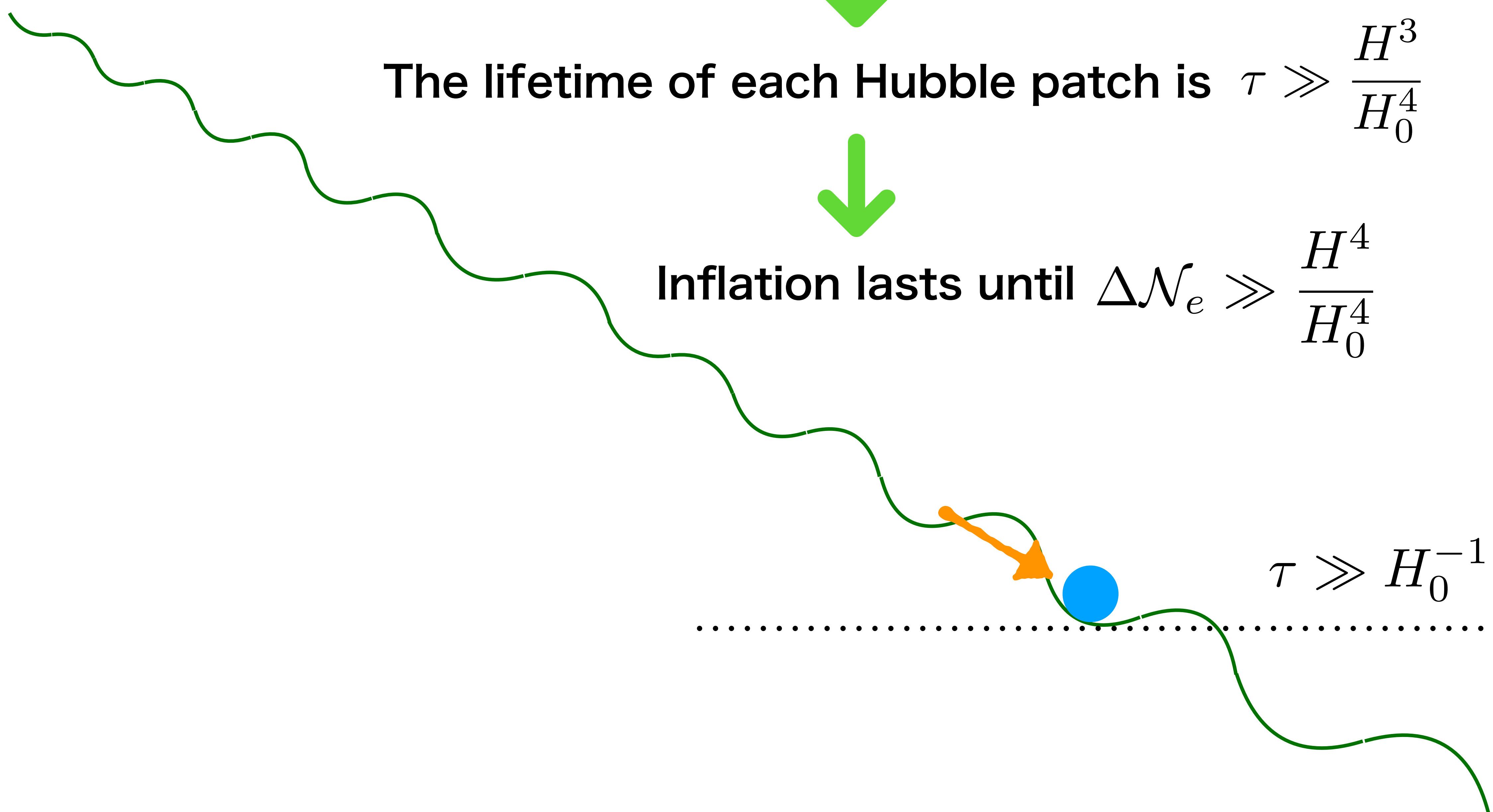
Bubble nucleation rate

↓

The lifetime of each Hubble patch is $\tau \gg \frac{H^3}{H_0^4}$

↓

Inflation lasts until $\Delta \mathcal{N}_e \gg \frac{H^4}{H_0^4}$



Timescale of tunneling

At the last tunneling, $\gamma \ll H_0^4$

Bubble nucleation rate

The lifetime of each Hubble patch is $\tau \gg \dfrac{H^3}{H_0^4}$

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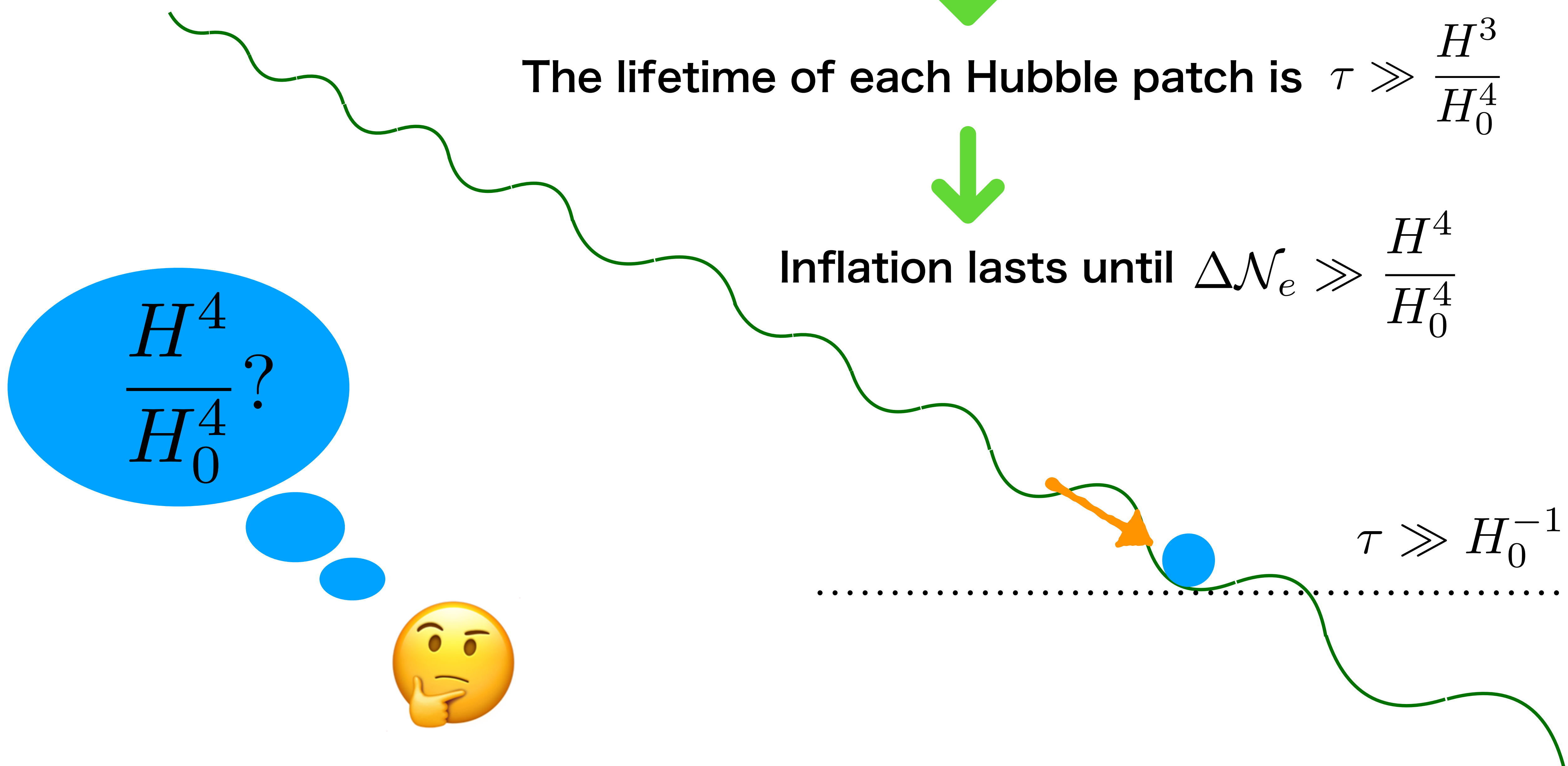


Too large e-folds

$$\mathcal{N}_e \gg \frac{H^4}{H_0^4} \simeq 10^{156} \left(\frac{H}{1 \text{ MeV}} \right)^4$$

It is argued that a too large number of e-folds will cause problems in inflation sector

e.g.) [K. Choi, S. H. Im; '16]
To avoid fine-tuning, we need $\mathcal{N}_e \lesssim 10^{24}$ for slow-roll inflation
(in the context of scanning time)

(possible solution may be N. Kitajima, Y. Tada, F. Takahashi' 19)

[H. Matsui, F. Takahashi; '18, K. Dimopoulos '18,⋯]

