

# **Radiative neutrino mass generation and Lepton flavor violation**

**Takaaki Nomura (KIAS)**

Based on T.N. and K. Yagyu, JHEP 1910 (2019) 105



## 1. Introduction

Neutrino mass is one of the mystery in particle physics

### □ Non-zero neutrino mass

- ❖ We need a mechanism to generate neutrino mass
- ❖ Also smallness of the mass should be explained

### □ Neutrino mixing

- ❖ Mixing is described by PMNS mixing matrix
- ❖ What is origin of flavor structure?

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \text{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$$

# 1. Introduction

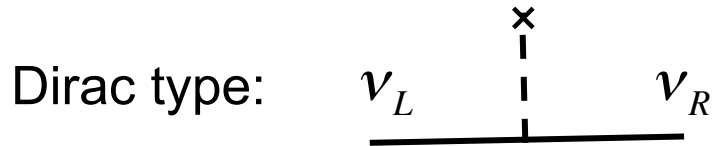
Neutrino mass is one of the mystery in particle physics

## □ Observables from neutrino oscillation experiments

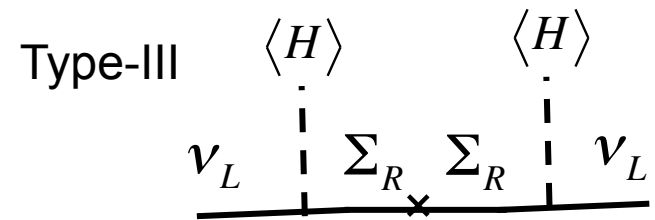
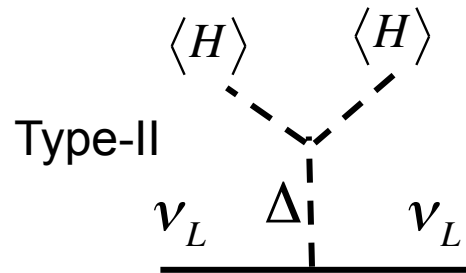
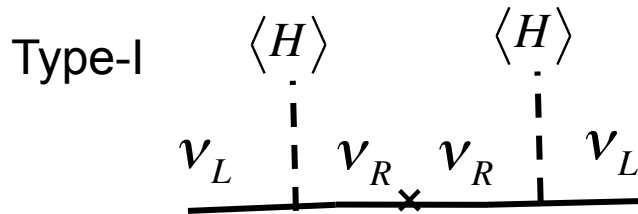
		Normal Ordering (best fit)		Inverted Ordering ( $\Delta\chi^2 = 9.3$ )	
		bfp $\pm 1\sigma$	$3\sigma$ range	bfp $\pm 1\sigma$	$3\sigma$ range
with SK-atm	$\sin^2 \theta_{12}$	$0.310^{+0.013}_{-0.012}$	0.275 $\rightarrow$ 0.350	$0.310^{+0.013}_{-0.012}$	0.275 $\rightarrow$ 0.350
	$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	31.61 $\rightarrow$ 36.27	$33.82^{+0.78}_{-0.75}$	31.62 $\rightarrow$ 36.27
	$\sin^2 \theta_{23}$	$0.582^{+0.015}_{-0.019}$	0.428 $\rightarrow$ 0.624	$0.582^{+0.015}_{-0.018}$	0.433 $\rightarrow$ 0.623
	$\theta_{23}/^\circ$	$49.7^{+0.9}_{-1.1}$	40.9 $\rightarrow$ 52.2	$49.7^{+0.9}_{-1.0}$	41.2 $\rightarrow$ 52.1
	$\sin^2 \theta_{13}$	$0.02240^{+0.00065}_{-0.00066}$	0.02044 $\rightarrow$ 0.02437	$0.02263^{+0.00065}_{-0.00066}$	0.02067 $\rightarrow$ 0.02461
	$\theta_{13}/^\circ$	$8.61^{+0.12}_{-0.13}$	8.22 $\rightarrow$ 8.98	$8.65^{+0.12}_{-0.13}$	8.27 $\rightarrow$ 9.03
	$\delta_{CP}/^\circ$	$217^{+40}_{-28}$	135 $\rightarrow$ 366	$280^{+25}_{-28}$	196 $\rightarrow$ 351
	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.39^{+0.21}_{-0.20}$	6.79 $\rightarrow$ 8.01	$7.39^{+0.21}_{-0.20}$	6.79 $\rightarrow$ 8.01
	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.525^{+0.033}_{-0.031}$	+2.431 $\rightarrow$ +2.622	$-2.512^{+0.034}_{-0.031}$	-2.606 $\rightarrow$ -2.413

# 1. Introduction

## Neutrino mass generation ?



### Seesaw mechanisms



Neutrino mass is generate at tree level

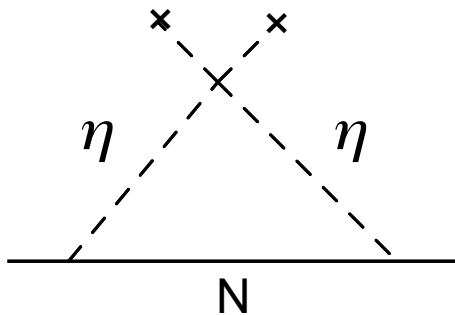
➡ Heavy masses and/or small couplings are required

**Alternative mechanism** ➡ **Mass generation @ loop level**

# 1. Introduction

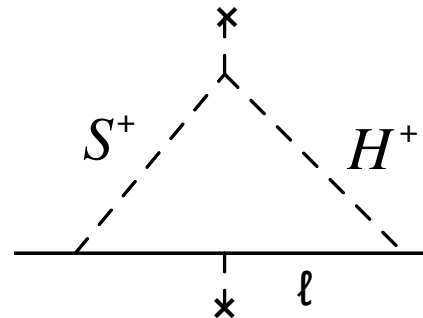
## Neutrino mass generation via loop diagram

### Examples of one-loop generation model



Scotogenic model ( $\eta$ : inert doublet)

Ma (2006)



Zee-model ( $S^+$ : charged scalar)

Zee (1980)

$\times$  : Higgs VEV

### □ Zee model

- It is based on two Higgs doublet model (THDM) + singlet charged scalar
- No right-handed neutrino

### □ Scotogenic model

- Tree level neutrino mass is forbidden by  $Z_2$  symmetry in scotogenic model
- The lightest  $Z_2$  odd neutral particle can be DM

