Weyl-symmetry Inspired Inflation and Dark Matter

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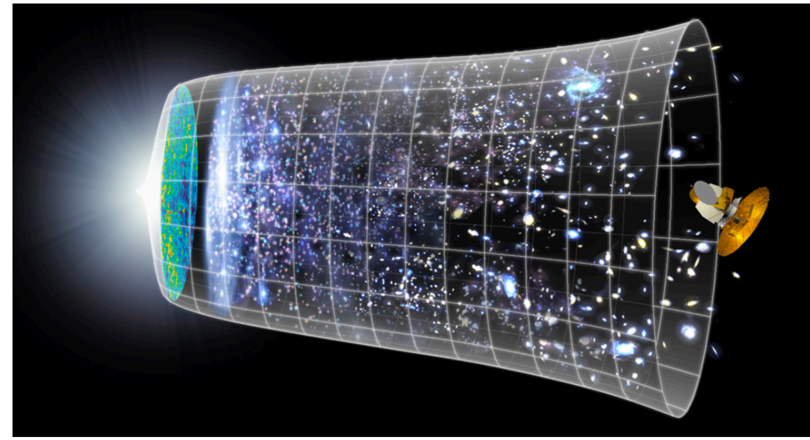
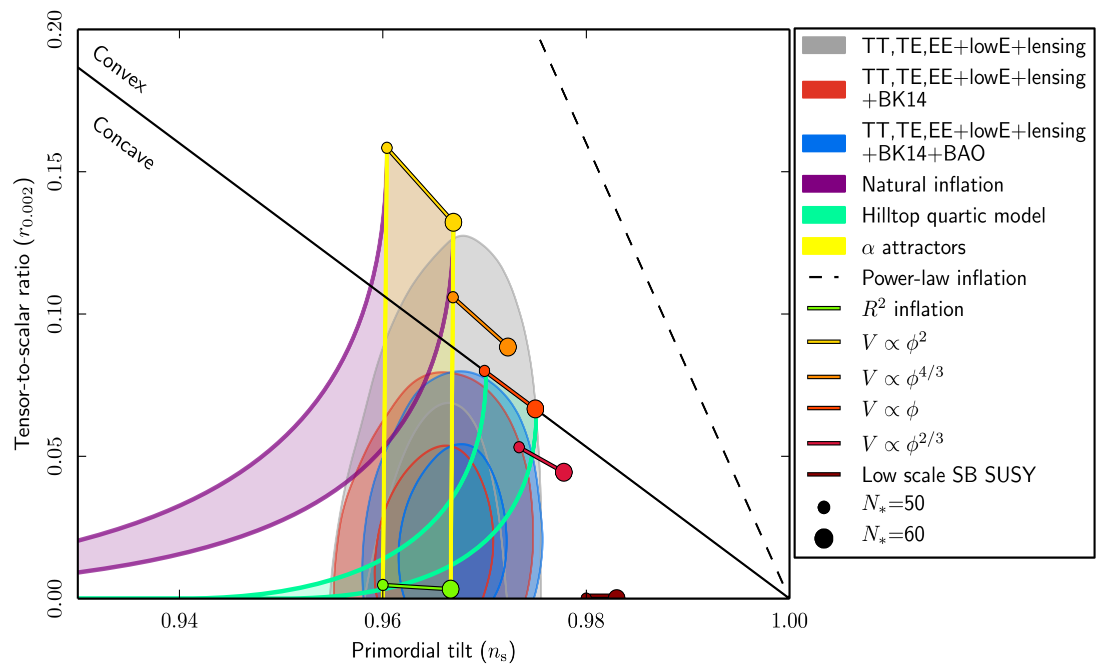
YT and Y.L. Wu, **1904.04493, 1805.08507**

Inflation

- Inflation is a very attraction solution to many cosmic problems,
  - flatness & horizon, ……
- Cosmological observations now have entered a precision era,
  - primordial GW,
  - power spectrum

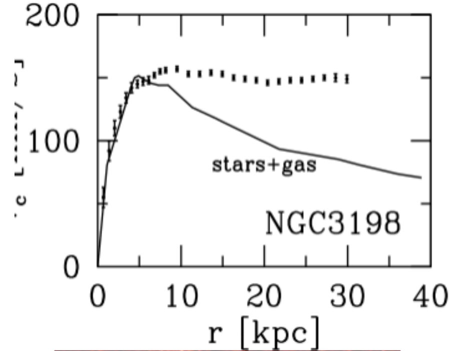


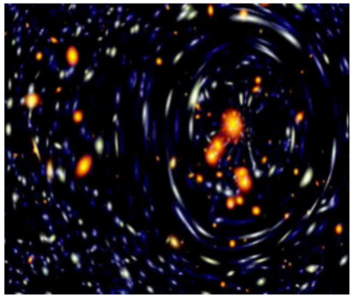


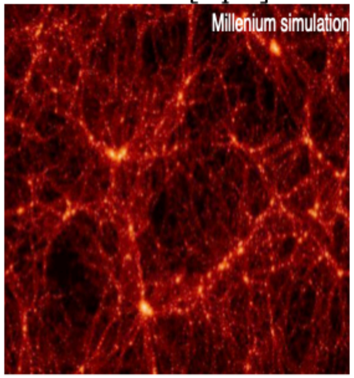
In future $r \sim 10^{-3}$

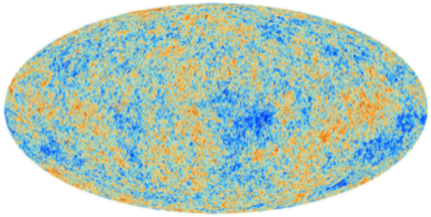
Dark Matter

- Dark Matter is challenging our standard model in particle physics
- So far only evidence for the gravitational interaction
- Extensions of Einstein's GR may give DM candidate









We propose a scenario based on Weyl symmetry,

⇒ Inflation and Dark Matter.

