Neutrino masses and Gravitational Waves

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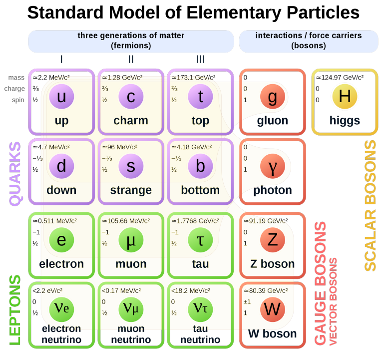
In collaboration with Graham White

Overview

Standard Model



Gauge symmetry group $\mathbb{G}_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}^Y = \bar{u} Y_u Q H + \bar{Q} Y_d d H + \text{h.c.}$$

$$\langle h^0 \rangle = v \to \quad m_f = Y_f v$$

Introduction/Motivation

- Neutrino masses (Dirac)
- Non-Abelian Dark Sector(self-interacting dark matter)
- Gravitational waves from early universe
- Neutrinogenesis
- Connect all of the above

Requirements for neutrino mass

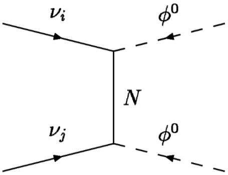
- Majorana or Dirac type?
- Tree level or radiative?
- New particles? (scalar, fermionic, vector)
- New gauge sectors? $(U(1), SU(2), SU(N))$

Tree level, Majorana

Weinberg Operator 1979: $\frac{LHLH}{\Lambda} = \frac{(l^- H^+ - \nu H^0)(l^- H^+ - \nu H^0)}{\Lambda}$

Add $N_R \sim (1,1,0)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = \bar{N} Y_D L H + m_N N_R N_R + \text{h.c.}$$



$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}.$$

$v \ll m_N \sim 10^{11}\text{GeV} \rightarrow m_\nu \ll v$ with $Y_D \sim 1$

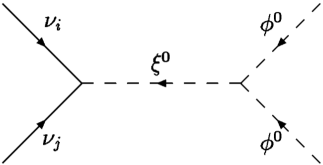
or $Y_D \ll 1 \rightarrow m_\nu \ll v$ with $m_N \sim O(10^{2-3} GeV)$

Seesaw−I

Seesaw-II

Add[1980] $\xi = (\xi^{++}, \xi^{+}, \xi^{0}) \sim (1,3,1)$ under $\mathbb{G}_{SM}$

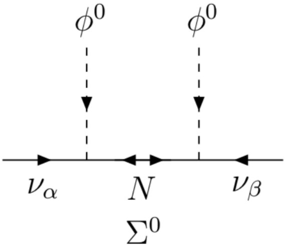
$$\mathcal{L}_{new} = YL\xi L - \mu H\xi H + \text{h.c.} \rightarrow m_\nu = Y\left\langle \xi^0 \right\rangle = -2\frac{Y\mu v^2}{M_\xi}$$



Seesaw-III

Add $\Sigma_R \sim (1,3,0)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = \bar{\Sigma} Y_D L H + m_N \Sigma_R \Sigma_R + \text{h.c.}$$



$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}$$

$$v \ll m_N \sim 10^{11}\text{GeV} \rightarrow m_\nu \ll v \text{ with } Y_D \sim 1$$

$$\text{or } Y_D \ll 1 \rightarrow m_\nu \ll v \text{ with } m_N \sim O(10^{2-3} GeV)$$

Seesaw variations

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$

Seesaw[1979] $m_\nu = -m_D^2/m_N$

Seesaw variations

$$\mathscr{M}_{\nu n} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix}$$

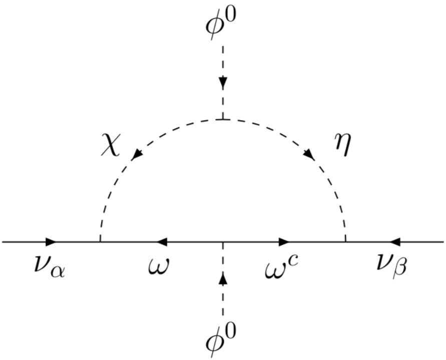
Inverse Seesaw[1986] $m_\nu = m_D^2 m_2/(m_N^2 - m_1 m_2)$

Seesaw variations

$$\mathscr{M}_{\nu N} = \begin{pmatrix} 0 & m_D & m'_D \\ m_D & 0 & m_N \\ m'_D & m_N & 0 \end{pmatrix}$$

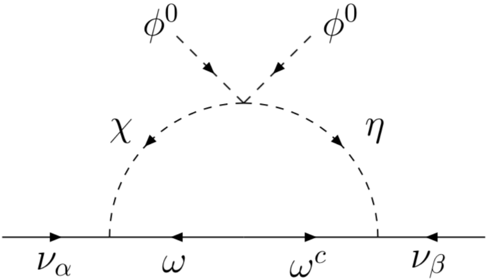
Linear Seesaw $m_\nu = -2 m_D m_{D'} / m_N$

