Light Mediators in Anomaly Free $U(1)_X$ Models

Based on 1905.03867 and 1905.03872

The 1^{st} AEI Workshop for BSM and 9^{th} Particle Physics and Cosmology

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Overview

Motivation - Discrepancies in the MeV regime

- Puzzle at low energies: $(g_\mu - 2)$ and Proton Radius Puzzle (Bound)
- Future Experiments (1608.03591):
  1. LHCb Run 3 (2021-2023) search for Dark Photons via $D^* \to D^0 A' (A' \to e^+ e^-)$;
  2. Mu3e Phase II (2018 -): muon decay channel $\mu \to e \nu_e \nu_\mu (A' \to e^+ e^-)$ for $10 < m_A[MeV] < 80$.
  3. DarkLight (2018 -): Electrons scattered off hydrogen gas to on-shell dark photons in $10 < m_A[MeV] < 100$.
  4. VEPP-3 (proposal): Positron beam on hydrogen gas target for $e^+ e^- \to \gamma A'$;
  5. E36 (J-PARC): $K_{\mu 2ee}$ decays.
- Dark Photons vs. Z': What are the consequences of axial-vector couplings and new decay modes?

Proton Radius

Estimation Comparison between a prediction (theoretical) and measurement of the Lamb shift in muonic and atomic Hydrogen.

Prediction

$$\Delta E|^l_{the} = \delta E^l_a + \delta E^l_b + \cdots + \lambda^l \langle r_p^2 \rangle|_l \quad (1)$$

where $l = \mu, e$. At leading order λ^l is given by

$$\lambda^l = \frac{2\alpha}{3a_l^3 n^3} \left(\delta_{P0} - \delta_{S0}\right) \quad (2)$$

where $n = 2$ for $2P - 2S$ and $a_l = (\alpha m_{lp})^{-1}$ is the Bohr radius of the system with reduced mass m_{lp}.

Proton Radius

$$\Delta E|^l_{the} = \Delta E|^l_{exp}; \qquad \Delta E|^\mu_{exp} = 202.3706(23)\ \text{meV} \quad (3)$$

At the theory side

$$\Delta E|^\mu_{the} = 206.0336(15) + 0.0332(20) - 5.2275 \langle r_p^2 \rangle \quad (4)$$

Discrepancy (N. Bezginov et.al Science 365 (2019))

$$\sqrt{\langle r_p^2 \rangle}|^0_\mu = 0.84087(39)\ \text{fm} \quad (5a)$$

$$\sqrt{\langle r_p^2 \rangle}|^0_e = 0.8758(77)\ \text{fm} \qquad \text{CODATA-2010} \quad (5b)$$

Guiding Principles

- **Non-Universality** : Selected puzzles as a signal of favored flavors;
- **Minimality** : Introducing the minimal set of new degrees of freedom;
- **Standard Model** features :
  1. Preserve fermion representations;
  2. Cancellation of anomalies per generation;
- **Low-Energy Phenomenology** (1103.0721):
  1. Interactions νe or νN not stronger than G_F;
  2. Absent of fundamental electrically charged particles with $m_p < 100$(GeV);
  3. QED and particle physics at the MeV.

Anomalies Requirement

- $U(1)_X^3$
- $U(1)_Y U(1)_X^2$
- $U(1)_Y^2 U(1)_X$
- $SU(2)^2 U(1)_X$
- $SU(3)^2 U(1)_X$
- $grav^2 U(1)_X$

Solutions per generation

$$X_D = 2X_Q - X_U, \quad X_L = -3X_Q, \quad X_l = -2X_Q - X_U, \quad X_\chi = X_U - 4X_Q$$

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$$SM \otimes U(1)_X$$

- Second Generation of Right-Handed fields;
- Two-Higgs Doublet Model;
- Scalar Singlet: New scale and breaking of residual $U(1)$;
- Phenomenology of light neutral gauge boson: Remaining fields around the decoupling limit;

Particle Content

- Three vector fields W^μ from $SU(2)_L$; One vector B^Y_μ from $U(1)_Y$ and B^X_μ from $U(1)_X$;
- Three independent coupling constants g, g_Y, g_X apart from a kinetic mixing term κ;
- Three generations of Weak Isospin doublets:

$$(L_L)_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}_L \qquad (Q_L)_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L \tag{6}$$

  with $i = 1, 2, 3$;
- Right-Handed $SU(2)_L$ singlets: $\chi_R, l_{iR}, u_{iR}, d_{iR}$;

