

# Muon $g-2$ in 2HDMs (g2HDM, Variant Axion Models)

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Michihisa Takeuchi (KMI, Nagoya)

based on arXiv:1907.09845 (with S. Iguro, Y. Omura)

arxiv:1807.00593 (with C.-W. Chiang, P.-Y. Tseng, T. T. Yanagida )

(and JHEP11(2015)057 [arXiv:1507.04354], PhysRevD.97.035015 [arXiv:1711.02993])



at the 1st AEI workshop for BSM, Jeju, on 6th Nov. 2019

# Muon g-2 : signature of BSM?

magnetic moment (potential term in a magnetic field)

$$\mathcal{H} = -\vec{\mu} \cdot \vec{B} \quad \vec{\mu} = -g \frac{e}{2m} \vec{S}$$

$$g = 2 \quad \text{tree level, Dirac equation}$$

$$g = 2.002\,331 \quad \text{QED, } \frac{\alpha}{\pi} = 0.00232\dots$$

$$g = 2.002\,331\,83 \quad \text{hadronic}$$

$$g = 2.002\,331\,836\,6 \quad \text{EW}$$

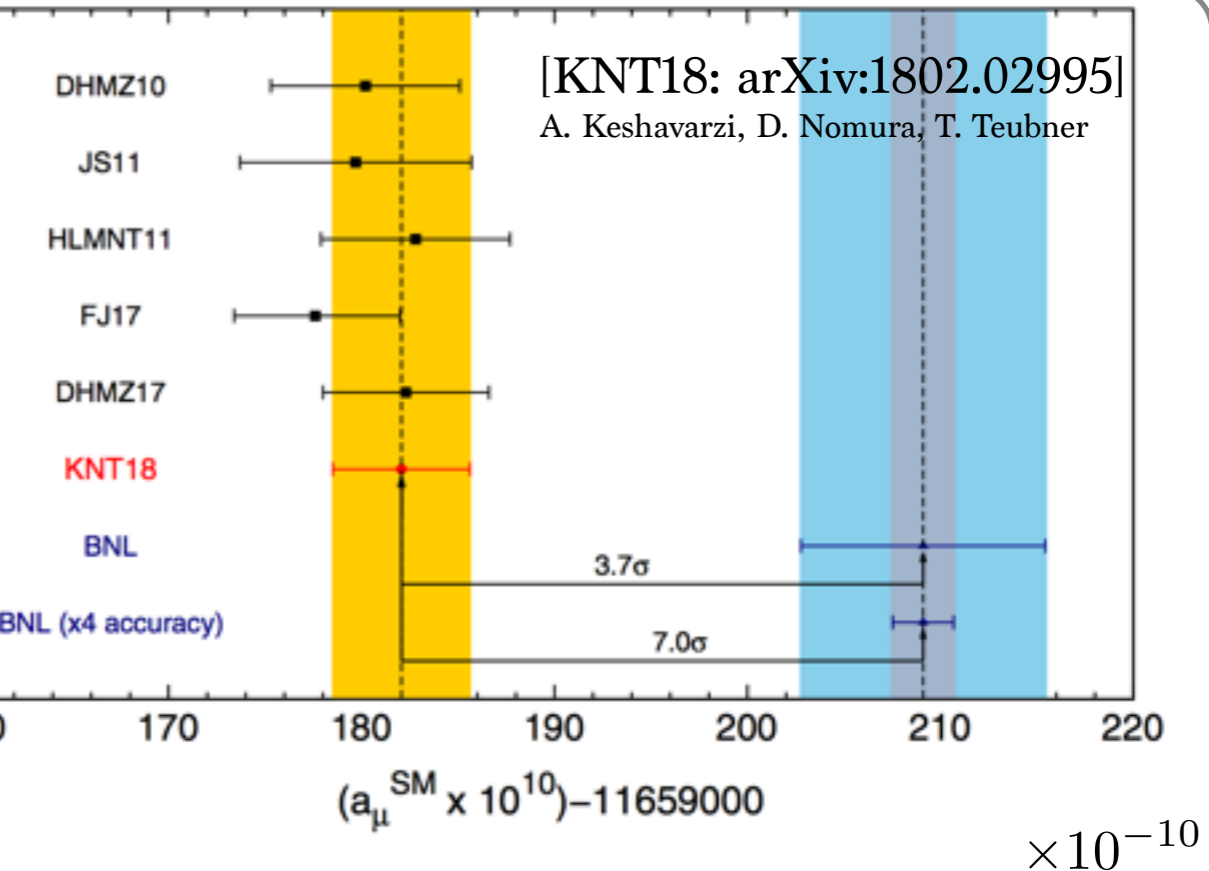
anomalous magnetic moment

$$a_\mu = (g_\mu - 2)/2$$

currently computed including 5-loop QED,  
up to 9th digit reliable

For long time, the  $3\sigma$  level discrepancy observed

$$\Delta a_\mu = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} \sim \Delta a_\mu^{\text{EW}} \sim \mathcal{O}(10^{-9})$$



Theory total	11659182.80 (4.94)	→	11659182.05 (3.56)
Experiment			11659209.10 (6.33)
Exp - Theory	26.1 (8.0)	→	27.1 (7.3)
Δa <sub>μ</sub>	3.3σ	→	3.7σ

last year, estimate of the uncertainty reduced  
the resulting significance increased

$$\Delta a_\mu^{\text{NP}} \sim \frac{g_{\text{NP}}^2}{16\pi^2} \frac{m_\mu^2}{m_{\text{NP}}^2} \quad \text{Hint for BSM?}$$

New physics at O(100GeV) ?

# Two Higgs Doublet Models (2HDM)

one additional Higgs doublet to the SM : new states  $H, A, H^\pm$

$$\Phi_1 = \begin{pmatrix} H_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + h_1 + ia_1) \end{pmatrix}, \Phi_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + h_2 + ia_2) \end{pmatrix}$$

$$v_1^2 + v_2^2 = v_{\text{SM}}^2 = (246\text{GeV})^2$$

$$\tan \beta = v_2/v_1$$

appear as a low energy EFT in many well-motivated models (MSSM, Axion Models (PQ sym))

Yukawa interactions in general for both higgs doublets

$$\begin{aligned} \mathcal{L} = & -\bar{Q}_L^i H_1 y_d^i d_R^i - \bar{Q}_L^i H_2 \rho_d^{ij} d_R^j - \bar{Q}_L^i (V^\dagger)^{ij} \tilde{H}_1 y_u^j u_R^j - \bar{Q}_L^i (V^\dagger)^{ij} \tilde{H}_2 \rho_u^{jk} u_R^k \\ & -\bar{L}_L^i H_1 y_e^i e_R^i - \bar{L}_L^i H_2 \rho_e^{ij} e_R^j + \text{h.c.} \end{aligned}$$

$$\tilde{H} = (i\sigma_2)H^*$$

to avoid tree-level FCNC, certain parity structure is often introduced (otherwise simultaneously not diagonalized)  
each type of fermions can couple to one higgs doublet

model	$u_R$	$d_R$	$e_R$	$\zeta_u$	$\zeta_d$	$\zeta_e$
Type I	$\Phi_2$	$\Phi_2$	$\Phi_2$	$\cot \beta$	$\cot \beta$	$\cot \beta$
Type II (MSSM-like)	$\Phi_2$	$\Phi_1$	$\Phi_1$	$\cot \beta$	$-\tan \beta$	$-\tan \beta$
Type X (Lepton-specific)	$\Phi_2$	$\Phi_2$	$\Phi_1$	$\cot \beta$	$\cot \beta$	$-\tan \beta$
Type Y (Flipped)	$\Phi_2$	$\Phi_1$	$\Phi_2$	$\cot \beta$	$-\tan \beta$	$\cot \beta$

$$\xi_f^h = s_{\beta-\alpha} + c_{\beta-\alpha}\zeta_f$$

$$\xi_f^H = c_{\beta-\alpha} - s_{\beta-\alpha}\zeta_f$$

$$\xi_f^A = \underline{(2T_f^3)\zeta_f}$$

\* tan beta enhancement always with the minus sign, the pseudo-scalar couplings depends on isospin

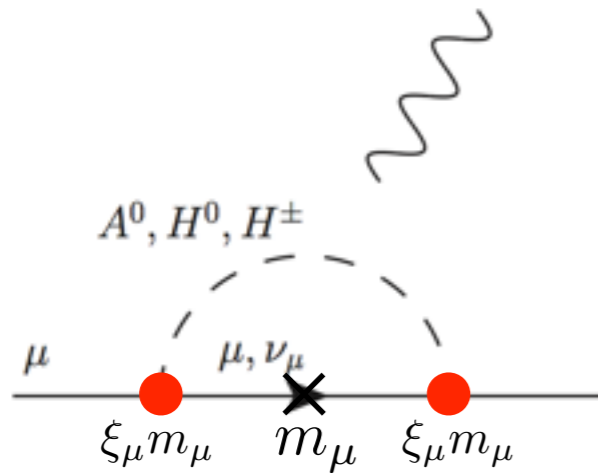
# g-2 in 2HDM

$$r_f^i = m_f^2/m_i^2$$

$$f_{h,H}(r) = \int_0^1 dx \frac{x^2(2-x)}{1-x+rx^2}, \quad f_A(r) = \int_0^1 dx \frac{-x^3}{1-x+rx^2}$$

$$f_{H^\pm}(r) = \int_0^1 dx \frac{-x(1-x)}{1-r(1-x)}$$

1-loop in 2HDM



suppressed by  $\frac{m_\mu}{v} \sim 10^{-3}$

$$\Delta a_\mu^{1\text{-loop}} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \sum_i^{h,H,A,H^\pm} (\xi_\mu^i)^2 \frac{m_\mu^2}{m_i^2} f_i(r_f^i) \sim 10^{-9} \sim 10^{-7} (m_H = 1\text{TeV})$$

$\mathcal{O}(10^{-9})$  contribution required

cf.) muon-specific 2HDM

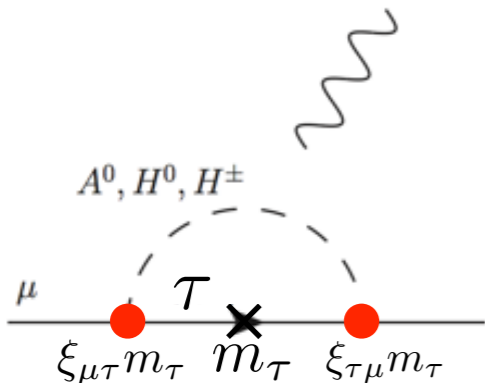
$$\xi_\mu \sim 3000$$

[T. Abe, R. Sato, K. Yagyu, arXiv:1705.01469]

$$m_H = 1\text{TeV}$$

introducing LFV coupling has an advantage

➔ LFV enhance with  $m_\tau^3/m_\mu^3 \sim 5000$ ,  $\xi_{\mu\tau} \sim \xi_{\tau\mu} \sim 50$  required  $m_H = 1\text{TeV}$



consider the case only LFV couplings  $\rho^{\mu\tau}, \rho^{\tau\mu}$  introduced for heavy higgses

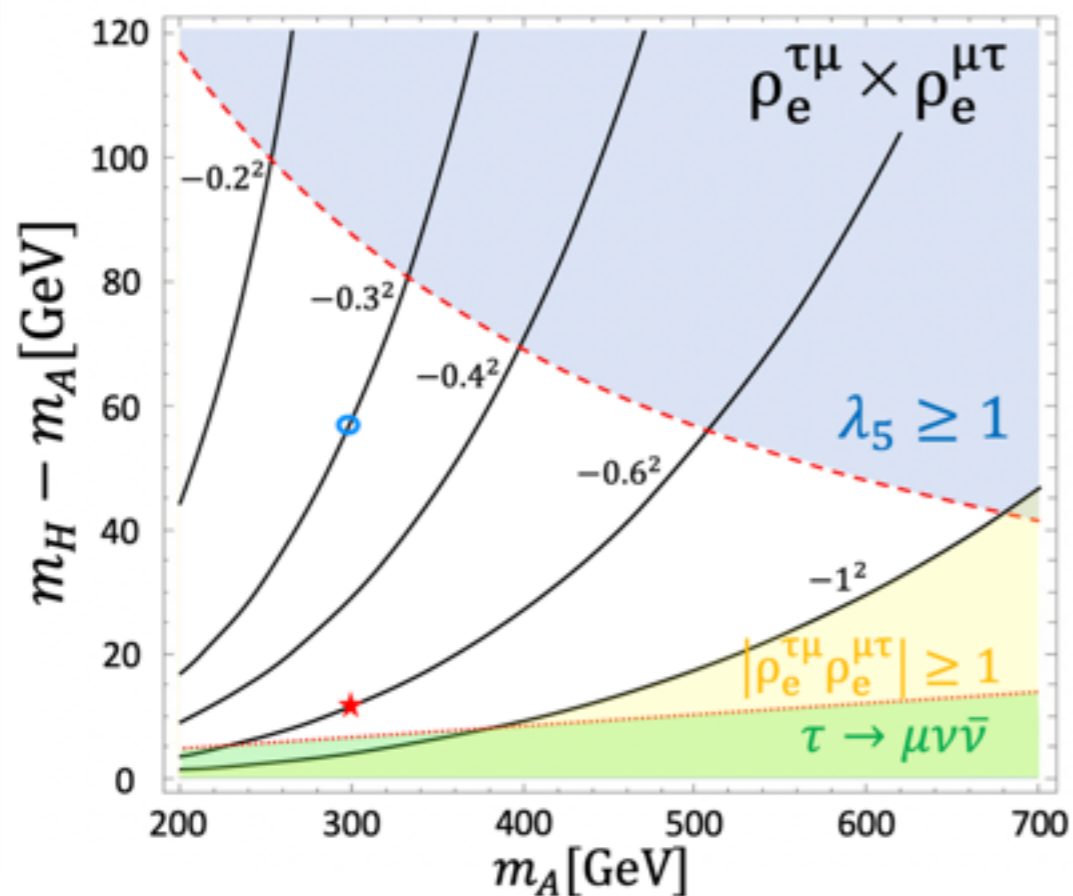
# g-2 via lepton flavor violation

[S.Iguro, Y. Omura, MT arXiv:1907.09845]

g2HDM (new Yukawa matrices : free parameters, phenomenological analysis)

we consider only  $\rho^{\mu\tau}, \rho^{\tau\mu}$  cf) [Y. Abe, T. Toma and K. Tsumura, arXiv:1904.10908]

