Potentially interesting aspects of the effective potential

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Ref.

- JhP, PRD**99**(2019)116013, 1902.04559

Effective potential

Essential object, e.g. for studying:

- symmetry breaking
- vacuum stability
- phase transitions at zero and finite temperatures
- cosmic inflation
- radiative corrections to Higgs mass

Generating functionals

"Partition function (Zustandssumme)"

$$Z[J] = \int \mathcal{D}\phi \exp\left[i\int d^4x\left(\mathcal{L}(x) + J(x)\phi(x)\right)\right]$$

$$= Z[0]\,\langle\Omega|\Omega\rangle_J = \exp\left(i\,W[J]\right)$$

= generating functional for correlation functions

Phase of vacuum to vacuum amplitude

$$W[J] = -i\ln Z[J] = -T E_\Omega[J]$$

= generating functional for connected correlation functions

1PI effective action

Legendre transform of vacuum energy functional

Jona-Lasinio, Nuovo Cim.34(1964)1790

$$\Gamma[\phi_{\rm cl}] = W[J] - \int d^4x\, J(x)\phi_{\rm cl}(x)$$

= generating functional for 1PI correlation functions

$$\frac{\delta W[J]}{\delta J(x)} = \frac{-i}{Z[J]}\frac{\delta Z[J]}{\delta J(x)} = \frac{\int \mathcal{D}\phi\, e^{i\int d^4x\,(\mathcal{L}+J\phi)}\phi(x)}{\int \mathcal{D}\phi\, e^{i\int d^4x\,(\mathcal{L}+J\phi)}}$$

$$= \frac{\langle \Omega|\phi(x)|\Omega\rangle_J}{\langle\Omega|\Omega\rangle_J} = \phi_{\rm cl}(x)$$

= Expectation value of $\phi(x)$ in the presence of $J(x)$

$$\frac{\delta\Gamma[\phi_{\rm cl}]}{\delta\phi_{\rm cl}(x)} = -J(x) \qquad = 0 \text{ if } \phi_{\rm cl} = \langle\phi\rangle$$

Effective potential or "free energy"

Effective action for x-independent ϕ_{cl}

$$V_{\text{full}}(\phi_{\text{cl}}) = -\frac{1}{VT}\Gamma[\phi_{\text{cl}}]\bigg|_{\phi_{\text{cl}}=\text{const.}}$$

Jackiw, PRD9(1974)1686

- real-valued by definition
- depends on gauge and renormalization scale

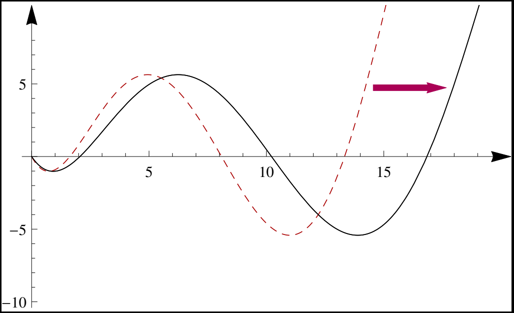


Fig. from Andreassen, Frost, Schwartz, PRD(2015)

Loop expansion of effective potential

$$V_{\text{full}} \simeq V_{\text{aprx}} = V_0 + V_1 + V_2 + \cdots$$

$$V_0(\phi_{\text{cl}}) = \text{tree-level potential in } \mathcal{L}$$

$$V_1(\phi_{\text{cl}}) = \frac{M^4(\phi_{\text{cl}})}{64\pi^2}\left(\ln\frac{M^2(\phi_{\text{cl}})}{\mu^2} - \frac{3}{2}\right) \quad \text{in } \overline{\text{MS}} \text{ scheme}$$

- develops imaginary part where V_0 is concave

Energy interpretation

Kurt Symanzik, CMP16(1970)48

$$V_{\text{full}}(\phi) = \frac{1}{V}\min_{\Omega}\frac{\langle\Omega|H|\Omega\rangle}{\langle\Omega|\Omega\rangle} \quad \text{s.t.} \quad \frac{\langle\Omega|\Phi|\Omega\rangle}{\langle\Omega|\Omega\rangle} = \phi$$

$V_{\text{full}}(\phi)$ is the minimum of the expectation value of the energy density for all states constrained by the condition that the scalar fields Φ have expectation values ϕ

