

1st AEI for BSM & 9th KIAS PP&C — Nov 7 2019

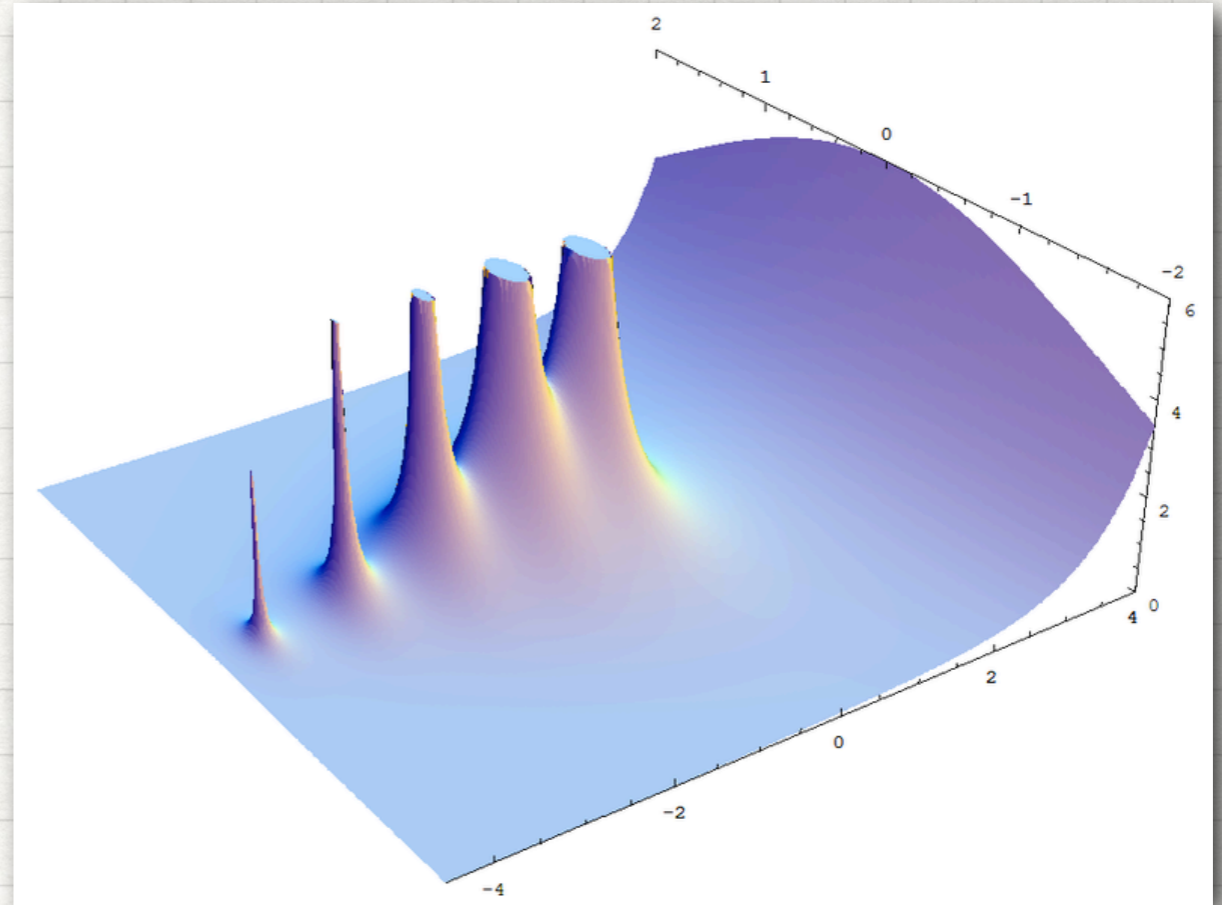
Rodrigo Alonso — Kavli IPMU

**AMPLITUDES,
RESONANCES & THE UV
COMPLETION OF GRAVITY**

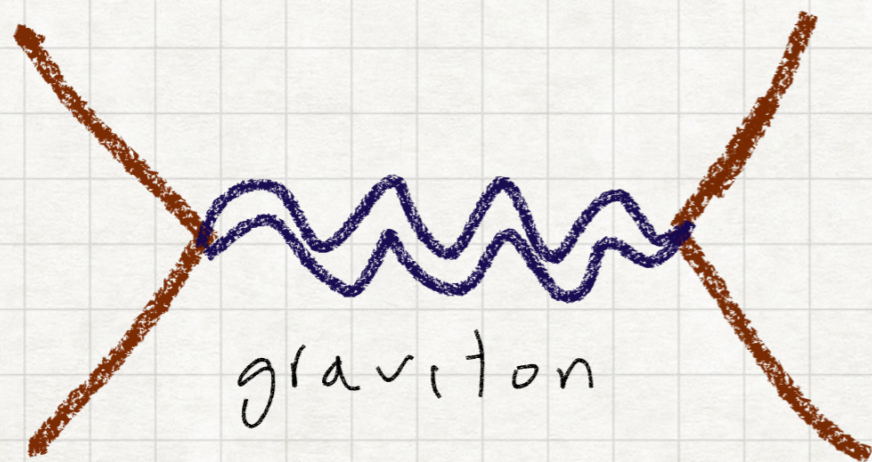
[R.A. & A. Urbano 1906.11687]