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+\phi_1+iG}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \frac{\phi_2+iA}{\sqrt{2}} \end{pmatrix} \quad \mathcal{L} = -\bar{\ell}_{Li} H_2 \rho^{ij} e_{Rj} + h.c.$$



$$\begin{aligned} \Delta a_\mu &\simeq -\frac{m_\mu m_\tau \rho_e^{\mu\tau} \rho_e^{\tau\mu}}{8\pi^2} \frac{\Delta_{H-A}}{m_A^3} \left( \ln \frac{m_A^2}{m_\tau^2} - \frac{5}{2} \right) \\ &\simeq -3 \times 10^{-9} \left( \frac{\rho_e^{\mu\tau} \rho_e^{\tau\mu}}{0.3^2} \right) \left( \frac{\Delta_{H-A}}{60[\text{GeV}]} \right) \left( \frac{300[\text{GeV}]}{m_A} \right)^3 \end{aligned}$$

$H, A$  contributions cancel each other, total contributions  $\propto \Delta_{H-A} = m_H - m_A$

controlled by Higgs potential,  $V(H_i) = \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) + \{ \frac{\lambda_5}{2} (H_1^\dagger H_2)^2 + h.c. \} + \dots$

$$m_H^2 \simeq m_A^2 + \lambda_5 v^2, \quad m_{H^\pm}^2 \simeq m_A^2 - \frac{\lambda_4 - \lambda_5}{2} v^2,$$

we assume  $m_A \leq m_H = m_{H^\pm}$  and require perturbativity, stability

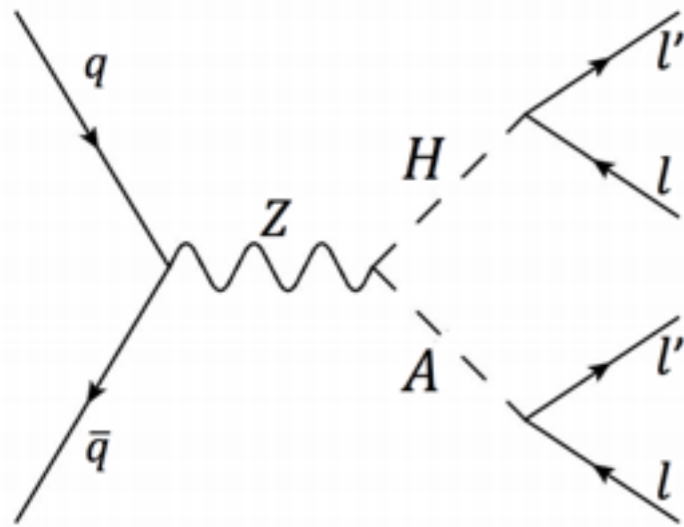
$$0 < \lambda_5 < 1 \quad |\rho^{\mu\tau}|, |\rho^{\tau\mu}| < 1$$

the parameter region available to explain g-2 is finite

$$m_A \lesssim 700\text{GeV} \quad \text{and} \quad 10\text{GeV} \lesssim \Delta_{H-A} \lesssim 100\text{GeV}$$

# g-2 via lepton flavor violation — LHC signatures

[S.Iguro, Y. Omura, MT arXiv:1907.09845]



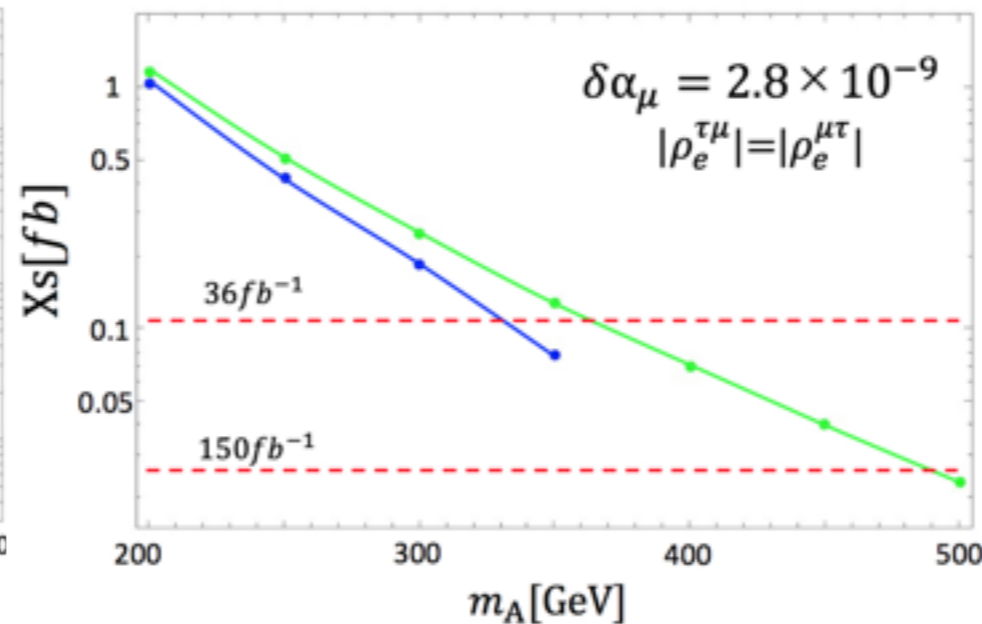
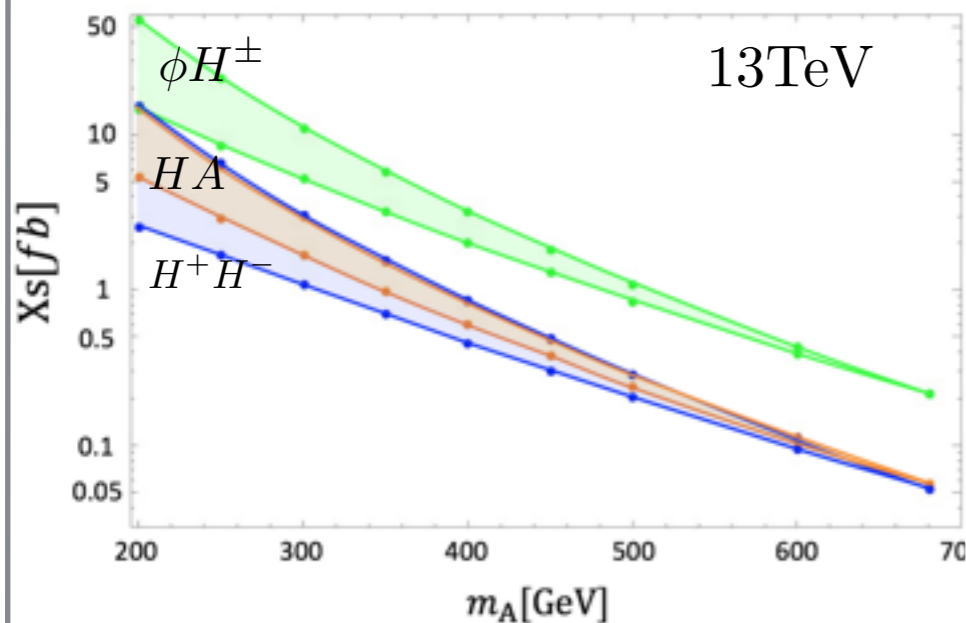
no QCD coupling : small but still sizable rate via SU(2) coupling

Heavy higgses produced in pair via Drell-Yan,  $HA$ ,  $\phi H^\pm$ , and  $H^+H^-$ , where  $\phi = H, A$ .

$$BR(\phi \rightarrow \tau^+ \mu^-) = BR(\phi \rightarrow \tau^- \mu^+) = 0.5,$$

$$BR(H^\pm \rightarrow \tau^\pm \nu) = 1 - BR(H^\pm \rightarrow \mu^\pm \nu) = \frac{|\rho_e^{\mu\tau}|^2}{|\rho_e^{\tau\mu}|^2 + |\rho_e^{\mu\tau}|^2} \equiv r.$$

they result in 4 leptons, 3 leptons, 2 leptons



multi-lepton  $2\mu 2\tau$  channels

Especially  $\mu^\pm \mu^\pm \tau^\mp \tau^\mp$

current data should already be sensitive at LHC up to 500 GeV

# g-2 via LFV — mass reconstruction at LHC

[S.Iguro, Y. Omura, MT arXiv:1907.09845]

in future at 14 TeV,  $\sim 2\text{fb}$  (300 GeV) with 3 ab  $\Rightarrow \sim 6000$  HA pair produced, other modes similarly produced

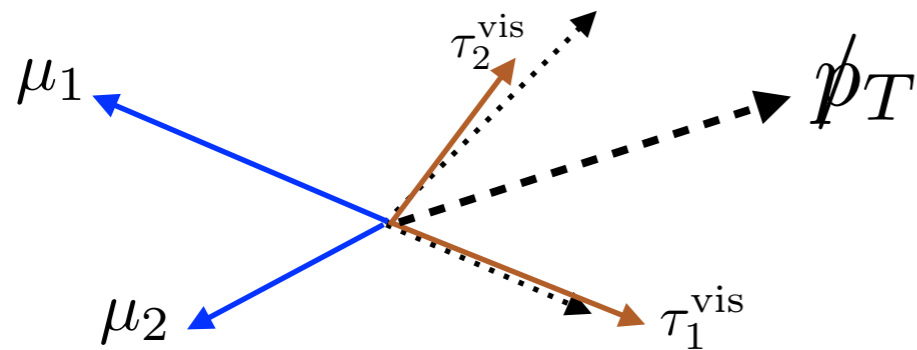
4 leptons from HA production

$\mu^\pm \mu^\pm \tau^\mp \tau^\mp$	same-sign di-muon di-tau (50%)	$\mathcal{O}(200 - 300)$ events for 3 ab $^{-1}$
$\mu^+ \mu^- \tau^+ \tau^-$	opposite-sign di-muon di-tau (50%)	OSOF pair gives the resonances (almost BG free)

$\tau$ -momentum : collinear approx.

$$\mathbf{p}_{\tau_i} = (1 + c_i) \mathbf{p}_{\tau_i}^{\text{vis}}$$

$$\not{p}_T = c_1 \mathbf{p}_{T,\tau_1}^{\text{vis}} + c_2 \mathbf{p}_{T,\tau_2}^{\text{vis}} \quad (c_1, c_2 > 0).$$



for  $\mu^\pm \mu^\pm \tau^\mp \tau^\mp$

two possible combinations :

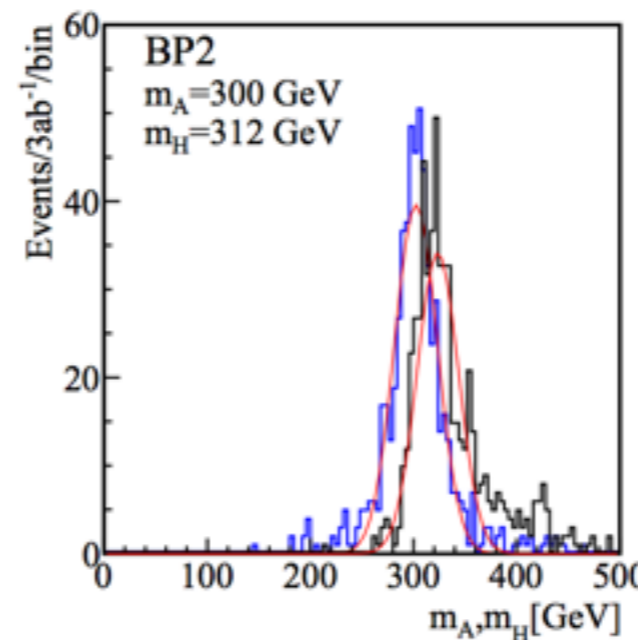
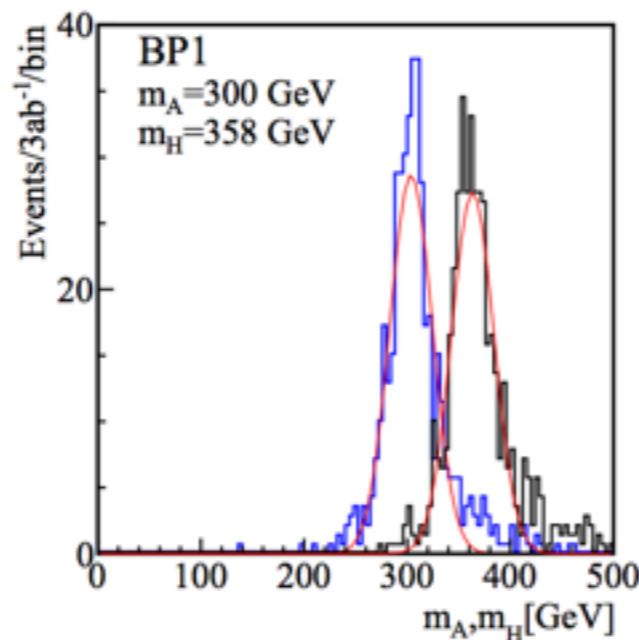
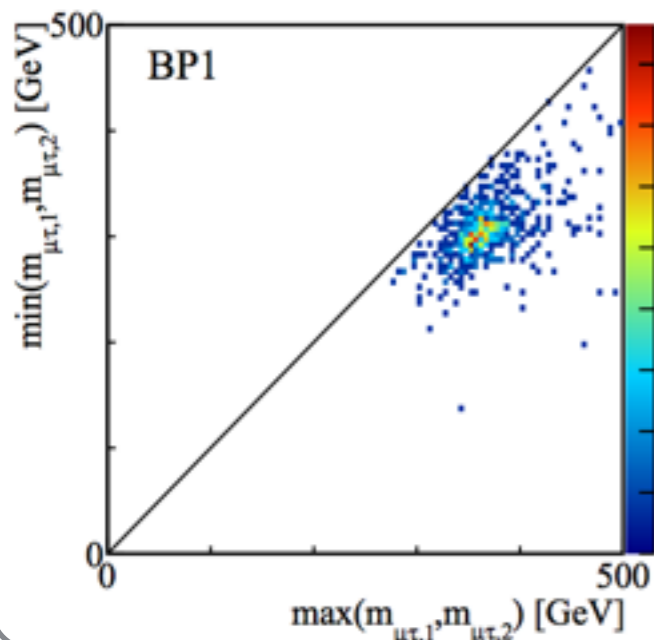
combination 1 :  $m_{\mu_1 \tau_1}$  and  $m_{\mu_2 \tau_2}$