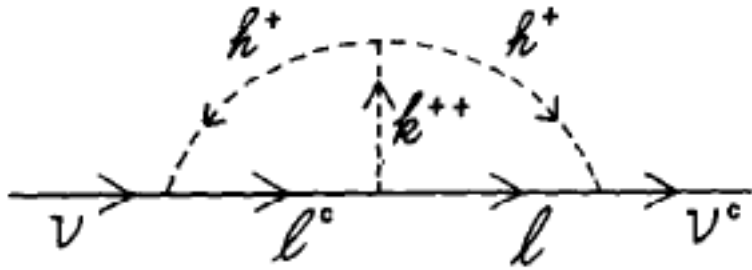
## 1. Introduction

# Neutrino mass generation via loop diagram

## Example of two-loop generation model

### □ Zee-Babu model

Babu (1986)



$$L \supset f_{ij} \bar{L}_i^c L_j h^+ + g_{ij} \bar{e}_R^i e_R^j k^{++} + h.c.$$

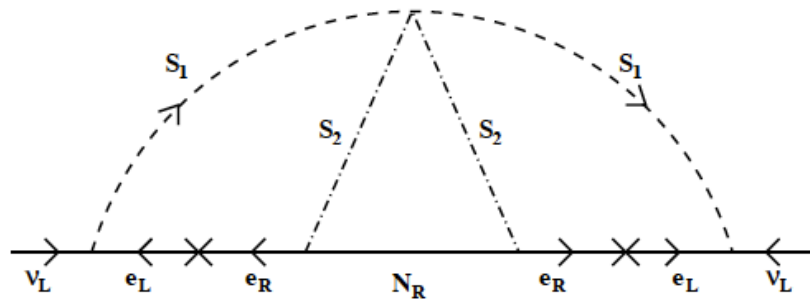
- Singly and doubly charged scalars are introduced
- No right-handed neutrino
- Majorana neutrino mass is induced

# 1. Introduction

## Neutrino mass generation via loop diagram

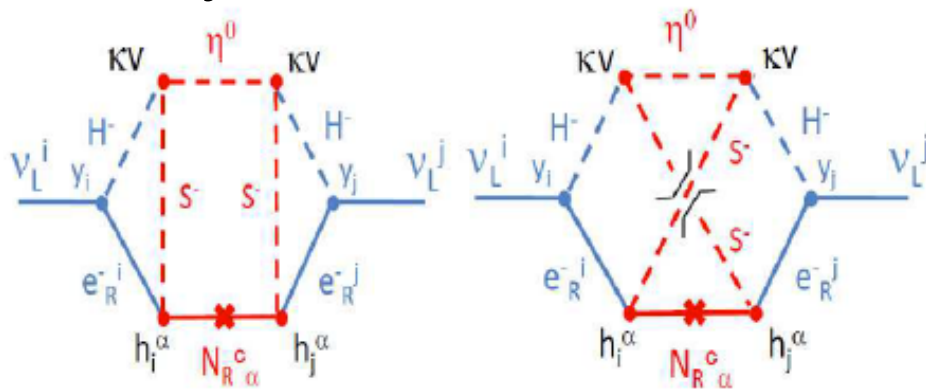
Example of three-loop generation model

□ Model by Krauss, Nasri and Trodden (2002)



With  $Z_2$  symmetry

□ Model by Aoki, Kanemura and Seto (2008)



$Z_2 \times Z_2$  symmetry

Higher loop generation  $\Rightarrow$  More symmetry and new particles

# 1. Introduction

## Neutrino mass generation via loop diagram

Example of three-loop generation model

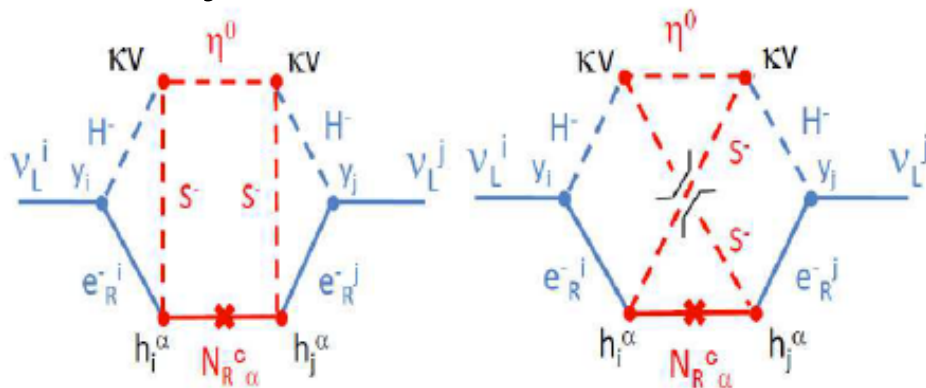
□ Model by Krauss, Nasri and Trodden (2002)

Four loop generation is also possible

T.N. and Hiroshi Okada PLB755 (2016)

T.N. and Hiroshi Okada PLB770 (2017)

□ Model by Aoki, Kanemura and Seto (2006)



$Z_2 \times Z_2$  symmetry

Higher loop generation  $\Rightarrow$  More symmetry and new particles

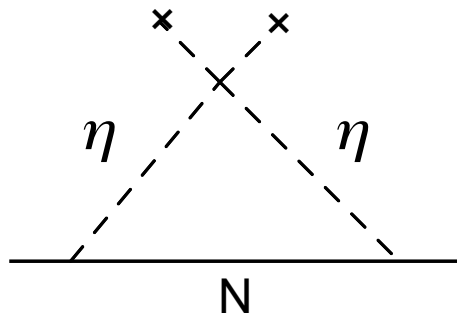


# 1. Introduction

## Radiative neutrino mass and LFV

In general we have LFV in radiative neutrino mass models

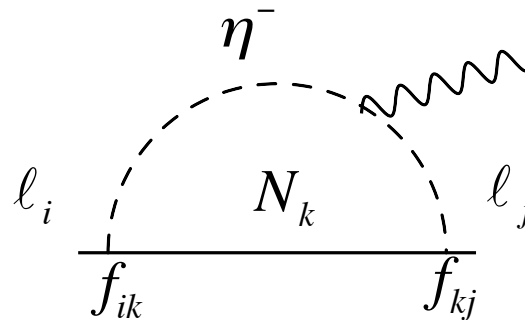
EX)



$$L \supset f_{ij} \bar{L}_i \eta N + h.c.$$

Flavor dependent coupling

Scotogenic model ( $\eta$ : inert doublet)



LFV decay is induced from Yukawa coupling

Experimental constraints should be taken into account

Ex)  $BR(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}$  MEG (2016)

## One interesting approach in controlling flavor

⇒ Applying flavor symmetry

### Radiative neutrino mass model + flavor symmetry

In this talk we discuss...