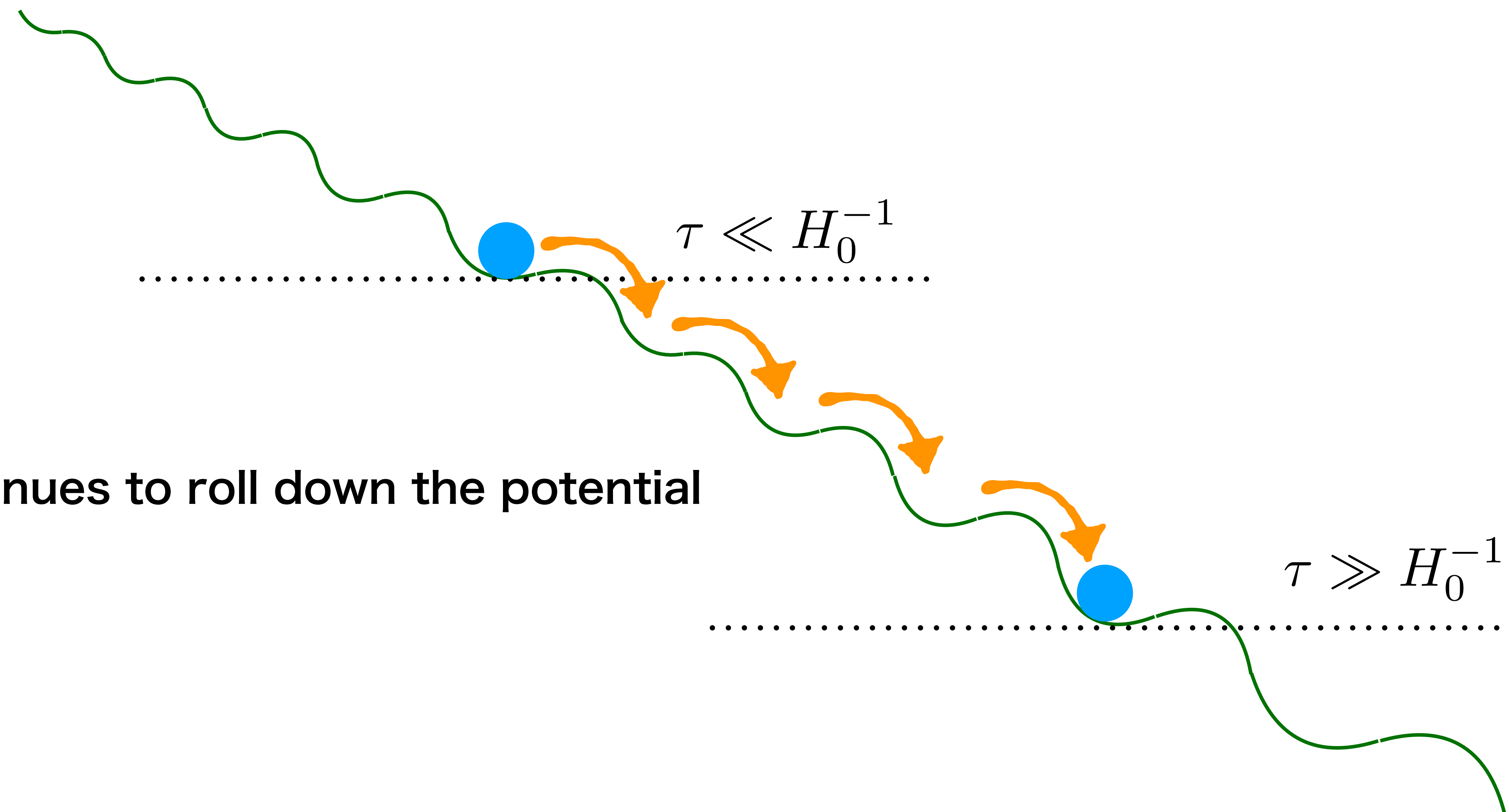
Eternal inflation is generically incompatible with the (refined) de Sitter swampland conjecture

+ Why not multiverse?

Fast-rolling relaxion

Fast-roll

The relaxion DOES NOT stop classically at $\dfrac{dV}{dX} = 0$



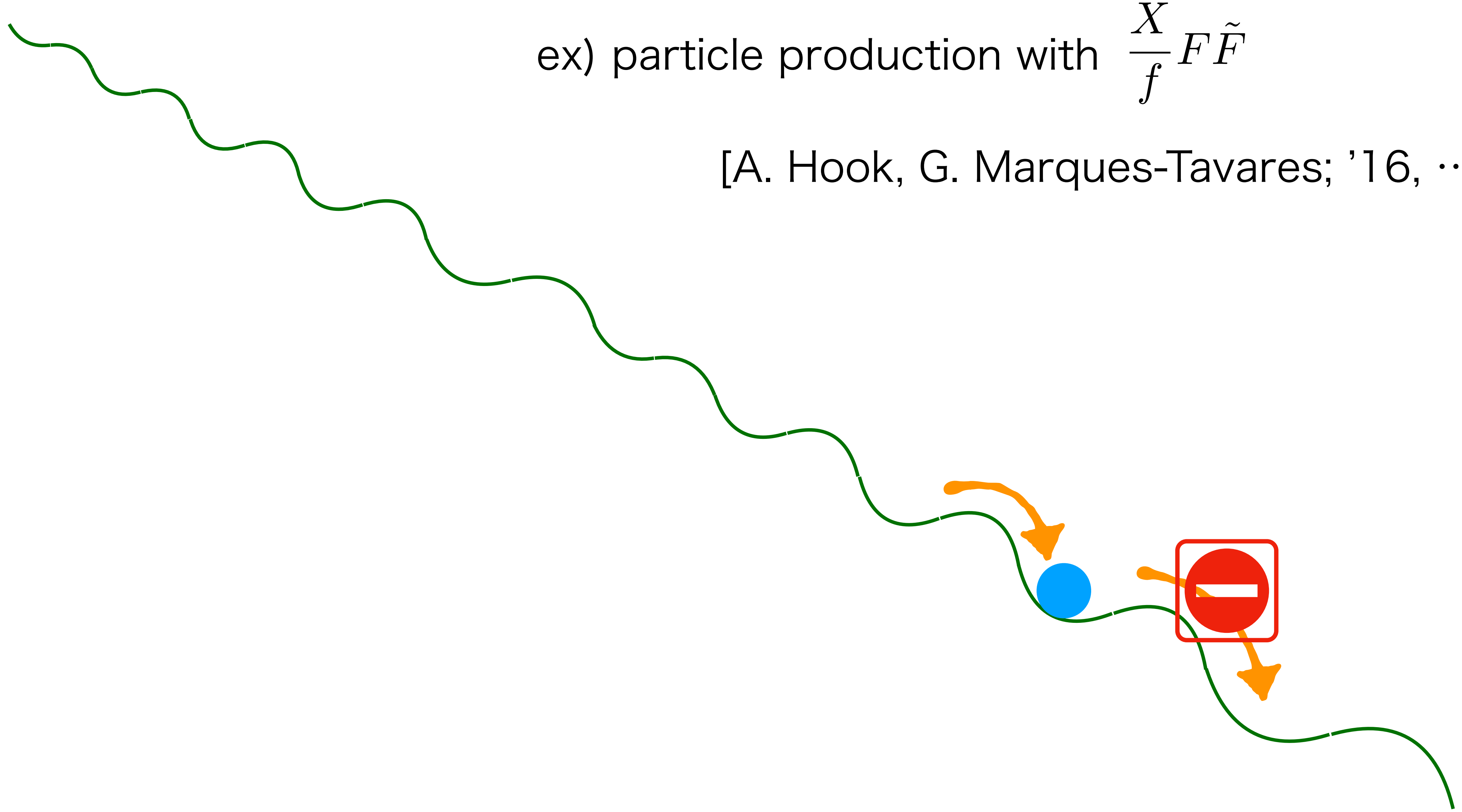
It continues to roll down the potential

Stopping mechanism

We need another stopping mechanism

ex) particle production with $\frac{X}{f}F\tilde{F}$

[A. Hook, G. Marques-Tavares; '16, ⋯]