combination 2 :  $m_{\mu_1 \tau_2}$  and  $m_{\mu_2 \tau_1}$

$\mu_1, \mu_2, \tau_1^{\text{vis}},$  and  $\tau_2^{\text{vis}}$  in  $p_T$ -order

select the one minimizing the sum of

$$\chi_i^2(m_A, m_H) = (m_{\mu\tau,i}^{\min} - m_A)^2 / \sigma_{\text{res}}^2 + (m_{\mu\tau,i}^{\max} - m_H)^2 / \sigma_{\text{res}}^2$$



can reconstruct  
two invariant masses  
 $m_A$  and  $m_H$

$$\sigma_{\text{res}} \sim 20\text{GeV}$$

cf.)  $10\text{GeV} \lesssim \Delta_{H-A} \lesssim 100\text{GeV}$

charged higgs mass  
from 3 and 2 lepton modes

# g-2 in 2HDM via 2-loop

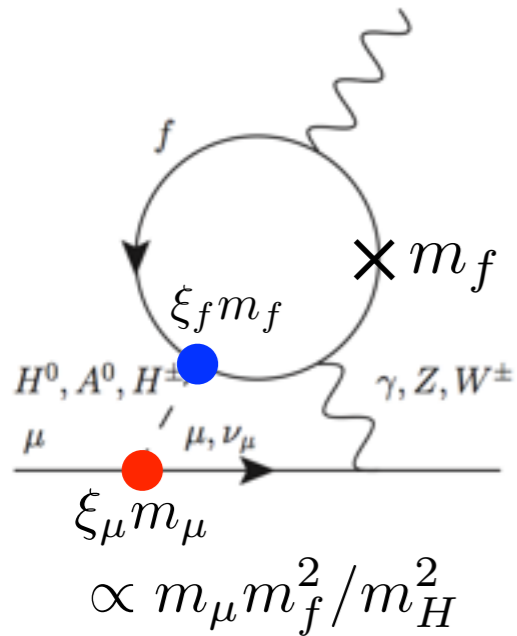
$$r_f^i = m_f^2/m_i^2$$

$$g_{h,H}(r) = \int_0^1 dx \frac{2x(1-x) - 1}{x(1-x) - r} \ln \frac{x(1-x)}{r}$$

$$g_A(r) = \int_0^1 dx \frac{1}{x(1-x) - r} \ln \frac{x(1-x)}{r}$$

2-loop (Barr-Zee) in 2HDM

enhanced by the **large yukawa** coupling for heavy fermions



$$\Delta a_\mu^{\text{BZ}} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \frac{\alpha_{\text{EM}}}{\pi} \sum_i^{h,H,A} \sum_f^{t,b,c,\tau} N_f^c Q_f^2 \xi_\mu^i \xi_f^i \frac{m_f^2}{m_i^2} g_i(r_f^i)$$

$\sim 10^{-9}$

	$\times \alpha N_f^c Q_f^2 / \pi$	Sign of $(\delta_H, \delta_A)$
t	$(-1.1, 1.5) \times 10^{-3}$	(-, -)
c	$(-5.9, 7.1) \times 10^{-7}$	(-, -)
u	$(-4.6, 5.4) \times 10^{-12}$	(-, -)
b	$(-1.1, 1.4) \times 10^{-6}$	(-, <b>+</b> )
$\tau$	$(-8.0, 9.6) \times 10^{-7}$	(-, <b>+</b> )

$\mathcal{O}(10^{-9})$  **positive** contribution required

positive sign only for A with down-type fermion

$\Rightarrow \tau$  is only the possibility  $\xi_\mu \xi_\tau / m_H^2 [\text{TeV}] \sim 10^6$  required

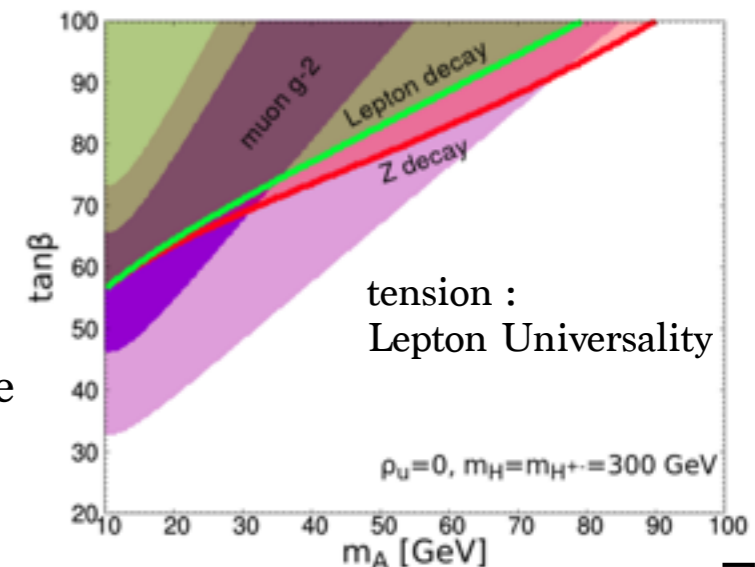
bottom (type II) disfavored by bbA at LHC and  $B_s \rightarrow \mu\mu$

In lepton-specific 2HDM model, where all leptons  $\tan\beta$  enhanced

$m_A \sim 30\text{GeV}$  and  $\tan\beta \sim 40$  will give an appropriate g-2 contribution

LHC constraints for additional higgs bosons suppressed with no QCD coupling

In future, Drell-Yan productions  $\Rightarrow$  multi-taus (4 tau, 3tau, 2tau) events would be sensitive



We extend this scenario to a well motivated model (Variant Axion Model)



# 2HDM as the solution for strong CP problem

Strong CP problem



PQ solution with axion



$\theta$ -vacuum

$$|\theta\rangle = \sum_{n=-\infty}^{\infty} e^{in\theta} |n\rangle \iff \mathcal{L}_\theta = \frac{g^2\theta}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

$\theta_{\text{eff}} = \theta + \arg \det[M^u M^d]$   
 Why  $\theta_{\text{eff}} < 10^{-11}$  ?

assume spontaneously broken U(1)  $\eta e^{i\theta_{PQ}} \sim \eta + ia$  to introduce axion field  
 triangle diagram (N: n. of coupled quarks),  $\delta\mathcal{L} = -\frac{g^2}{32\pi^2} N \frac{a}{\eta} G^{\mu\nu} \tilde{G}_{\mu\nu}$  induced  
 after QCD PT,  $\langle G^{\mu\nu} \tilde{G}_{\mu\nu} \rangle \sim \Lambda_{\text{QCD}}^4$  the potential

$$\theta_{\text{eff}} = \theta + \arg \det[M^u M^d] + \frac{\langle a \rangle}{F_a}$$

very attractive,  $a$  also play a good CDM role

invisible axion models

**KSVZ**

heavy Q introduced  
 (no problem but no low energy phenomenology, not interesting)

$$\mathcal{L}_Q = -y_Q \bar{Q}_L \Phi Q_R + \text{h.c.}$$

$$N_{DM} = 1$$

(Kim 1979, Shifman, Vainshtein, Zakharov 1980)

**ZDFS**

two Higgs doublet model

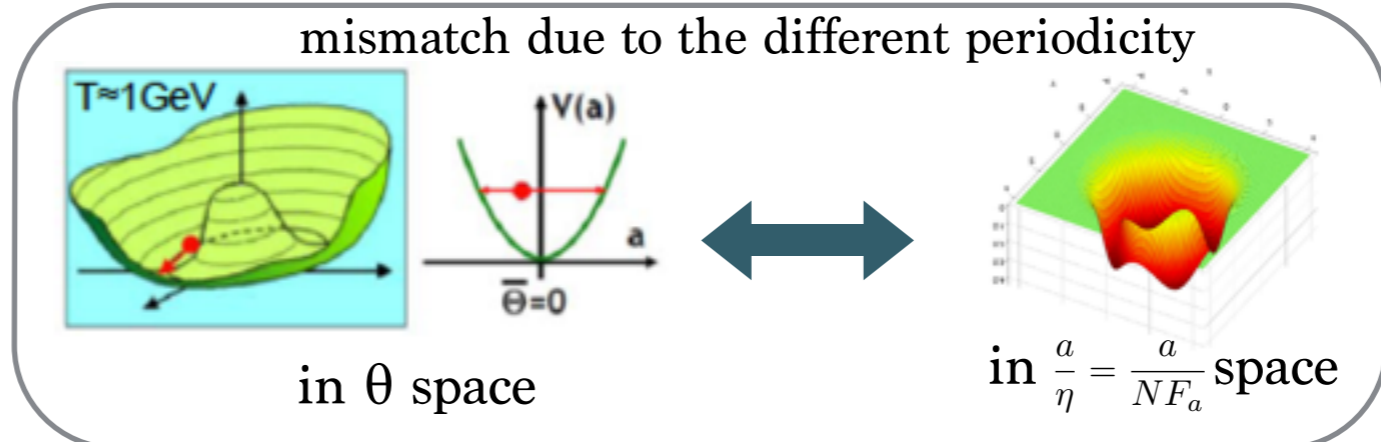
$$\Phi_1^\dagger \Phi_2 \sigma^2$$

$$m \Phi_1^\dagger \Phi_2 \sigma$$

$$N_{DM} = 6$$

$$N_{DM} = 3$$

(Zhitnitsky 1980,  
 Dine, Fischler, Srednicki 1981)



Variant Axion model

domain wall problem absent if only 1 quark coupled to PQ-Higgs

$$N_{DM} = 1$$

[R.D. Peccei, T.T. Wu and T. Yanagida, Phys. Lett. B172, 435 (1986)]

# $g-2$ in Lepton-specific 2HDM with VAM

[arxiv:1807.00593, C.-W. Chiang, MT, P.-Y. Tseng, T. T. Yanagida]

VAM is a 2HDM at low energy, there is a choice which one quark is PQ charged.

quark sector : domain wall problem  $\Rightarrow$  only one  $q_R$  PQ charged

lepton sector : lepton yukawa has to be enhanced for muon  $g-2 \Leftrightarrow$  corresponding VEV is small ( $\tan\beta \gg 1$ )

(lepton sector is irrelevant to domain wall problem)

$e$	$\Phi_1(\text{PQ} = +1)$	$u_R, d_R$
$\mu$		$c_R, s_R$
$\tau$	$\Phi_2(\text{PQ} = 0)$	$t_R, b_R$

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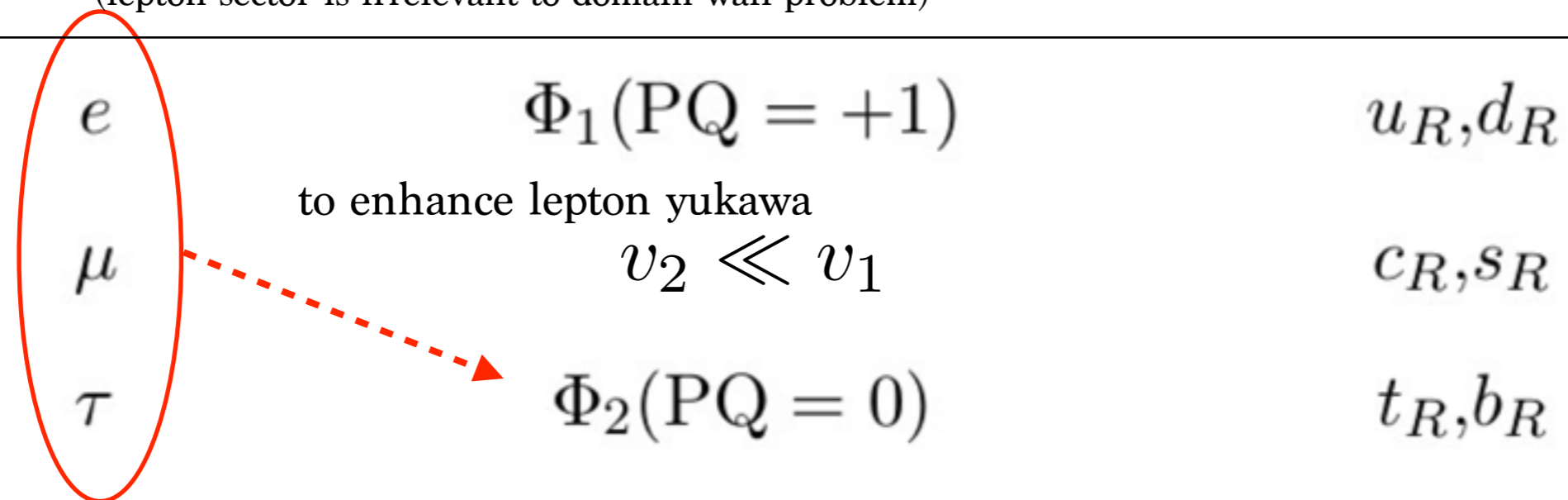
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VAM is a 2HDM at low energy with various PQ charge assignments.