Radiative neutrino mass, Majorana



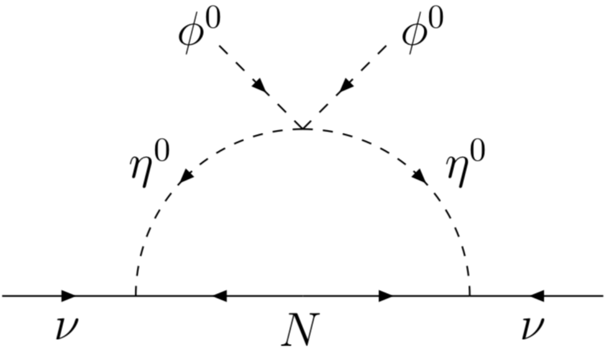
Zee[1986]

Radiative neutrino mass, Majorana



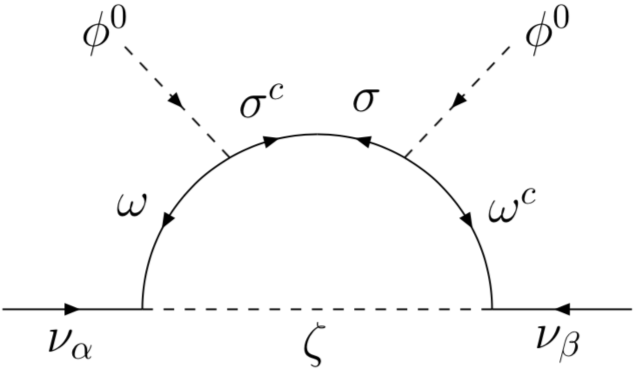
Ma[2006]

Scotogenic radiative neutrino mass



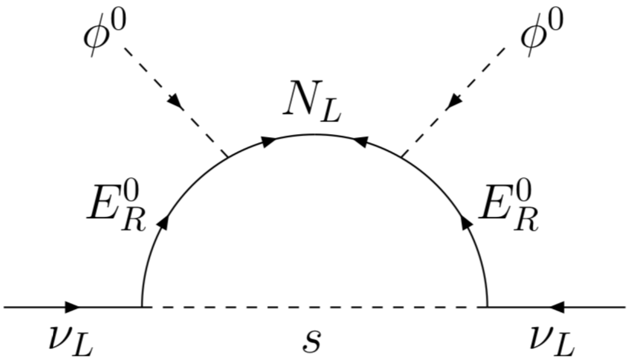
Add $\mathbb{Z}_2$ symmetry under which $\eta \sim (1, 2, 1/2)$ and N_R are odd

Radiative neutrino mass, Majorana



Fraser,Ma,OP[2014]

Radiative inverse seesaw neutrino mass, Majorana



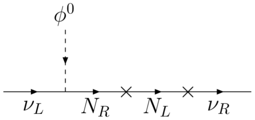
Add $\mathbb{Z}_2$ symmetry under which real singlet scalar and $E_{L,R} \sim (1, 2, 1/2)$ and $N_L \sim (1, 1, 0)$ are odd

Dirac neutrinos

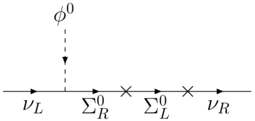
- Add $N_R \sim (1,1,0)$ under G_{SM}
- N_R MUST transfor under some other symmetry non-trivially
- New symmetry S is discrete, global, gauged, dark?

Tree Dirac case

- Insert a Dirac fermion singlet N which does not transform under $\mathcal{S}$, then break $\mathcal{S}$ softly by the dimension-three $\bar{\nu}_R N_L$ term.

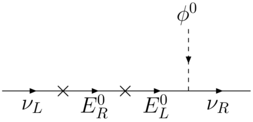


- Insert a Dirac fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ which does not transform under $\mathcal{S}$, then break $\mathcal{S}$ and $SU(2)_L \times U(1)$ together spontaneously to obtain the dimension-three $\bar{\nu}_R \Sigma^0_L$ term.

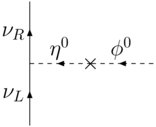


Dirac case

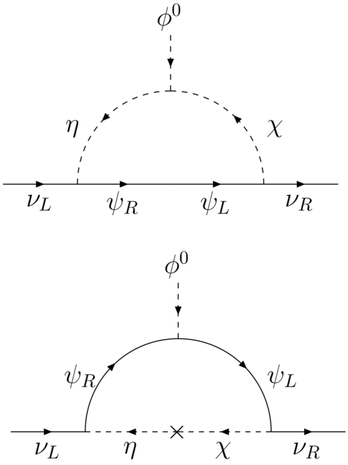
- Insert a Dirac fermion doublet (E^0, E^-) which transforms as ν_R under $\mathcal{S}$, then break $\mathcal{S}$ softly by the dimension-three $(\bar{E}^0\nu_L + E^+e^-)$ term.



- Insert a scalar doublet (η^+, η^0) which transforms as ν_R under $\mathcal{S}$, then break $\mathcal{S}$ softly by the dimension-two $(\eta^-\phi^+ + \bar{\eta}^0\phi^0)$ term.