Weyl Symmetry

- Weyl symmetry was referred to

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = e^{\theta(x)} g_{\mu\nu}(x)$$
$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu \theta(x)$$

- First proposed by Weyl around 1919 in order to unify general relativity and electromagnetic interaction

Unsuccessful

- Later Weyl modified it to

$$\psi(x) \to \psi'(x) = e^{i\theta(x)} \psi(x)$$
$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu \theta(x)$$

Gauge symmetry

- To describe electron after quantum mechanics.

Einstein's Gravity

- Einstein-Hilbert action, $S = \int d^4x \mathcal{L}$

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_p^2}{2}R - \Lambda\right]$$

$$R^\rho_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma}$$

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}\right)$$

- Under Weyl/conformal transformation

$$g_{\mu\nu}(x) \to \bar{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x)$$

- Then we have

$$\sqrt{-g} = \Omega^{-4}(x)\sqrt{-\bar{g}}$$

$$R = \Omega^2\left[\bar{R} - 6\bar{\Box}\ln\Omega + 6\bar{g}^{\mu\nu}\partial_\mu \ln\Omega \partial_\nu \ln\Omega\right],$$

$$\bar{\Box}\ln\Omega = \frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\sqrt{-\bar{g}}\bar{g}^{\mu\nu}\partial_\nu \ln\Omega\right)$$

$$\Longrightarrow \mathcal{L}[g_{\mu\nu}] \neq \bar{\mathcal{L}}[\bar{g}_{\mu\nu}]$$

Einstein's Gravity -> Weyl Invariant

- Modified to

$$\sqrt{-g}\left[\frac{M_p^2}{2}R-\Lambda\right] \rightarrow \sqrt{-g}\left[\alpha(\phi^2 R - 6\partial_\mu\phi\partial^\mu\phi) - \beta\phi^4\right]$$

$$\phi \rightarrow \bar{\phi} = \Omega^{-1}(x)\phi$$

- Weyl-invariant, but no new degree of freedom
- So no new dynamics, no new predictions.
- We can not add $\partial_\mu\phi\partial^\mu\phi/2$
  - Downgrade into global symmetry

$$\sqrt{-g}\left[\alpha\phi^2 R + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \beta\phi^4\right]$$

Jordan-Brans-Dicke

……

$$\beta\phi^4 \Rightarrow \beta(\phi^2 - v^2)^2$$

A. Zee

Variants

- Conformal/Weyl gravity

$$\mathcal{L} = \frac{1}{2\alpha^2}\sqrt{-g}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} \Rightarrow \frac{1}{\alpha^2}\sqrt{-g}\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right)$$

- Weyl tensor

$$\begin{aligned} C_{ik\ell m} =& R_{ik\ell m} + \frac{1}{n-2}\left(R_{im}g_{k\ell} - R_{i\ell}g_{km} + R_{k\ell}g_{im} - R_{km}g_{i\ell}\right) \\ &+ \frac{1}{(n-1)(n-2)}R\left(g_{i\ell}g_{km} - g_{im}g_{k\ell}\right) \end{aligned}$$

- Usually with higher-derivative terms
  - Renormalizable
  - Ghosts, Unitarity
- Formulate gravity as gauge theory

Ref. Y.L. Wu, 1506.01807, 1712.04537

Gauge Symmetry

- With Weyl gauge field W_μ

$$\sqrt{-g}\left[\alpha\left(\phi^2 R - 6\partial_\mu\phi\partial^\mu\phi\right) + \frac{1}{2}D_\mu\phi D^\mu\phi - \beta\phi^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