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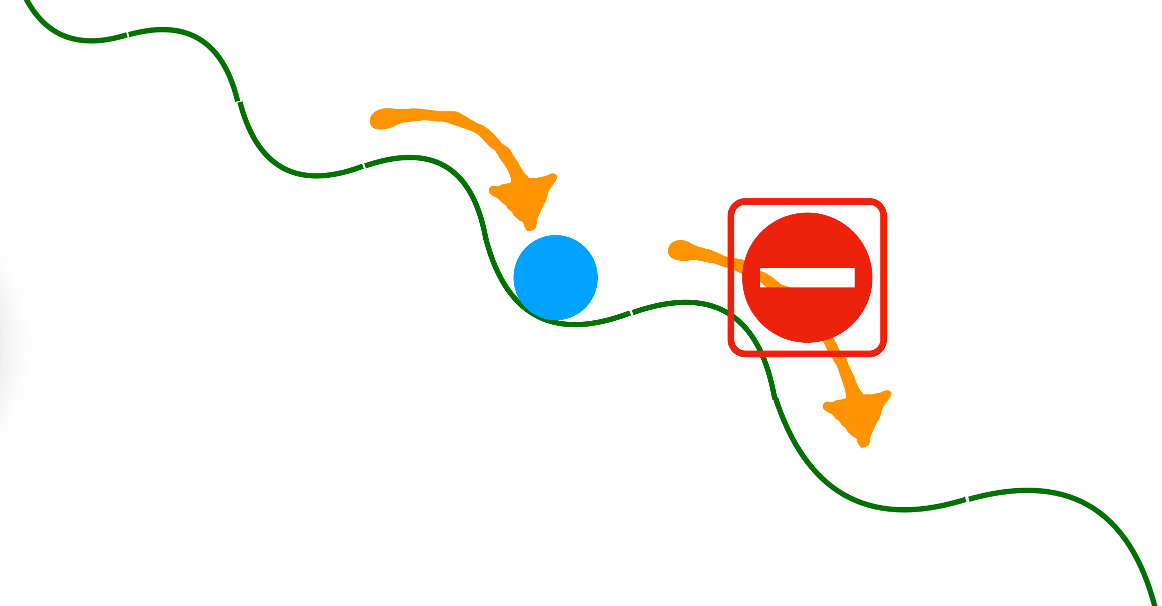
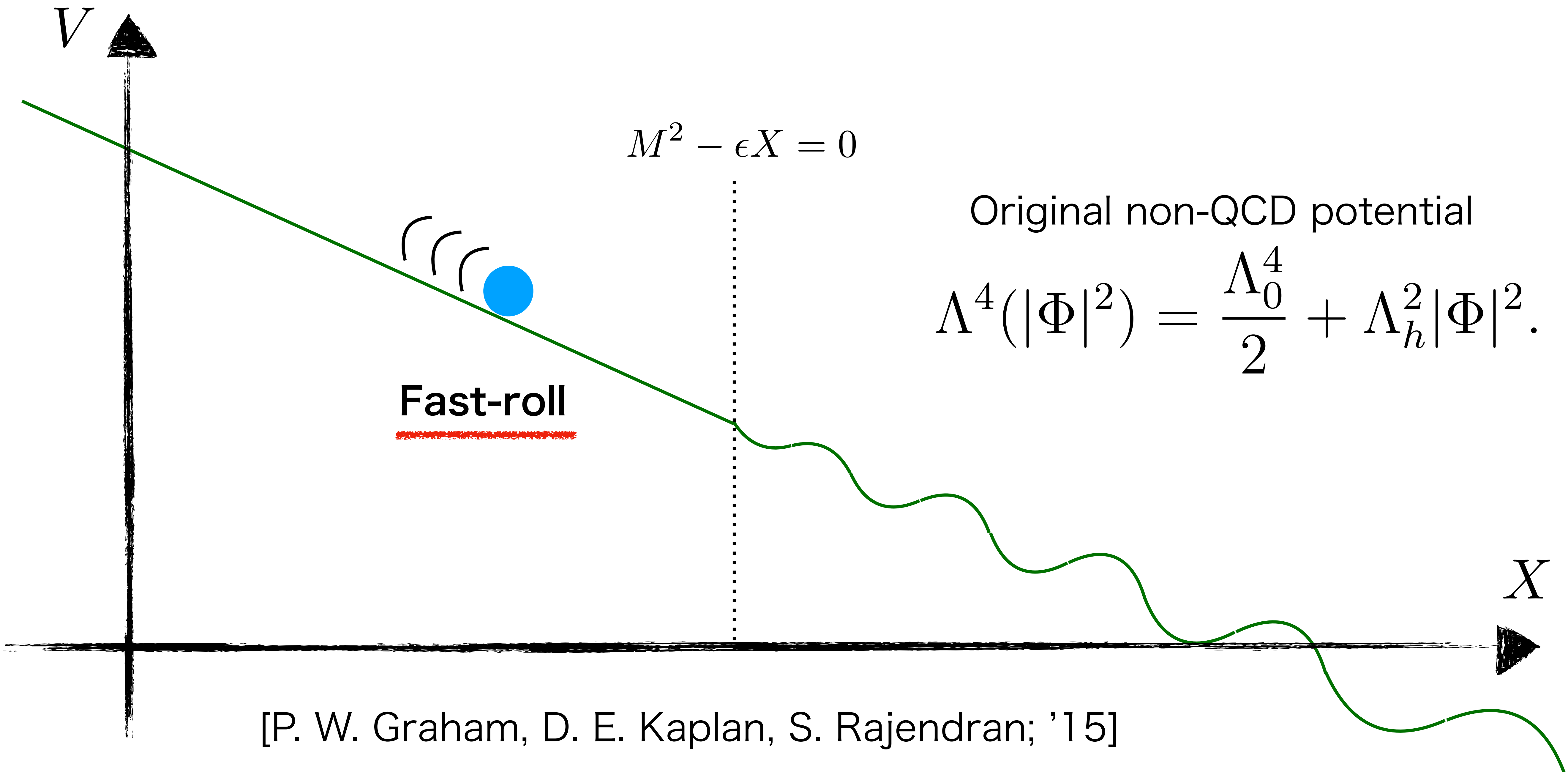


But, we want something

Model

No extension!

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$



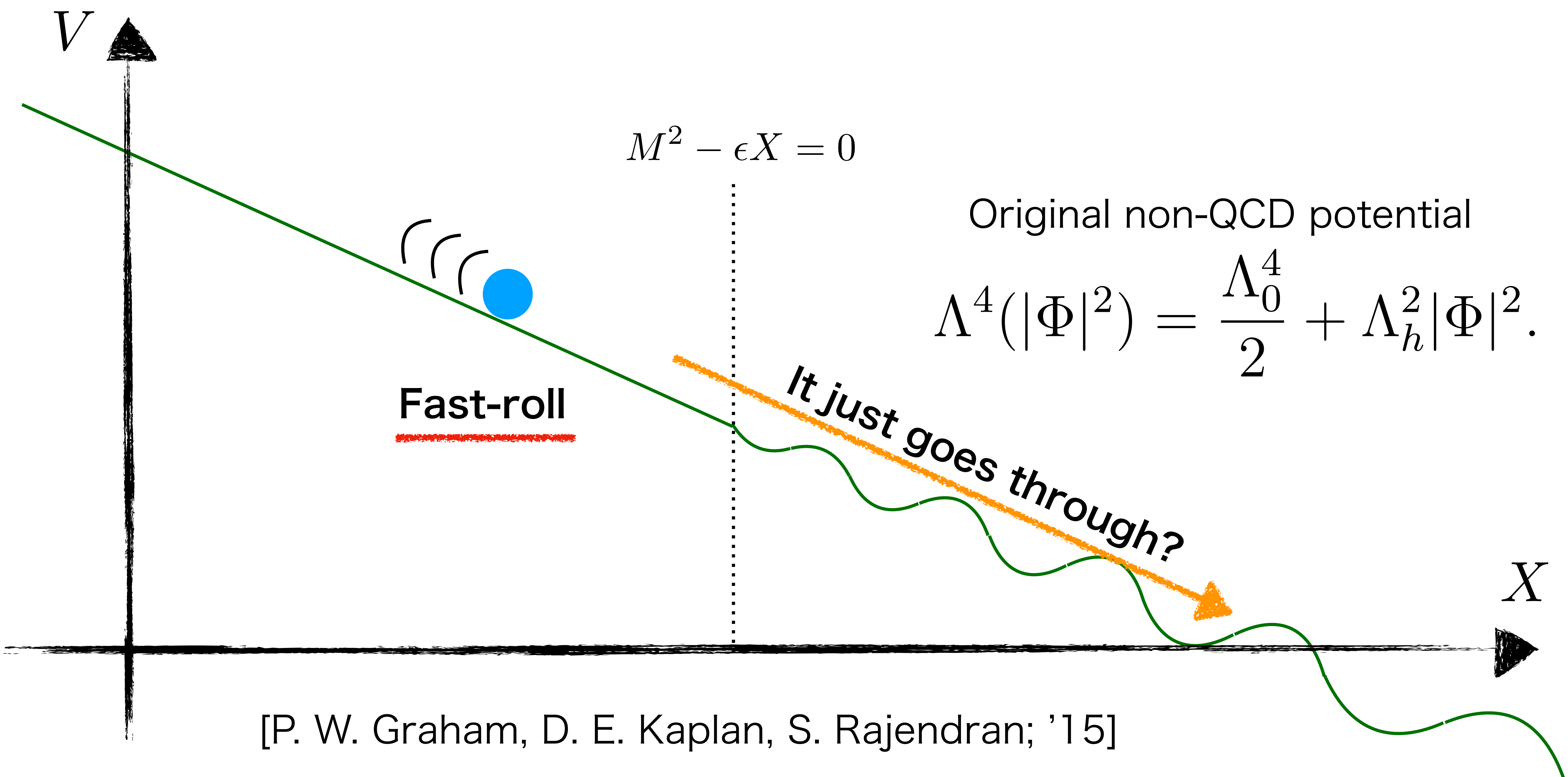
Original non-QCD potential

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2.$$

[P. W. Graham, D. E. Kaplan, S. Rajendran; '15]

No extension!