Dirac case



$U(1)_{B-L}$ *case*

- Add 3 $N_R \sim (1,1,0)$ which carry $L = (1,1,1)$
- Add 3 $N_R \sim (1,1,0)$ which carry $L = (4,4,-5)$
- Other variations are possible
- Makes $U(1)_{B-L}$ anomaly free.
- $U(1)_{B-L}$ can global or gauged
- Global: softly or spontaneously broken(Majorana, Dirac)
- Gauged: spontaneously broken(Majorana, Dirac)

Models

Model*	Field	$SU(2)_L$	$U(1)_Y$	$SU(N)$	Flavor
1,2	ν_R	1	0	$\square$	N_f
1a	$N_{R,L}$	1	0	1	N_f
	ϕ	1	0	$\square$	1
1b	$\Sigma_{R,L}$	3	0	1	N_f
1b	ϕ	3	0	$\square$	1
1c	$E_{R,L}$	2	$-\frac{1}{2}$	$\square$	N_f
1c	ϕ	1	0	$\square$	1
2	η	2	$\frac{1}{2}$	$\square$	1
2	ϕ	1	0	$\square$	1

*10.1016/j.physletb.2016.11.027

Anomalies

$SU(N)$	fermion irreps	$\sum A(R) = 0$	mass invariants	Remarks
$SU(2)$	$\mathbf{2}$	$N_f \in$even	$\nu_R^a m_{ab} \nu_R^b$	$m_{ab} = -m_{ba}$
$SU(3)$	$\mathbf{3}, \mathbf{6}$	$\frac{N_f}{2} - \frac{7}{2} N_{\mathbf{6}} = 0$	$\nu_R \phi \Psi_6 \sim 1$, $\nu_R \nu_R \phi \sim 1$	
$SU(4)$	$\mathbf{4}, \bar{\mathbf{20}}, \bar{\mathbf{10}}$	$\frac{1}{2} + \frac{7}{2} - \frac{8}{2} = 0$		extra scalars?
$SU(5)$	$\mathbf{5}, \bar{\mathbf{10}}$	$\frac{1}{2} - \frac{1}{2} = 0$	$\nu_R \phi \Psi_{\bar{\mathbf{10}}} \sim 1$, $\Psi_{\bar{\mathbf{10}}} \Psi_{\bar{\mathbf{10}}} \phi^\dagger \sim 1$	same as in $SU(5)$ GUT
$SU(6)$	$\mathbf{6}, \bar{\mathbf{15}}$	$N_f \frac{1}{2} - X = 0$	$\nu_R \phi \Psi_{\bar{\mathbf{15}}} = 1$	$N_f \in$even
$SU(7)$	$\mathbf{7}, \bar{\mathbf{21}}, \mathbf{35}$	$\frac{1}{2} - \frac{3}{2} + 1 = 0$	$\mathbf{7}_S \mathbf{7}_F \bar{\mathbf{21}}_F \sim 1$, $\mathbf{35}_F \mathbf{35}_F \mathbf{7}_S \sim 1$	

Models' Lagrangians

$$-\mathcal{L}_a = N_R^\dagger Y_L L H + \phi \nu_R^\dagger Y_R N_L + N_R^\dagger M_N N_L + \text{h.c.}$$

$$-\mathcal{L}_b = H \Sigma_R^\dagger Y_L L + \nu_R^\dagger Y_R Tr[\Sigma_L \phi] + Tr[\Sigma_R^\dagger M_\Sigma \Sigma_L] + \text{h.c.}$$

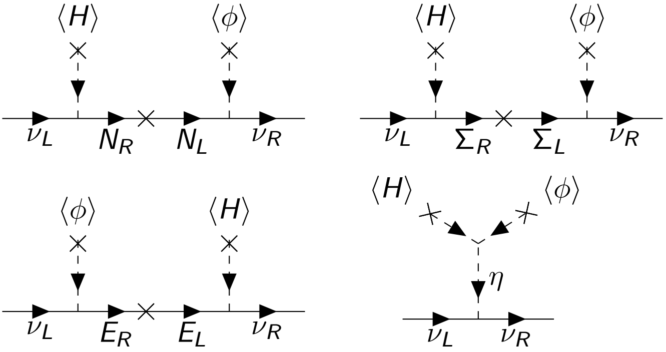
$$-\mathcal{L}_c = E_R^\dagger Y_L L \phi + \nu_R^\dagger Y_R E_L H + E_R^\dagger M_E E_L + \text{h.c.}$$

$$-\mathcal{L}_2 = \nu_R^\dagger Y_\nu L \eta + \text{h.c.}$$

Models' Potentials

$$
\begin{aligned}
V_1 = &-\mu_H^2 H^\dagger H + \frac{\lambda_H}{2}(H^\dagger H)^2 - m_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2}(\phi^\dagger \phi)^2 \\
&+ \lambda_{H\phi}(H^\dagger H)(\phi^\dagger \phi) + \delta_{\mathrm{SU}(2)} \lambda'_{H\phi} H^\dagger \phi \phi^\dagger H \\
V_2 = &-\mu_H^2 H^\dagger H + \frac{\lambda_H}{2}(H^\dagger H)^2 - \mu_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2}(\phi^\dagger \phi)^2 \\
&+ \mu_\eta^2 \eta^\dagger \eta + \frac{\lambda_\eta}{2}(\eta^\dagger \eta)^2 + \frac{\lambda'_\eta}{2}(\eta^{\dagger, i\alpha}\eta_{j\alpha})(\eta^{\dagger, j\beta}\eta_{i\beta}) \\
&+ \lambda_{H\phi}(H^\dagger H)(\phi^\dagger \phi) + \lambda_{H\eta}(H^\dagger H)(\eta^\dagger \eta) \\
&+ \lambda'_{H\eta}(H^\dagger \eta)(\eta^\dagger H) + \lambda_{\eta\phi}(\eta^\dagger \eta)(\phi^\dagger \phi) + \lambda'_{\eta\phi}(\phi^\dagger \eta)(\eta^\dagger \phi) \\
&+ \sqrt{2}\mu \eta^\dagger H \phi + \mathrm{h.c.}
\end{aligned}
$$