Zee model with global  $U(1)$  flavor symmetry

T.N. and K. Yagyu, JHEP 1910 (2019) 105

**1. Introduction**

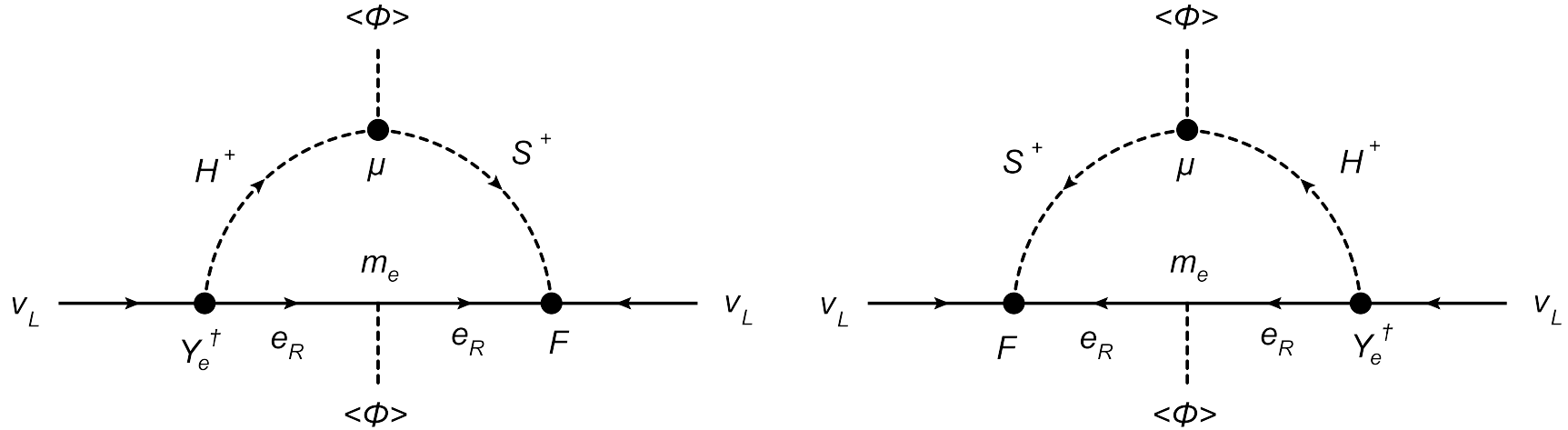
**2. Zee-model with flavor symmetry**

**3. Prediction for LFV**

**4. Summary and discussion**

## 2. Zee-model with flavor symmetry

# Original Zee-model for neutrino mass



One-loop mass generation

Realized by SM + second Higgs doublet + charged scalar

Softly-broken  $Z_2$  symmetry is assigned to forbid FCNC:  $\Phi_2 \rightarrow -\Phi_2$

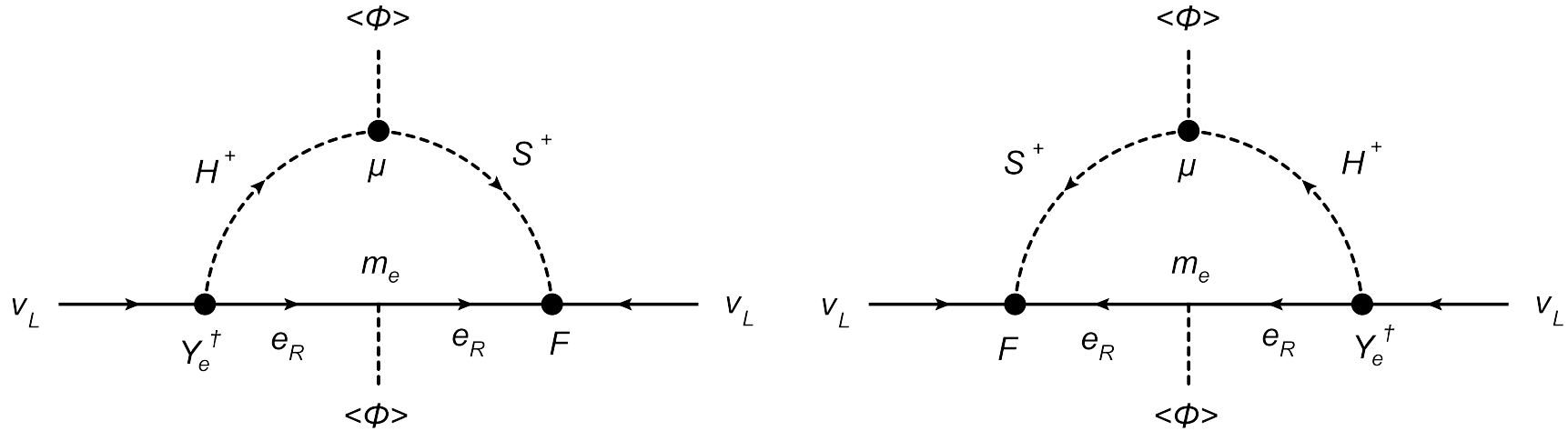
	$Q_L^i$	$u_R^i$	$d_R^i$	$L_L^i$	$\ell_R^i$	$\Phi_1$	$\Phi_2$	$S^+$
$SU(3)_c$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	2	2	1
$U(1)_Y$	1/6	2/3	-1/3	-1/2	-1	1/2	1/2	1

$$L \supset F_{ij} \bar{L}_L^i L_L^j S^+ + \frac{\sqrt{2} m_\ell}{v} \cot \beta \bar{\nu}_L e_R H^+ + \mu \left[ \Phi_1^T (i\sigma_2) \Phi_2 (S^+)^* + h.c. \right]$$

Zee (1980)

## 2. Zee-model with flavor symmetry

# Original Zee-model for neutrino mass



One-loop mass generation

Realized by SM + second Higgs doublet + charged scalar

Softly broken  $Z_2$  symmetry is assigned to forbid FCNC:  $\Phi_2 \rightarrow -\Phi_2$

⇒ Current neutrino oscillation data can not be fitted

- ✓ We relax  $Z_2$  symmetry to get more general structure of couplings
- ✓ To avoid FCNC in quark sector, we introduce global  $U(1)$  symmetry

## 2. Zee-model with flavor symmetry

# Zee-model with global U(1) symmetry

T.N. and K. Yagyu, JHEP 1910 (2019) 105

□ We assign global U(1) charge for leptons and scalars

	$Q_L^i$	$u_R^i$	$d_R^i$	$L_L^i$	$\ell_R^i$	$\Phi_1$	$\Phi_2$	$S^+$
$SU(3)_c$	3	3	3	1	1	1	1	1
$SU(2)_L$	2	1	1	2	1	2	2	1
$U(1)_Y$	1/6	2/3	-1/3	-1/2	-1	1/2	1/2	1
$U(1)'$	0	0	0	$q_L^i$	$q_R^i$	$q$	0	$q_S$