On the Kinetic Mixing

$$\mathcal{L} \supset -\frac{1}{4}\mathbf{W}^{\mu\nu}\cdot\mathbf{W}_{\mu\nu} - \frac{1}{4}B^{Y\mu\nu}B^{Y}_{\mu\nu} - \frac{1}{4}B^{X\mu\nu}B^{X}_{\mu\nu} + \frac{\epsilon}{2}B^{Y\mu\nu}B^{X}_{\mu\nu}$$

Field redefinition:

$$B^Y_\mu \to B^Y_\mu + \epsilon B^X_\mu \tag{7}$$

or

$$\mathcal{L}_{k.m.} \supset -\frac{1}{2}(B^Y_\mu + \epsilon B^X_\mu)\hat{\mathcal{O}}^{\mu\nu}(B^Y_\nu + \epsilon B^X_\nu) - \frac{1}{2}B^X_\mu \hat{\mathcal{O}}^{\mu\nu} B^X_\nu + \epsilon\, B^X_\mu \hat{\mathcal{O}}^{\mu\nu}(B^Y_\nu + \epsilon B^X_\nu) \tag{8}$$

such that, up to order $\mathcal{O}(\epsilon)$,

$$\mathcal{L}_{k.m.} \supset -\frac{1}{2}B^Y_\mu \hat{\mathcal{O}}^{\mu\nu} B^Y_\nu - \frac{1}{2}B^X_\mu \hat{\mathcal{O}}^{\mu\nu} B^X_\nu + \mathcal{O}(\epsilon^2) \tag{9}$$

i.e. the crossed terms vanishes and the mixing effect is converted into the Covariant Derivative:

$$D_\mu \to D_\mu = \partial_\mu - ig\mathbf{W}_\mu\cdot\tau - ig_Y B^Y_\mu Y^p - i(\kappa Y^p + g_X X^p)B^X_\mu \tag{10}$$

where $\epsilon g_Y \equiv \kappa$.

Couplings and Masses

- Mixing Matrix:

$$\mathbb{V} = \begin{pmatrix} s_\theta c_\phi & -s_\theta s_\phi & -c_\theta \\ s_\phi & c_\phi & 0 \\ c_\theta c_\phi & -c_\theta s_\phi & s_\theta \end{pmatrix} \tag{11}$$

- Photon Couplings:

$$e\mathbb{Q} = g s_\phi \tau^3 + g_Y c_\phi Y \tag{12}$$

and by applying it to the standard fields, it can be extracted

$$g s_\phi = g_Y c_\phi = e \tag{13}$$

- Z Couplings:

$$g_Z = c_\theta g_Z^{SM} + s_\theta(\kappa Y + g_X X) \tag{14}$$

- X Couplings:

$$g_R = s_\theta g_Z^{SM} - c_\theta(\kappa Y + g_X X) \tag{15}$$

where $g_Z^{SM} = \frac{g}{c_\phi}(\tau_3 - s_\phi^2 \mathbb{Q})$.

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where $g_Z^{SM} = \frac{g}{c_\phi}(\tau_3 - s_\phi^2 \mathbb{Q})$.

- Neutral Vector Masses: If $g_X, \kappa \ll \bar{g}$

$$m_Z^2 \to \frac{v^2}{4}\bar{g}^2, \qquad m_X^2 \to \frac{v^2}{4}a_1 \tag{16}$$

where

$$\bar{g}^2 = g_Y^2 + g^2\,, \qquad a_1 = 4\left[g_X^2\frac{\bar{v}^2}{v^2} - g_X\kappa c_\beta^2\right] + \kappa^2$$

- Mixing Angle:

$$s_\theta \approx \frac{|2g_X c_\beta^2 - \kappa|}{\bar{g}}\left[1 - \frac{m_X^2}{m_Z^2}\right]^{-1} \tag{17}$$

Fermion Gauge Interactions

$$\mathcal{L}_{kin} \supset i\left[\overline{L}_{\alpha L} \not{D} L_{\alpha L} + \overline{Q}_{\alpha L} \not{D} Q_{\alpha L} + \bar{l}_{\alpha R} \not{D} l_{\alpha R} + \overline{d}_{\alpha R} \not{D} d_{\alpha R} + \overline{u}_{\alpha R} \not{D} u_{\alpha R} + \overline{\chi}_R \not{D} \chi_R\right] \quad (18)$$

with $\alpha = 1, 2, 3$, $\beta = 1, 2$. The Covariant Derivative in terms of the mass eigenstates can be written like

$$D_\mu = \partial_\mu - ig(W^+ \mathbb{I}_+ + W^- \mathbb{I}_-) - ie\mathbb{Q}A_\mu - ig_Z Z_\mu - ig_R X_\mu \quad (19)$$