Models



Neutrino masses

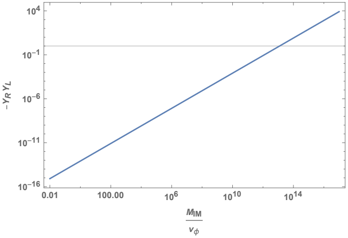
$$m_\nu^{(a)} \cong -\boldsymbol{Y_R M_N^{-1} Y_L} \frac{v v_\phi}{2}$$

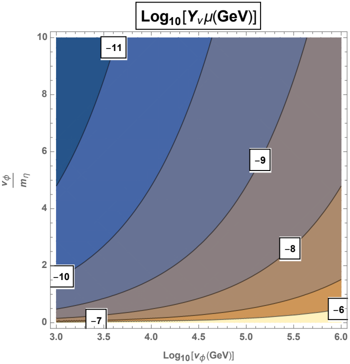
$$m_\nu^{(b)} \cong -\boldsymbol{Y_R M_\Sigma^{-1} Y_L} \frac{v v_\phi}{2}$$

$$m_\nu^{(c)} \cong -\boldsymbol{Y_R M_E^{-1} Y_L} \frac{v v_\phi}{2}$$

$$m_\nu \cong -Y_\nu \frac{v_\eta}{\sqrt{2}} \cong \frac{Y_\nu \mu v v_\phi}{\sqrt{2} 4 m_\eta^2}$$

Neutrino Masses





Yukawas

$$\boldsymbol{Y_L} = -\imath \mathcal{D}_{\sqrt{N,E,\Sigma}} R \mathcal{D}_{\sqrt{\kappa}} \mathcal{U}^{\dagger}_{PMNS}$$

$$\boldsymbol{Y_R}^{\dagger} = \imath \mathcal{D}^{*}_{\sqrt{N,E,\Sigma}} R \mathcal{D}^{*}_{\sqrt{\kappa}} \mathcal{U}^{\dagger}_{R}$$

$$\boldsymbol{Y_\nu} = \frac{\sqrt{24} m_\eta^2}{\mu v v_\phi} \mathcal{U}_R \mathrm{Diag}\left[m_1, m_2, m_3\right] \mathcal{U}^{\dagger}_{PMNS}$$

$$\mathcal{D}_\kappa = \frac{1}{v v_\phi} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix}$$

Spontaneous Symmetry Breaking

$$SM: SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{H} SU(3)_c \times U(1)_{em}$$

$$DS: SU(N) \xrightarrow{\phi} SU(N-1)$$

$$m_{W_D^{\pm}} = \frac{1}{2} g_D^2(v_\phi) v_\phi^2 \times (N-1) complex$$

$$m_{Z_D} = g_D^2(v_\phi) v_\phi^2 \frac{N-1}{2N}$$

Thermal potential

$$V_0 = -\mu_H^2 H^\dagger H + \frac{\lambda_H}{2}(H^\dagger H)^2 - m_\phi^2 \phi^\dagger \phi + \frac{\lambda_\phi}{2}(\phi^\dagger \phi)^2$$
$$+ \lambda_{H\phi}(H^\dagger H)(\phi^\dagger \phi) + \delta_{\mathrm{SU(2)}} \lambda'_{H\phi} H^\dagger \phi \phi^\dagger H$$
$$V_{CW} = n_{GB} \frac{m_{GB}^4}{64\pi^2}\left(\log\left[\frac{m_{GB}^2}{\mu^2}\right] - \frac{5}{6}\right) + \sum_{i \neq GB} n_i \frac{m_i^4}{64\pi^2}\left(\log\left[\frac{m_i^2}{\mu^2}\right] - \frac{3}{2}\right)$$
$$V_{T\neq 0} = \sum_i \frac{T^4}{2\pi^2} J_B\left(\frac{m_i^2 + \Pi_i}{T^2}\right)$$
$$V(\phi, T) = V_0(\phi) + V_{CW}(\phi, \mu) + V_{T\neq 0}(\phi, T)$$