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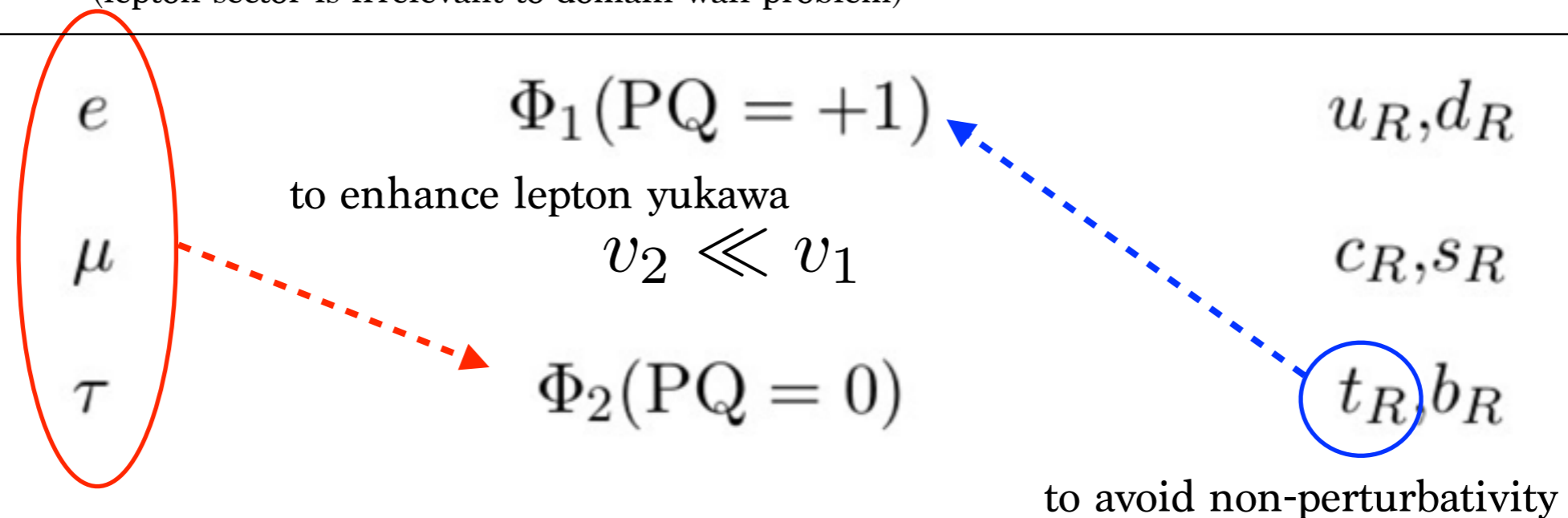
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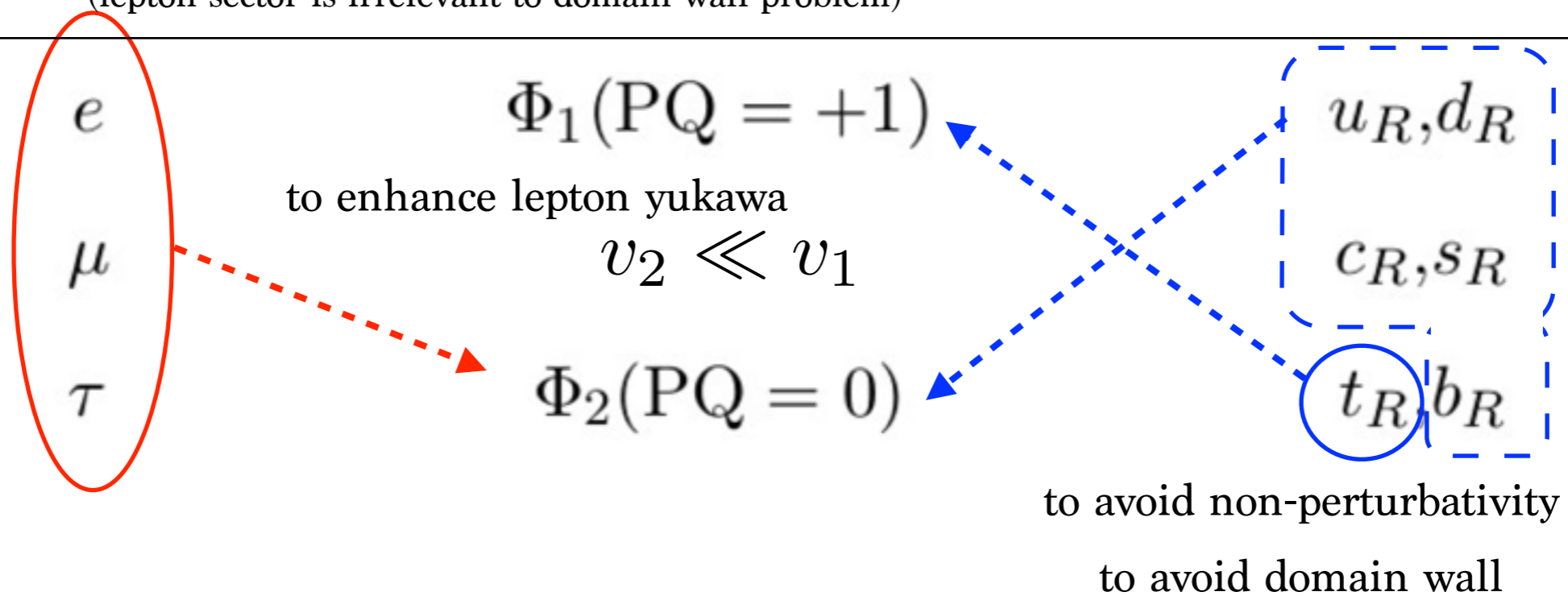
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the 3rd gen. part becomes identical to the type II 2HDM  $\Rightarrow$  very constrained by LHC via  $bbA$  production

also by  $B_s \rightarrow \mu\mu$

$\Rightarrow$  not viable possibility

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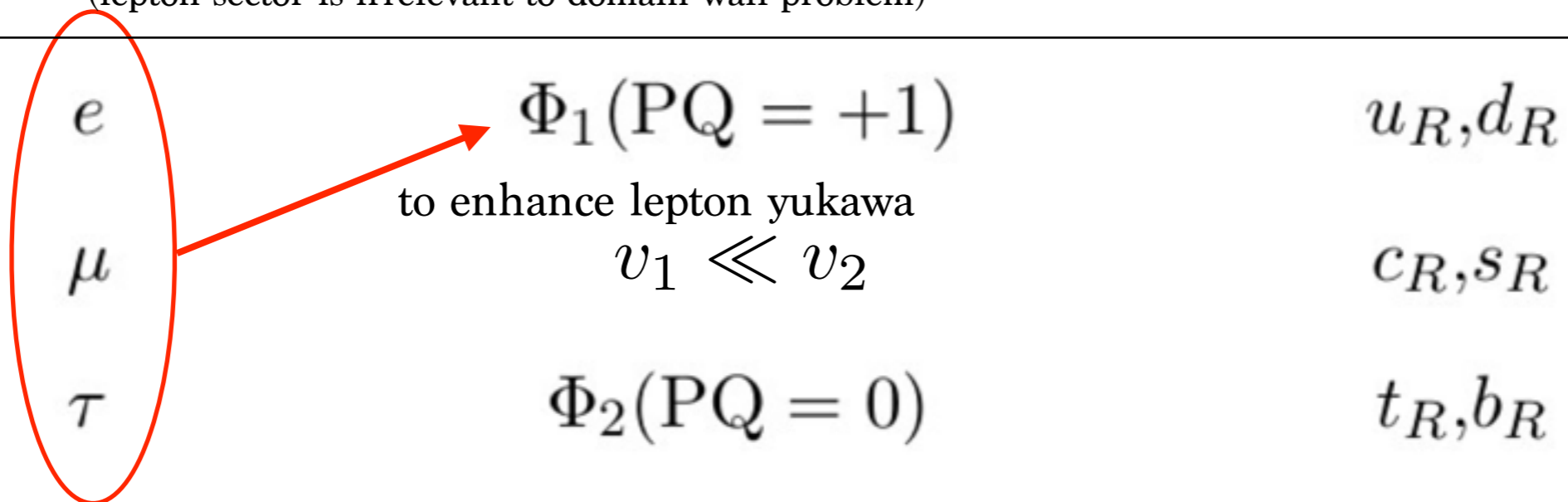
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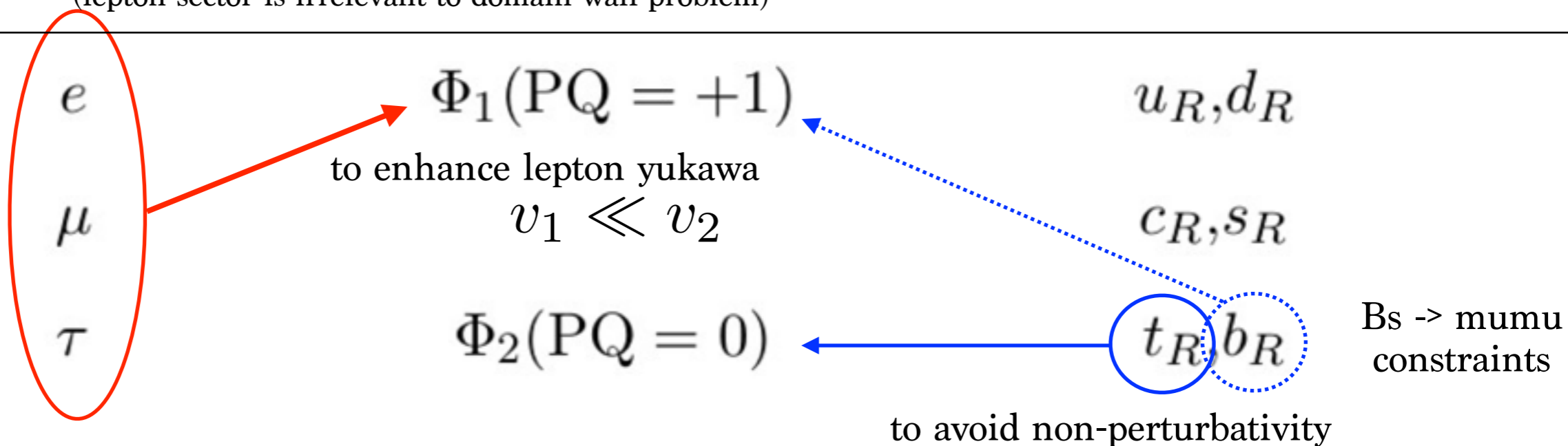
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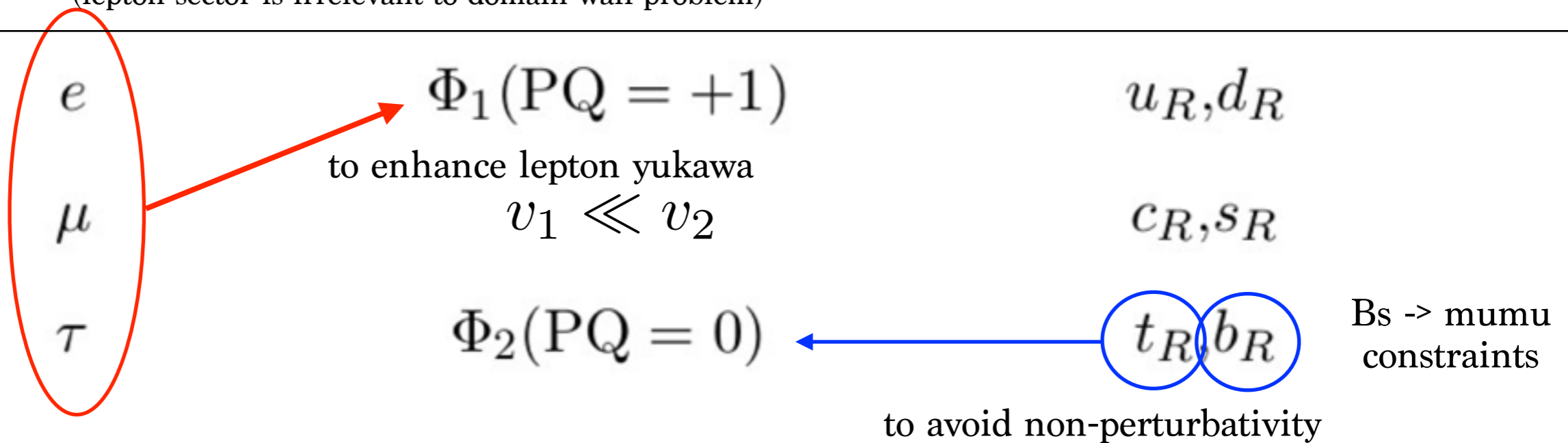
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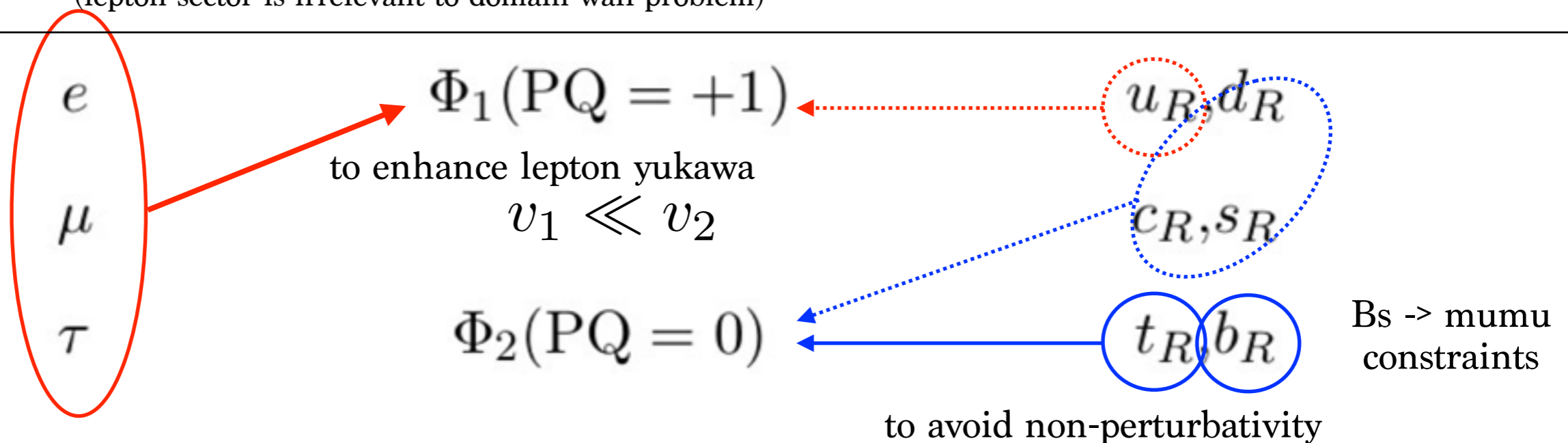
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(lepton sector is irrelevant to domain wall problem)



several choices, but up-specific is most interesting possibility

charm-specific : opposite sign for g-2

down/strange-specific : very constrained by Kaon physics

# up-type specific Variant Axion model

[arxiv:1807.00593, C.-W. Chiang, MT, P.-Y. Tseng, T. T. Yanagida]

$$\begin{array}{cccccccc} \Phi_1 & \Phi_2 & u_R & c_R & t_R & d_R & Q_L & \ell_R & L_L \\ + & - & - & + & + & + & + & - & + \end{array}$$

when we take up-type VAM,  
top/charm/up FCNC is the generic prediction

For up-specific VAM,

$$L^u = -\Phi_1 \bar{u}_{Ra} [Y_{u1}]_{ai} Q_i - \Phi_2 \bar{u}_{R3} [Y_{u2}]_{i3} Q_i + \text{h.c.}$$