Flavor Violating processes in both Z and X interactions are exclusive to RH sector. Defining the vector of fermion fields $f = (f_1, f_2, f_3)$ and rotating the system to the mass basis, $f_R \to V_{fR} f'_R \equiv V_{fR} f_R$, the general currents depending on the X charges can be fully separated via:

$$\mathcal{L}_{kin} \supset -c_\theta g_X \left[\overline{u}_R \mathbb{F}^U \gamma^\mu u_R \; + \overline{d}_R \mathbb{F}^D \gamma^\mu d_R \; + \bar{l}_R \mathbb{F}^l \gamma^\mu l_R\right] X_\mu \quad (20)$$

apart from the s_θ-dependent universal contribution. The matrices

$$\mathbb{F}^f \equiv V^\dagger_{fR} \mathbb{X}^f V_{fR}, \qquad \text{where} \qquad (\mathbb{X}^f)_{ij} \equiv X^f \delta_{2i} \delta_{2j} \quad (21)$$

or

$$(\mathbb{F}^f)_{ij} = X^f (V^\dagger_{fR})_{i2} (V_{fR})_{2j} \quad (22)$$

summarizes the amount of flavor violation and fermion non-universality in the model.

Fermion Gauge Interactions and Non-Universality

$$(\mathbb{F}^f)_{ij} = X^f (V_{fR}^\dagger)_{i2} (V_{fR})_{2j} \tag{23}$$

- By unitarity the trace of $\mathbb{F}^f$ is equal to X^f:

$$\begin{aligned} \text{Tr}[\mathbb{F}^f] &= \text{Tr}[V_{fR}^\dagger \mathbb{X}^f V_{fR}] \\ &= \text{Tr}[\mathbb{X}^f] \\ &= X^f \end{aligned} \tag{24}$$

- In the scenario where flavor is aligned to mass eigenstates, i.e. when the absolute value of diagonal elements of V_{fR} are larger than the non-diagonal ones, the flavor violating processes also will favor second generation in the final state.

$$|\mathbb{F}^f| \equiv X^f \begin{pmatrix} |V_{fR}|_{21}^2 & |V_{fR}|_{21}|V_{fR}|_{22} & |V_{fR}|_{21}|V_{fR}|_{23} \\ |V_{fR}|_{21}|V_{fR}|_{22} & |V_{fR}|_{22}^2 & |V_{fR}|_{22}|V_{fR}|_{23} \\ |V_{fR}|_{21}|V_{fR}|_{23} & |V_{fR}|_{22}|V_{fR}|_{23} & |V_{fR}|_{23}^2 \end{pmatrix} \tag{25}$$

- X hypercharges (P.Ko, Y.Omura, C.Yu, PLB 717(2012)202-206)

$$X_L = 0; \quad X_Q = 0; \quad X_{e2} = 1; \quad X_\chi = -1; \quad X_{u2} = -1; \quad X_{d2} = 1 \tag{26}$$

with the remaining RH fields uncharged

Parameter Space - $U(1)_X$

Parameter Space P: Initially the set P is given by

$$P := [\kappa, g, g_Y, g_X, v_X, v_0, v_s, \mathbb{F}] \tag{27}$$

To reproduce the Electroweak interactions, both g and g_Y can be solved in terms of the remaining elements. The m_W and m_Z pole mass can, in addition, solve v_0 and v_X. However, in the asymptotic limit both masses depends only on v such that it may be convenient to preserve c_β in the analysis. Finally, the scale v_s can be *replaced* by m_X. We end up with a five-dimensional parameter space, namely

$$P := [c_\beta, \kappa, g_X, m_X, \mathbb{F}] \tag{28}$$

The kinetic mixing variable is independent and can be replaced by the new mixing angle. Accordingly, there must be a region for κ where the SM Z interactions are exactly reproduced, i.e. $s_\theta = 0$.

Constraints

Most stringent from previous work (PRD 94, 115023 (2016)):

- ρ parameter;
- Proton Puzzle in the $U(1)_X$;
- χ Fermion - Mixing Energy Considerations and Relic Abundance;
- Kaon Leptonic Decays $K_{\mu 2ee}$;
- $(g_l - 2)$, $l = e, \mu$;
- Neutrino Trident Production;
- Parity Non-Conserving Processes.