Thermal Potential

$$m^2_{W_D}(\phi) = \frac{1}{2} g_D^2 \phi^2 \times (N-1) complex \qquad m^2_{Z_D}(\phi) = \frac{1}{2} g_D^2 \phi^2 \frac{N-1}{N}$$

$$m^2_{Re[\phi]}(\phi) = -\mu^2 + 3\frac{\lambda_\phi}{2}\phi^2 \qquad m^2_H(\phi) = -\mu_H^2 + \lambda_{H\phi}\phi^2/2$$

$$m^2_{Im[\phi]}(\phi) = -\mu^2 + \frac{\lambda_\phi}{2}\phi^2 \qquad m^2_{\phi_{N-1}}(\phi) = -\mu^2 + \frac{\lambda_\phi}{2}\phi^2$$

$$\Pi_{W_D} = \frac{11}{3} g_D^2 T^2 \qquad \Pi_{Z_D} = \frac{20+N}{6N} g_D^2 T^2$$

$$\Pi_{Re[\phi]} = \frac{T^2}{12}\left(2N\frac{\lambda_\phi}{2} + 4\lambda_{H\phi} + 3g_D^2\left(2(N-1) + \frac{N-1}{N}\right)\right)$$

$$\Pi_H = \frac{T^2}{12}\left(4\lambda_H + N\lambda_{H\phi}\right)$$

$$\Pi_{Im[\phi]} = \frac{T^2}{12}\left(N\frac{\lambda_\phi}{2} + 4\lambda_{H\phi} + 3g_D^2\left(2(N-1) + \frac{N-1}{N}\right)\right)$$

$$\Pi_{\phi_{N-1}} = \frac{T^2}{12}\left(N\frac{\lambda_\phi}{2} + 4\lambda_{H\phi} + 3g_D^2\left(2(N-1) + \frac{1}{(N-1)N}\right)\right)$$

Thermal parameters

$$\frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r} = \frac{\partial V(\phi, T)}{\partial\phi} \quad \phi'(0) = 0 \quad \phi(\infty) = 0$$

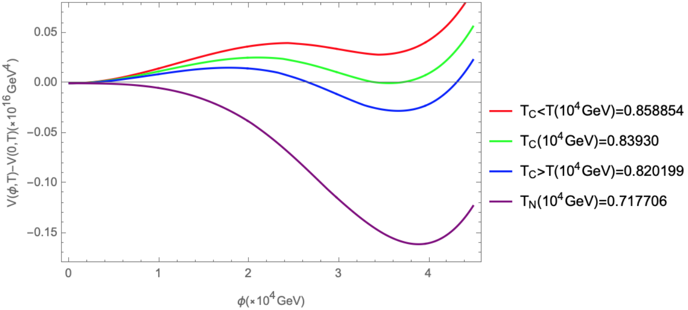
$$\mathcal{S}_E = 4\pi \int_0^\infty r^2 dr \left[\left(\frac{\partial\phi}{\partial r}\right)^2 + V(\phi, T) \right]$$

$$p(t_N)t_N^4 \approx 1 \quad p(T) \approx T^4 e^{-\mathcal{S}_E(\mathcal{S}_b;T)/T} \quad T^2 t = \sqrt{\frac{45}{16\pi^3}}\frac{M_p}{\sqrt{g_\star}}$$

$$\alpha = \left.\frac{\Delta\left(V - T\frac{\partial V}{\partial T}\right)}{\pi^2 g_\star T^4/30}\right|_{T_N}$$

$$\frac{\beta}{H} = T\left.\left(\frac{d(\mathcal{S}_E/T)}{dT}\right)\right|_{T_N}$$

Thermal parameters



$$\lambda_\phi = 0.0009,\ \lambda_{h\phi} = 0.05,\ g_D = 0.5,\ N = 5,\ \frac{\phi_C}{T_C} = 4.268,\ \alpha = 0.278,\ \beta/H = 1942$$

$$V(0, T_C) = V(\phi_C, T_C) \quad \frac{\phi_C}{T_C} > 1$$

Gravitational waves

$$h^2\Omega_{sw}(f) = 8.5 \times 10^{-6}\left(\frac{100}{g_\star}\right)^{1/3}\Gamma^2\bar{U}_f^4\left(\frac{H_\star}{\beta}\right)v_w S_{sw}(f)^\dagger$$

$$\Gamma = 4/3, \quad \bar{U}_f^2 = \frac{3}{4}\kappa_f\alpha_{T_\star}, \quad S_{sw}(f) = \left(\frac{f}{f_{sw}}\right)^3\left(\frac{7}{4+3(f/f_{sw})^2}\right)^{7/2}$$

$$f_{sw} = 8.9\mu Hz\frac{1}{v_w}\left(\frac{\beta}{H_\star}\right)\left(\frac{z_p}{10}\right)\left(\frac{T_\star}{100\text{GeV}}\right)\left(\frac{g_\star}{100}\right)^{1/6}$$

$$h^2\Omega_{turb}(f) = 3.35 \times 10^{-4}\left(\frac{H_\star}{\beta}\right)\left(\frac{\kappa_{turb}(\alpha_{T_\star})\alpha_{T_\star}}{1+\alpha_{T_\star}}\right)^{3/2}\left(\frac{100}{g_\star}\right)^{1/3}v_w S_{turb}(f)$$

$$S_{turb}(f) = \frac{(f/f_{turb})^3}{[1+(f/f_{turb})]^{11/3}(1+8\pi f/h_\star)} \quad h_\star = 16.5\mu Hz\left(\frac{T_\star}{100\text{GeV}}\right)\left(\frac{g_\star}{100}\right)^{1/6}$$

$$f_{turb} = 27\mu Hz\frac{1}{v_w}\left(\frac{\beta}{H_\star}\right)\left(\frac{T_\star}{100\text{GeV}}\right)\left(\frac{g_\star}{100}\right)^{1/6}$$