$\Phi_2$  only couple with  $u_R, e_R$

$$Y_{u1} = \begin{pmatrix} * & * & * \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } Y_{u2} = \begin{pmatrix} 0 & 0 & 0 \\ * & * & * \\ * & * & * \end{pmatrix},$$

mix with  $\beta$

Higgs basis  
(only  $\Phi^{\text{SM}}$  has a VEV)

$$\mathcal{L} = -\Phi^{\text{SM}} \bar{u}_{Rj} [Y_u^{\text{SM}}]_{ji} Q_i - \Phi' \bar{u}_{Ra} [Y'_u]_{ji} Q_i$$

$$Y'_u = -s_\beta Y_{u1} + c_\beta Y_{u2} = \begin{pmatrix} -\tan \beta & & \\ & \cot \beta & \\ & & \cot \beta \end{pmatrix} Y_u^{\text{SM}}$$

$$\begin{aligned} Y'_e &= -\tan \beta Y_e^{\text{SM}}, \\ Y'_d &= \cot \beta Y_d^{\text{SM}}. \end{aligned}$$

diagonalizing mass matrix

$$Y_u'^{\text{diag}} = \begin{pmatrix} -\tan \beta & & \\ & \cot \beta & \\ & & \cot \beta \end{pmatrix} Y_u^{\text{diag}} + (\tan \beta + \cot \beta) H_u Y_u^{\text{diag}},$$

$$H_u = V_u \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} V_u^\dagger - \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} = \begin{pmatrix} \frac{\cos \rho_u - 1}{2} & 0 & \frac{\sin \rho_u}{2} \\ 0 & 0 & 0 \\ \frac{\sin \rho_u}{2} & 0 & \frac{1 - \cos \rho_u}{2} \end{pmatrix}.$$

consider u-t mixing for simplicity

$$\zeta_{uu} \equiv -\tan \beta - (\tan \beta + \cot \beta) \frac{\cos \rho_u - 1}{2},$$

$$\zeta_{cc} \equiv \cot \beta,$$

$$\zeta_{tt} \equiv \cot \beta - (\tan \beta + \cot \beta) \frac{1 - \cos \rho_u}{2},$$

$$\zeta_{ut} = \zeta_{tu} = (\tan \beta + \cot \beta) \frac{\sin \rho_u}{2}.$$

$$\xi_{ff'}^h \equiv s_{\beta-\alpha} \delta_{ff'} + c_{\beta-\alpha} \zeta_{ff'},$$

$$\xi_{ff'}^H \equiv c_{\beta-\alpha} \delta_{ff'} - s_{\beta-\alpha} \zeta_{ff'},$$

$$\xi_{ff'}^A \equiv (2T_3^f) \zeta_{ff'},$$

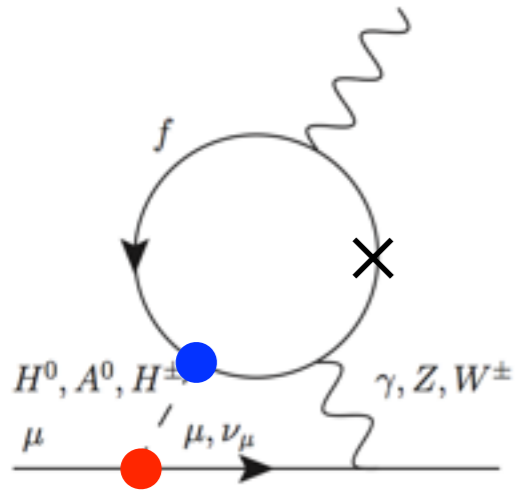
leptons and up: tan beta enhanced

$$\text{FV} \propto \sim \sin \rho \tan \beta$$

$$\text{mixing eff.} : \zeta_{uu} : -\tan \beta \nearrow, \zeta_{tt} : \cot \beta \searrow -\tan \beta$$

# g-2 in Lepton-specific 2HDM with VAM

[arxiv:1807.00593, C.-W. Chiang, MT, P.-Y. Tseng, T. T. Yanagida]



$$\Delta a_\mu^{\text{BZ}} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \frac{\alpha_{\text{EM}}}{\pi} \sum_i^{h,H,A} \sum_f^{t,b,c,\tau} N_f^c Q_f^2 \xi_\mu^i \xi_f^i \frac{m_f^2}{m_i^2} g_i(r_f^i) \sim 10^{-9}$$

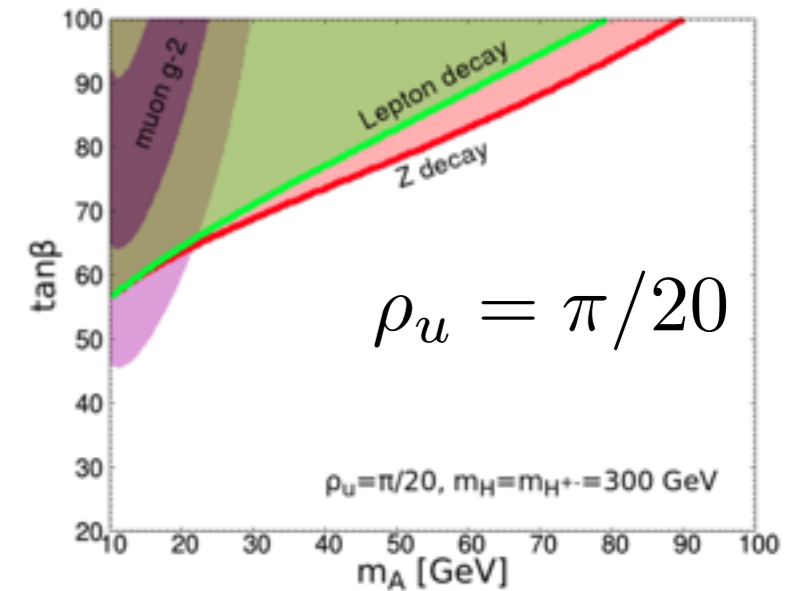
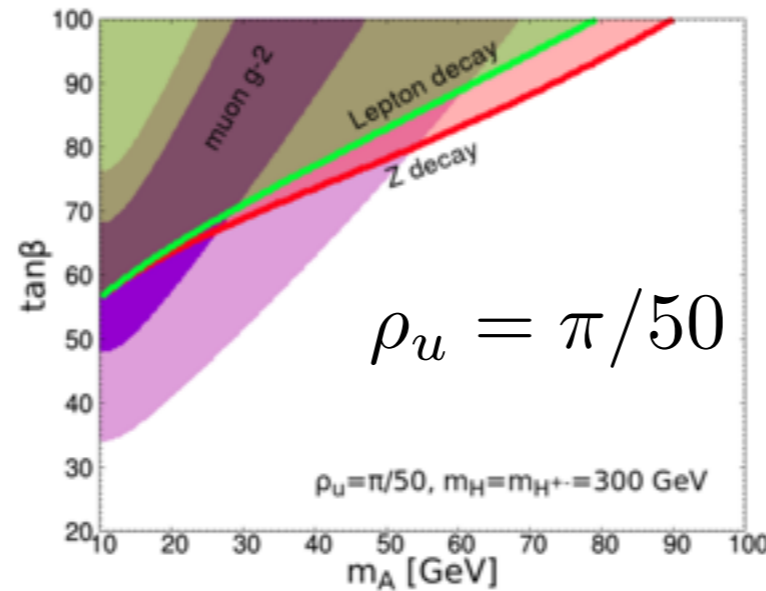
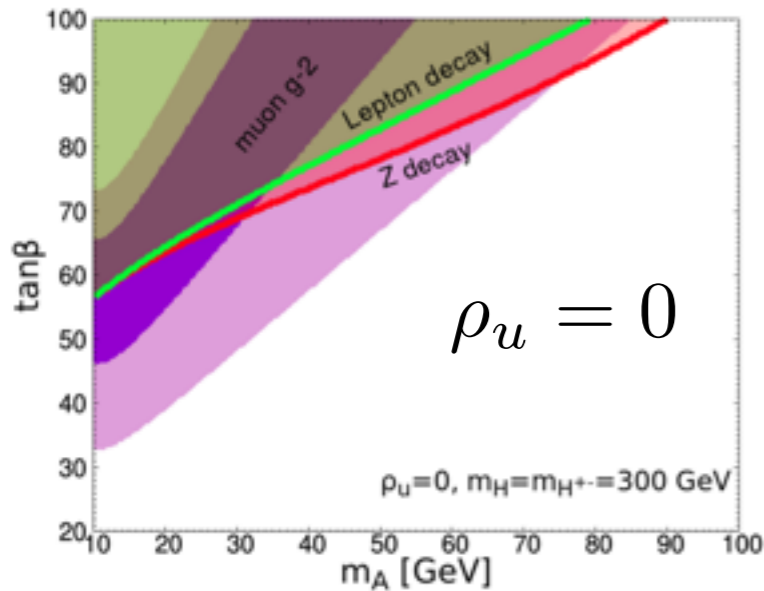
	$\times \alpha N_f^c Q_f^2 / \pi$	Sign of $(\delta_H, \delta_A)$
t	$(-1.1, 1.5) \times 10^{-3}$	(-, -)
c	$(-5.9, 7.1) \times 10^{-7}$	(-, -)
u	$(-4.6, 5.4) \times 10^{-12}$	(-, -)
b	$(-1.1, 1.4) \times 10^{-6}$	(-, +)
$\tau$	$(-8.0, 9.6) \times 10^{-7}$	(-, +)

$$\propto m_\mu m_f^2 / m_H^2$$

opposite sign contributions  $-\tan \beta$  enhanced for up-type  $\Rightarrow$  only up negligible  
 LFV doesn't contribute directly to g-2, but affects the diagonal elements

$$\text{FV} \propto \sim \sin \rho \tan \beta$$

$$\text{mixing eff.} : \zeta_{uu} : -\tan \beta \nearrow, \zeta_{tt} : \cot \beta \searrow -\tan \beta$$



switching on LFV coupling induces negative top-loop contribution  $\Rightarrow$  rather disfavored by g-2