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

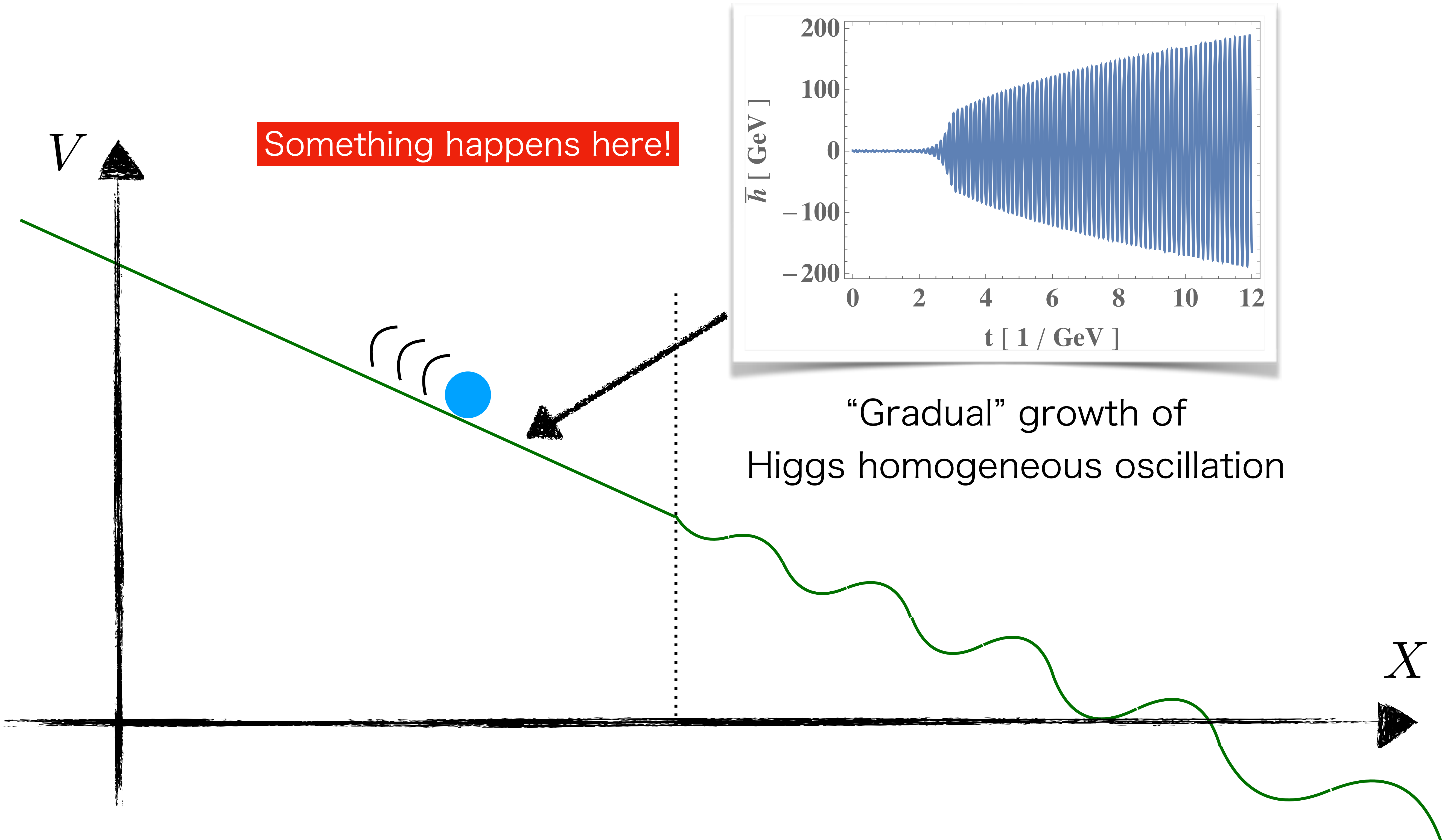


Original non-QCD potential

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2.$$

[P. W. Graham, D. E. Kaplan, S. Rajendran; '15]

Higgs homogeneous oscillation



“Gradual” growth of
Higgs homogeneous oscillation

Contents

- Introduction
- Edge solution
- Stopping mechanism
- Constraints
- Summary

Edge solution

Equations of motion

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon\left(rM^2 + \frac{\bar{h}^2}{2}\right) + \frac{\cancel{\Lambda_0^4} + \Lambda_h^2\bar{h}^2}{2f}\sin\frac{\bar{X}}{f} \ ,$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} = -(M^2 - \epsilon\bar{X})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2\bar{h}\cos\frac{\bar{X}}{f} \ ,$$

$\bar{X}$: Relaxion homogeneous mode

$\bar{h}$: Higgs homogeneous mode

Equations of motion

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon\left(rM^2 + \frac{\bar{h}^2}{2}\right) + \frac{\cancel{\Lambda_0^4} + \Lambda_h^2 \bar{h}^2}{2f} \sin\frac{\bar{X}}{f} \,,$$

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Terminal velocity

$$\dot{\bar{X}} \simeq \frac{\epsilon r M^2}{3H}$$

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$\bar{h}$: Higgs homogeneous mode

Equations of motion

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon\left(rM^2 + \frac{\bar{h}^2}{2}\right) + \frac{\cancel{\Lambda_6^4} + \Lambda_h^2\bar{h}^2}{2f}\sin\frac{\bar{X}}{f}\ ,$$

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Terminal velocity

$$\dot{\bar{X}} \simeq \frac{\epsilon r M^2}{3H}$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h\cos\omega t$$

$$\delta m^2 = f\omega = \frac{\epsilon r M^2}{3H}$$

$$m^2 = M^2 - \epsilon\bar{X}(0)$$

$\bar{X}$: Relaxion homogeneous mode

$\bar{h}$: Higgs homogeneous mode

Oscillatory solution

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos\omega t$$

Oscillatory solution

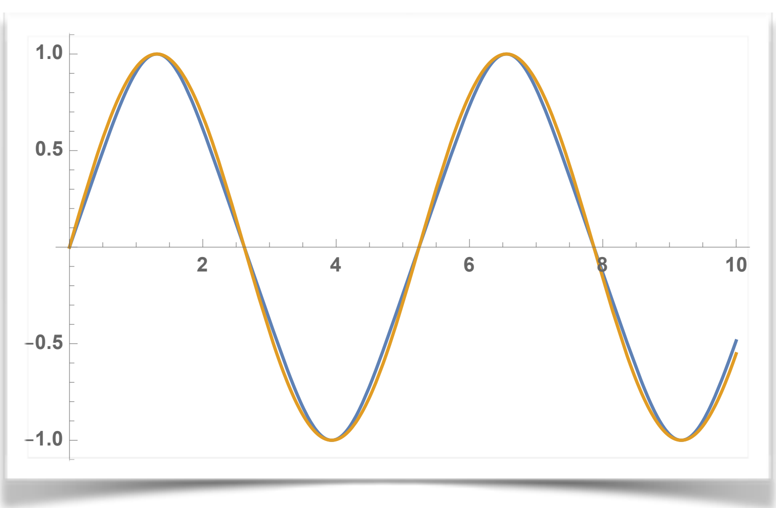
$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \cancel{\Lambda_h^2 h \cos\omega t}$$

Oscillatory solution

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \cancel{\Lambda_h^2 \bar{h} \cos\omega t}$$

$$\bar{h}(t) = \mathcal{A}\,\mathrm{sn}\left(\sqrt{m^2 + \frac{\lambda}{8}\mathcal{A}^2}t, -\frac{\mathcal{A}^2\lambda}{8m^2 + \mathcal{A}^2\lambda}\right)$$

$$\simeq \mathcal{A}\sin(\overline{m}(\mathcal{A})t)\ ,$$



$\mathcal{A}$: Amplitude of oscillation

Effective mass

$$\overline{m}^2(\mathcal{A}) = \frac{\pi^2(8m^2 + \mathcal{A}^2\lambda)}{32\left[K\left(-\frac{\mathcal{A}^2\lambda}{8m^2+\mathcal{A}^2\lambda}\right)\right]^2} \simeq m^2 + \frac{3}{16}\lambda\mathcal{A}^2$$

Parametric resonance

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \underline{\Lambda_h^2 h \cos\omega t}$$

$$\underbrace{(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3}_{\overline{m}^2(\mathcal{A})\bar{h}}$$

Parametric resonance

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \underline{\Lambda_h^2 h \cos\omega t}$$

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Mathieu equation

$$\rightarrow \quad \ddot{\bar{h}} \simeq -\overline{m}^2(\mathcal{A})\bar{h} - \Lambda_h^2 h \cos\omega t$$

Parametric resonance

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \cancel{\delta m^2 t})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \underline{\Lambda_h^2 h \cos\omega t}$$

$$\overline{m}^2(\mathcal{A})\bar{h}$$

Mathieu equation

$$\Rightarrow \quad \boxed{\ddot{\bar{h}} \simeq -\overline{m}^2(\mathcal{A})\bar{h} - \Lambda_h^2 h \cos\omega t}$$

Exponential growth of the amplitude occurs when

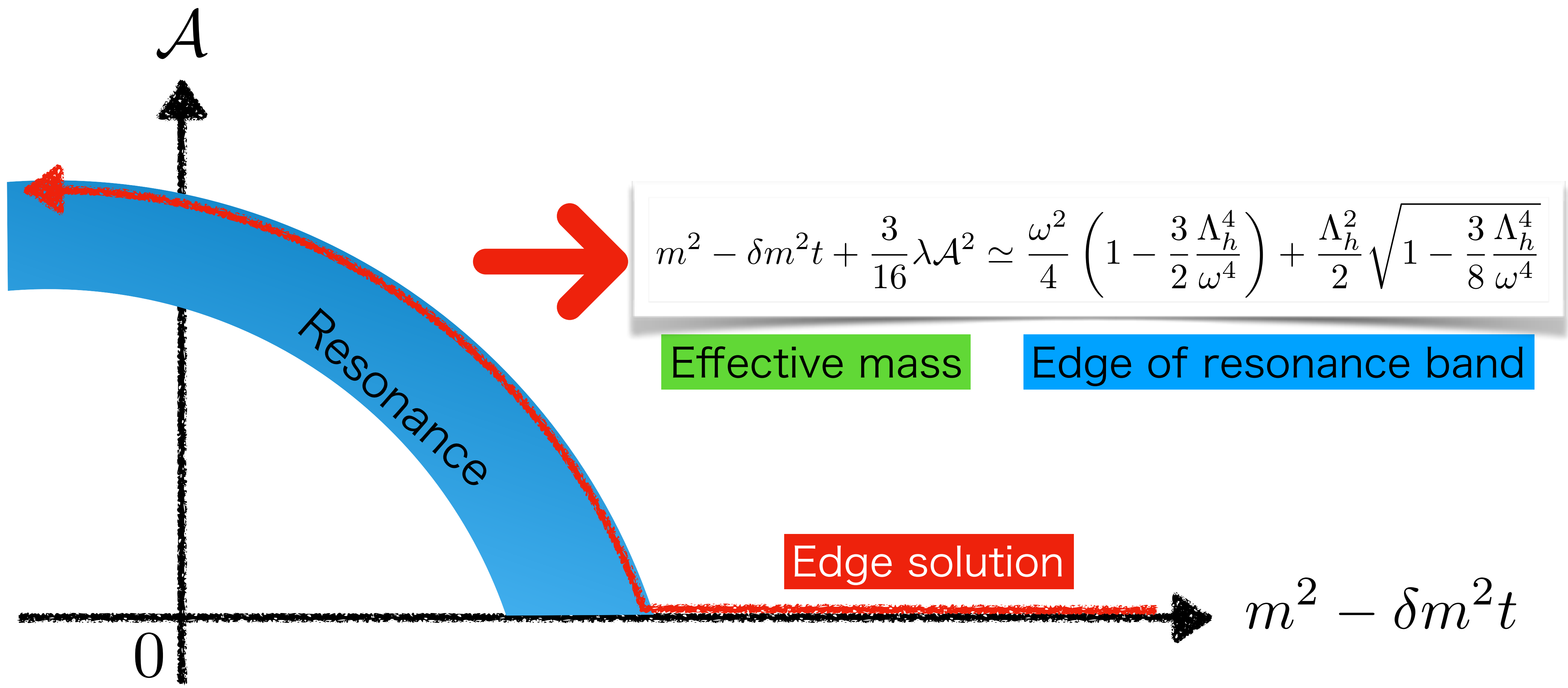
$$\frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) - \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}} \lesssim \overline{m}^2(\mathcal{A}) \lesssim \frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) + \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}}$$