ρ Parameter

The ρ parameter is a quantity defined by three observables, namely m_W, m_Z and the weak mixing angle through the expression

$$\rho = \frac{m_W^2}{m_Z^2 c_w^2} \tag{29}$$

In the SM these parameters are connected by a natural relation and results in $\rho = 1$ at tree-level. In order to verify how the parameter will escape from the unity, in first approximation we can rewrite the Z mass

$$m_Z^2 \approx \frac{v^2}{4}\bar{g}^2\left(1 + s_\theta^2\right) \tag{30}$$

where the X_μ light mass condition has been used. It follows that

$$\rho_X^{tree} \approx c_\theta^2 \tag{31}$$

which cannot touch the central value of the experimental measurement

$$\rho \in 1.00040(24) \tag{32}$$

At two sigmas we can demand $0.99992 < c_\theta^2 \leq 1$ or

$$s_\theta^2 < 8 \cdot 10^{-5} \tag{33}$$

Proton Radius in $U(1)_X$

The discrepancy can be accommodated

$$\Delta E|^l_{the} = \delta E^l_0 + \delta E^l_X + \lambda^l \langle r_p^2\rangle|^X_l \to \langle r_p^2\rangle|^X_\mu = \langle r_p^2\rangle|^X_e \tag{34}$$

The difference between the "X" and "0" frameworks can be expressed as a small deviation like

$$\langle r_p^2\rangle|^X_l = \langle r_p^2\rangle|^0_l - \delta^X_l \quad \text{where} \quad \delta^X_l \equiv \frac{\delta E^l_X}{\lambda^l} \tag{35}$$

In summary, a proton radius constraint is imposed by

$$\delta^X_e - \delta^X_\mu = \langle r_p^2\rangle|^0_e - \langle r_p^2\rangle|^0_\mu \tag{36}$$

The correction δ^X_l originates from a contribution to the Coulomb potential due to the exchange of a massive vector boson X_μ

$$V^l_X(r) = \frac{g_l g_p}{e^2}\frac{\alpha e^{-m_X r}}{r} \tag{37}$$

with a correspondent shift in $2P - 2S$

$$\begin{aligned} \delta E^l_X &= \int dr\, V^l_X(r)\left(|R_{21}(r)|^2 - |R_{20}(r)|^2\right) r^2 \\ &= -\frac{\alpha}{2a_l^3}\left(\frac{g_l g_p}{e^2}\right)\frac{f(a_l m_X)}{m_X^2} \end{aligned} \tag{38}$$

For $m_X > 10$ MeV we can take $f(x) = \frac{x^4}{(1+x)^4} \sim 1$.

Proton Radius in $U(1)_X$

A *Proton curve* is defined by

$$6\frac{g_p}{e^2}\frac{(g_e - g_\mu)}{m_X^2} = \langle r_p^2\rangle|_e^0 - \langle r_p^2\rangle|_\mu^0 \tag{39}$$

which in principle can be solved by an attractive force (i.e. $\text{sgn} g_p = -\text{sgn} g_l$) strongly coupled with muons. In the $U(1)_X$ framework, and under the limit where $f(x) \sim 1$, the sgn g_p must be opposite only to the non-universal part of the X^μ coupling. The couplings g_p and g_l are given by:

$$g_p = -c_\phi^2\kappa; \qquad g_l = \frac{x_V^l}{2} \tag{40}$$

For simplicity $\mathbb{F}_{\tau\tau}$ may be taken zero such that $\mathbb{F}_{\mu\mu} + \mathbb{F}_{ee} = 1$, what reduces the Proton curve to

$$6\frac{g_p g_X}{e^2}\frac{2\mathbb{F}_{\mu\mu} - 1}{m_X^2} = 0.060(13)\ \text{fm}^2 \tag{41}$$

$(g_e - 2)$

The correction to a_e due to the presence of X_μ corresponds to a shift of the fine-structure constant:

$$d\alpha = 2\pi a_e^X \quad \rightarrow \quad \frac{d\alpha^{-1}}{\alpha^{-1}} = -\frac{2\pi a_e^X}{\alpha} \tag{42}$$

The r.h.s is the relative correction to the measurement of α^{-1} which should not exceed 0.5 ppb. The dipole function can be written like

$$a_e^X = \frac{m_e^2}{4\pi^2}\left[(x_V^e)^2 I_V(m_X^2) + (x_A^e)^2 I_A(m_X^2)\right] \tag{43}$$

where

$$\begin{aligned} I_V(m_X^2) &= \int_0^1 dz \frac{z^2(1-z)}{[m_l^2 z^2 + m_X^2(1-z)]} \xrightarrow{m_X \gg m_l} \frac{1}{3m_X^2} \\ I_A(m_X^2) &= \int_0^1 dz \frac{z(1-z)(z-4) - \left(2\frac{m_l^2}{m_X^2}\right)}{[m_l^2 z^2 + m_X^2(1-z)]} \xrightarrow{m_X \gg m_l} -\frac{5}{3m_X^2} \end{aligned} \tag{44}$$