- Covariant derivative

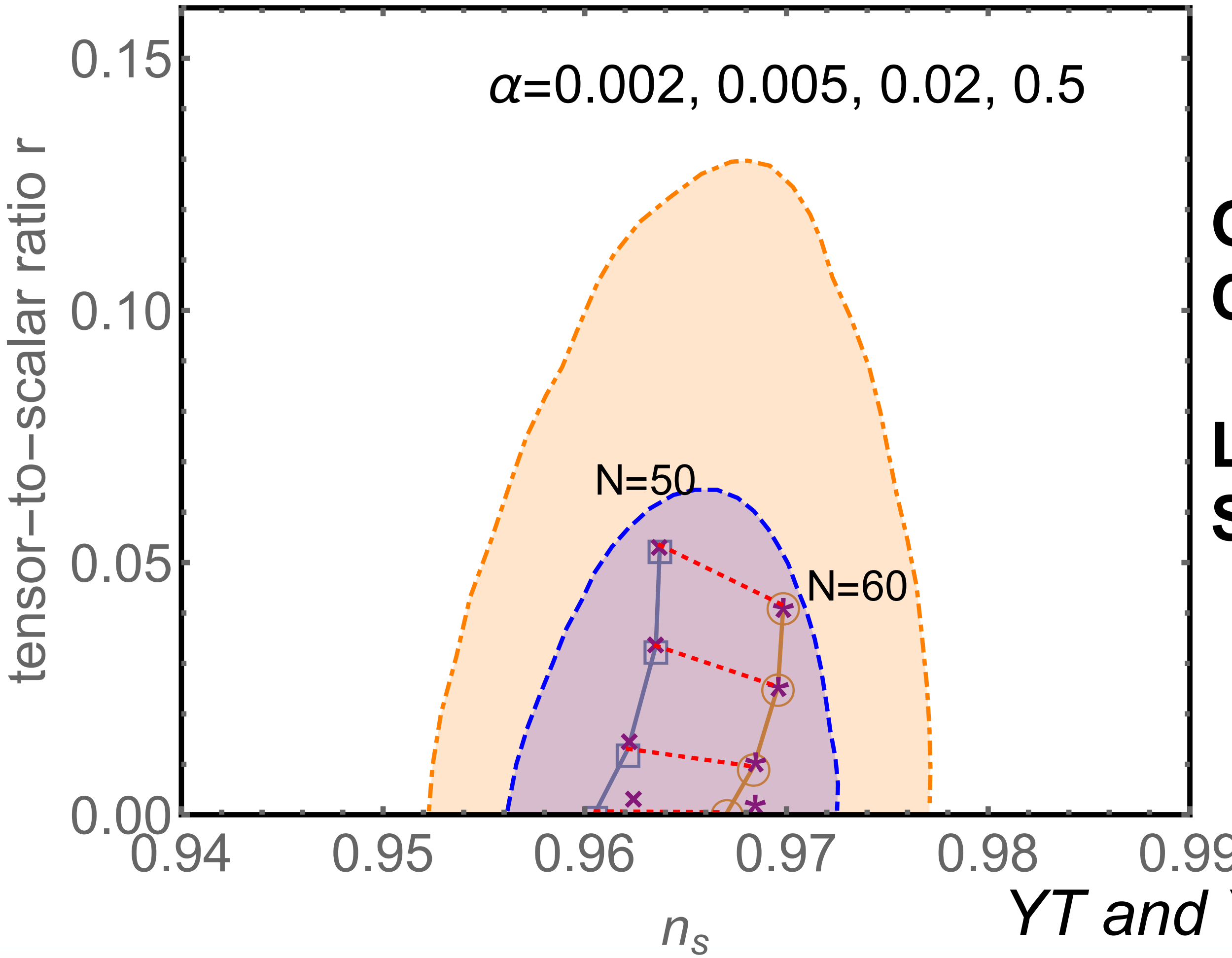
$$D_\mu = \partial_\mu - g_W W_\mu$$

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = \lambda^2(x)\, g_{\mu\nu}(x)\,,$$
$$\phi(x) \to \phi'(x) = \lambda^{-1}(x)\,\phi(x)\,,$$
$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu \ln\lambda(x)/g_W$$

- There is no i in front of $g_W W_\mu$
- Once a "frame" is fixed, $\phi \to \frac{M_P}{\sqrt{2\alpha}}$
    - Weyl gauge boson gets a mass, $m_W = \frac{g_W M_P}{\sqrt{2\alpha}}$
    - ϕ is absorbed as the longitudinal mode, a physical degree of freedom, but hidden
    - unless explicitly broken, mass and $\beta\phi^4 \Rightarrow \beta(\phi^2 - v^2)^2$

Global vs Local

- Global $\alpha\phi^2 R + \partial_\mu\phi\partial^\mu\phi/2 - \beta(\phi^2 - v^2)^2$
- Local $\alpha\left(\phi^2 R - 6\partial_\mu\phi\partial^\mu\phi\right) + \partial_\mu\phi\partial^\mu\phi/2 - \beta(\phi^2 - v^2)^2$



Global:
Crosses and stars

Local:
Squares and Circles

YT and Y.L.Wu, **1805.08507**

Lagrangian—Version-2.0

$$\mathcal{L} \supset \sqrt{-g}\Big[\alpha\left(\varphi^2 R - 6\partial_\mu\varphi\partial^\mu\varphi\right) + \beta\left(\phi^2 R - 6\partial_\mu\phi\partial^\mu\phi\right)$$
$$+\frac{\zeta_1}{2}D_\mu\varphi D^\mu\varphi + \frac{\zeta_2}{2}D_\mu\phi D^\mu\phi - V(\phi,\varphi)$$
$$+\frac{i}{2}\left(\overline{\psi}\gamma^\mu D_\mu\psi - \overline{D_\mu\psi}\gamma^\mu\psi\right) - f\,\phi\overline{\psi}\psi - y\,\varphi\overline{\psi}\psi$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big],$$

YT and Y.L.Wu, **1904.04493**

- R: Ricci scalar
- φ and ϕ : scalars, $V(\varphi,\phi) = \sum c_i\varphi^i\phi^{4-i}$, i=0,…,4
- $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, W_μ: Weyl gauge boson
- ψ : fermion

Weyl Symmetry

- Invariant under the following transformation

$$g_{\mu\nu}(x) \to g'_{\mu\nu}(x) = \lambda^2(x)\, g_{\mu\nu}(x)\,,$$
$$\varphi(x) \to \varphi'(x) = \lambda^{-1}(x)\, \varphi(x)\,,$$
$$\phi(x) \to \phi'(x) = \lambda^{-1}(x)\, \phi(x)\,,$$
$$\psi(x) \to \psi'(x) = \lambda^{-3/2}(x)\, \psi(x)\,,$$
$$W_\mu(x) \to W'_\mu(x) = W_\mu(x) - \partial_\mu \ln \lambda(x)/g_W\,,$$

- ψ does **not** couple to W_μ
  - W_μ a dark matter candidate? H.Cheng, PRL1988
  - Not stable, actually **vs. Vector Dark Matter**

An illustration

$$\mathcal{L} \supset \sqrt{-g}\Big[\alpha\left(\varphi^2 R - 6\partial_\mu\varphi\partial^\mu\varphi\right) + \beta\left(\phi^2 R - 6\partial_\mu\phi\partial^\mu\phi\right)$$
$$+\frac{\zeta_1}{2}D_\mu\varphi D^\mu\varphi + \frac{\zeta_2}{2}D_\mu\phi D^\mu\phi - V(\phi,\varphi)$$
$$+\frac{i}{2}\left(\overline{\psi}\gamma^\mu D_\mu\psi - \overline{D_\mu\psi}\gamma^\mu\psi\right) - f\,\phi\overline{\psi}\psi - y\,\varphi\overline{\psi}\psi$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\Big],$$

- We illustrate with $\beta = 0, \zeta_1 = 1, \zeta_2 \equiv \zeta$
- The potential $V(\varphi, \phi) = c\left(\varphi^2 - \phi^2\right)^2$