but acceptable as long as a small mixing

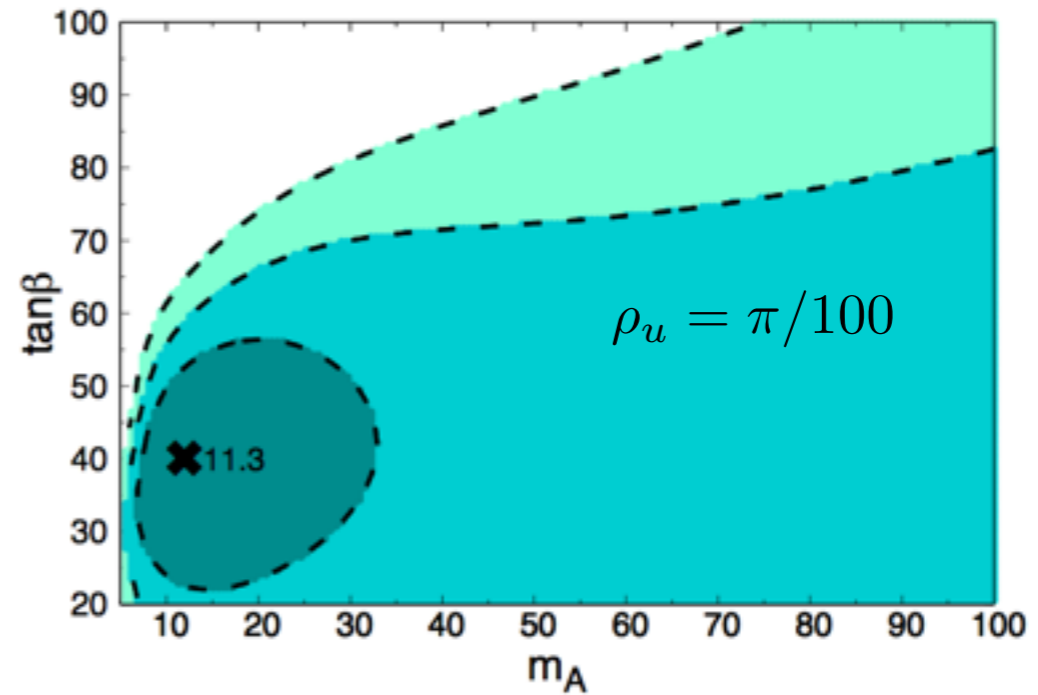
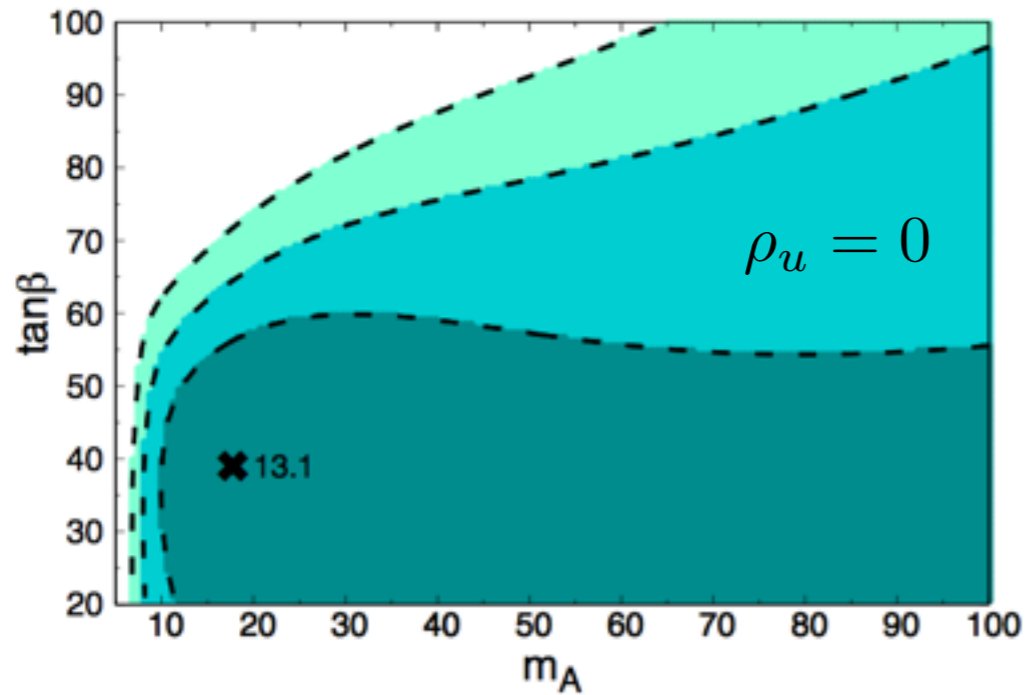
# g-2 in Lepton-specific 2HDM with VAM

[arxiv:1807.00593, C.-W. Chiang, MT, P.-Y. Tseng, T. T. Yanagida]

$B_s \rightarrow \mu\mu$  observation exhibit a slight deficit from the SM prediction

$$\bar{R}_{s\mu} \equiv \frac{\overline{\text{BR}}(B_s^0 \rightarrow \mu^+\mu^-)_{\text{EXP}}}{\overline{\text{BR}}(B_s^0 \rightarrow \mu^+\mu^-)_{\text{SM}}} = 0.79 \pm 0.20$$

$$\frac{\mathcal{M}}{\mathcal{M}^{\text{SM}}} = 1 + \frac{\mathcal{M}^{u\text{VAM}}}{\mathcal{M}^{\text{SM}}} \sim 1 - 0.21 \xi_{tt}^A \xi_{\mu\mu}^A \left( \frac{15\text{GeV}}{m_A} \right)^2 \sim 1.21 - 0.05 \rho_u^2 \tan^2 \beta$$



for combined  $\chi^2$ -fit including  $B_s \rightarrow \mu\mu$ , small mixing  $\rho_u = \pi/100$  slightly improves the fit

$m_A \sim 15\text{GeV}$ ,  $\tan\beta \sim 40$ ,  $\rho_u \sim 0.03$  will give a best fit

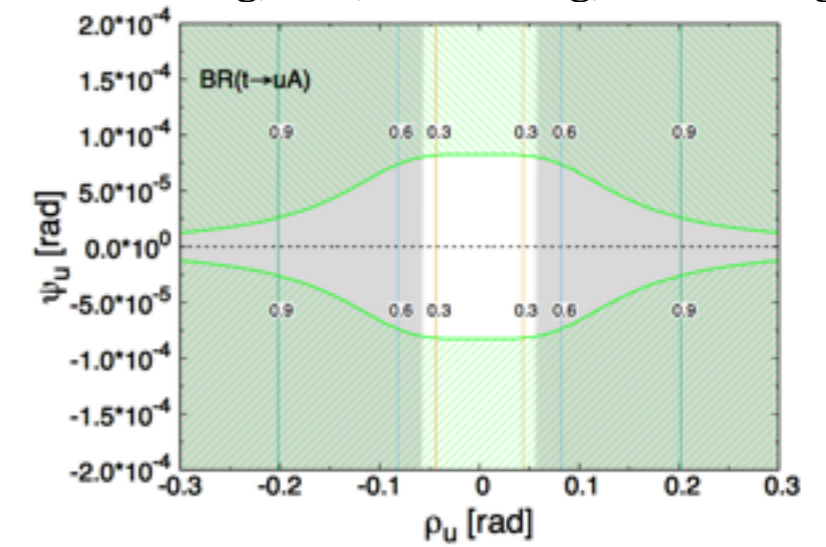
$$t \rightarrow u A, A \rightarrow \tau \tau$$

even for a slight mixing  $\rho \sim 0.03$  induces large  $BR(t \rightarrow uA) \sim O(10\%)$

$$\Gamma_{t \rightarrow uA/cA} \propto \sin^2 \rho_u \tan^2 \beta$$

$A$  decays dominantly to  $\tau\tau$  about 100%

important signal from top pair production :  $t\bar{t} \rightarrow t\bar{u}A, A \rightarrow \tau\tau$



$$\Gamma_{t,\text{tot}} \leq 2.5\text{GeV} \rightarrow BR(t \rightarrow uA/cA) \lesssim 40\% \quad |\rho_u| \lesssim 0.06$$

recast the LHC searches for  $bbA, A \rightarrow \tau\tau$ , in the context of MSSM (type II)

(CMS at 8TeV in  $\mu\tau, e\tau, e\mu$  modes)

kinematics is different between  $tuA$  and  $bbA$

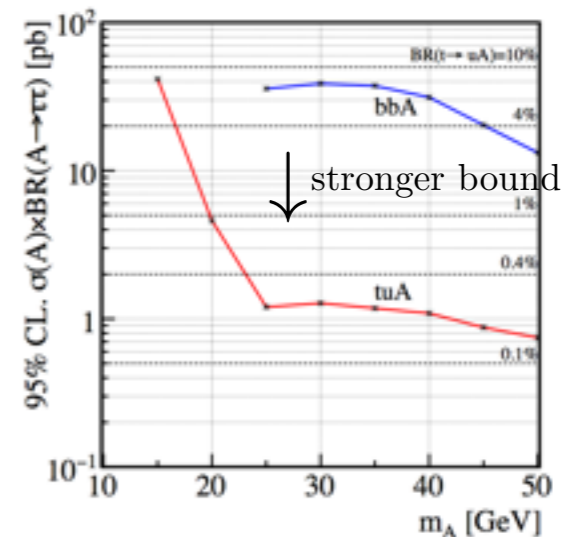
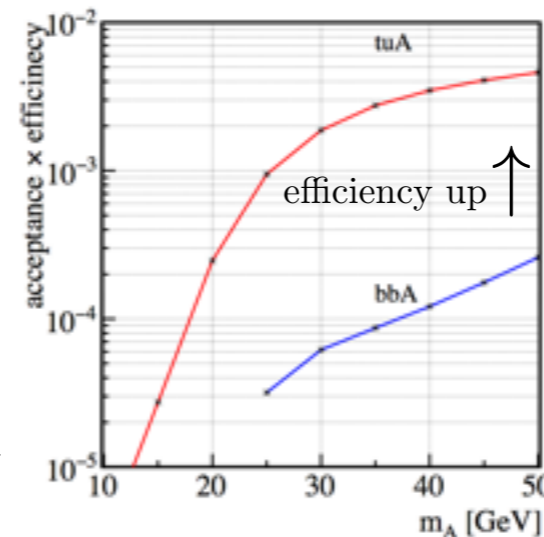
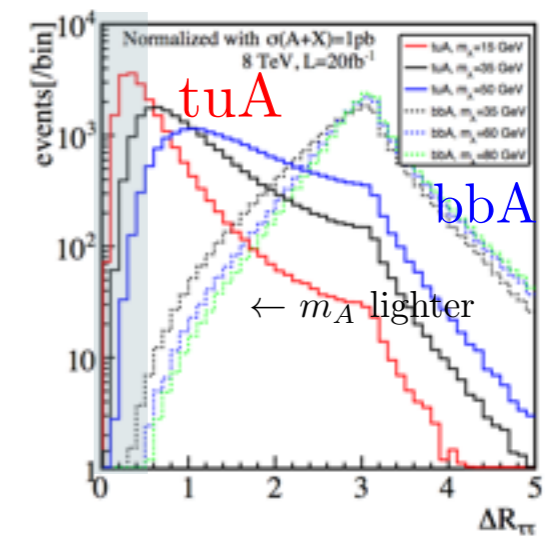
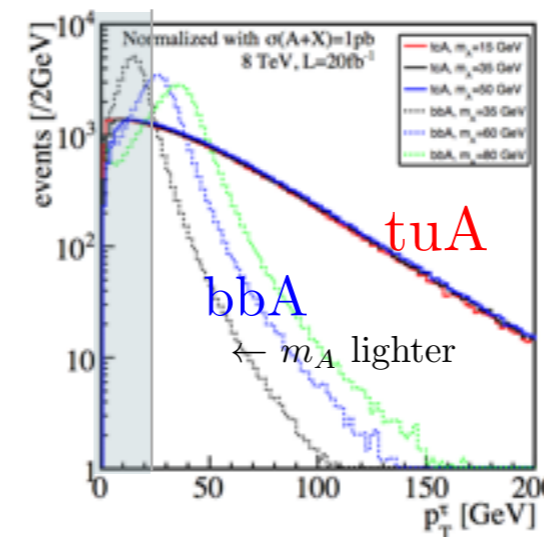
– efficiency for  $tuA$

higher due to  $p_{T,\tau}$  cut

quickly goes down as  $m_A \rightarrow 0$  due to  $\Delta R$  cut

we estimate 8 TeV sensitivity,

$$BR(t \rightarrow uA) < 0.2\% \quad (m_A > 25\text{GeV}), 10\% \quad (m_A = 15\text{GeV}) : \text{marginal}$$

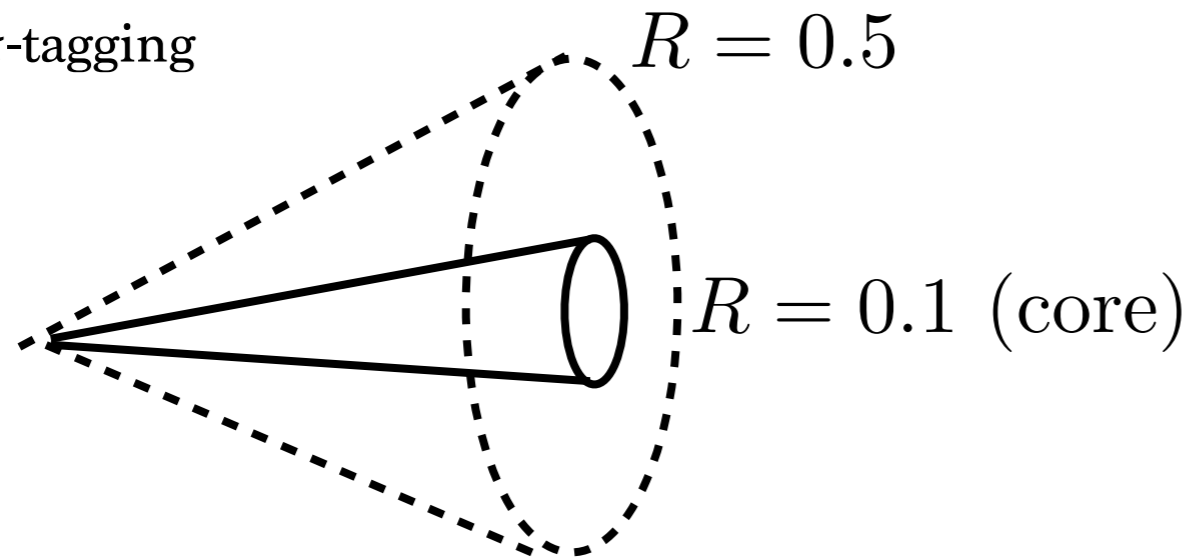


# boosted $A \rightarrow \tau \tau$

[arxiv:1807.00593, C.-W. Chiang, MT, P.-Y. Tseng, T. T. Yanagida]

The reason for rapid drop of the efficiency is due to the overlapping  $\tau$ 's due to the boost