Since the limit $m_X \gg m_e$ is valid in our region we can set the bounding curve

$$f\left(\frac{m_e^2}{m_X^2}\right) \equiv \left(\frac{m_e^2}{m_X^2}\right)\frac{1}{6\pi\alpha}|(x_V^e)^2 - 5(x_A^e)^2| < 0.5\text{ppb} \tag{45}$$

Parameter Space facing Selected Process

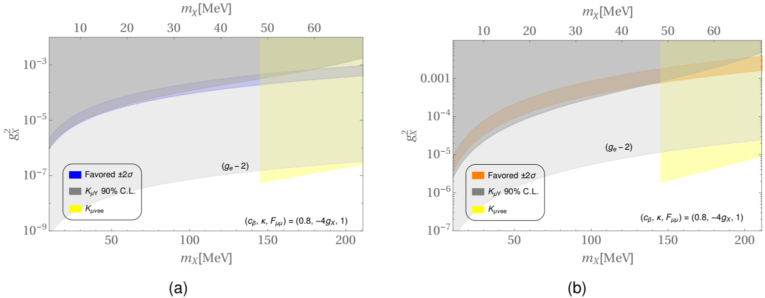


Figure: The favored region for the proton radius anomaly explanation facing the selected bounds. Under the Narrow-Width approximation the vector X_μ decays into a lepton pair $\bar{l}l$ for $l = e, 3\nu, \tau$. Here $m_X = 3m_\chi$ while $\mathbb{F}_{\tau\tau} = 0$.

Parameter Space - Fine Tuning

We must deal with the task of fixing a plane of a five-dimensional parameter space under the assumption that the model must explain, for instance, the proton puzzle. For that particular discrepancy one needs

$$\mathrm{sgn} g_X = -\mathrm{sgn}\kappa \tag{46}$$

In the examples depicted in the previous figures one can verify how stringent $(g-2)_e$ bounds are. A possible strategy to loose these lines is to look in their definition and work with the interference between vector and axial-vector couplings. For instance, in the region around the root

$$|(x_V^e)^2 - 5(x_A^e)^2| = 0 \tag{47}$$

for some fixed $\mathbb{F}$, the bound would be approximately absent. For instance, for $\mathbb{F}_{ee} = 0$ the solutions are

$$n \in \left[-\frac{7}{5}, \frac{3}{2}, 3\right] c_\beta^2 \tag{48}$$

for $\kappa = n g_X$. Hence, only one value can satisfy the condition of Eq.(46).

Parameter Space - Fine Tuning

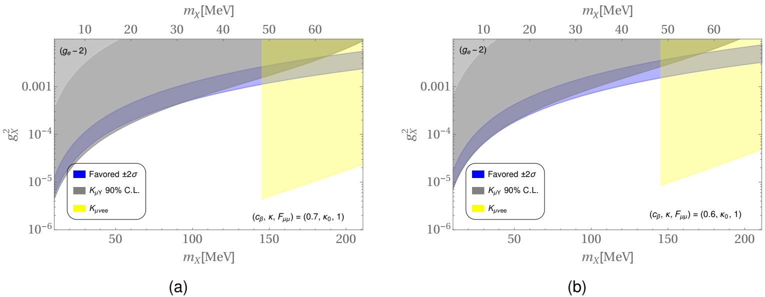
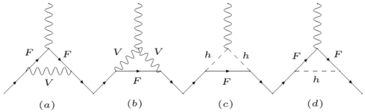


Figure: Close to the root for the $(g_e - 2)$ bound one can reduce the discrepancy of the proton puzzle from 5σ to 2σ.

$(g_\mu - 2)$



$$\frac{\Delta(g_\mu - 2)}{2} \sim 288(80) \times 10^{-11} \tag{49}$$

- X Boson Contribution

$$\mathcal{L} = \frac{1}{2} \sum_F \bar{\mu}[x_V \gamma^\rho + x_A \gamma^\rho \gamma^5] F \, X_\rho \tag{50}$$

- Neglecting flavor violating vertex, i.e. $F = \mu$.

$$[a_\mu]_a = \frac{m_\mu^2}{16\pi^2} \int_0^1 dz \, \frac{\left[x_V^2 [(z - z^2) z] + x_A^2 [(z - z^2)(z - 4) - 2 \frac{m_\mu^2}{m_X^2} z^3] \right]}{m_\mu^2 x^2 + m_X^2 (1 - x)} \tag{51}$$

In the very large Higgs mass assumption only $[a_\mu]_a$ contributes. However, for $c_\beta < .9$ it leads to negative sign to the dipole function, thus forbidding the explanation.