Edge solution

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos\omega t$$

Edge solution

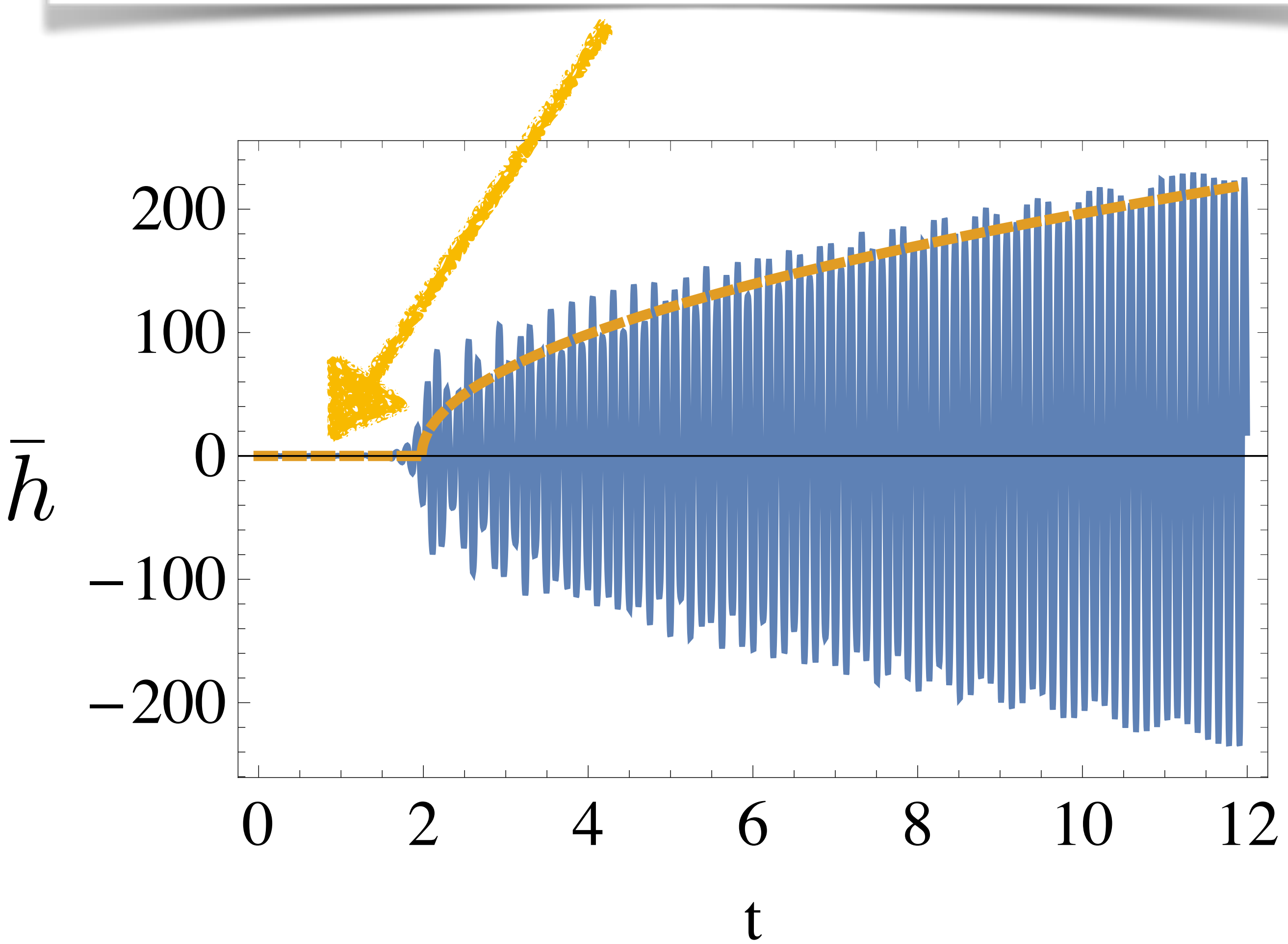
$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos\omega t$$



Numerical check

$$\ddot{\bar{h}} + \cancel{3H\dot{\bar{h}}} \simeq -(m^2 - \delta m^2 t)\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2 h \cos\omega t$$

$$m^2 - \delta m^2 t + \frac{3}{16}\lambda\mathcal{A}^2 \simeq \frac{\omega^2}{4}\left(1 - \frac{3}{2}\frac{\Lambda_h^4}{\omega^4}\right) + \frac{\Lambda_h^2}{2}\sqrt{1 - \frac{3}{8}\frac{\Lambda_h^4}{\omega^4}}$$

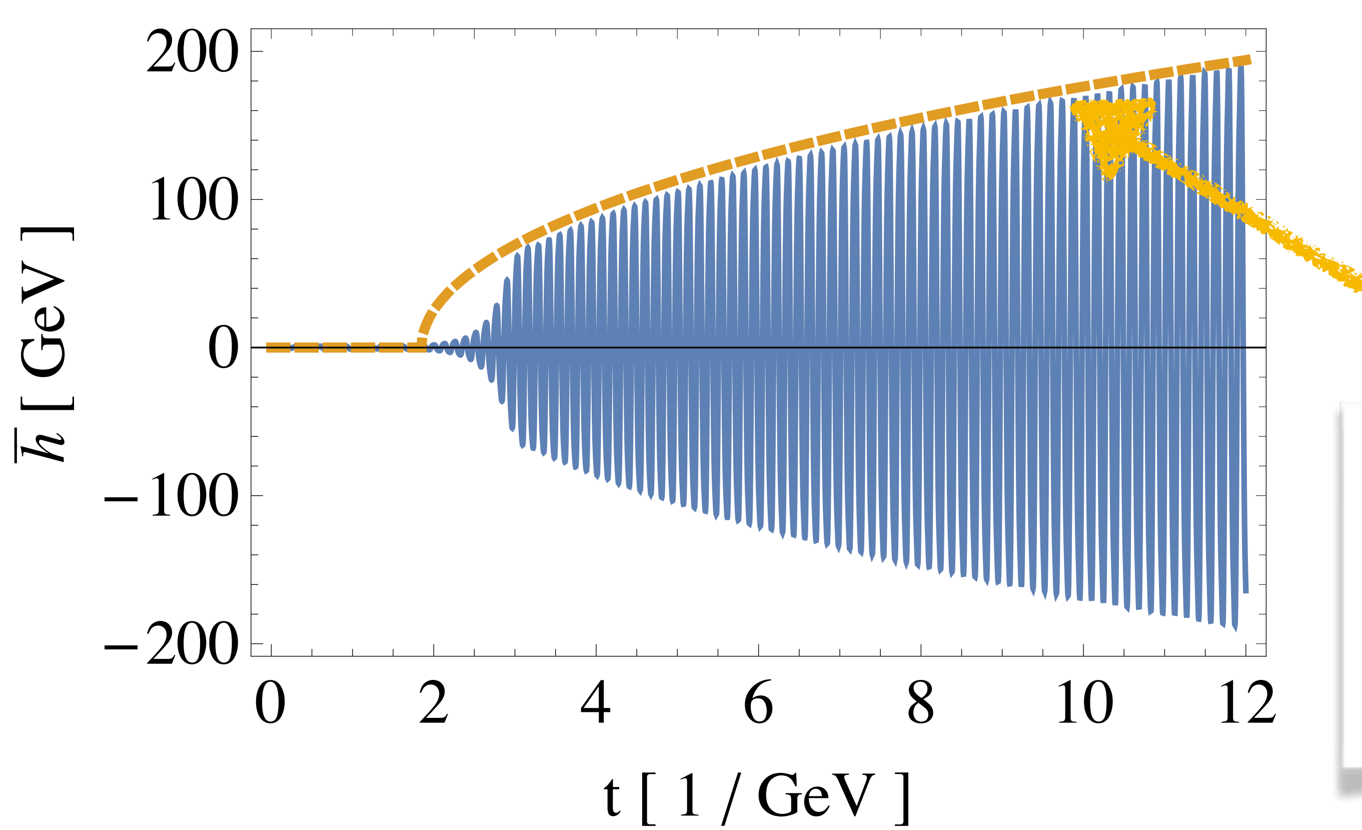


Edge solution (full)

Hubble friction, Back reactions, …

$$\ddot{\bar{X}} + 3H\dot{\bar{X}} = \epsilon\left(rM^2 + \frac{\bar{h}^2}{2}\right) + \frac{\Lambda_0^4 + \Lambda_h^2\bar{h}^2}{2f}\sin\frac{\bar{X}}{f} \ ,$$

$$\ddot{\bar{h}} + 3H\dot{\bar{h}} = -(M^2 - \epsilon\bar{X})\bar{h} - \frac{\lambda}{4}\bar{h}^3 - \Lambda_h^2\bar{h}\cos\frac{\bar{X}}{f} \ ,$$