Continued

$$\mathcal{L} \supset \sqrt{-g}\left[\alpha\left(\varphi^2 R - 6\partial_\mu\varphi\partial^\mu\varphi\right) + \frac{1}{2}D_\mu\varphi D^\mu\varphi + \frac{\zeta}{2}D_\mu\phi D^\mu\phi - c(\varphi^2-\phi^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\psi}\gamma^\mu\partial_\mu\psi - f\,\phi\overline{\psi}\psi - y\,\varphi\overline{\psi}\psi\right],$$

- Once a frame is chosen $\phi \rightarrow v$

$$M_p = \sqrt{2\alpha}v \simeq 2.4\times 10^{18}\mathrm{GeV},$$

$$\mathcal{L} \supset \sqrt{-g}\left[\alpha\left(\varphi^2 R - 6\partial_\mu\varphi\partial^\mu\varphi\right) + \frac{1}{2}D_\mu\varphi D^\mu\varphi - c(\varphi^2-v^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\zeta}{2}g_W^2 v^2 W_\mu W^\mu + i\overline{\psi}\gamma^\mu\partial_\mu\psi - f v\,\overline{\psi}\psi - y\,\varphi\overline{\psi}\psi\right],$$

- Both Weyl boson and fermion get a mass
- Scalar is not minimally coupled
- Jordan frame ⇒ Einstein frame

Einstein frame

- Transformations

$$\bar{g}_{\mu\nu} = \lambda^2 g_{\mu\nu},\ \lambda \equiv \frac{\varphi}{v},\ \Psi = \lambda^{-3/2}\psi,\ S = \frac{\sigma}{\xi},\ \sigma = v\ln\left(\frac{\varphi}{v}\right),$$

$$\overline{W}_\mu = W_\mu - \frac{g_W v}{M_W^2}\partial_\mu\sigma,\ \xi \equiv \sqrt{\frac{\zeta+1}{\zeta}},\qquad \zeta < -1 \text{ or } \zeta > 0$$

- Rewrite in the canonical form

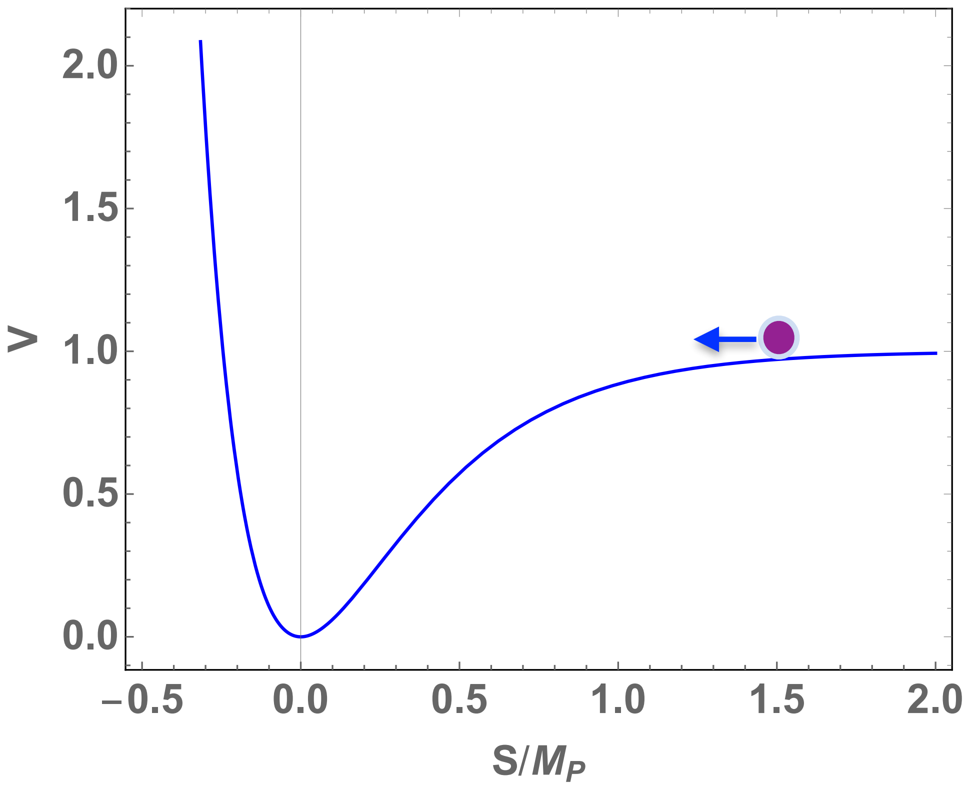
$$\frac{\mathcal{L}}{\sqrt{-\bar{g}}} \supset \frac{M_p^2}{2}\bar{R} + \frac{1}{2}\partial_\mu S\partial^\mu S - cv^4\left[1 - \exp\left(-\frac{2\xi}{v}S\right)\right]^2$$
$$+i\overline{\Psi}\gamma^\mu\partial_\mu\Psi - m_\Psi\overline{\Psi}\Psi - f\xi S\overline{\Psi}\Psi$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_W^2\overline{W}_\mu\overline{W}^\mu + \zeta g_W^2\left(\xi^2S^2 - v\xi S\right)W_\mu W^\mu$$

$$M_p = \sqrt{2\alpha}v \simeq 2.4\times10^{18}\mathrm{GeV},\ M_W^2 = g_W^2v^2(\zeta+1),\ m_\Psi = v(f+y).$$

Inflation

- Potential for inflation *S*

$$\mathcal{V}(S) = \bar{c}\frac{M_p^4}{4\bar{\alpha}^2}\left[1-\exp\left(-\frac{2\sqrt{2\bar{\alpha}}}{M_p}S\right)\right]^2 \quad \bar{\alpha}\equiv\xi^2\alpha,\ \bar{c}\equiv\xi^4 c$$