$\tau$ -tagging

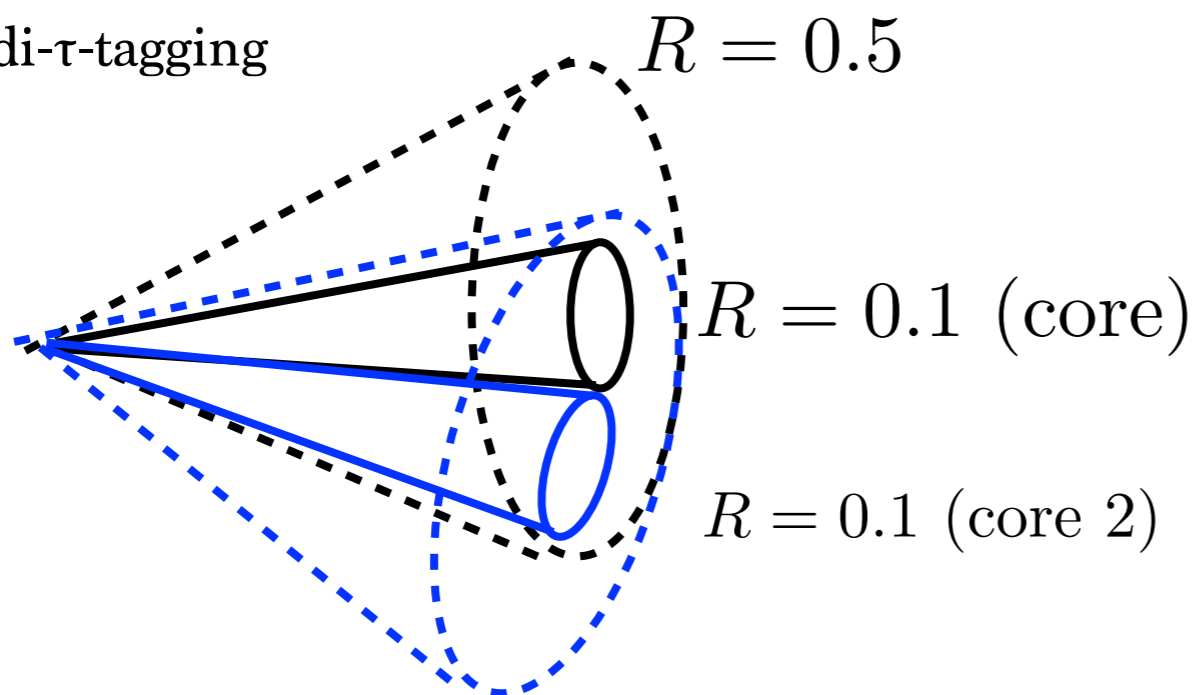


require energy deposit in the core part

$$f = \frac{E(R = 0.1)}{E(R = 0.5)} > 0.95$$

for boosted tau pair the usual isolation fails

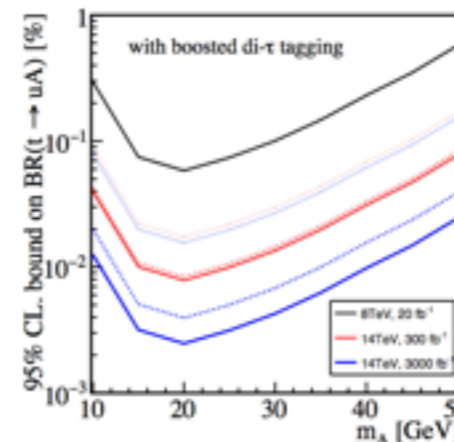
di- $\tau$ -tagging



mutual isolation

[A. Katz, M. Son, B. Tweedie, PRD 83, 114033(2011).]

if core 1 is removed, the rest is  $\tau$ -tagged  
if core 2 is removed, the rest is also  $\tau$ -tagged



For  $m_A=15\text{GeV}$

$$BR(t \rightarrow uA) < 0.08\%$$

(10% by CMS study)

0.003-0.01% in future

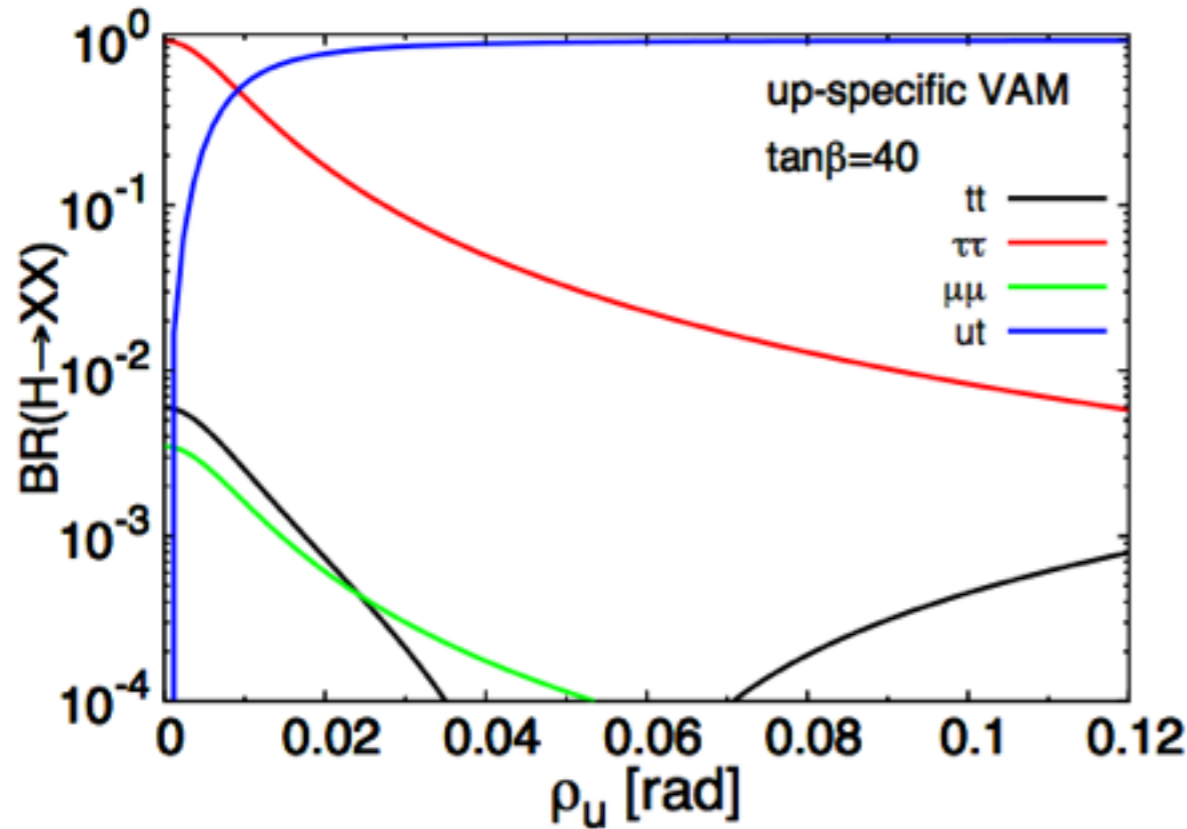
# Flavor violating Heavy higgs decays

[arxiv:1807.00593, C.-W. Chiang, MT, P.-Y. Tseng, T. T. Yanagida]

For  $m_H \gg m_t$  and  $\tan \beta \gg 1$ , we have

$$\frac{BR(H \rightarrow tu)}{BR(H \rightarrow \tau\tau)} \sim \frac{m_t^2}{m_\tau^2} \frac{3 \sin^2 \rho_u}{2} \simeq (120 \cdot \sin \rho_u)^2$$

$$\mathcal{L} \supset \sum_{f,f'}^{u,c,t,d,s,b,e,\mu,\tau} -\frac{m_{f'}}{v} (\xi_{ff'}^h h \bar{f}_R f'_L + \xi_{ff'}^H H \bar{f}_R f'_L + i \xi_{ff'}^A A^0 \bar{f}_R f'_L) + \text{h.c.},$$



$$\xi_{ff'}^h \equiv s_{\beta-\alpha} \delta_{ff'} + c_{\beta-\alpha} \zeta_{ff'},$$

$$\xi_{ff'}^H \equiv c_{\beta-\alpha} \delta_{ff'} - s_{\beta-\alpha} \zeta_{ff'},$$

$$\xi_{ff'}^A \equiv (2T_3^f) \zeta_{ff'},$$

$$\zeta_{ff'} = \begin{cases} \cot \beta \delta_{ff'} & (\text{for } f = d, s, b), \\ -\tan \beta \delta_{ff'} & (\text{for } f = e, \mu, \tau) \end{cases}$$

$$\zeta_{uu} \equiv -\tan \beta - (\tan \beta + \cot \beta) \frac{\cos \rho_u - 1}{2},$$

$$\zeta_{cc} \equiv \cot \beta,$$

$$\zeta_{tt} \equiv \cot \beta - (\tan \beta + \cot \beta) \frac{1 - \cos \rho_u}{2},$$

$$\zeta_{ut} = \zeta_{tu} = (\tan \beta + \cot \beta) \frac{\sin \rho_u}{2}.$$

the flavor-violating decay  $H \rightarrow tu$  dominates for  $\rho_u \gtrsim 1/120$ .

very striking signature of the up-specific Variant Axion Model

# Conclusions

muon  $g-2$  : long standing puzzle, the new updates coming soon

to explain the anomaly in the muon  $g-2$  in 2HDMs

LFV in g2HDM or lepton-specific 2HDM

Lepton Flavor Violation in g2HDM

$$m_A < 700 \text{ GeV}, 10 \text{ GeV} < m_H - m_A < 100 \text{ GeV}$$

Drell-Yan production provide LFV tau-mu resonances, which would be sensitive at LHC

a well motivated extension of lepton-specific 2HDM

strong CP problem  $\Rightarrow$  domain wall problem

$\Rightarrow$  variant axion models (only 1 right-handed quark PQ charged)

lepton sector has freedom for PQ charge assignments and muon  $g-2$  anomaly can be accommodated

by assign PQ charge to all leptons  $m_A \sim 15 \text{ GeV}, \tan\beta \sim 40$

(by assign PQ charge only to muon  $m_A \sim 1 \text{ TeV}, \tan\beta \sim 3000$ )

For light  $A$  case,  $t \rightarrow uA, A \rightarrow \tau\tau$  current constraints marginal

using boosted di-tau-tagging improves sensitivity significantly

For both cases, flavor violating heavy higgs decays ( $H \rightarrow \tau\mu, tu$ ) would be the distinctive signatures at LHC



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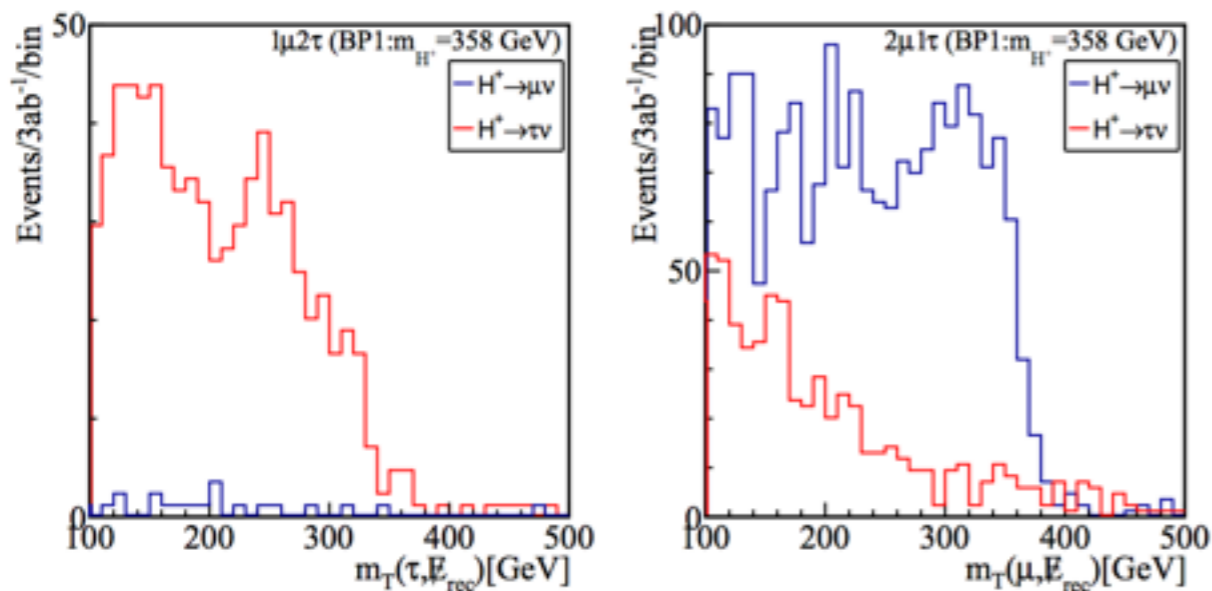
Backup

# g-2 via LFV — mass reconstruction at LHC

charged Higgs mass can be reconstructed via 3 leptons from  $\phi H^\pm$  production  $\mu^\pm \tau^\mp \tau \nu$  and  $\mu^\pm \tau^\mp \mu \nu$

ratio controlled by  $BR(H^\pm \rightarrow \tau^\pm \nu) = 1 - BR(H^\pm \rightarrow \mu^\pm \nu) = \frac{|\rho_c^{\mu\tau}|^2}{|\rho_c^{\tau\mu}|^2 + |\rho_c^{\mu\tau}|^2} \equiv r$ .

part of  $\tau$ -mode contribute to  $\mu$ -mode



4 combinatorics : (production  $\Phi=A, H$ ) x (2  $\tau\mu$  combinations)

$$\mathbf{p}_{\tau_i}^{\text{rec}} = (1 + c_{\tau_i\phi})\mathbf{p}_{\tau_i}^{\text{vis}}, \quad (c_{\tau_i\phi} > 0).$$

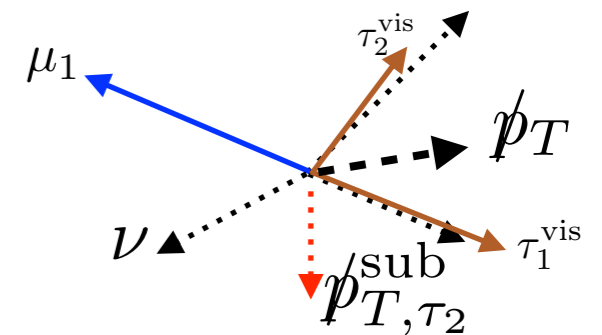
$$m_{\mu\tau_i}^{\text{rec}2} = (p_\mu + p_{\tau_i}^{\text{rec}})^2 = m_\phi^2,$$

$$\mathbf{p}_{T,\tau_i\phi}^{\text{sub}} = \mathbf{p}_T - c_{\tau_i\phi}\mathbf{p}_{T,\tau}^{\text{vis}}$$