Particle contents are the same as the original Zee-model

← Lepton flavor dependent

### ◆ Yukawa interactions

$$-L_Y = \left(\tilde{Y}_u\right)_{ij} \bar{Q}_L^i \Phi_2^c u_R^j + \left(\tilde{Y}_d\right)_{ij} \bar{Q}_L^i \Phi_2 d_R^j + \left(\tilde{Y}_\ell^1\right)_{ij} \bar{L}_L^i \Phi_1 \ell_R^j + \left(\tilde{Y}_\ell^2\right)_{ij} \bar{L}_L^i \Phi_2 \ell_R^j + \tilde{F}_{ij} \bar{L}_L^{ci} (i\sigma_2) L_L^j S^+ + h.c.$$

### ◆ Relevant term for neutrino mass generation in Higgs potential

$$V \supset \mu \left[ \Phi_1^T (i\sigma_2) \Phi_2 (S^+)^* + h.c. \right]$$

## 2. Zee-model with flavor symmetry

### Structure of Yukawa coupling with global U(1)

➤ **Class I :**  $q_R = (0, 0, -q)$ ,  $q_L = q_S = 0$

➔ 
$$\tilde{Y}_\ell^1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \quad \tilde{Y}_\ell^2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \quad \tilde{F} = \begin{pmatrix} 0 & \times & \times \\ & 0 & \times \\ & & 0 \end{pmatrix},$$

➤ **Class II :**  $q_R = 0$ ,  $q_L = (0, 0, q)$ ,  $q_S = -q$

➔ 
$$\tilde{Y}_\ell^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \quad \tilde{Y}_\ell^2 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{F} = \begin{pmatrix} 0 & 0 & \times \\ & 0 & \times \\ & & 0 \end{pmatrix}$$

➤ **Class III :**  $q_R = (0, 0, q)$ ,  $q_L = (0, 0, -2q)$ ,  $q_S = q$

➔ 
$$\tilde{Y}_\ell^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad \tilde{Y}_\ell^2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{F} = \begin{pmatrix} 0 & 0 & \times \\ & 0 & \times \\ & & 0 \end{pmatrix}$$

## 2. Zee-model with flavor symmetry

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$$\tilde{Y}_\ell^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \quad \tilde{Y}_\ell^2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{F} = \begin{pmatrix} 0 & 0 & \times \\ & 0 & \times \\ & & 0 \end{pmatrix}$$



## 2. Zee-model with flavor symmetry

### Charged lepton mass and relevant couplings

**Charged lepton mass:**  $M_\ell = \frac{v}{\sqrt{2}} (c_\beta \tilde{Y}_\ell^1 + s_\beta \tilde{Y}_\ell^2)$

$$\ell_{L,R} \rightarrow U_{L,R} \ell_{L,R}$$

$$U_L^\dagger M_\ell U_R = (m_e, m_\mu, m_\tau)$$

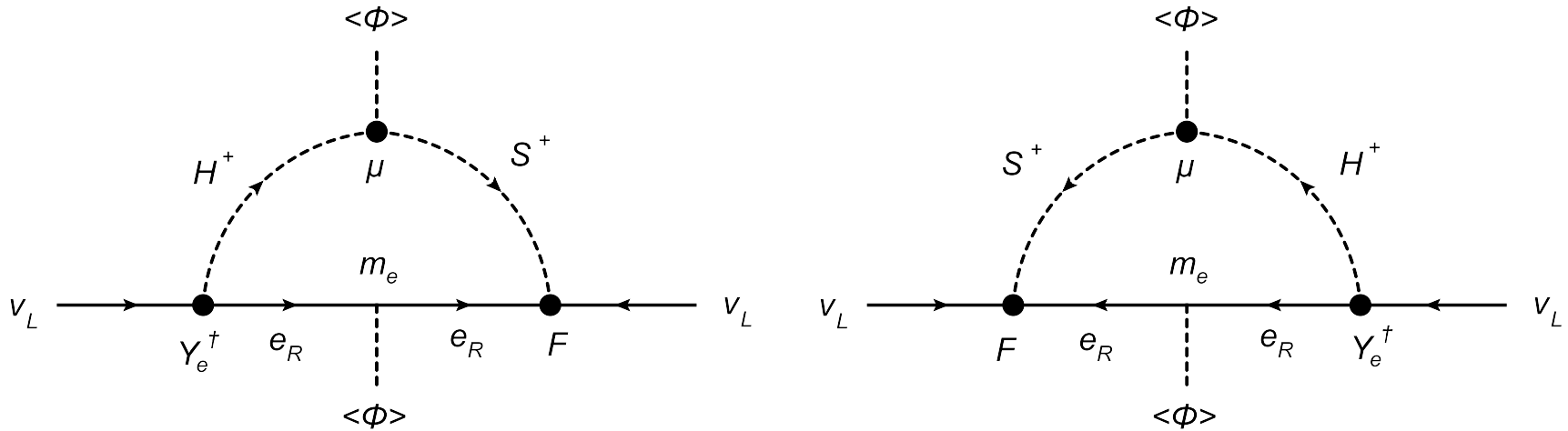
### Couplings in charged lepton mass basis

$$-L_Y \supset \left( \begin{array}{c} \bar{\nu}_L \\ \bar{\ell}_L \end{array} \right) \left[ \frac{\sqrt{2}}{v} \left( \begin{array}{c} M_\ell U_R G^+ \\ m_\ell \Phi^0 \end{array} \right) + \left( \begin{array}{c} Y_\ell H^+ \\ Y_\ell^0 \Phi'^0 \end{array} \right) \right] \ell_R - 2F \bar{\nu}^c \ell_R S^+ + h.c.$$

$$\left( \begin{array}{l} Y_\ell^0 = U_L^\dagger \tilde{Y}_\ell U_R, \quad Y_e = \tilde{Y}_\ell U_R, \quad (\tilde{Y}_\ell = -s_\beta \tilde{Y}_\ell^1 + c_\beta \tilde{Y}_\ell^2) \\ F = \tilde{F} U_R \end{array} \right)$$