$$h^2\Omega(f) = h^2\Omega_{sw}(f) + h^2\Omega_{turb}(f)$$

† 10.1098/rsta.2017.0126

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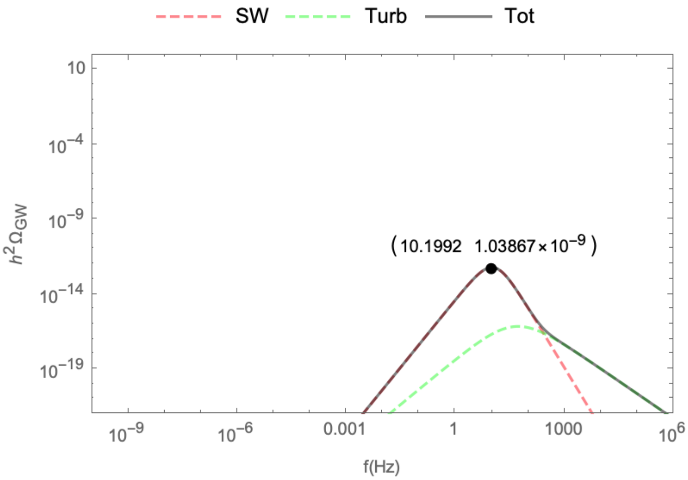
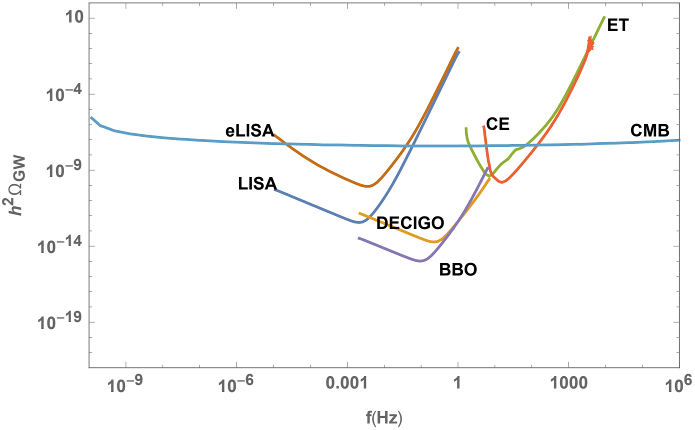


$\alpha = 0.1,\ \beta/H = 1500,\ T_C = 10^5$ GeV, $v_w \approx 1$

Gravitational waves



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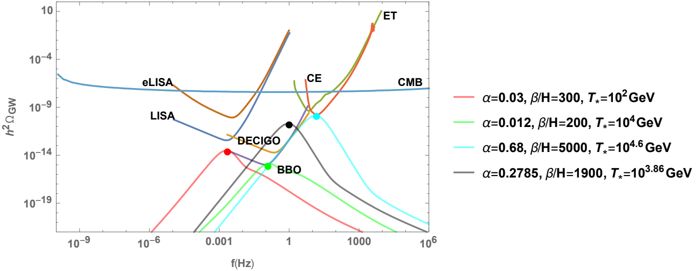
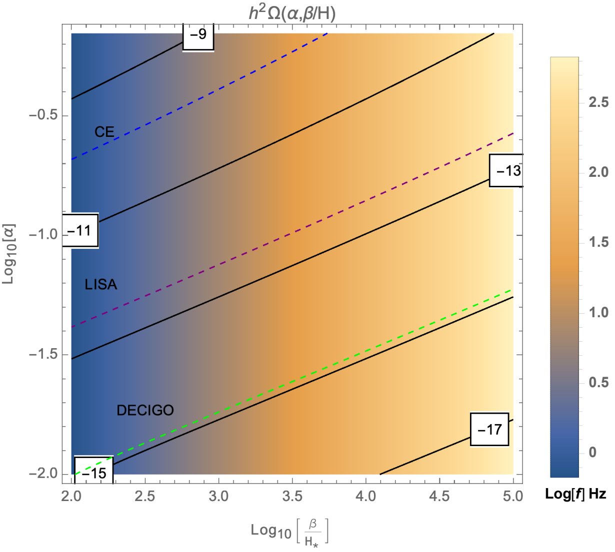
Gravitational waves

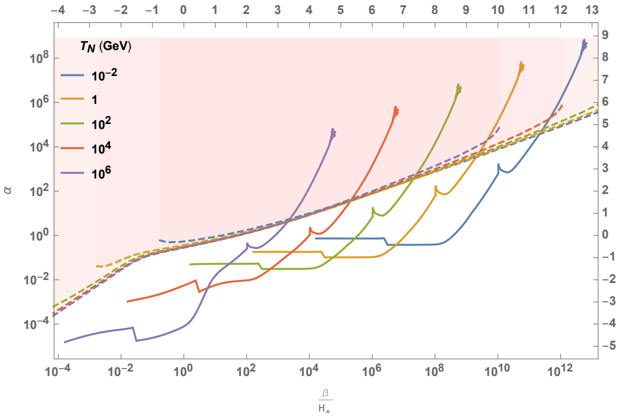


$\mu_\phi(10^4$ GeV)	$\mu_H(10^4$ GeV)	λ_ϕ	λ_H	$\lambda_{h\phi}$	g_D	N	$g_\star$
0.212168	1.58143	0.0009	3.04	0.05	0.5	5	184.25
$\frac{\phi_C}{T_C}$	$T_C(10^4$ GeV)	$T_N(10^4$ GeV)	α	$\frac{\beta}{H_\star}$			
4.268	0.8393	0.717706	0.278	1942			