OUTLINE

1. ON SHELL AMPLITUDES
2. UNITARITY
3. RESONANCES FOR GRAVITY
4. ANALYSIS

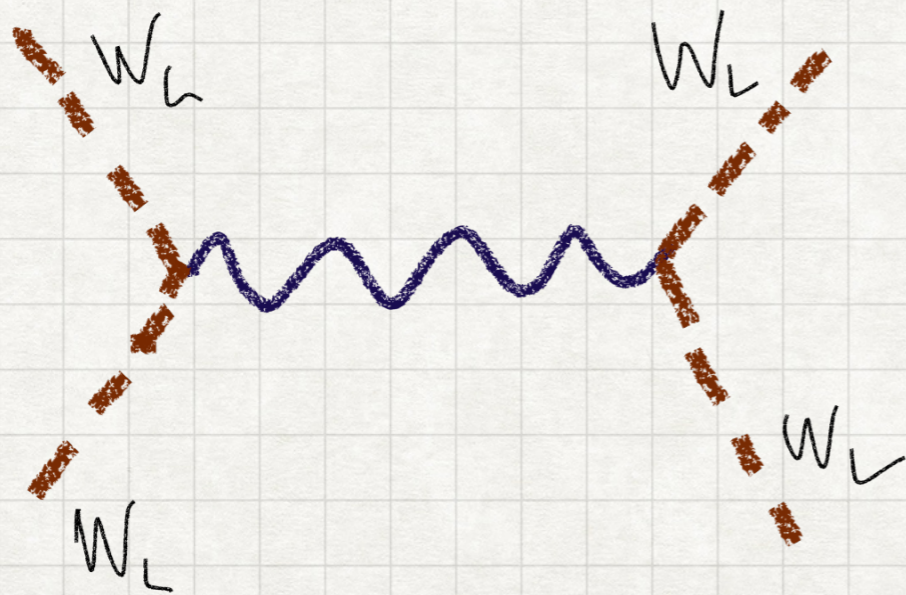


This talk in a nutshell



$$\mathcal{A} \sim \frac{S}{M_{\text{pl}}^2}$$

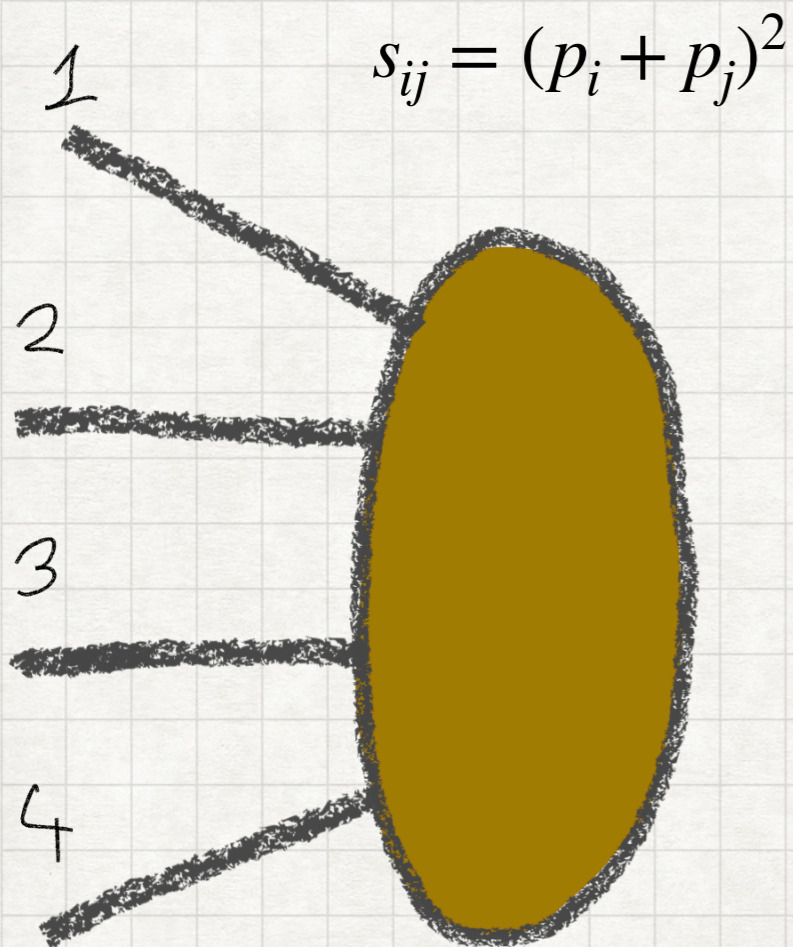
Let's add
Resonances
to fix this



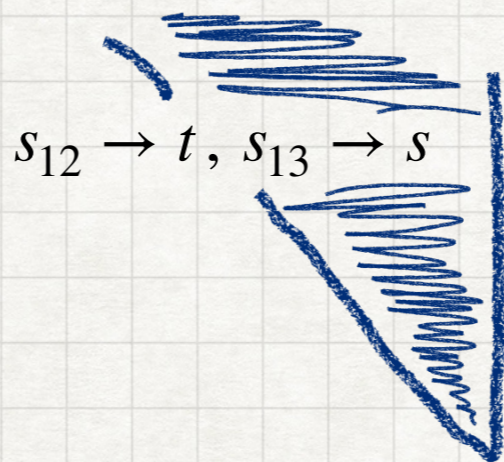
$$\mathcal{A} \sim \frac{S}{v_{\text{ew}}^2}$$

The Higgs

Setting the stage



$\mathcal{A}(s_{12}, s_{13})$



$p_{2,4} \rightarrow -p_{2,4}$

$p_{3,4} \rightarrow -p_{3,4}$

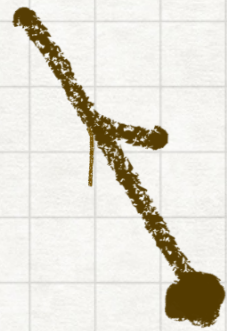
$p_{2,3} \rightarrow -p_{2,3}$

Helicity

Flip

$s_{12} + s_{13} + s_{14} = \sum m_i^2$

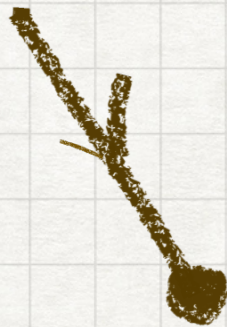
Helicity scaling or Little group Reps.



$\alpha |p\rangle$

$$SO(4); |p\rangle \rightarrow i\Lambda^{\mu\nu} \sigma_{\mu\nu} |p\rangle$$

$$U(1)_{LG}; |p\rangle \rightarrow |p\rangle e^{-i\phi/2}$$



$\dot{\alpha} |p]$

$$SO(4); |p] \rightarrow i\Lambda^{\mu\nu} \bar{\sigma}_{\mu\nu} |p]$$

$$U(1)_{LG}; |p] \rightarrow |p] e^{i\phi/2}$$

(Just Weyl spinors)



$$\epsilon_+^\mu = \frac{\langle \xi | \sigma^\mu | p]}{\sqrt{2} \langle \xi p \rangle}$$