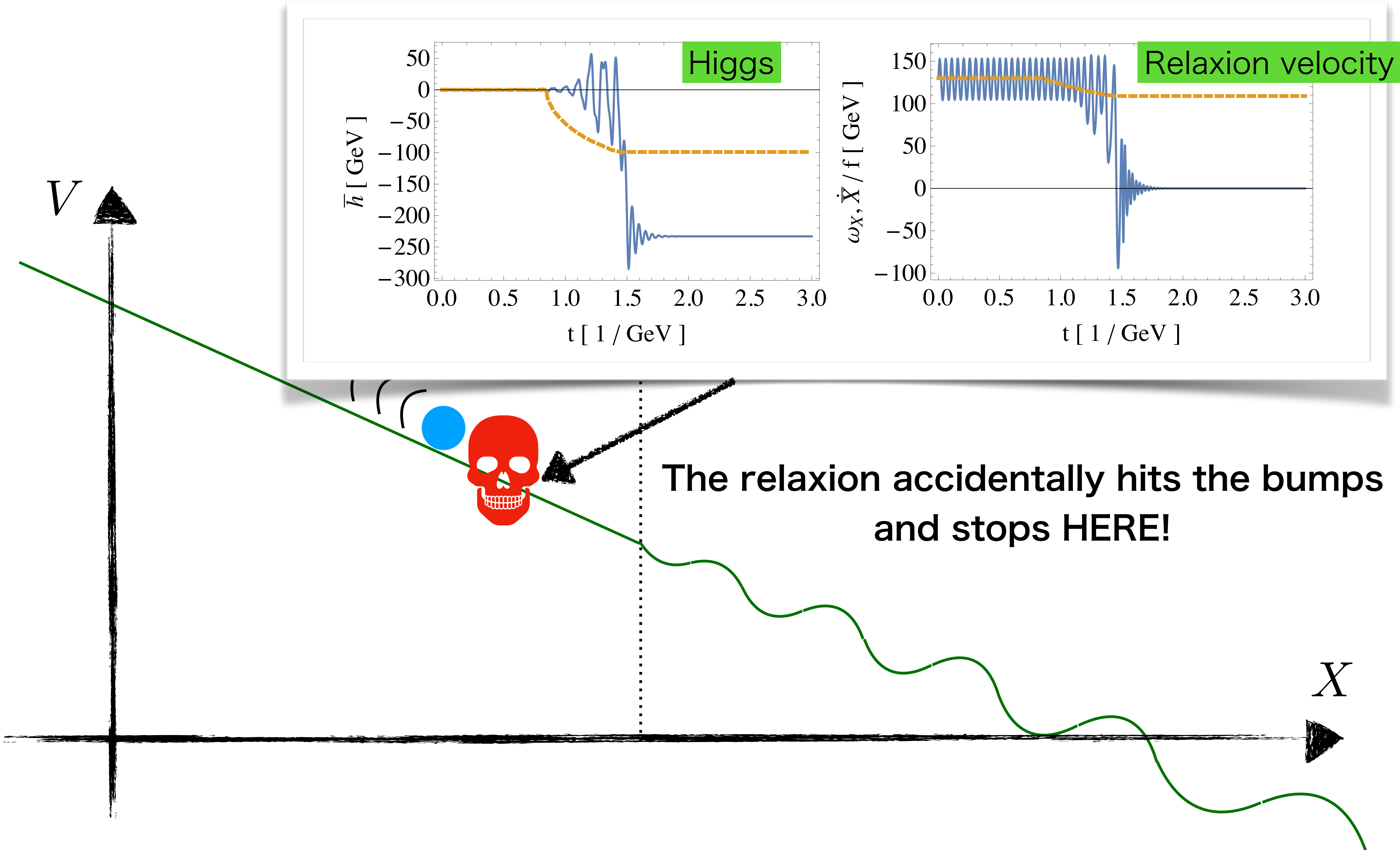
Analytic expression

$$m_\Phi^2 + \frac{3\lambda}{16}\mathcal{A}_h^2 \simeq \frac{\omega_X^2}{4} + \frac{\Lambda_h^2}{2} - \frac{\Lambda_h^2(32\Lambda_0^4 + 17\Lambda_h^2\mathcal{A}_h^2)}{128f^2\omega_X^2} - \frac{(8\Lambda_h^2 - 3\lambda\mathcal{A}_h^2)(8\Lambda_h^2 - \lambda\mathcal{A}_h^2)}{512\omega_X^2} - \frac{9\omega_X^2}{4\Lambda_h^2}H^2 \ ,$$

$$\omega_X \simeq \frac{r\epsilon M^2}{3Hf} - \frac{\mathcal{A}_h^2\omega_X}{8f^2} - \frac{\lambda\mathcal{A}_h^4}{128f^2\omega_X} + \frac{3\mathcal{A}_h^2\omega_X^3}{16f^2\Lambda_h^4}H^2 \ ,$$