Gravitational waves



Gravitational waves



Connection between SSB and confinement

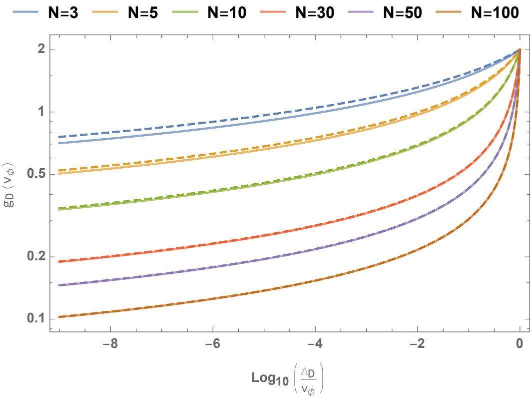
$$\alpha_d^{-1}(m_2) = \alpha_d^{-1}(m_1) + \frac{-b_d}{2\pi}\ln\left(\frac{m_2}{m_1}\right)$$

$$\beta_d(g_d) = \frac{-g_d^3}{(4\pi)^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}\kappa_f S_2(R_F) - \frac{1}{3}\kappa_s S_2(R_S) + \frac{2\kappa_f}{(4\pi)^2}Y_4(R_F)\right]$$

$$b_d = -\frac{11}{3}(N-1) + \frac{1}{3}N_f + \frac{1}{6}N_s$$

§10.1103/PhysRevD.67.065019

Connection between SSB and confinement



Correlation between $g_d(v_\phi)$ and $\frac{\Lambda_D}{v_\phi}$ for different values of N where for solid(dashed) curves $N_f = 2(5)$. If $\Lambda_D > 0.2$ MeV (BBN) and $v_\phi \sim O(10^{4-6}\text{GeV})$ than $\frac{\Lambda_D}{v_\phi}$ can be as low as $O(10^{-8} - 10^{-10})$.

Confinement of $SU(N-1)$

Gravitational waves from confinement:

- $N_C = 2$ and $N_F \geq 2$ ¶
- $N_C \geq 3$ and $N_F \geq 3$ ‖
- $N_F = 0$ **
- $\mu_{B,D} \gtrsim \Lambda_{\rm conf}$ ††

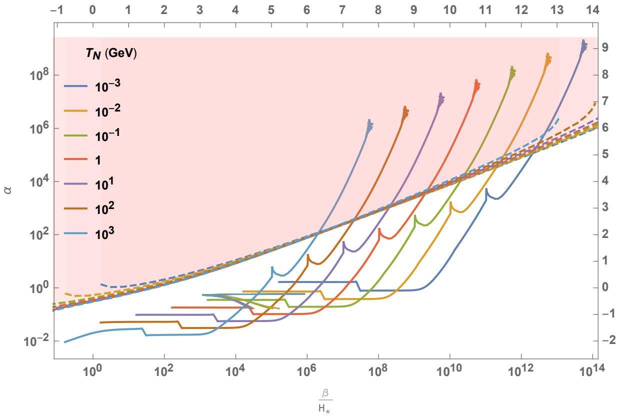
¶ 10.1103/PhysRevD.62.045012

‖ 10.1103/PhysRevD.29.338

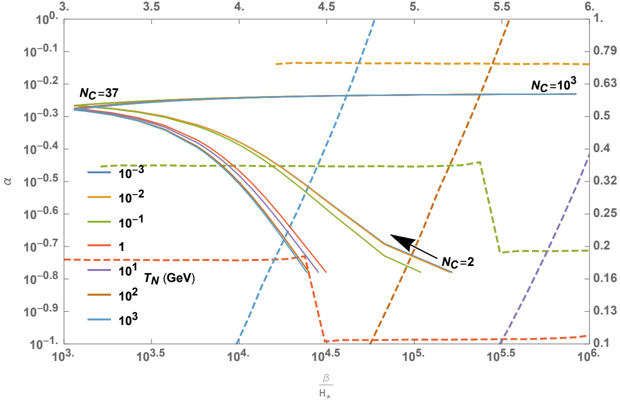
** 10.1088/1126-6708/2005/02/033;10.1103/PhysRevD.82.114505;10.1103/PhysRevD.92.055034

†† 10.1142/S0217751X05027965

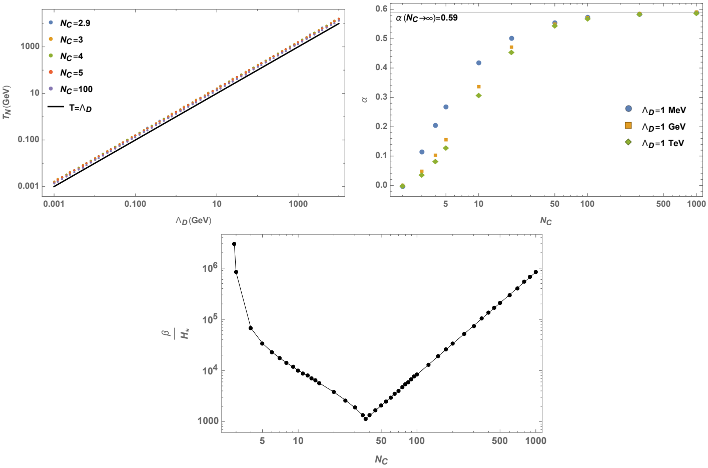
N =odd case (dark glueball condensate)