## 2. Zee-model with flavor symmetry

### Neutrino mass generation at one loop level

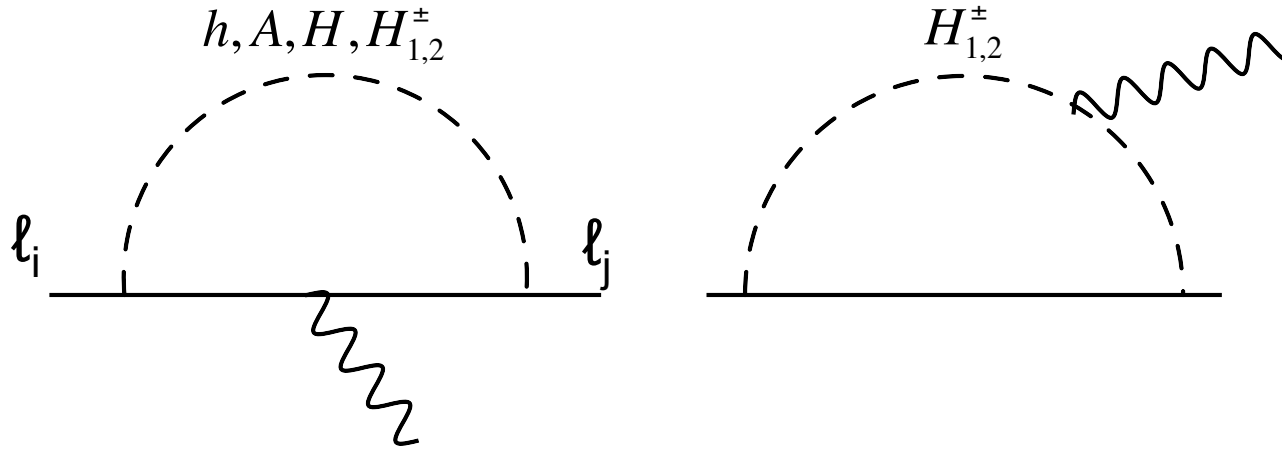


$$M_{\nu}^{ij} = \frac{1}{16\pi^2} \frac{\sqrt{2}v\mu}{m_{H_2^\pm}^2 - m_{H_1^\pm}^2} \ln \left( \frac{m_{H_2^\pm}^2}{m_{H_1^\pm}^2} \right) \left( F m_\ell Y_\ell^\dagger \right) + (i \leftrightarrow j)$$

✓ Two charged scalar mass eigenstate :  $H_1^\pm, H_2^\pm$

## 2. Zee-model with flavor symmetry

### □ LFV decay of charged lepton $\ell_i \rightarrow \ell_j \gamma$



$$\frac{BR(\ell_i \rightarrow \ell_j \gamma)}{BR(\ell_i \rightarrow \ell_j \nu_i \bar{\nu}_j)} \approx \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{\ell_i}^2} \left( \left| \sum_{\phi} (a_R^{\phi})_{ij} \right|^2 + \left| \sum_{\phi} (a_L^{\phi})_{ij} \right|^2 \right) \quad \phi = h, H, A, H_1^{\pm}$$

$\left[ a_{L,R}^{\phi} : \text{Amplitude estimated from diagrams and given by Yukawa couplings} \right]$

- 1. Introduction**
- 2. Zee-model with flavor symmetry**
- 3. Prediction for LFV**
- 4. Summary and discussion**

### 3. Prediction for LFV

## Numerical analysis

Searching for Yukawa couplings with mixing  $U_{L,R}$  satisfying:

$$\{F_{ij}, (Y_\ell)_{ij}\}$$

- Charged lepton mass
- Neutrino mass matrix which can fit neutrino oscillation data



Calculate BRs for CLFV processes by allowed parameters

- Imposing constraint on  $\ell_i \rightarrow \ell_j \gamma$  :

$$\text{BR}(\mu \rightarrow e\gamma) < 4.2 \times 10^{-13}, \quad \text{BR}(\tau \rightarrow e\gamma) < 3.3 \times 10^{-8}, \quad \text{BR}(\tau \rightarrow \mu\gamma) < 4.4 \times 10^{-8},$$

MEG(2016)

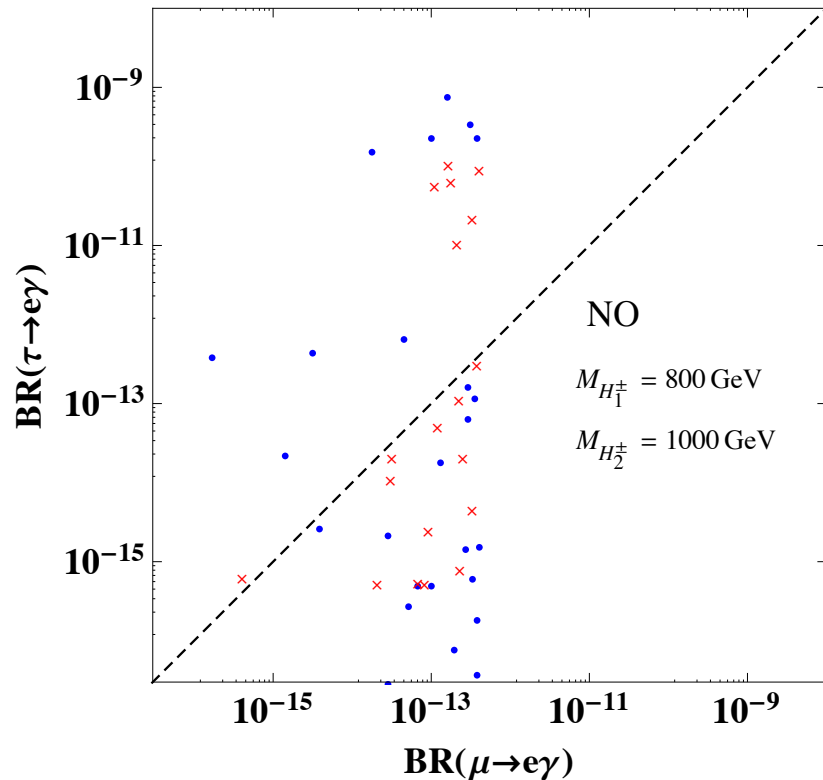
Belle(2008) Babar(2010)