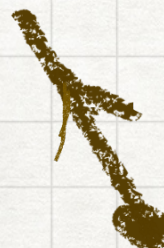
$$U(1)_{LG}; \frac{\langle \xi | \sigma^\mu | p] e^{i\phi/2}}{\sqrt{2} \langle \xi p \rangle e^{-i\phi/2}}$$

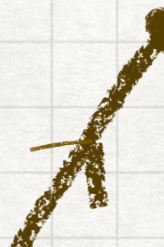
In this talk all external lines \rightarrow massless

Learn more

[Dixon 9601359, Elvang & Huang 1308.1697]

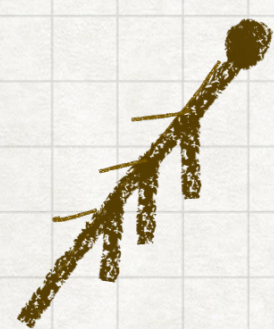
Helicity scaling or Little group Reps. Massive


$${}_{\alpha} |p_I\rangle \quad SO(4); \quad |p_I\rangle \rightarrow i\Lambda^{\mu\nu} \sigma_{\mu\nu} |p_I\rangle \quad SU(2)_{LG}; \quad |p\rangle \rightarrow |p_J\rangle i[T_a]_{IJ} \phi_a$$


$${}^{\dot{\alpha}} |p_I] \quad SO(4); \quad |p_I] \rightarrow i\Lambda^{\mu\nu} \bar{\sigma}_{\mu\nu} |p_I] \quad SU(2)_{LG}; \quad |p] \rightarrow |p_J] i[T_a]_{IJ} \phi_a$$

$$u(p) = \begin{pmatrix} |p_I\rangle \\ |p_I] \end{pmatrix}$$

Just half of
Dirac
Spinors



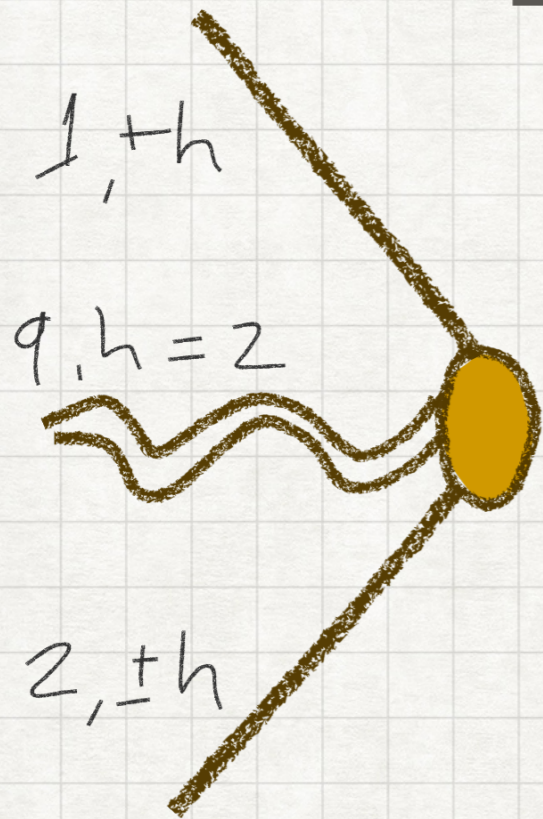
Spin J
Massive
State

$$\otimes_{\text{sym}}^{2J} |p] = |p_{I_1}] \times |p_{I_2}] \times \dots \times |p_{I_{2J}}] \quad (2J + 1 \text{ elements})$$

Learn more

[Arkani-Hamed & Huang(x2), 1709.04891]

3 point amplitude for gravity



$$\mathcal{A}_3^{\text{GR}} = C |1|^{2h} \times |2|^{\pm 2h} \times |q|^4 = C [12]^a [1q]^b [2q]^c$$