→ Inflaton mass

$$m_S = 2\sqrt{\frac{\bar{c}}{\bar{\alpha}}}M_p \sim 10^{13}\mathrm{GeV}$$

→ slow-roll parameter

$$\epsilon = \frac{1}{2}M_p^2\left(\frac{\mathcal{V}_S(S)}{\mathcal{V}(S)}\right)^2,\ \eta = M_p^2\frac{\mathcal{V}_{SS}(S)}{\mathcal{V}(S)}$$

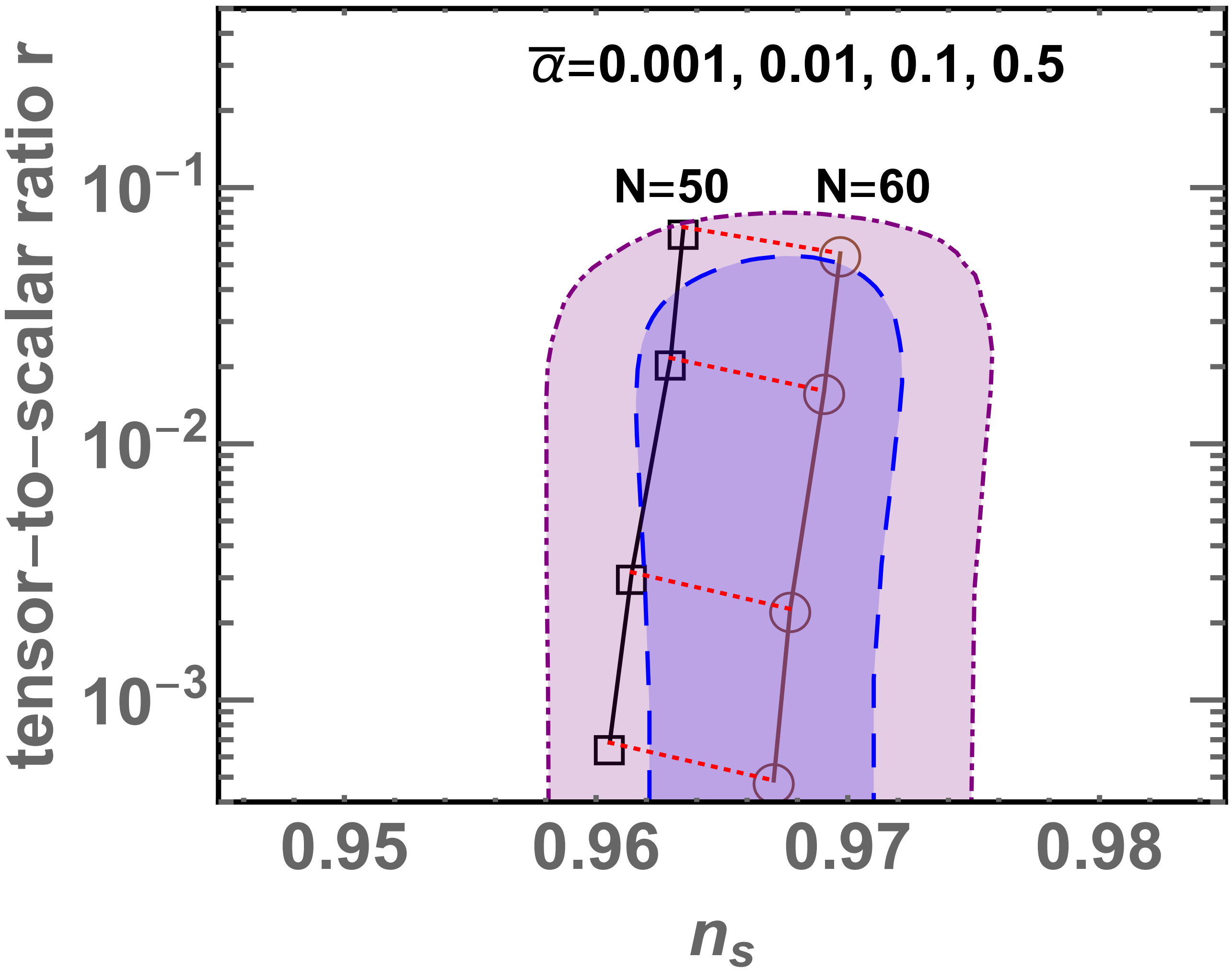
$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_*}\right)^{n_{\rm s}-1},\ \mathcal{P}_{\rm t}(k) = A_{\rm t}\left(\frac{k}{k_*}\right)^{n_t}$$

$$A_{\rm s}\approx\frac{\mathcal{V}}{24\pi^2 M_{\rm pl}^4\epsilon},\ A_{\rm t}\approx\frac{2\mathcal{V}}{3\pi^2 M_{\rm pl}^4}$$

$$n_s = 1-6\epsilon+2\eta,\ n_t=-2\epsilon,\ r\equiv\frac{A_{\rm t}}{A_{\rm s}}=16\epsilon,$$

Observables

- $r - n_s$ diagram $n_s = 1 - 6\epsilon + 2\eta,\ r \equiv \frac{A_t}{A_s} = 16\epsilon$



For $\bar{\alpha} \gtrsim \mathcal{O}(0.1)$

$$n_s \simeq 1 - \frac{2}{N}$$

$$r \simeq \frac{1}{\bar{\alpha} N^2}$$

N: **e-fold number**

Starobinsky:

$$R + \frac{R^2}{6M^2}$$

$$n_s \simeq 1 - \frac{2}{N}$$

$$r \simeq \frac{12}{N^2}$$

Reheating

- The reheating can proceed as standard since the inflaton can decay into fermion pair

$$S \to \Psi + \overline{\Psi}$$

with decay width

$$\Gamma_S \sim m_S f^2 \xi^2 / 8\pi$$

- So the reheating temperature is

$$H \sim \frac{T_R^2}{M_P} \simeq \Gamma_s \Rightarrow \; T_R \simeq \sqrt{\Gamma_S M_p} \simeq 1.5 f\xi \times 10^{16} \mathrm{GeV}$$