- Prediction for LFV decay of heavy Higgs H

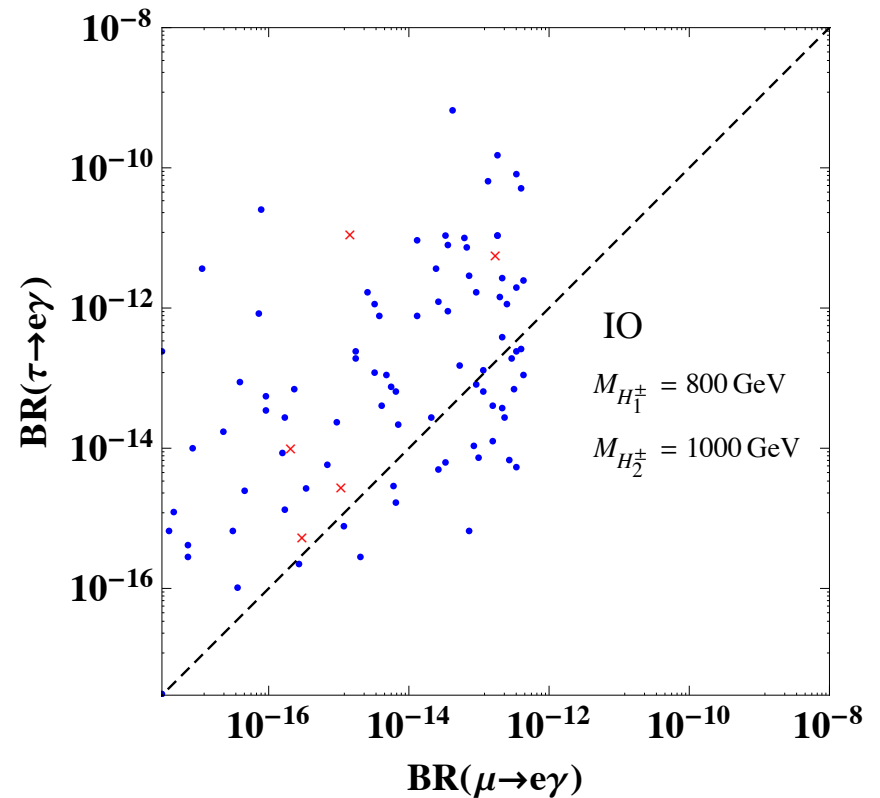
### 3. Prediction for LFV

## ◆ Correlation between BR of $\mu \rightarrow e\gamma$ and $\tau \rightarrow e\gamma$

Normal ordering



Inverted ordering



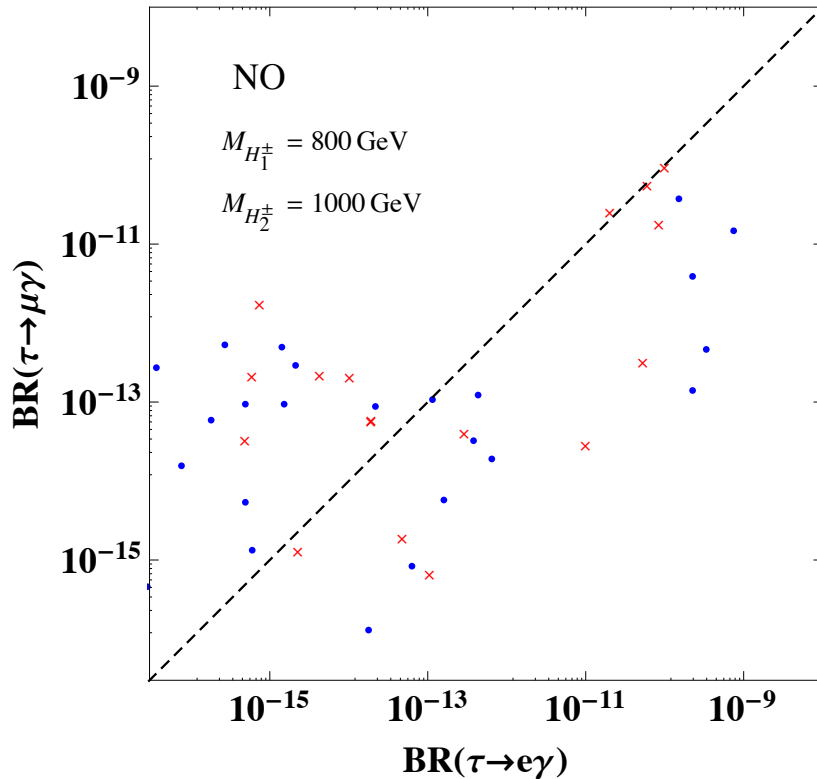
Blue dot :  $1 < \tan\beta < 10$ , Red cross :  $10 < \tan\beta < 30$