h, h

$h, -h$

~~$[12]^{2h-2} [1q]^2 [2q]^2,$~~

$[12]^{-2} [1q]^{2+2h} [2q]^{2-2h}.$

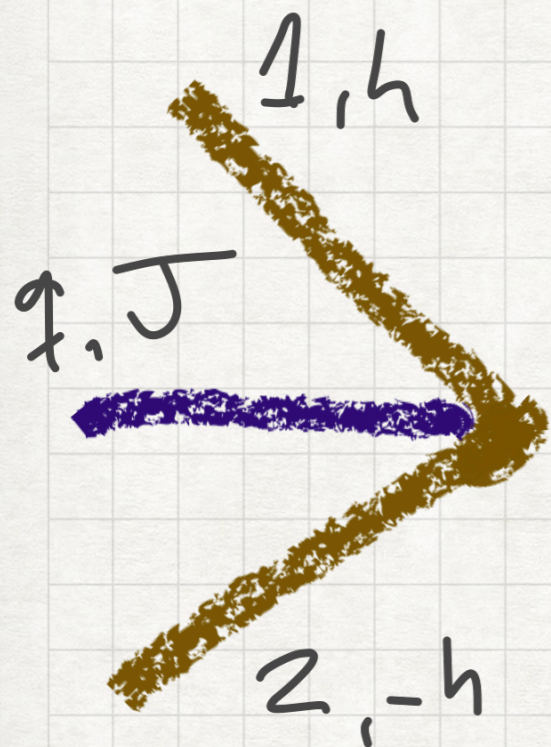
Mass dimension of C

$$\dim(\mathcal{A}_n) = 4 - n \quad \Rightarrow \quad [C] = \begin{pmatrix} 1 - 2h - 2 \\ 1 - 2 \end{pmatrix} \quad C = \frac{\sqrt{8\pi}}{M_{\text{pl}}}$$