- Once can introduce additional gauge coupling between SM fermions and Ψ to reheat SM at T_h

Dark Matter

- Lagrangian

$$\frac{\mathcal{L}}{\sqrt{-\bar{g}}} \supset \frac{M_p^2}{2}\bar{R} + \frac{1}{2}\partial_\mu S \partial^\mu S - c v^4 \left[1 - \exp\left(-\frac{2\xi}{v} S\right)\right]^2$$
$$+ i\overline{\Psi}\gamma^\mu \partial_\mu \Psi - m_\Psi \overline{\Psi}\Psi - f\xi S \overline{\Psi}\Psi$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}M_W^2 \overline{W}_\mu \overline{W}^\mu + \zeta g_W^2 \left(\xi^2 S^2 - v\xi S\right) W_\mu W^\mu$$

- Z_2 symmetry, $W_\mu \to -W_\mu$
- Does not couple to fermion → Dark Matter?
  - Accidental symmetry
  - Explicitly broken when we consider SM Higgs
  - Decaying Dark Matter if g_W and M_W are small

SM Higgs

- The starting Weyl-symmetric

$$\mathcal{L}_H \supset \sqrt{-g}\left[(\mathcal{D}_\mu H)^\dagger \mathcal{D}^\mu H - \lambda_H \left(H^\dagger H - \lambda_{\phi H}\phi^2/2\right)^2\right],$$

- We can replace $H \to (0, v_H + h)^T/\sqrt{2}$ $\lambda_{\phi H} \equiv v_H^2/v^2$

$$\mathcal{L}_H \supset \sqrt{-\bar{g}}\left[\frac{1}{2}\partial^\mu \bar{h}\partial_\mu \bar{h} - \frac{1}{2}m_{\bar{h}}^2\bar{h}^2 - \frac{1}{4}\overline{F}_{\mu\nu}\overline{F}^{\mu\nu} + \frac{1}{2}m_W^2\overline{W}^\mu\overline{W}_\mu + L_{\rm int}\right],$$

$$L_{\rm int} = \frac{1}{2}g_W^2\left(\frac{2v_H\bar{h}}{C_H} + \frac{\bar{h}^2}{C_H^2}\right)\left(\overline{W}^\mu\overline{W}_\mu + \frac{2g_W v_H}{m_W^2 C_H}\overline{W}_\mu\partial^\mu\bar{h} + \frac{g_W^2 v_H^2}{m_W^4 C_H^2}\partial_\mu\bar{h}\partial^\mu\bar{h}\right) - \frac{g_W}{C_H^2}\left(\overline{W}^\mu\bar{h}\partial_\mu\bar{h} + \frac{g_W v_H}{m_W^2 C_H}\bar{h}\partial_\mu\bar{h}\partial^\mu\bar{h}\right) - \frac{\lambda_H\bar{h}^3}{C_H^3}\left(v_H + \frac{\bar{h}}{4C_H}\right).$$

- New mass contribution

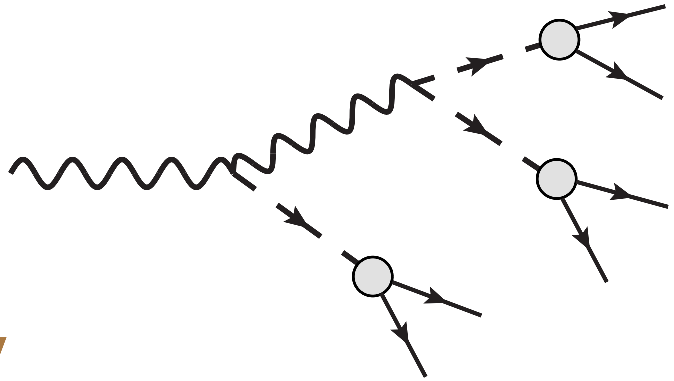
$$\overline{W}_\mu = W_\mu - \frac{g_W}{m_W^2}\left(\xi v\partial_\mu S + v_H\partial_\mu h\right)$$

$$m_W^2 = M_W^2 + g_W^2 v_H^2,\ m_{\bar{h}} = v_H\sqrt{2\lambda_H}/C_H,\ \bar{h} = C_H h,\ C_H \equiv \sqrt{1 - \frac{g_W^2 v_H^2}{m_W^2}}.$$

Continued

- New interactions

$$L_{\rm int} = \frac{1}{2} g_W^2 \left(\frac{2 v_H \bar{h}}{C_H} + \frac{\bar{h}^2}{C_H^2} \right) \left(\overline{W}^\mu \overline{W}_\mu + \frac{2 g_W v_H}{m_W^2 C_H} \overline{W}_\mu \partial^\mu \bar{h} + \frac{g_W^2 v_H^2}{m_W^4 C_H^2} \partial_\mu \bar{h} \partial^\mu \bar{h} \right) - \frac{g_W}{C_H^2} \left(\overline{W}^\mu \bar{h} \partial_\mu \bar{h} + \frac{g_W v_H}{m_W^2 C_H} \bar{h} \partial_\mu \bar{h} \partial^\mu \bar{h} \right) - \frac{\lambda_H \bar{h}^3}{C_H^3} \left(v_H + \frac{\bar{h}}{4 C_H} \right).$$