Convexity of effective potential

Kurt Symanzik, CMP16(1970)48

$$V_{\rm full}\big(x\phi_1 + (1-x)\phi_2\big) \le x V_{\rm full}(\phi_1) + (1-x) V_{\rm full}(\phi_2)$$

for $0 \le x \le 1$

- Can be understood by taking linear combination of states

$$\sqrt{x}\,|\Omega_1\rangle + \sqrt{1-x}\,|\Omega_2\rangle \quad \text{with} \quad \langle\Omega_{1,2}|\Omega_{1,2}\rangle = 1$$

$$\langle\Omega_{1,2}|\Phi|\Omega_{1,2}\rangle = \phi_{1,2}, \quad \langle\Omega_{1,2}|H|\Omega_{1,2}\rangle/V = V_{\rm full}(\phi_{1,2})$$

  or by imagining a state with volumes in different phases
- Convexity of Gibbs free energy is also known in statistical mechanics

Section 11.3 of Peskin & Schroeder

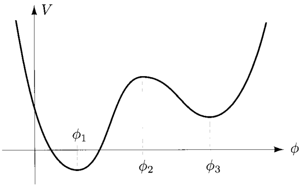


Figure 11.6. A possible form for the effective potential in a scalar field theory. The extrema of the effective potential occur at the points $\phi_{\rm cl} = \phi_1, \phi_2, \phi_3$. The true vacuum state is the one corresponding to ϕ_1. The state ϕ_2 is unstable. The state ϕ_3 is metastable, but it can decay to ϕ_1 by quantum-mechanical tunneling.

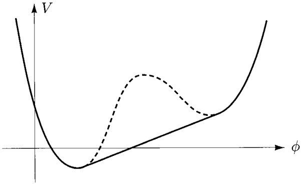


Figure 11.7. Exact convex form of the effective potential for the system of Fig. 11.6.

type shown in Fig. 11.6. The Maxwell construction must be performed by hand to yield the final form of $V_{\rm eff}(\phi_{\rm cl})$. Fortunately, the absolute minimum of $V_{\rm eff}$ is not affected by this nicety.

We have now solved the problem that we posed at the beginning of this

“Maxwell construction”

- $V_{\rm full}(\phi)$ is linear between two local minima of $V_{\rm aprx}(\phi)$

Fujimoto, O’Raifeartaigh, Parravicini, NPB212(1983)268

- Approximation by linear interpolation

$$V_{\rm full}\big(x\phi_1 + (1-x)\phi_2\big) \simeq xV_{\rm aprx}(\phi_1) + (1-x)V_{\rm aprx}(\phi_2)$$

for $0 < x < 1$

guaranteed to be real

- QFT analogue of Maxwell construction for free energies in thermodynamics

dS swampland conjecture

- A low energy effective theory belongs to landscape if it has string theory as its UV completion
- Otherwise it belongs to swampland
- Scalar potential of a low energy effective theory in landscape satisfies

Obied, Ooguri, Spodyneiko, Vafa (2018)

$$M_{\mathrm{Pl}} \left| \nabla V \right| > c\, V, \quad 0 < c \sim \mathcal{O}(1)$$

- Excludes de Sitter extrema
- Refined to include alternative condition

Ooguri, Palti, Shiu, Vafa (2018)

$$M_{\mathrm{Pl}}^2 \min(\nabla_i \nabla_j V) \leq -c'\, V, \quad 0 < c' \sim \mathcal{O}(1)$$

Old dS swampland conjecture vs HEP models

- Quintessence could accommodate observed cosmological constant and local maximum of Higgs potential

  Denef, Hebecker, Wrase (2018)

- Severely constrained by a long-range force and time dependence of proton-to-electron mass ratio

  Hamaguchi, Ibe, Moroi (2018)

- Quintessence + pion extremum requires $c < 1.4 \times 10^{-2}$

  K. Choi, D. Chway, C. S. Shin (2018)

- QCD axion becomes difficult

  Murayama, Yamazaki, Yanagida (2018)

- Root of problems is exclusion of positive local maxima of V

QFT questions about dS conjectures

- Which V? V_{full}? V_0? V_{aprx}?
- How to interpret inequalities

$$M_{\text{Pl}}\,|\nabla V| > c\,V, \quad 0 < c \sim \mathcal{O}(1)$$
$$M_{\text{Pl}}^2 \min(\nabla_i \nabla_j V) \le -c'\,V, \quad 0 < c' \sim \mathcal{O}(1)$$

if $\quad \operatorname{Im} V \neq 0$?