✓ Normal ordering case tends to give larger  $BR(\mu \rightarrow e\gamma)$

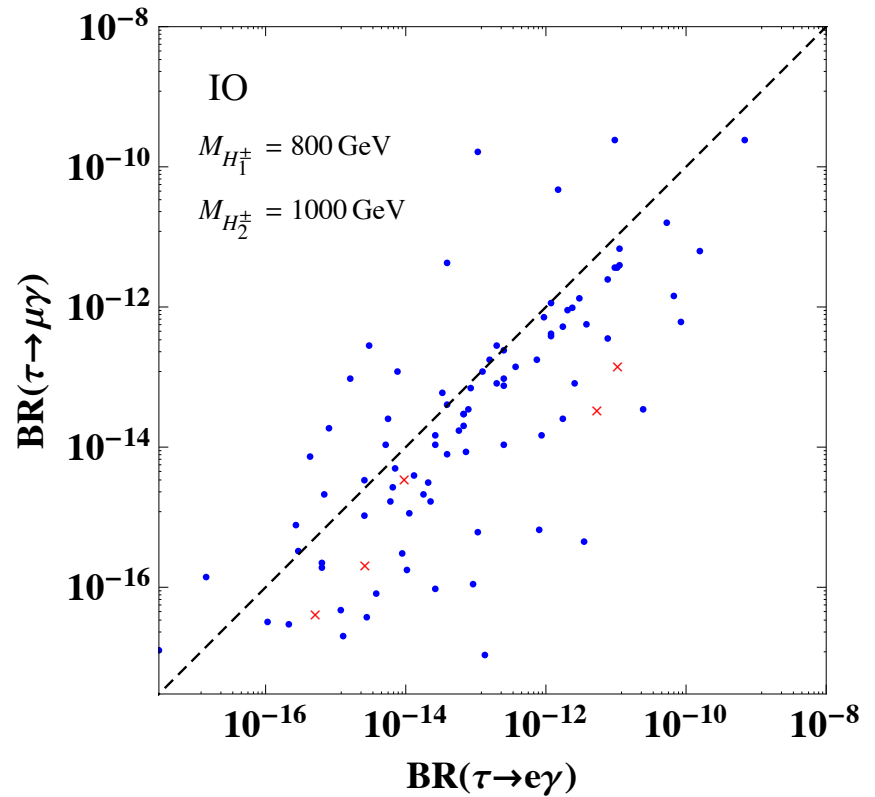
### 3. Prediction for LFV

## ◆ Correlation between BR of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$

Normal ordering



Inverted ordering

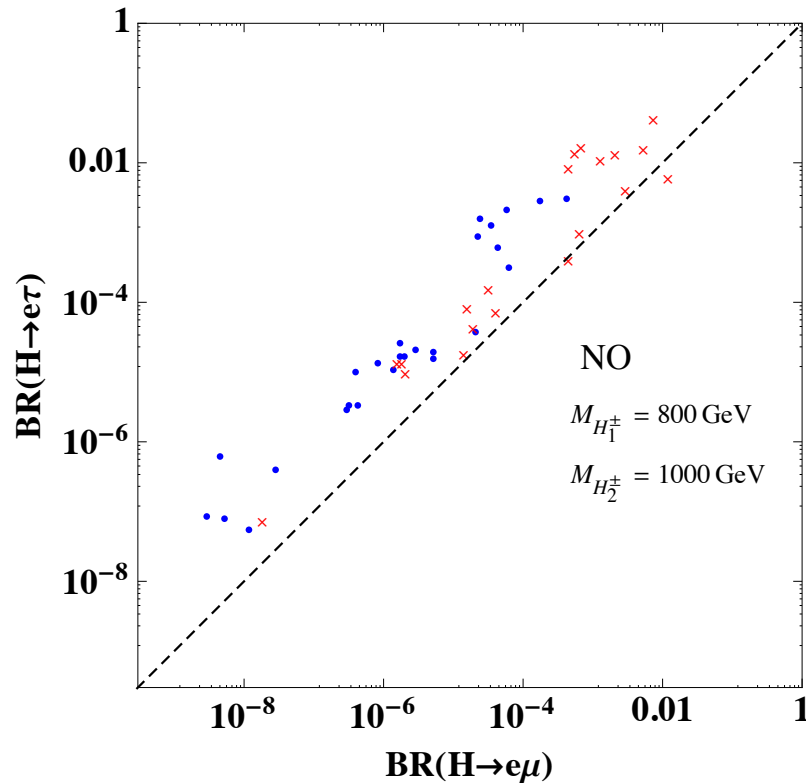


✓ BRs are more correlated for inverted ordering case

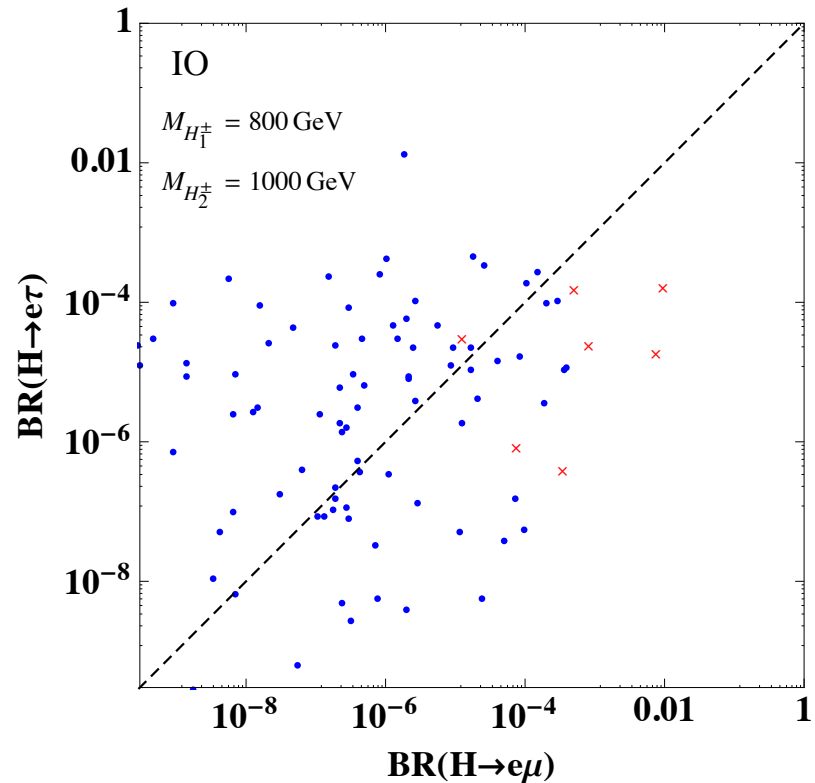
### 3. Prediction for LFV

## ◆ Correlation between BR of $H \rightarrow e\mu$ and $H \rightarrow e\tau$

Normal ordering



Inverted ordering



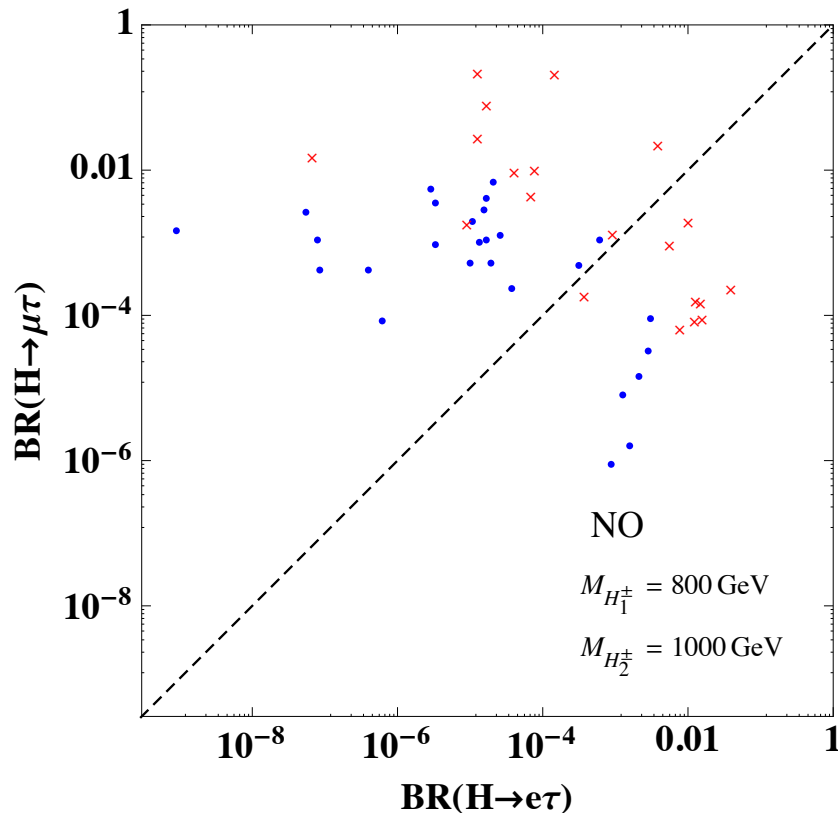
- ✓ Correlation in normal ordering case
- ✓ Inverted ordering case is less correlated