- No Z_2 symmetry any more
- W_μ can decay into 3-body,…
- Higgs couplings are rescaled by

$$C_H^n, \ \bar{h} = C_H h, \ C_H \equiv \sqrt{1 - \frac{g_W^2 v_H^2}{m_W^2}}.$$

Phenomenology

- Invisible decay if kinematically allowed

$$\frac{g_W^2 v_H}{C_H} \bar{h} \overline{W}^\mu \overline{W}_\mu$$

- Decay width

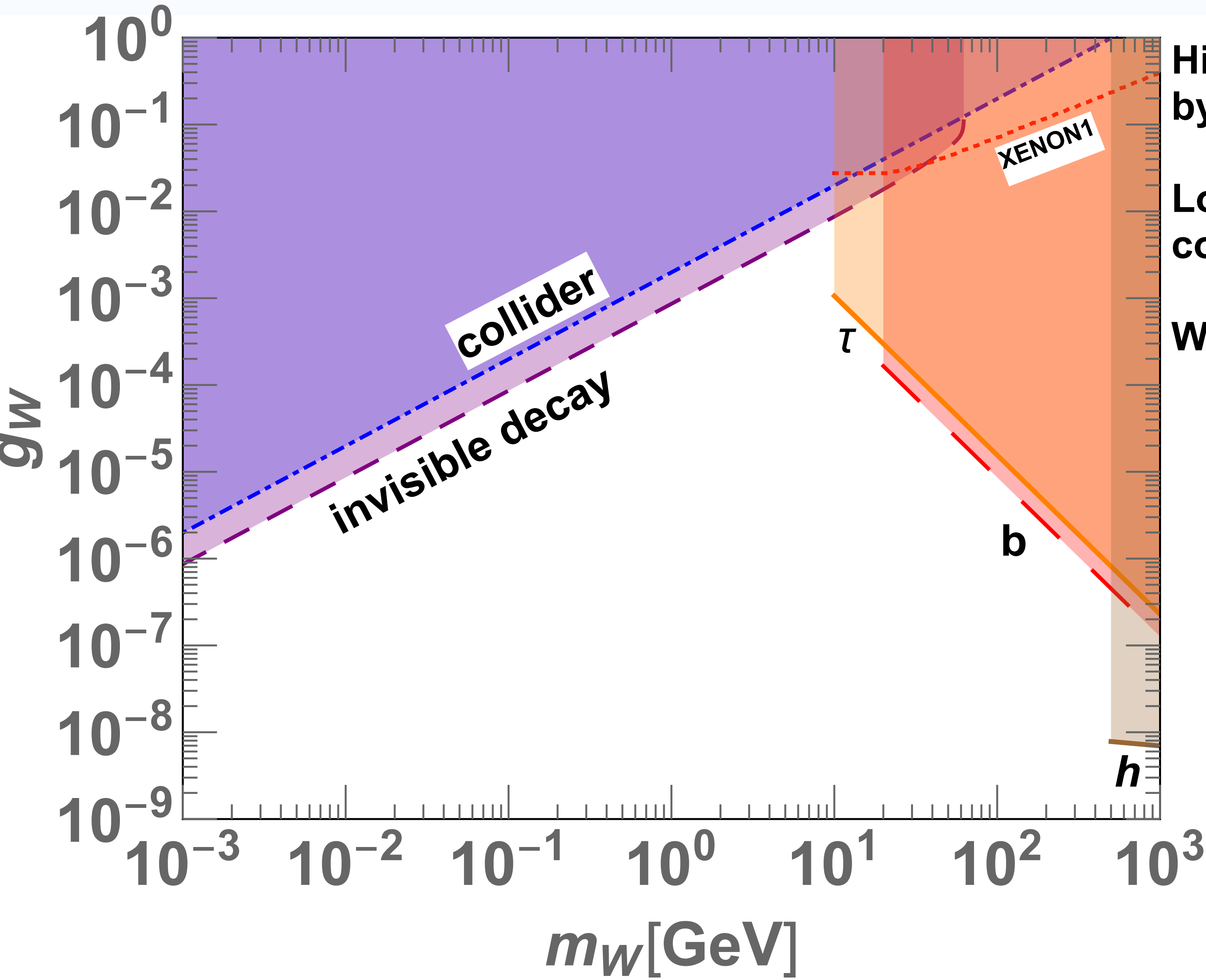
$$\Gamma\left(\bar{h} \to W_\mu + W_\mu\right) = \frac{g_W^4}{32\pi} \frac{v_H^2 m_{\bar{h}}^3}{m_W^4 C_H^2} \sqrt{1 - x_W} \left(1 - x_W + \frac{3}{4} x_W^2\right)$$

- If being DM, direct searches constraints.
- Signal strength at LHC is also modified, enhanced by $1/C_H^2$

$$\mu = 1.09 \pm 0.11 \Rightarrow C_H^2 \gtrsim \frac{1}{1.31}, \frac{m_W}{g_W} \gtrsim 505\text{GeV}$$