N =odd case (dark glueball condensate)



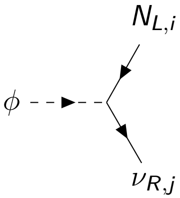
N =odd case (dark glueball condensate)



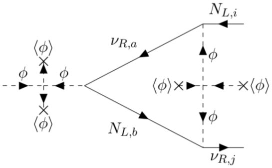
Neutrino mass models and gravitational waves

- $h^2\Omega \propto \frac{v_\phi}{m_\phi}$
- $h^2\Omega \propto g_D$
- $h^2\Omega \propto n_{f_i}$ where $m_{f_i} \propto v_\phi$
- $h^2\Omega \propto N$
- $h^2\Omega \propto g_\star^{-1}$
- Some models: Pati-Salam, LR, Dark Sector, GUT

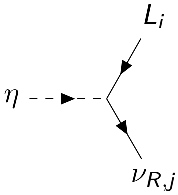
Neutrinogenesis



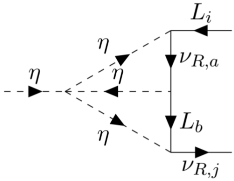
$\sim Y_R^{ij}$



$\sim \frac{1}{16\pi^2} Y_R^{ia} Y_R^{*ab} Y_R^{bj} \lambda_\phi^2 \langle\phi\rangle^4$



$\sim Y_\nu^{ij}$



$\sim \frac{1}{(16\pi^2)^2} Y_\nu^{ia} Y_\nu^{*ab} Y_\nu^{bj} \lambda_\eta^{\rm eff}$

Neutrinogenesis

$$n_B = n_L = O(1) n_{\nu_R} = O(1)\varepsilon_A n_{\phi,\eta}{}^{\ddagger\ddagger} \rightarrow \varepsilon_A \sim O(10^{-8})$$

$$\varepsilon_A = \frac{\Gamma(\phi \rightarrow N_L^\dagger \nu_R) - \Gamma(\phi^\dagger \rightarrow N_L \nu_R^\dagger)}{\Gamma(\phi \rightarrow N_L^\dagger \nu_R) + \Gamma(\phi^\dagger \rightarrow N_L \nu_R^\dagger)}$$

$$= \frac{Im[Z] Im[\mathcal{F}]}{16\pi^2}\left[\frac{\left|X^{ij}\right|^2 + \left|X^{i\alpha}X^{\star\alpha\beta}X^{\beta j}\right|^2 |\mathcal{F}|^2 (16\pi^2)^{-2}}{2} + \frac{Re[Z]Re[\mathcal{F}]}{16\pi^2}\right]^{-1}$$

$$Z = X^{ij}\left(X^{i\alpha}X^{\star\alpha\beta}X^{\beta j}\right)^\star \qquad X^{ij} = R^i_{N\alpha}\frac{Y_R^{\alpha\beta}}{\sqrt{2}}U^j_{R\beta}$$

$$\mathcal{F} = F(x_i, x_R, x_I, x_N) = (1-x_i)^{-1}\left[(1-x_I)\log\left(\frac{x_I}{x_I - 1}\right) - (1-x_R)\log\left(\frac{x_R}{x_R-1}\right) - \log\left(\frac{x_I}{x_R}\right)\right.$$
$$\left. + (x_N - x_i x_I) C_0(0, m^2_{N_f}, m^2_{s_i}; m_N, m_{S_I}, 0) - (x_N - x_i x_R) C_0(0, m^2_{N_f}, m^2_{s_i}; m_N, m_{S_R}, 0)\right]$$

$$x_i = \left(\frac{m_{s_i}}{m_{N_f}}\right)^2 \qquad x_{R,I} = \left(\frac{m_{s_{R,I}}}{m_{N_f}}\right)^2 \quad x_N = \left(\frac{m_N}{m_{N_f}}\right)^2 \qquad m_{s_I} > m_{s_R} > m_{N_f}, m_N$$

$R \neq \mathbb{1}$

‡‡10.1103/PhysRevLett.84.4039

Parameters and Constraints

Parameter	Constrained by
N, N_F	GW, DM
g_D	DM, GW, DS
λ_H, v	SM
v_ϕ	GW, m_ν, Direct searches
μ_H, μ_ϕ	V minimization
$\lambda_\phi, \lambda_{H\phi}$	GW, m_ϕ
$Y_{R,L,\nu}$	m_ν, Dirac leptogenesis, PMNS
M_{IM}	m_ν, Dirac leptogenesis, DM

Future tasks

- Correlation between GW signals and DM dynamics
- GUT completions (3 possible GW signals)
- Radiative neutrino models

Conclusion

- Minimal Dirac neutrino models
- Correlated GW signals
- Confined Dark Sector
- Baryon asymmetry via neutrinogenesis
- Probing neutrino models via GW

Thanks for your attention!