$(g_\mu - 2)$

We include the contributions from Light Higgs to the dipole function in the region where the asymptotic approximation to the integrals is still valid $m_h > 20 m_\mu$.

- General Yukawa Lagrangian

$$\mathcal{L}_Y = \sum_{h,F} \bar{\mu}[C_S + C_P \gamma_5] F\ h \tag{52}$$

- **Asymptotic Limit of the Integrals** : For $m_{h^+}, m_{h^0} >> m_\mu$

$$[a_\mu]_c \quad \rightarrow \quad \frac{m_\mu^2}{8\pi^2}(|C_S^+|^2 + |C_P^+|^2)\left(-\frac{1}{3}\right) \tag{53}$$

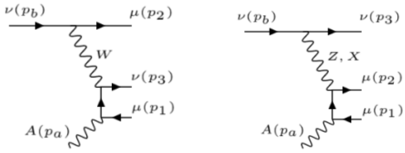
$$[a_\mu]_d^S \quad \rightarrow \quad \frac{m_\mu^2}{m_{h_0}^2}\frac{|C^0|_S^2}{8\pi^2}\left[\log\left[\frac{m_{h_0}^2}{m_\mu^2}\right] - \frac{7}{6}\right] \tag{54}$$

$$[a_\mu]_d^P \quad \rightarrow \quad \frac{m_\mu^2}{m_{h_0}^2}\frac{|C^0|_P^2}{8\pi^2}\left[\log\left[\frac{m_{h_0}^2}{m_\mu^2}\right] - \frac{11}{6}\right] \tag{55}$$

- Charged scalars cannot contribute to the correct sign;
- $c_\beta > .9$: Scalars allowed to stay in the decoupling region;
- For $c_\beta < 0.7$ (small v_X), light neutral scalars with $m_{h^0} \in (10 - 100) m_\mu$ might be required to restore $g_\mu - 2$, depending on κ. Charged scalars are disfavored in the low-energy regime.

Neutrino Trident Production

Clean tests for leptonic couplings.



(a) (b)

Figure: The trident production in the equivalent photon approximation (EPA). In addition, there are the reciprocal diagrams where the real photon is attached to μ^-.

In the CHARM-II experiment, a neutrino beam with the mean energy $E_\nu \sim 20$ GeV is scattered by a glass target ($Z = 10$). We require that the contribution coming from the interference of the SM and the $U(1)_X$ SM extension to the total cross-section should be inside the one standard deviation region, i.e.

$$|\sigma^{int}_{\text{SM+X}}| < 0.57\,\sigma_{\text{SM}}. \tag{56}$$

For the SM prediction, averaged over both neutrino and antineutrino scattering, we obtained $\sigma_{\text{SM}} = 1.8 \times 10^{-41}\text{cm}^2$.

Relic Abundance - χ fermion

- Approximate formula for the WIMP relic density (P.Gondolo et al., NPB360 (1991))

$$\Omega h^2 \approx \frac{3 \times 10^{-27} \mathrm{cm}^3 s^{-1}}{\langle \sigma_{ann} v \rangle}. \tag{57}$$

with the thermal average computed at the freeze-out temperature $T_{f.o.} \backsimeq \frac{m_\chi}{20}$.

- General case

$$\frac{dY}{dx} = -\left(\frac{45}{\pi M_P^2}\right)^{-1/2} \frac{g_*^{1/2} m_\chi}{x^2} \langle \sigma v \rangle (Y^2 - Y_{eq}^2), \tag{58}$$

by describing the evolution of the comoving abundance Y.

- The variable $x \equiv \frac{m_\chi}{T}$, where T is the photon temperature. It is commonly taken from $x = 1$, which defines the boundary for the condition $Y = Y_{eq}$, to the present value.
- The abundance Y_0 is related to the WIMP relic density through

$$\Omega_\chi h^2 = 2.755 \times 10^5 Y_0 \frac{m_\chi}{MeV} \tag{59}$$

and must be consistent with the current measurement $\Omega_{CDM} h^2 = 0.1131(34)$ (J.Silk et al., Cambridge U.Press, 2010.)