ATLAS&CMS

Constraints



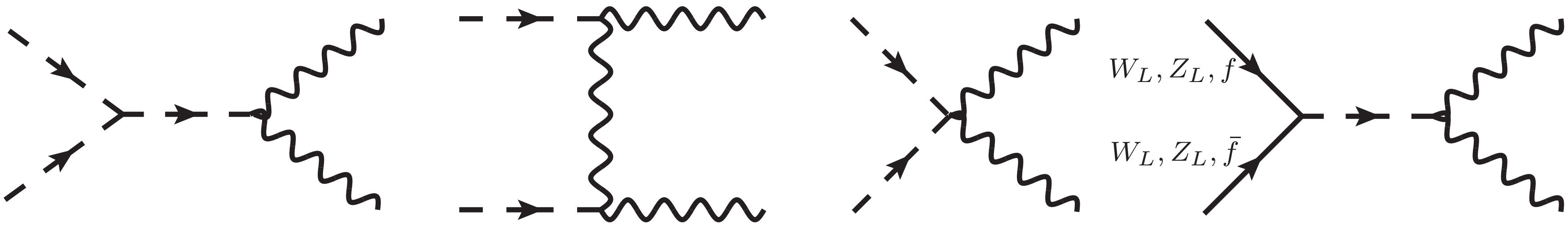
High mass region by cosmic rays

Low mass by collider searches

White part is allowed

Relic abundance

- We consider two contributions
  - One from thermal production, freeze-in



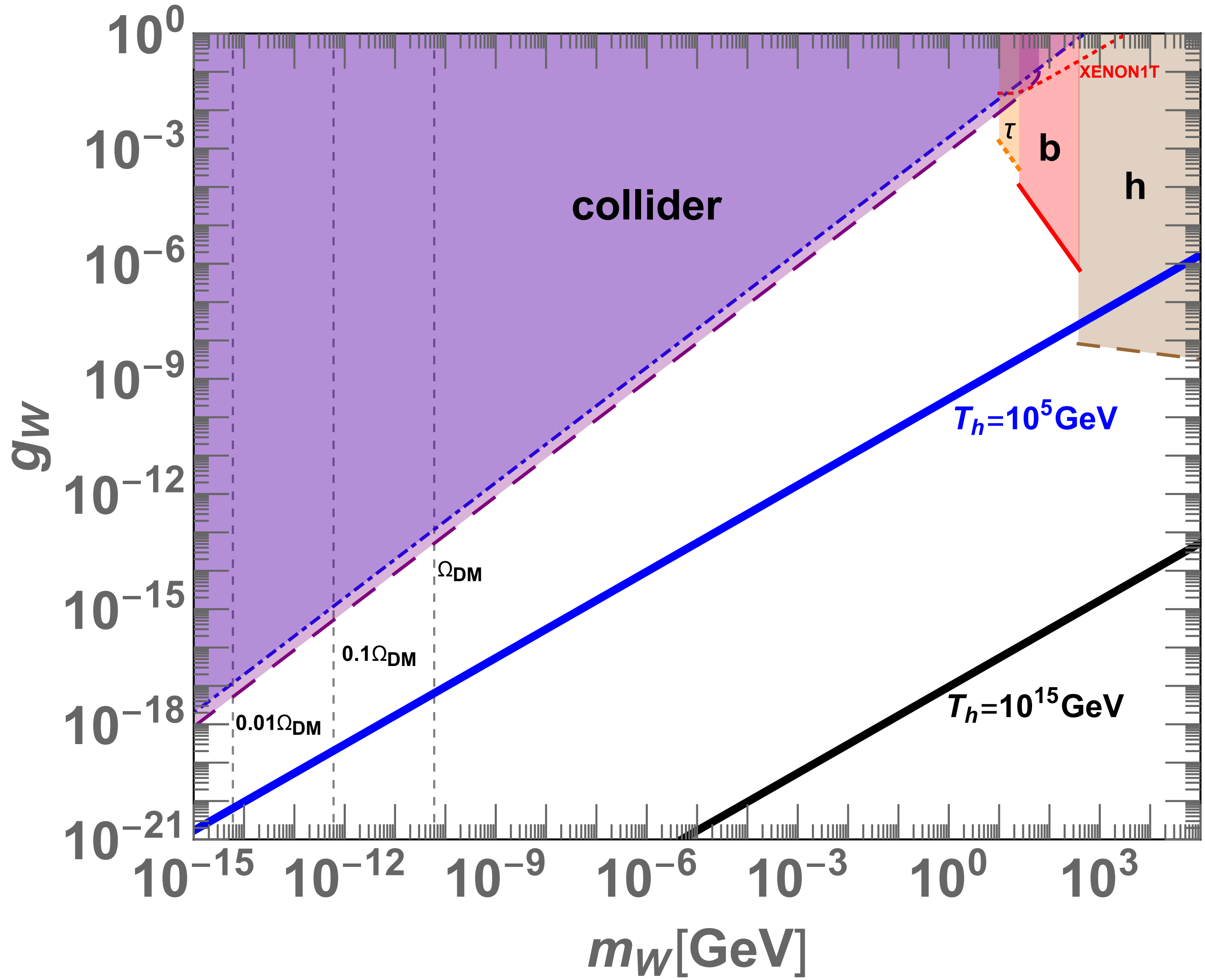
$$\Omega_W \simeq \Omega_{\rm DM} \times \left(\frac{g_W}{3\times 10^{-7}}\right)^4 \left(\frac{10^2 {\rm GeV}}{m_W}\right)^3 \left(\frac{T_h}{10^3 {\rm GeV}}\right)^3,$$

  - One from vacuum fluctuation, $g_W \to 0$

$$\Omega_W \simeq \Omega_{\rm DM} \times \sqrt{\frac{m_W}{6\times 10^{-11}{\rm GeV}}} \times \left(\frac{\mathcal{H}}{10^{13}{\rm GeV}}\right)^2,$$

Graham, Mardon&Rajendran, 1504.02102
Ema, Nakayama&Tang, 1903.10973

Combined Plot



Open Questions

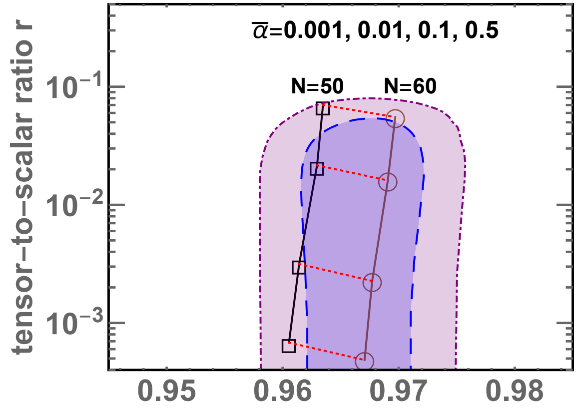
- The symmetry is not exact, origin of the scale v?
- The mass of Weyl gauge boson is around Planck scale, unless the coupling is very small or there is a dedicated cancellation
- The production from vacuum fluctuation for general gauge coupling
- New ideas for searches are needed
- For more general parameter sets

Summary

- We have investigated the inflation dynamics, and Higgs and dark matter phenomenology, based on a theory with Weyl symmetry.
- For inflation,

$$n_s \simeq 1 - \frac{2}{N}$$

$$r \simeq \frac{1}{\bar{\alpha} N^2}$$



- For Higgs and DM
  - Collider
  - Cosmic rays
  - Relic density