Stopping mechanism

Stopping mechanism



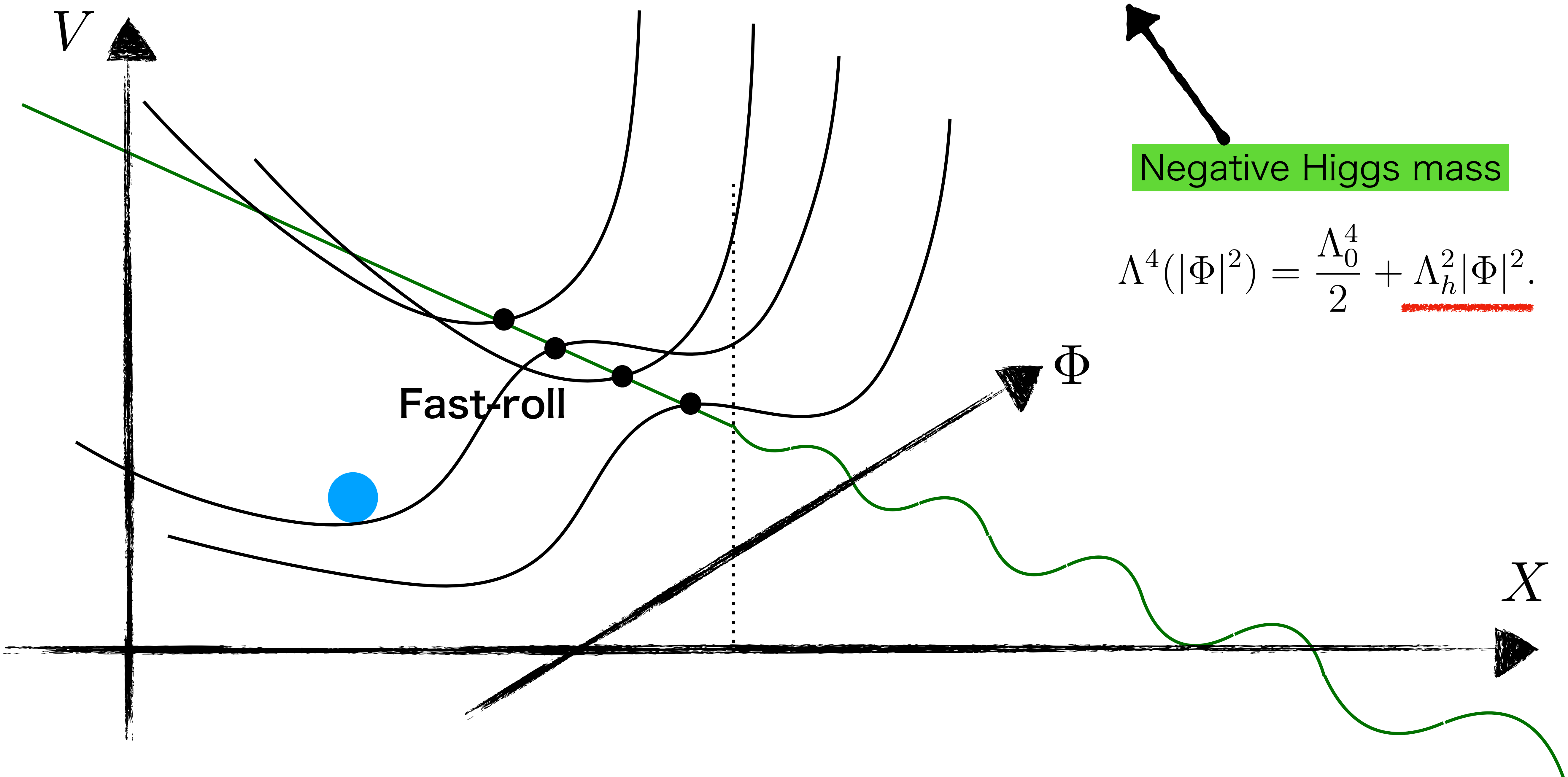
Negative Higgs mass

Positive but small

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

Negative Higgs mass

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2.$$



Parameter space

Conditions

Relaxion rolling

The relaxion does not slow-roll

The relaxion can go over
the cosine potential
with its terminal velocity

Classical rolling dominates
over quantum fluctuations

Stopping mechanism

The edge solution grows

Oscillation starts before
the relaxion pass through
the EW scale

The Higgs boson obtains
a large enough VEV
after the relaxion stops

EW vacuum

The EW vacuum is a stationary point

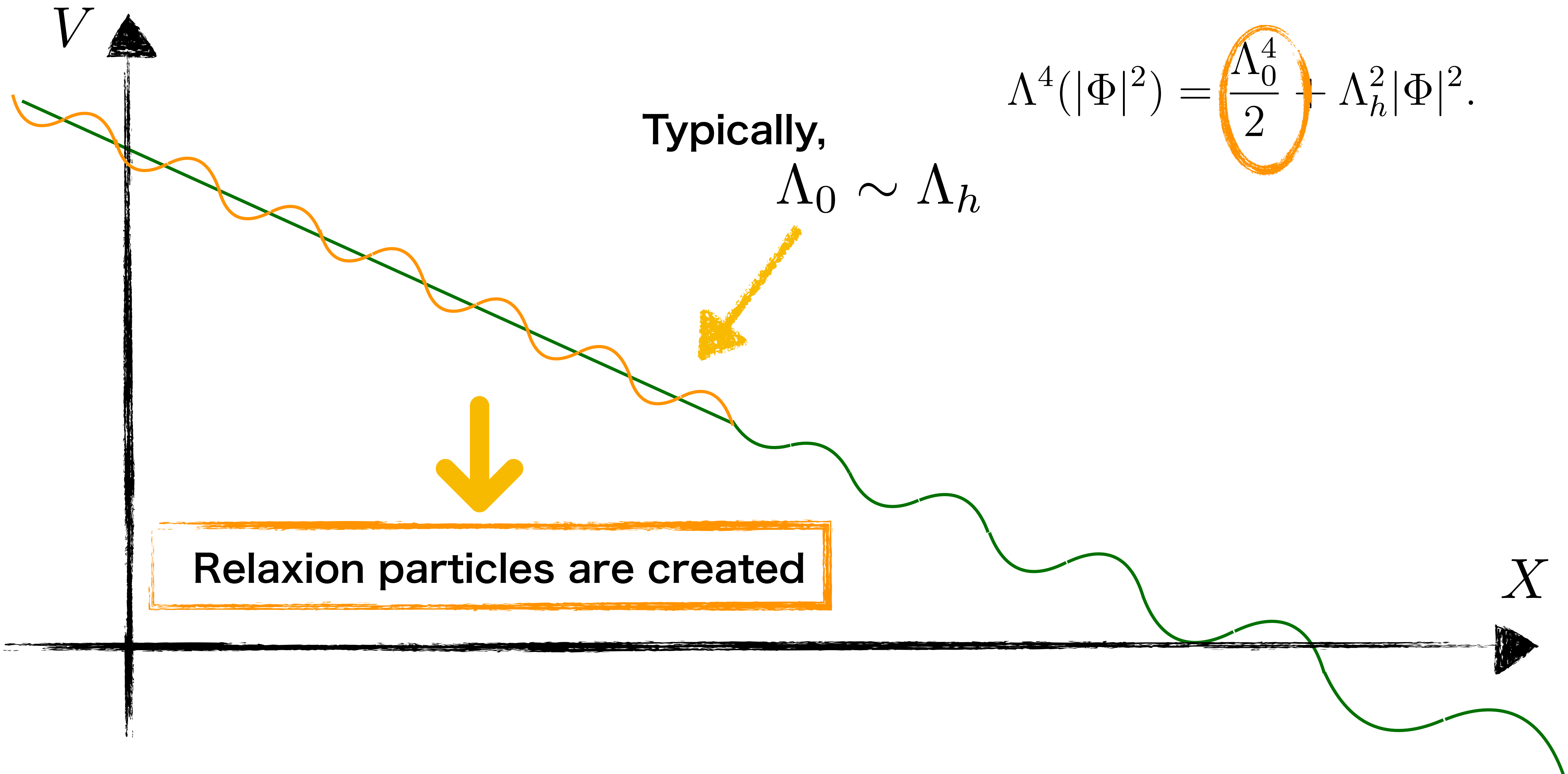
The stability of the EW vacuum

There are enough number of vacua around the EW scale

Particle production

$$V(\Phi, X) = (M^2 - \epsilon X)|\Phi|^2 - r\epsilon M^2 X + \Lambda^4(|\Phi|^2)\cos\frac{X}{f} + \frac{\lambda}{4}|\Phi|^4,$$

$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2.$$



Particle production

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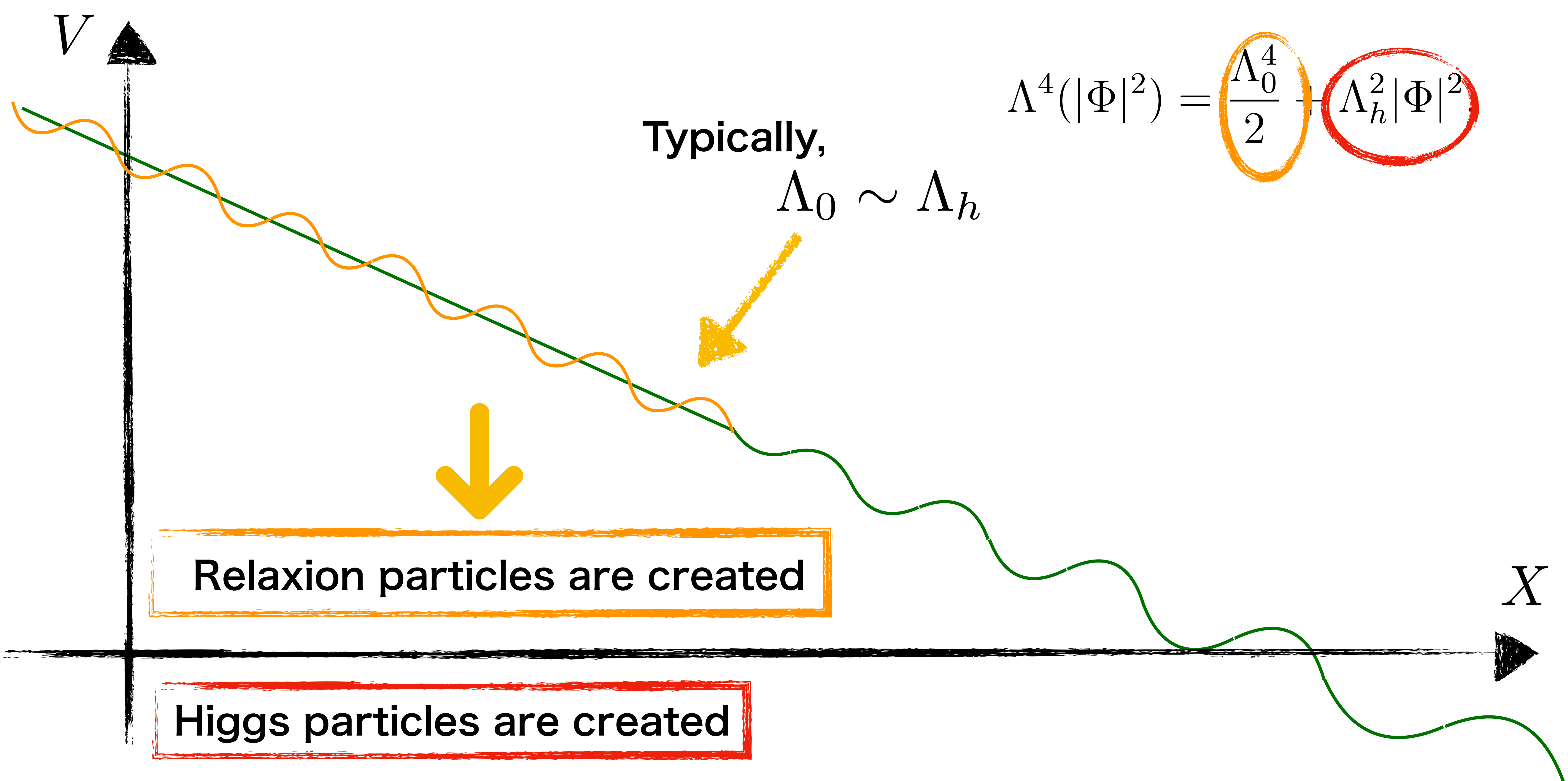
$$\Lambda^4(|\Phi|^2) = \frac{\Lambda_0^4}{2} + \Lambda_h^2|\Phi|^2$$