Only opposite helicity case in GR

$$\frac{\sqrt{8\pi}}{M_{\text{pl}}} \frac{[1q]^{2+2h} [2q]^{2-2h}}{[12]^2}$$

3 point amplitude massive spin J

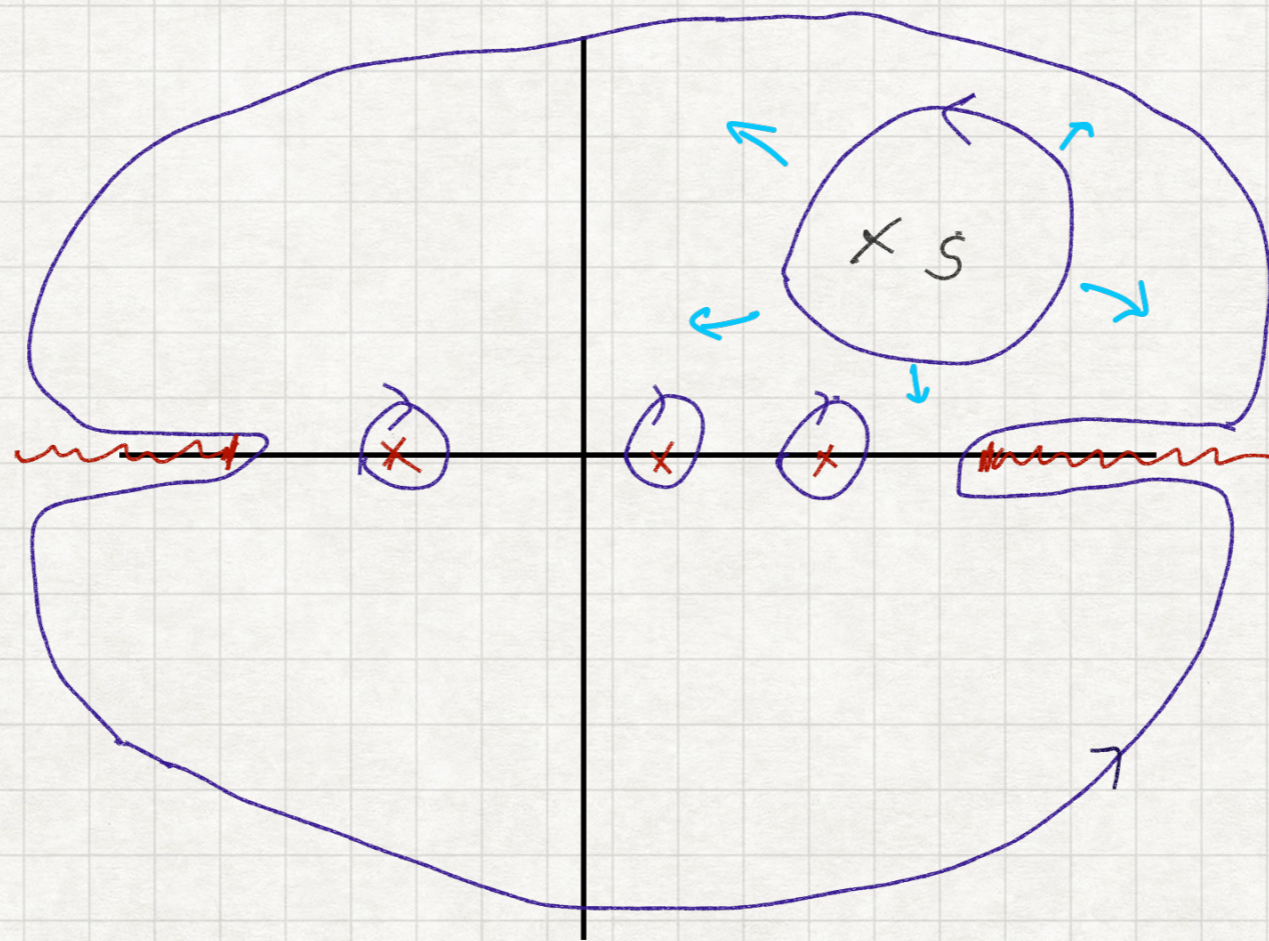


$$gM \frac{(\langle q_I \hat{P}_{12} q_I \rangle)^{J-2h} ([1q_I] \langle 2q_I \rangle)^{2h}}{M^{2J}}$$