- Gauge and scale dependence of V

Could free energy interpretation save old dS conjecture?

Kobakhidze, 1901.08137; JhP, 1902.04559

Suppose V is V_{full} in dS swampland conjecture

$$M_{\text{Pl}}\,|\nabla V_{\text{full}}| > c\,V_{\text{full}}, \quad 0 < c \sim \mathcal{O}(1) \tag{1}$$

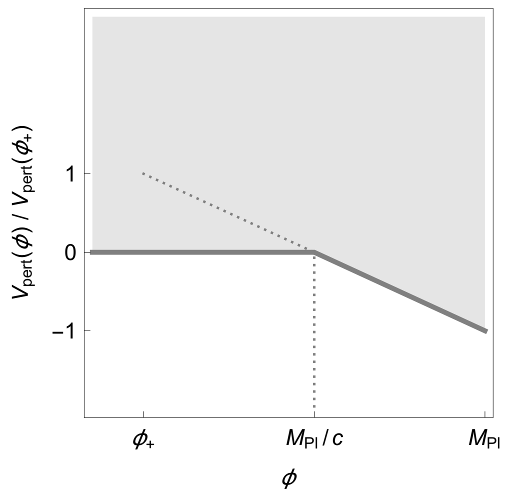
- V_{full} is real-valued, as quantum as possible
- More permissive than old and refined dS criteria as local maxima of V_{aprx} are flattened in V_{full}
- Global minima still need to be AdS
- We are not living in a true vacuum, i.e. we are living in

```
!(true && vacuum) == (false || !vacuum)
```

Option !vacuum: living on a slope

- Quintessence solution to cosmological constant with $|\nabla V_{\text{aprx}}| > 0$
- Enough to add quintessence terms to Higgs potential as V_{full} has no local maximum
- No problem with a long-range force or time dependence of proton-to-electron mass ratio

Option `false vacuum`: V_{aprx} with only two minima



Use Maxwell construction to represent condition (1) by

$$V_{\mathrm{aprx}}(\phi_+) < \frac{M_{\mathrm{Pl}}}{c}\left|\frac{V_{\mathrm{aprx}}(\phi_+) - V_{\mathrm{aprx}}(\phi_-)}{\phi_+ - \phi_-}\right|, \qquad V_{\mathrm{aprx}}(\phi_-) < 0$$

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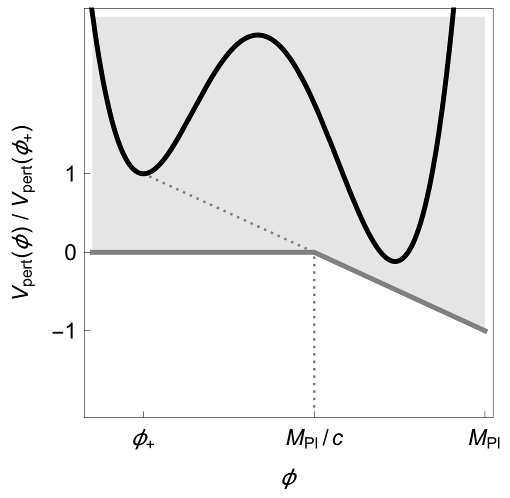

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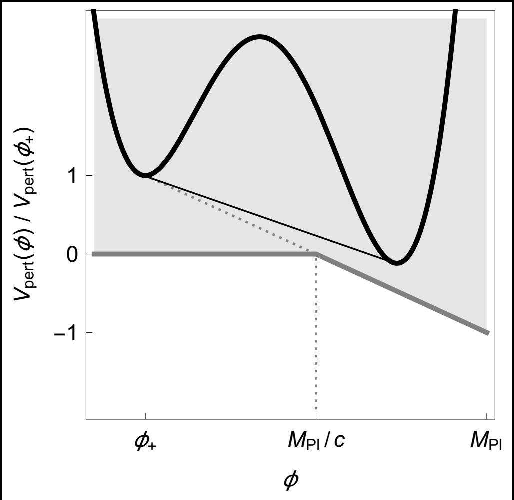