Typically,

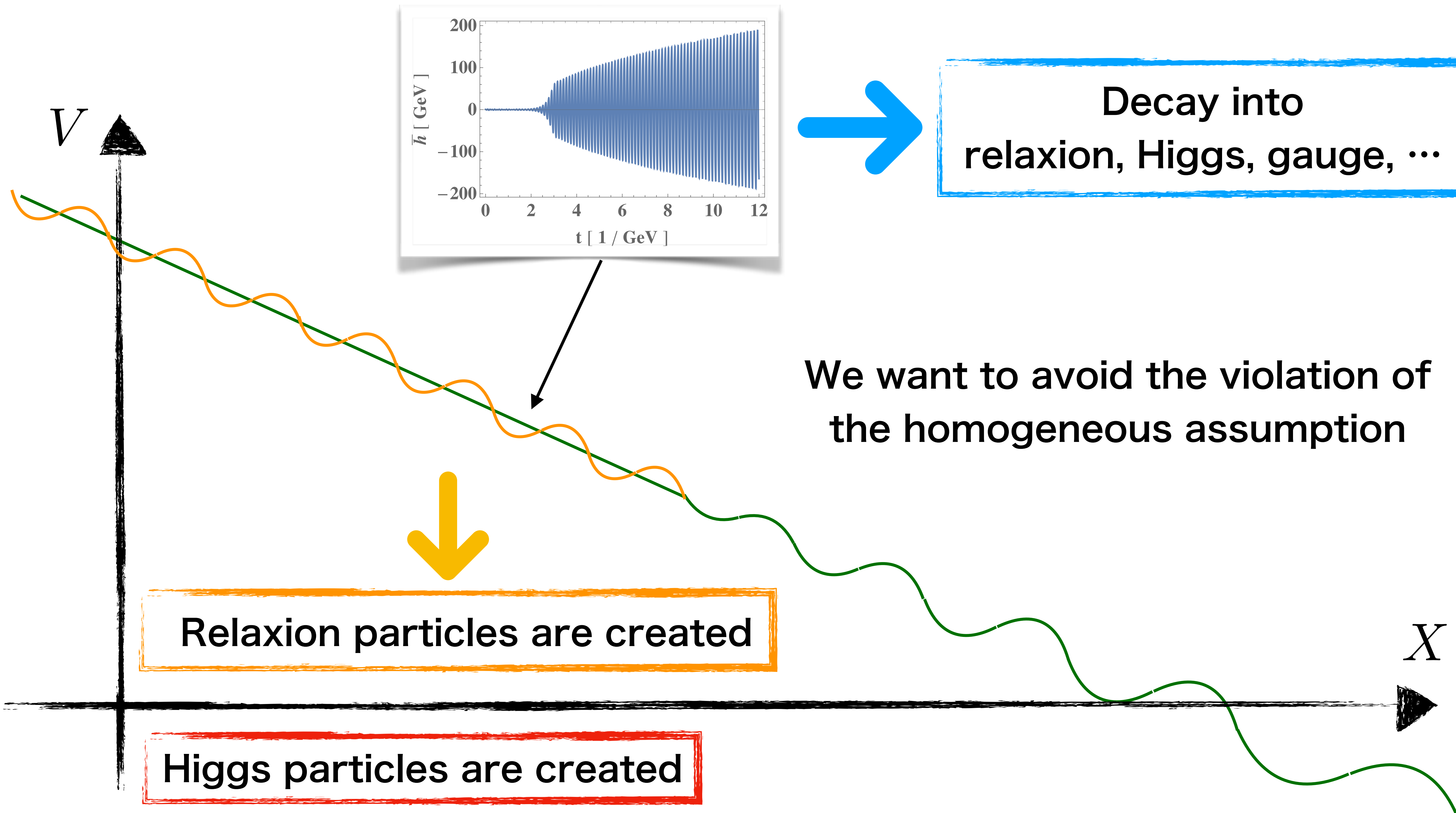
$$\Lambda_0 \sim \Lambda_h$$



Relaxion particles are created

Higgs particles are created

Particle production



Decay into
relaxion, Higgs, gauge, ···

We want to avoid the violation of the homogeneous assumption

Relaxion particles are created

Higgs particles are created

Relaxion-Higgs mixing

Mass matrix

Higgs　　Relaxion

$$\begin{pmatrix} \frac{\lambda v^2}{2} & \frac{\Lambda_h^2 v}{f} s_X \\ \frac{\Lambda_h^2 v}{f} s_X & \frac{\Lambda_0^4+\Lambda_h^2 v^2}{2f^2} c_X \end{pmatrix}$$

Large off-diagonal due to small f

$$s_X \equiv -\sin\frac{\langle X\rangle}{f} = \frac{2fr\epsilon M^2}{\Lambda_0^4+\Lambda_h^2 v^2}\ , \quad c_X \equiv -\cos\frac{\langle X\rangle}{f} = \sqrt{1-s_X^2}\ ,$$

Mixing angle

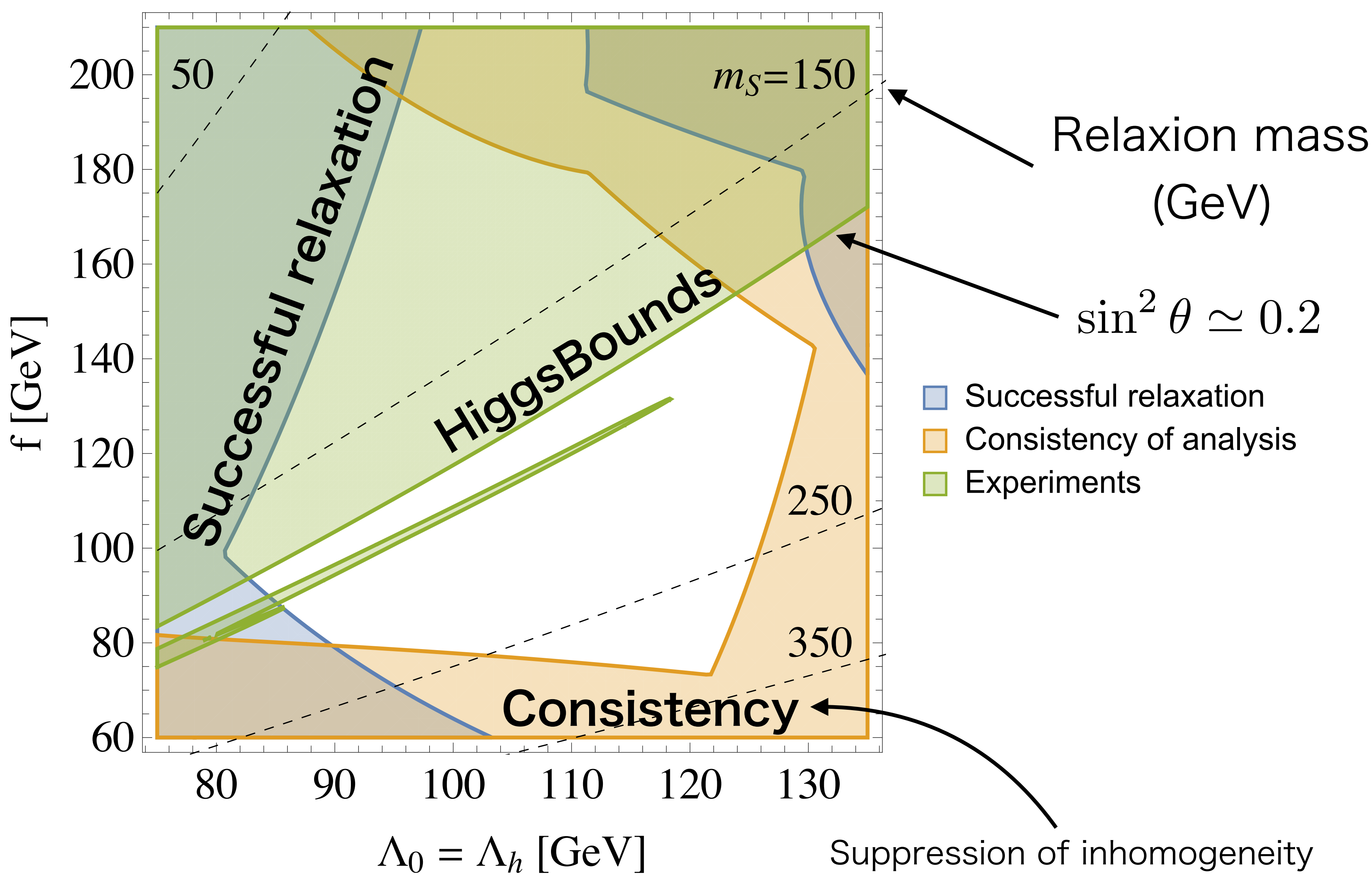
$$\theta \simeq \frac{-2fv\Lambda_h^2 s_X}{(\Lambda_0^4+v^2\Lambda_h^2)c_X - \lambda f^2 v^2}$$

Mixing angle is not suppressed
if the masses are comparable

The relaxion can be produced at colliders
The Higgs couplings are modified

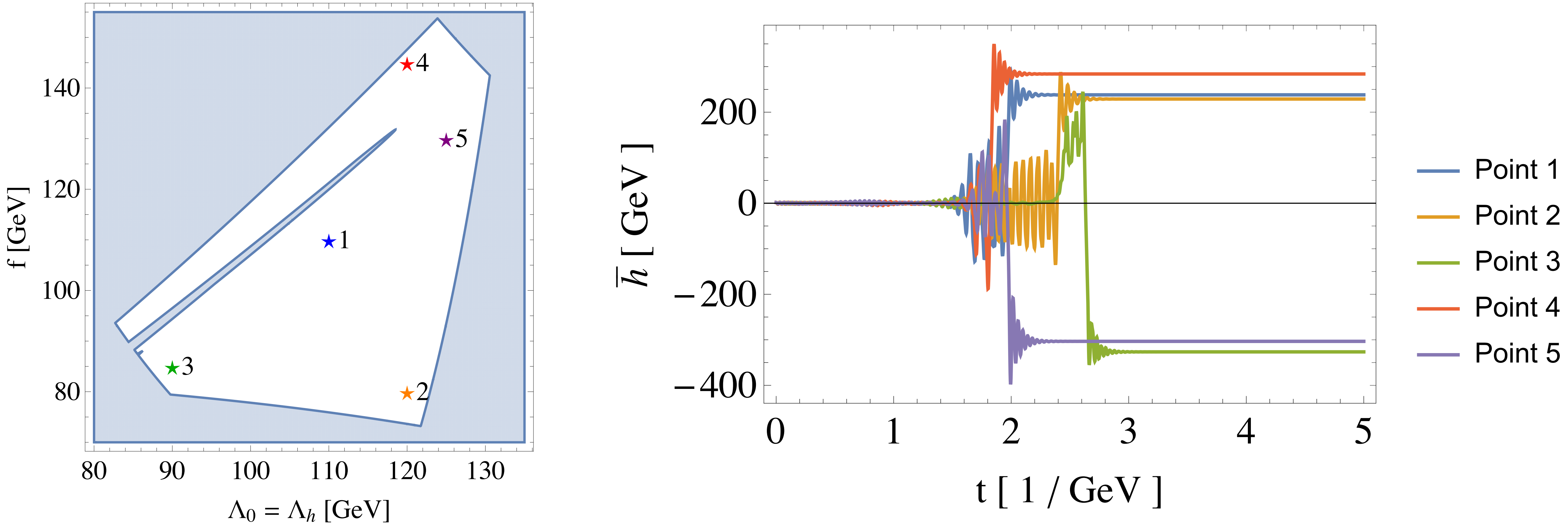
Parameter space

$$H = 10 \text{ GeV}, \ \epsilon = 0.8 \text{ GeV}, \ r = 0.002, \ M = 20 \text{ TeV}.$$



It works!

$$H = 10\ \text{GeV},\ \epsilon = 0.8\ \text{GeV},\ r = 0.002,\ M = 20\ \text{TeV}.$$



Summary

- In the original relaxion model, the tunneling phase requires an unacceptably large number of e-folds.

- The fast-roll relaxion can easily solve this problem, but we can not use the original stopping mechanism.

- We propose a mechanism to stop the relaxion without extending the original model.

- The mechanism predicts a relaxion that has a mass of O(100) GeV and mixes with the Higgs boson. It improves the testability of the model.