$$\hat{P} = \sigma_\mu P^\mu \quad P_{ij} = p_i - p_j$$

On shell methods @ tree level

$$2\pi i \mathcal{A}(s, t) = \oint \frac{\mathcal{A}(s', t)}{s' - s} ds'$$

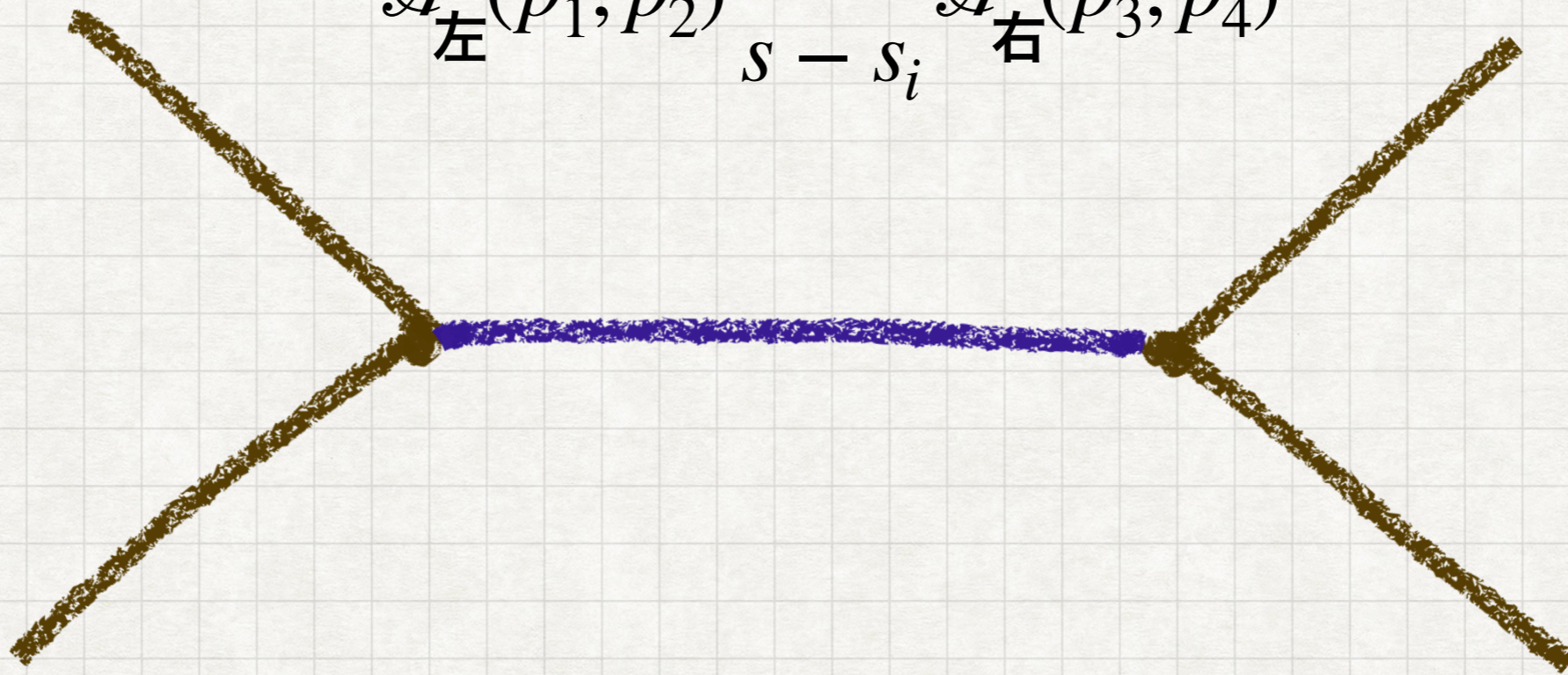


$$2\pi i \mathcal{A}(s, t) = - \oint_{\text{Poles}} \frac{\mathcal{A}(s', t)}{s' - s} ds' + \oint_{\infty} \frac{\mathcal{A}(s', t)}{s' - s} ds'$$

On shell part of the amplitude

$$2\pi i \mathcal{A}(s, t) = - \oint_{\text{Poles}} \frac{\mathcal{A}(s', t)}{s' - s} ds' + B = - \frac{\text{Res}(\mathcal{A})_{s=s_i}}{s_i - s} + B$$

$$\mathcal{A}_{\text{左}}(p_1, p_2) \frac{1}{s - s_i} \mathcal{A}_{\text{右}}(p_3, p_4)$$



4 pt amplitude from 3 pt in Gravity



$$\frac{\sqrt{8\pi}}{M_{\text{pl}}} \frac{[1q]^{2+2h}[2q]^{2-2h}}{[12]^2}$$

$$\frac{1}{q^2}$$

$$\frac{\sqrt{8\pi}}{M_{\text{pl}}} \frac{\langle 3q \rangle^{2+2h'} \langle 4q \rangle^{2-2h'}}{\langle 34 \rangle^2}$$

Using on-shell relations like

$$|q\rangle[q] = \bar{\sigma}_\mu q^\mu = -\bar{\sigma}_\mu p_1^\mu - \bar{\sigma}_\mu p_2^\mu$$

$$\hat{P} = \sigma_\mu P^\mu$$

We obtain:

$$= \frac{8\pi}{M_{\text{pl}}^2} c_r \left(\frac{s_{14}}{s_{13}} \right)^r \frac{(s_{13}s_{14})^{1-h'}}{s_{12}} ([14]\langle 23 \rangle)^{2h} (\langle 3\hat{P}_{12}4 \rangle)^{2h'-2h}$$

$$h' - 1 < r < 1 - h'$$

$$h - 1 < r < 1 - h$$

Ambiguity in fermions and scalars

-> Scalar

$$\frac{8\pi}{M_{\text{pl}}^2} \left(\frac{s_{13}s_{14}}{s_{12}} - as_{12} \right)$$