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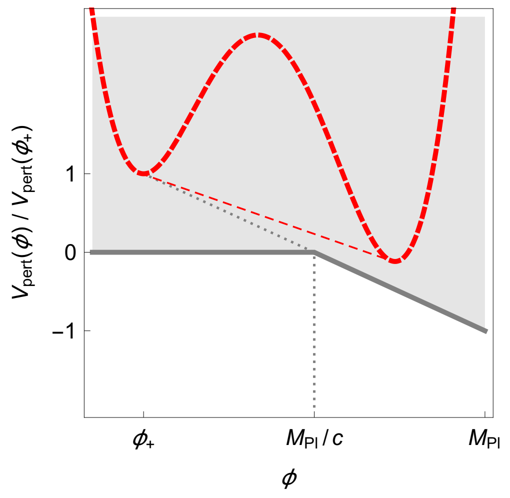

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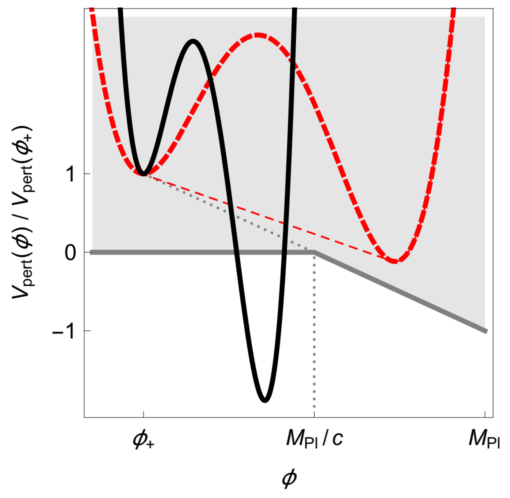

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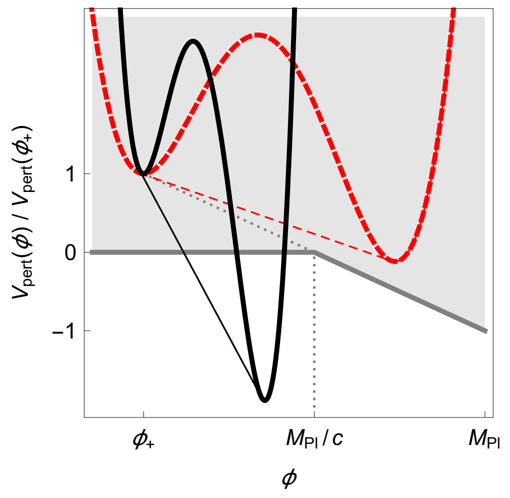

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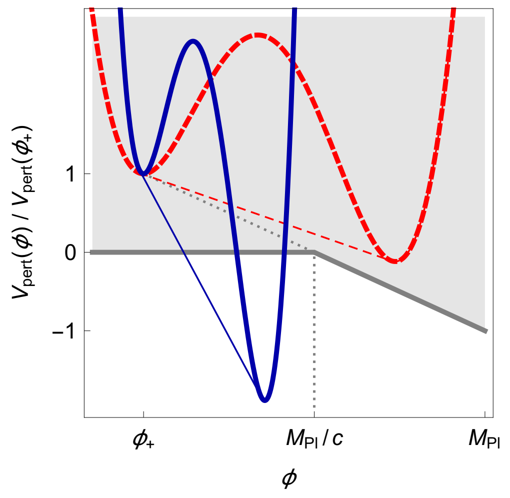

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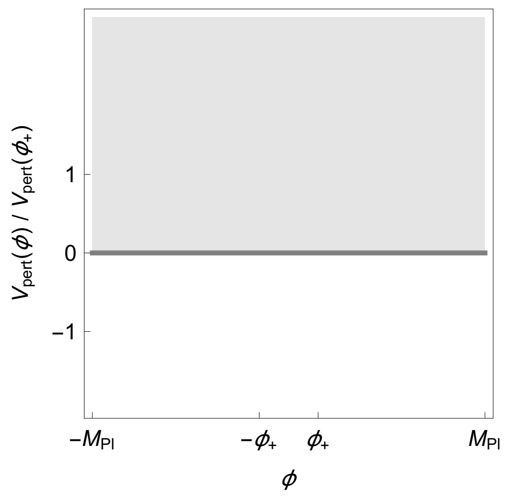
Option `false vacuum`: symmetric V_{aprx}