Relic Abundance - χ fermion

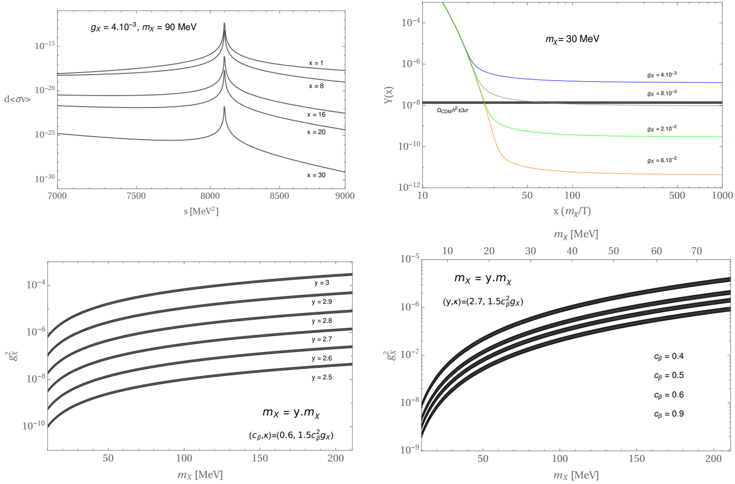


Figure: (a) The differential thermal average dominated by a narrow resonance. In the example, $m_\chi = 30$ MeV. In (b), the horizontal black band presents the 3σ region allowed by the current measurement of cold dark matter density.

Parameter Space

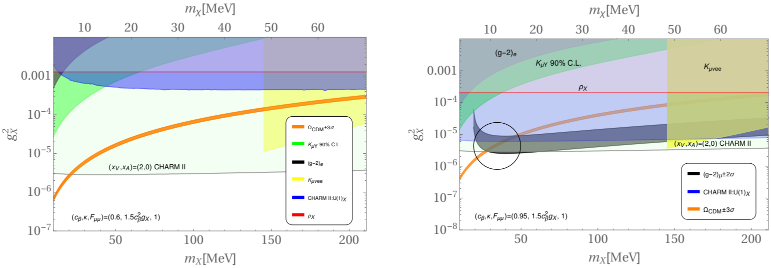


Figure: The parameter space for $\kappa = \frac{3}{2} c_\beta^2 g_X$. Notice, in (a), the excluded region for the dark photon (A')(light-blue) is presented in comparison to the dark gauge boson (Z') region (dark-blue). In (b), the difference is reduced when $c_\beta = 0.95$ which, on the other hand, produces a possible solution both for the $(g-2)_\mu$ discrepancy and the Ω_{CDM} (circled region).

Outlook - Parity Non-Conserving Observables

LEP phenomenology should be repeated.

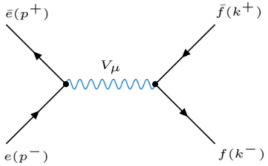


Figure: Forward-Backward Asymmetries in $e^+e^- \to \bar{f}f$ are important tests for axial-vector couplings. The model would predict non-universality in for $f = \mu$ and $f = \tau$. Here $V = \gamma, Z, X$.

The Forward-Backward Asymmetry is defined like

$$A(\theta) \equiv \frac{d\sigma(\theta) - d\sigma(\pi - \theta)}{d\sigma(\theta) + d\sigma(\pi - \theta)} \tag{60}$$

Here we will focus on the energy region distant of both Z and X peaks, i.e. $2m_\mu \ll \sqrt{s} \ll m_Z$ and we must compute the generic diagram of Fig.6 for $V = \gamma, Z, X$. For convenience, the generic vertex is written like

$$\bar{f}fV_\mu : \quad ie\gamma_\mu(v_f^V - a_f^V\gamma_5) \tag{61}$$

For instance, $(v_f^\gamma, a_f^\gamma) = (-q_f, 0)$ where $q_e = -1, q_u = \frac{2}{3}, q_d = -\frac{1}{3}$.

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The amplitude can then be expressed like

$$\mathcal{M}_V = \frac{e^2}{s - m_V^2}[\bar{v}(p^+)\gamma^\mu(v_e^V - a_e^V\gamma_5)u(p^-)][\bar{u}(k^-)\gamma_\mu(v_f^V - a_f^V\gamma_5)v(k^+)] \qquad (62)$$

with $|\mathcal{M}|^2 = |\sum_{V=\gamma,Z,X}\mathcal{M}_V|^2$.

$$A(\theta) \approx \frac{[d\sigma^{\gamma Z}(\theta) + d\sigma^{\gamma X}(\theta)] - [d\sigma^{\gamma Z}(\pi-\theta) + d\sigma^{\gamma X}(\pi-\theta)]}{d\sigma^{\gamma}(\theta) + d\sigma^{\gamma}(\pi-\theta)} \qquad (63)$$