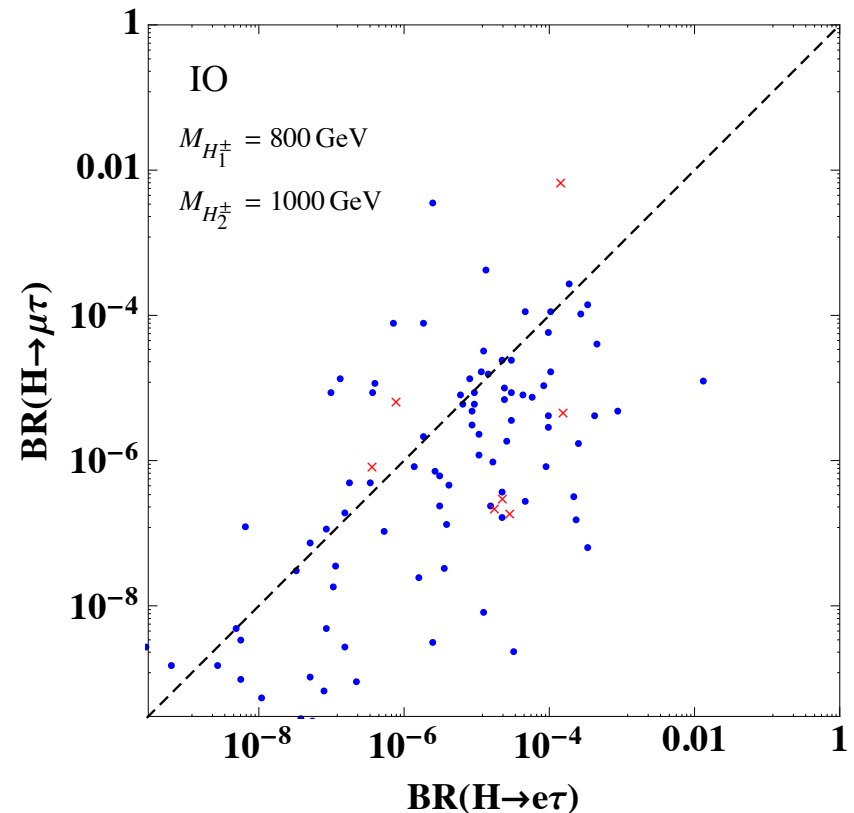
### 3. Prediction for LFV

## ◆ Correlation between BR of $H \rightarrow e\tau$ and $H \rightarrow \mu\tau$

Normal ordering



Inverted ordering



- ✓ BRs are more correlated for inverted ordering case
- ✓ These LFV H decay could be searched for at the LHC

# Summary and discussion

- Radiative neutrino mass generation is reviewed
- Zee-model with flavor symmetry
  - It connects neutrino mass generation and LFV pattern
  - LFV decay of Higgs boson is predicted
  - We also show LFV branching ratio of heavy Higgs

Thanks for listening !

# CLFV amplitudes in Zee-model

$$(a_R^{H_1^\pm})_{ij} = \frac{1}{16\pi^2} \sum_{k=1}^3 \left[ (Y_\ell)_{kj}^* (Y_\ell)_{ki} c_\chi^2 F_1(m_{\ell_i}, m_{\ell_j}, m_{H_1^\pm}) - F_{kj}^* F_{ki} s_\chi^2 F_2(m_{\ell_i}, m_{\ell_j}, m_{H_2^\pm}) \right], \quad (C1)$$

$$(a_L^{H_1^\pm})_{ij} = \frac{1}{16\pi^2} \sum_{k=1}^3 \left[ (Y_\ell)_{kj}^* (Y_\ell)_{ki} c_\chi^2 F_2(m_{\ell_i}, m_{\ell_j}, m_{H_1^\pm}) - F_{kj}^* F_{ki} s_\chi^2 F_1(m_{\ell_i}, m_{\ell_j}, m_{H_2^\pm}) \right], \quad (C2)$$

$$(a_R^{H_2^\pm})_{ij} = (a_R^{H_1^\pm})_{ij} \Big|_{c_Y^2 \leftrightarrow s_Y^2}, \quad (a_L^{H_2^\pm})_{ij} = (a_L^{H_1^\pm})_{ij} \Big|_{c_Y^2 \leftrightarrow s_Y^2}. \quad (C3)$$

$$F_{1[2]}(m_1, m_2, m_3) = \int [dX] \frac{xzm_2[xy m_1]}{(x^2 - x)m_1^2 + xz(m_1^2 - m_2^2) + (y + z)m_3^2},$$

$$(a_R^\varphi)_{ij} = \frac{1}{8\pi^2} \sum_{k=1}^3 \int [dX] \frac{xym_{\ell_i} f_\varphi^{jk} f_\varphi^{ki} + xzm_{\ell_j} g_\varphi^{jk} g_\varphi^{ki} + (1-x)m_{\ell_k} f_\varphi^{jk} g_\varphi^{ki}}{-x(1-x)m_{\ell_i}^2 - xz(m_{\ell_j}^2 - m_{\ell_i}^2) + (z+y)m_{\ell_k}^2 + xm_\varphi^2},$$

$$(a_L^\varphi)_{ij} = \frac{1}{8\pi^2} \sum_{k=1}^3 \int [dX] \frac{xzm_{\ell_i} f_\varphi^{jk} f_\varphi^{ki} + xym_{\ell_j} g_\varphi^{jk} g_\varphi^{ki} + (1-x)m_{\ell_k} g_\varphi^{jk} f_\varphi^{ki}}{-x(1-x)m_{\ell_i}^2 - xz(m_{\ell_j}^2 - m_{\ell_i}^2) + (z+y)m_{\ell_k}^2 + xm_\varphi^2},$$

$$f_h^{ij} = \frac{\sqrt{2}m_{\ell_i}}{v} s_{\beta-\alpha} \delta_{ij} + \frac{1}{\sqrt{2}} (Y_\ell^0)_{ji}^* c_{\beta-\alpha}, \quad g_h^{ij} = \frac{\sqrt{2}m_{\ell_i}}{v} s_{\beta-\alpha} \delta_{ij} + \frac{1}{\sqrt{2}} (Y_\ell^0)_{ij} c_{\beta-\alpha}$$

$$f_H^{ij} = \frac{\sqrt{2}m_{\ell_i}}{v} c_{\beta-\alpha} \delta_{ij} - \frac{1}{\sqrt{2}} (Y_\ell^0)_{ji}^* s_{\beta-\alpha}, \quad g_H^{ij} = \frac{\sqrt{2}m_{\ell_i}}{v} c_{\beta-\alpha} \delta_{ij} - \frac{1}{\sqrt{2}} (Y_\ell^0)_{ij} s_{\beta-\alpha}$$

$$f_A^{ij} = -\frac{i}{\sqrt{2}} (Y_\ell^0)_{ji}^* c_{\beta-\alpha}, \quad g_A^{ij} = \frac{i}{\sqrt{2}} (Y_\ell^0)_{ij} c_{\beta-\alpha}.$$