Disallowed

Allowed



Maxwell construction is a constant between global minima

Option `false vacuum`: symmetric V_{aprx}

Disallowed
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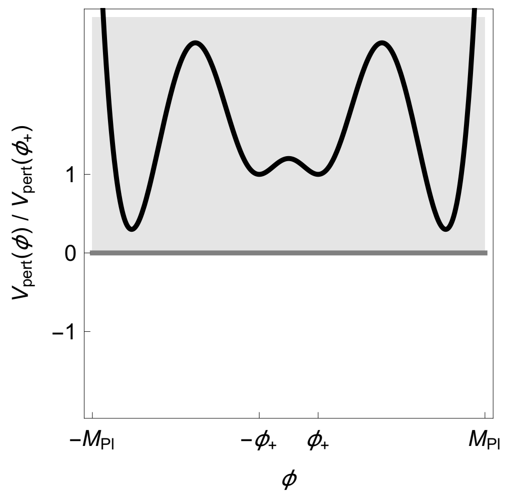

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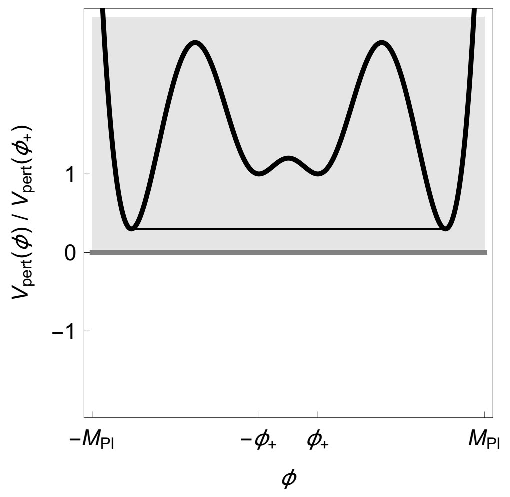

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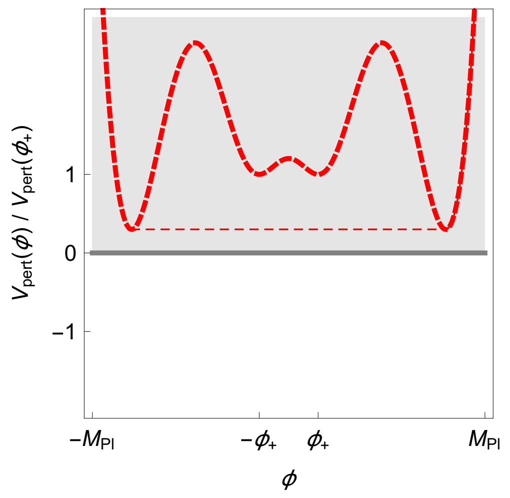

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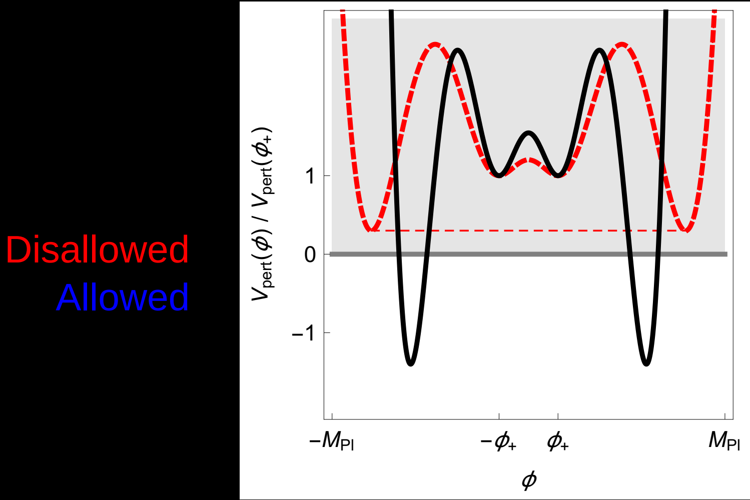