In the CM reference frame it results in

$$A(\theta) \approx \frac{8sc_\theta|\mathbf{k}|\sqrt{s}}{4c_\theta^2|\mathbf{k}|^2 + 4m_f^2 + s}\left[\frac{a_e^X a_f^X}{s - m_X^2} + \frac{a_e^Z a_f^Z}{s - m_Z^2}\right] \qquad (64)$$

Here c_θ is the scattering angle, and $\mathbf{k}$ the 3-momenta of the products. In the region $\sqrt{s} \gg m_\mu$ the contribution from X exchange can be represented by $\delta A^X(\theta) \propto \frac{a_e^X a_f^X}{s}$ or

$$A(\theta) \propto \left[\frac{a_e^X a_f^X}{s} - \frac{a_e^Z a_f^Z}{m_Z^2}\right] \qquad (65)$$

Once $a^Z \sim g$ and $a^X \sim g_X$, where for instance $\sqrt{s} \sim \frac{m_Z}{10}$ the region $g_X \sim 10^{-1}g$ would be highly constrained.

Leptonic Meson Decays: $M \to l' \nu_{l'} l l$

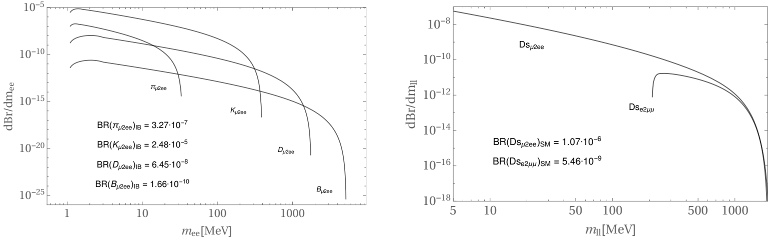

Figure: Differential branching ratio as a function of the di-lepton invariant mass in the SM. In the plot (a), the IB diagrams are dominant. In (b), the IB and SD contributions are presented for D_s decays.

Leptonic Meson Decays: $M \to l' \nu_{l'} ll$

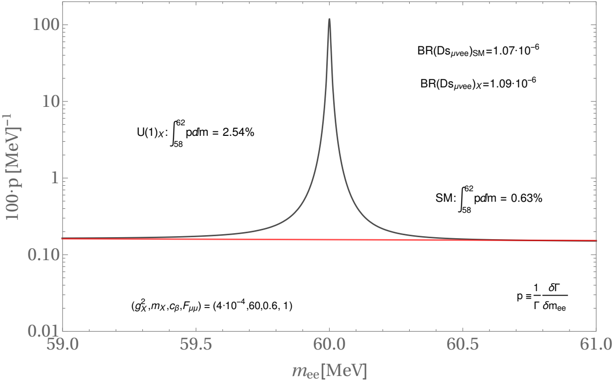

Figure: The normalized differential branching ratio corresponds to the probability $P = 2.54\%$ of measuring the di-lepton mass in the interval 58 MeV $< m_{ee} <$ 62 MeV at the resonance $(g_X^2, m_X) = (4 \times 10^{-4}, 60)$. $P = 0.63\%$, in the SM framework.

Lepton Flavor Universality

- Possible test for lepton flavor universality:

$$R(f) = \frac{Br(M_{f2\mu\mu})}{Br(M_{f2ee})}, \tag{66}$$

  in a kinematic region far from resonances. In the SM the ratio is close to the unity for $q^2 >> (2m_\mu)^2$ and different leptons in the final state. This case might be potentially interesting, and it is kinetically allowed only for $B_{\tau 2ll}$.

- For m_{ll}^2 far from the X_μ pole, any non-universality effects are negligible if compared with the SM prediction.

For instance, for $(1500)^2$ MeV$^2 < m_{ll}^2 < (1600)^2$ MeV^2, we find

$$R_X(\tau) = \begin{matrix} 0.93 & m_X = 1550 \text{ MeV}, \\ 0.99 & m_X = 60 \text{ MeV}, \end{matrix} \tag{67}$$

while for the SM value, $R_{SM}(\tau) = 0.9998$. In the region where the invariant mass of the di-lepton pair is in $(300)^2$ MeV$^2 < m_{ll}^2 < (400)^2$ MeV^2 we find

$$R_{SM}(\tau) = 0.933, \qquad R_X(\tau) = \begin{matrix} 0.90 & m_X = 350 \text{ MeV}, \\ 0.931 & m_X = 60 \text{ MeV}. \end{matrix} \tag{68}$$

Conclusions

1. Light Z' and RH currents;
2. Dark Photons vs. Light Z': Axial vector couplings may provide a larger room in the parameter space;
3. Proton Puzzle must face $(g_\mu - 2)$;
4. Sensibility in Meson Leptonic Decays;