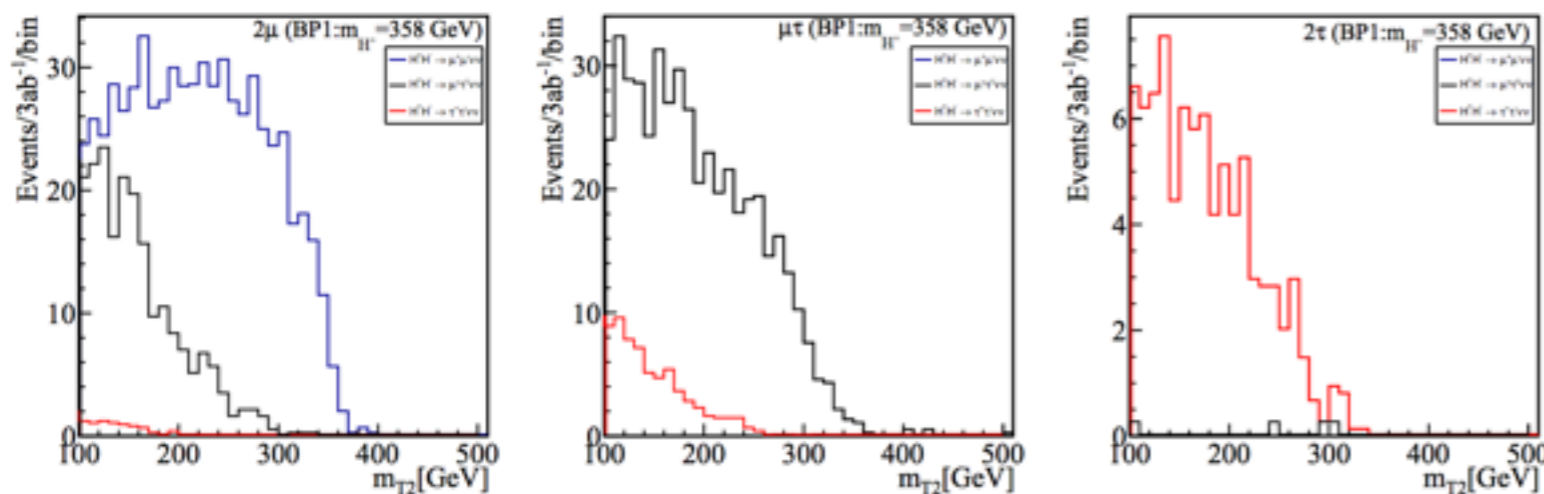
$$m_{T,\tau_i\phi} = m_T(\mathbf{p}_{\tau_i}^{\text{vis}}, \mathbf{p}_{T,\tau_i\phi}^{\text{sub}}),$$

taking the minimum of the 4 possibilities

$$m_{T,\tau}^{\text{min}} = \min(m_{T,\tau_1 A}, m_{T,\tau_1 H}, m_{T,\tau_2 A}, m_{T,\tau_2 H}).$$



also via 2 leptons from  $H^+ H^-$  production



$m_H, m_A, m_{H^\pm}$  reconstructed by 4,3,2 lepton events

event number ratios among various modes sensitive to the BR

$$m_{T2}(\mathbf{p}_{\ell_1}, \mathbf{p}_{\ell_2}, \mathbf{p}_T) = \min_{\mathbf{p}'_T = \mathbf{p}'_{T,1} + \mathbf{p}'_{T,2}} \{ \max[m_T(\mathbf{p}_{\ell_1}, \mathbf{p}'_{T,1}), m_T(\mathbf{p}_{\ell_2}, \mathbf{p}'_{T,2})] \}$$

# u-specific VAM with muon-specific lepton sector

An extreme model: muon-specific 2HDM to accommodate muon  $g-2$  [T. Abe, R. Sato, K. Yagyu JHEP 1707, 012 (2017)]

only muon yukawa is  $\tan\beta$  enhanced  $\sim 3000$

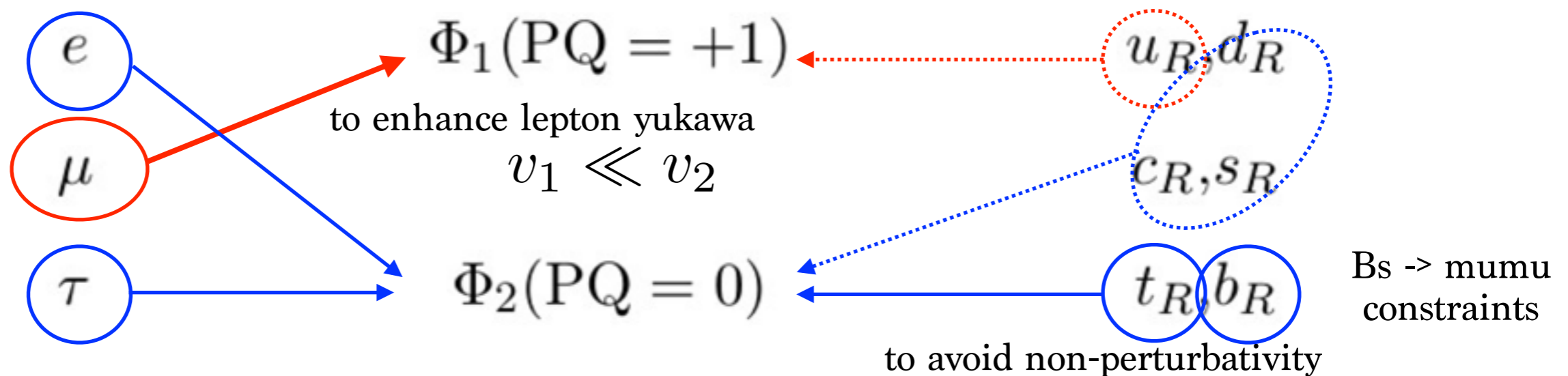
better fit against the lepton universality constraints

constrained by multi-muon searches at LHC ( $A/H \rightarrow \mu\mu$  100%)

VAM is essentially just a 2HDM with various PQ charge assignments (only one  $q_R$  PQ charged)

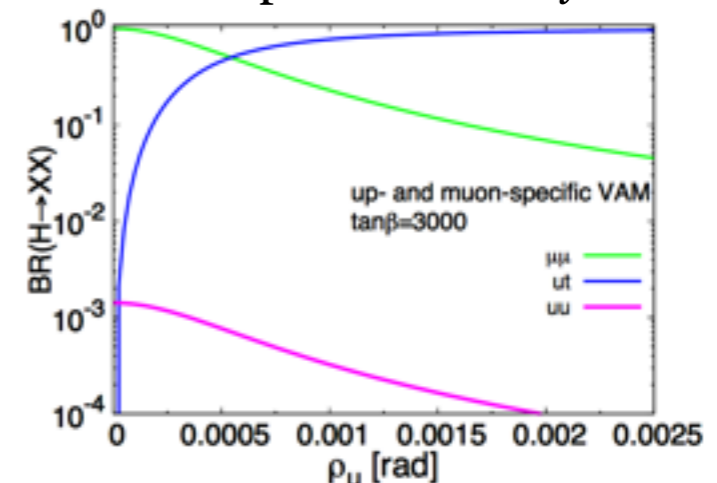
lepton sector is irrelevant to the strong CP problem nor domain wall problem

muon yukawa has to be enhanced to accommodate muon  $g-2 \Leftrightarrow$  corresponding VEV is small ( $\tan\beta \gg 1$ )



in this setup, suppressed muon BR to accommodate LHC constraints

$$\frac{BR(A \rightarrow tu)}{BR(A \rightarrow \mu\mu)} \simeq \frac{BR(H \rightarrow tu)}{BR(H \rightarrow \mu\mu)} \simeq \frac{m_t^2}{m_\mu^2} \frac{3 \sin^2 \rho_u}{2} \simeq (2000 \cdot \rho_u)^2$$



# g-2 in 2HDM

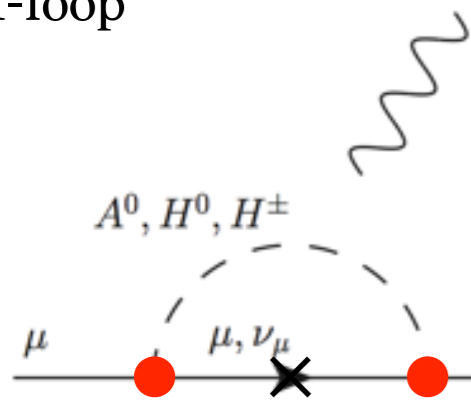
$\mathcal{O}(10^{-9})$  positive contribution required  
 $2.6 \times 10^{-9}$

Flavor dependent contribution : yukawa type

chirality flip required

$$\mathcal{L} = a_\mu \frac{e}{4m_\mu} \bar{\psi} \sigma_{\mu\nu} \psi F^{\mu\nu}$$

1-loop



$$\Delta a_\mu^{\text{VAM,1-loop}} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \sum_i (h, H, A, H^\pm) (\xi_{\mu\mu}^i)^2 r_\mu^i f_i(r_\mu^i) \sim 10^{-9} \sim 10^{-7} \quad m_H = 1\text{TeV}$$

cf.) muon-specific 2HDM  $\xi_\mu \sim 3000$   
 [T. Abe, R. Sato, K. Yagyu, arXiv:1705.01469]

$\propto m_\mu^3/m_H^2 \rightarrow$  LFV enhance with  $m_\tau^3/m_\mu^3 \sim 5000, \xi_{\mu\tau}^2$   $\xi_{\mu\tau}\xi_{\tau\mu}/m_H^2 [\text{TeV}] \sim 10^4$  required

$$r_f^i = m_f^2/m_i^2$$

$$f_{h,H}(r) = \int_0^1 dx \frac{x^2(2-x)}{1-x+rx^2}, \quad f_A(r) = \int_0^1 dx \frac{-x^3}{1-x+rx^2}$$

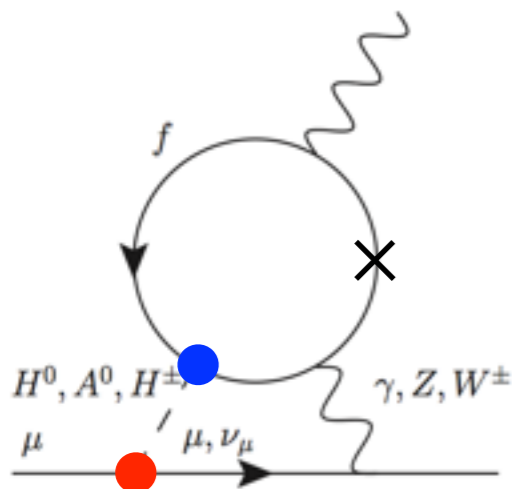
$$f_{H^\pm}(r) = \int_0^1 dx \frac{-x(1-x)}{1-r(1-x)}$$

$$g_{h,H}(r) = \int_0^1 dx \frac{2x(1-x)-1}{x(1-x)-r} \ln \frac{x(1-x)}{r}$$

$$g_A(r) = \int_0^1 dx \frac{1}{x(1-x)-r} \ln \frac{x(1-x)}{r}$$

2-loop (Barr-Zee)

$$\Delta a_\mu^{\text{VAM,BZ}} = \frac{G_F m_\mu^2}{4\sqrt{2}\pi^2} \frac{\alpha_{\text{em}}}{\pi} \sum_i (h, H, A, t, b, c, \tau) \sum_f N_f^c Q_f^2 \xi_{\mu\mu}^i \xi_{ff}^i r_f^i g_i(r_f^i)$$



heavy fermion contributions enhance at 2-loop

$\xi_\mu \xi_\tau / m_H^2 [\text{TeV}] \sim 10^6$  required

$$\propto m_\mu m_f^2 / m_H^2$$

	Fermion	$(g_f^H, g_f^A)$	$(r_f^H g_f^H, r_f^A g_f^A)$	$\times \alpha N_f^c Q_f^2 / \pi$	Sign of $(\delta_H, \delta_A)$
One loop	$\mu$	(17, -16)	$(1.9, -1.8) \times 10^{-7}$	$(1.9, -1.8) \times 10^{-7}$	(+, -)
	$t$	(-12, 15.9)	$(-3.6, 4.7) \times 10^{-1}$	$(-1.1, 1.5) \times 10^{-3}$	(-, -)
	$c$	(-118, 140)	$(-1.9, 2.3) \times 10^{-4}$	$(-5.9, 7.1) \times 10^{-7}$	(-, -)
Two loop	$u$	(-282, 330)	$(-1.5, 1.7) \times 10^{-9}$	$(-4.6, 5.4) \times 10^{-12}$	(-, -)
	$b$	(-87, 105)	$(-1.5, 1.8) \times 10^{-3}$	$(-1.1, 1.4) \times 10^{-6}$	(-, <span style="border: 1px solid red; padding: 2px;">+</span> )
	$\tau$	(-109, 130)	$(-3.4, 4.1) \times 10^{-4}$	$(-8.0, 9.6) \times 10^{-7}$	(-, <span style="border: 1px solid red; padding: 2px;">+</span> )

# g-2 via lepton flavor violation — other elements

Other Yukawa elements : 1st, 2nd generations severely constrained

$$\rho_e^{\tau\tau}, \rho_u^{tt}, \rho_u^{tc}, \rho_u^{ct} \text{ and } \rho_d^{bb}.$$

$$BR(\tau \rightarrow \mu\gamma) \text{ sets } \underset{\text{2-loop}}{|\rho_u^{tt}|} < 0.05 \text{ and } \underset{\text{1-loop}}{|\rho_e^{\tau\tau}|} < 0.06.$$

$$|\rho_u^{tc}| < 0.11: \text{ lepton univ. in } B \rightarrow D\ell\nu$$

$$\epsilon_K \text{ measurements provide a severe constraint as } |\rho_u^{ct}| < 0.04$$

$$|\rho_d^{bb}| < 0.22 \text{ is obtained by the flavor observables including } BR(B \rightarrow \mu\nu)$$

$$BR(H/A \rightarrow \mu^\pm \tau^\mp) \text{ is diluted by } H/A \rightarrow b\bar{b}$$