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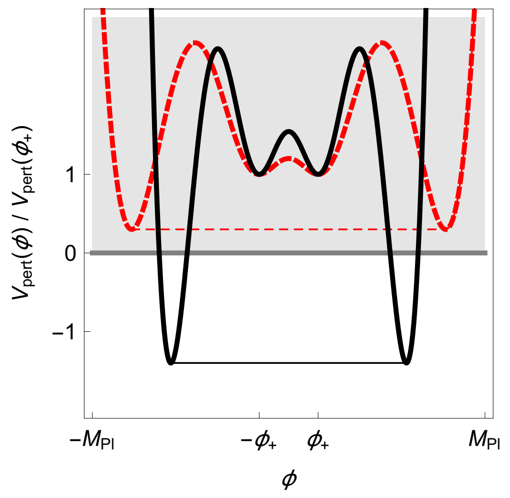

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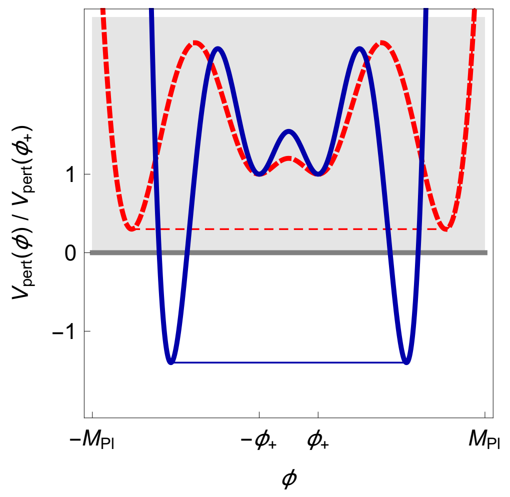

Maxwell construction is a constant between global minima

Implications for V_{aprx}

- Our EW vacuum is required to be metastable
- Maybe due to an extra field such as in:
  - supersymmetry with CCB minima

    Claudson, Hall, Hinchliffe (1983)
  - metastable supersymmetry breaking sectors

    Intriligator, Seiberg, Shih (2006)
  - relaxion mechanism

    Graham, Kaplan, Rajendran (2015)
  - scalar extensions of the Higgs sector
- Otherwise metastability is due to SM Higgs

Near-criticality of Higgs potential

- RG-improved effective potential

$$V_{\text{aprx}}(h) = \frac{\lambda(\mu = |h|)}{4} h^4$$

- Condition (1) + distance conjecture + UV cutoff of SM suggest $\lambda(\mu)$ should turn negative at $\mu \lesssim M_{\text{Pl}}$

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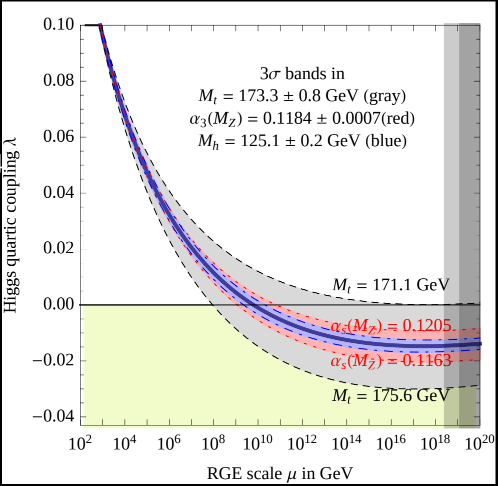

Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (2013)

Connection to string theory?

- Hard to judge if condition (1) has something to do with string theory
- Admits false dS vacua contrary to original motivation for dS criteria
- Conjectures have not been proved

Summary

Yet another refinement of dS swampland conjecture

- Form is identical to original but effective potential integrates all (non)perturbative quantum effects
- V_{full} is real $\rightsquigarrow$ no inequality on complex numbers
- V_{full} is convex $\rightsquigarrow$ allows local maxima and false dS vacua in V_{aprx} if slope of V_{full} is everywhere steep enough $\rightsquigarrow$ compatible with HEP models
- True vacua must still be AdS $\rightsquigarrow$ quintessence or metastable EW vacuum
- Reason for near-criticality of SM Higgs potential?