Contact terms

-> Fermion

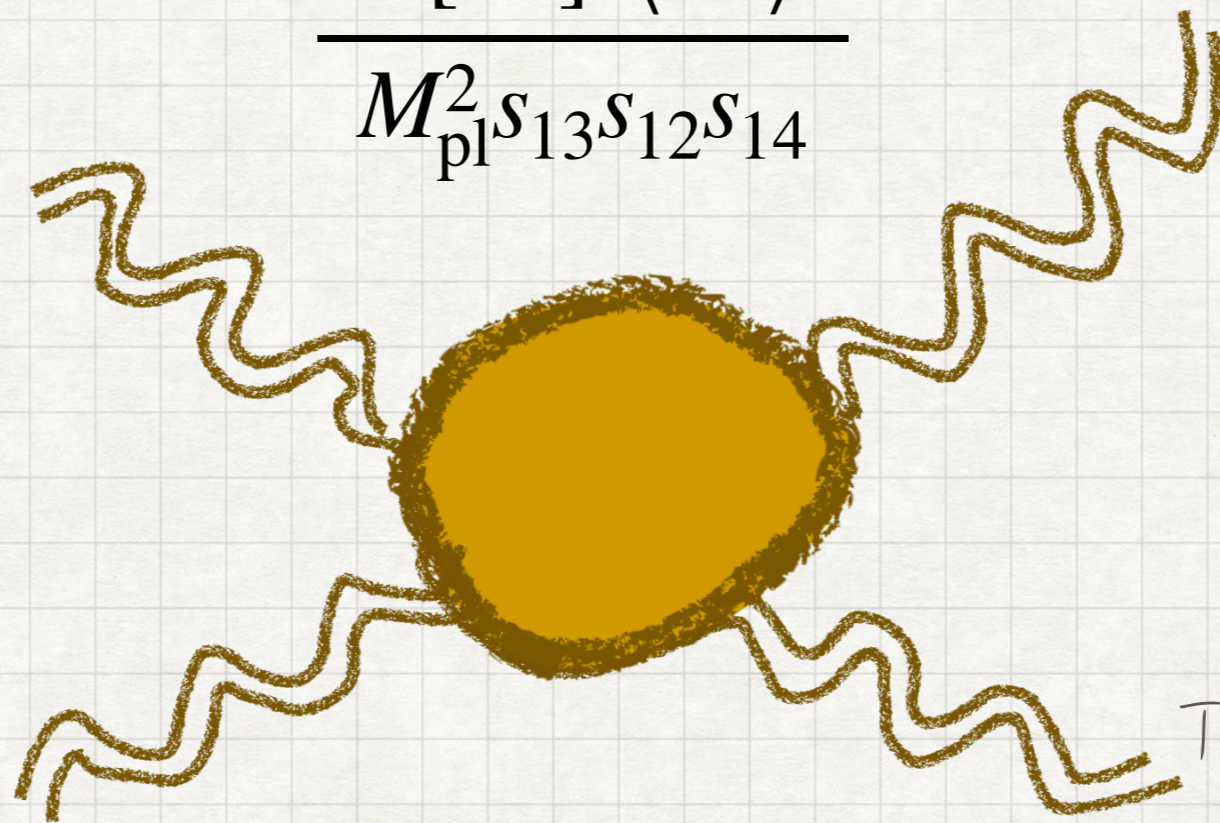
$$\frac{8\pi[14]\langle 23 \rangle}{M_{\text{pl}}^2} \left(\frac{s_{13}}{s_{12}} + \frac{b}{2} \right)$$

We have computed more than we know!

$$\mathcal{A} = \frac{8\pi}{M_{\text{pl}}^2} \frac{(s_{13}s_{14})^{1-h'}}{s_{12}} ([14]\langle 23 \rangle)^{2h} (\langle 3\hat{P}_{12}4 \rangle)^{2h'-2h}$$

$$\rightarrow \underline{h = h' = 2}$$

$$\frac{8\pi [14]^4 \langle 23 \rangle^4}{M_{\text{pl}}^2 s_{13} s_{12} s_{14}}$$



The Feynman rule
computation has ~ 1000 terms!

[B. DeWitt, Phys. Rev. 162 1239, 1967]

These are all of them

$\mathcal{A}_{1h_2-h_3-h'_4h'}$ ^{GR}	Scalar	Fermion	Vector	Graviton
Scalar	$\frac{8\pi}{M_{Pl}^2} \left(\frac{s_{13}s_{14}}{s_{12}} - as_{12} \right)$	$\frac{8\pi \langle 3 \hat{P}_{12} 4 \rangle}{M_{Pl}^2} \left(\frac{s_{13}-s_{14}}{2s_{12}} \right)$	$-\frac{8\pi \langle 3 \hat{P}_{12} 4 \rangle^2}{M_{Pl}^2 s_{12}}$	$\frac{8\pi \langle 3 \hat{P}_{12} 4 \rangle^4}{M_{Pl}^2 s_{12}s_{13}s_{14}}$
	$\frac{8\pi}{M_{Pl}^2} \left(\frac{s_{13}s_{14}}{s_{12}} + \frac{s_{12}s_{14}}{s_{13}} + \frac{s_{13}s_{12}}{s_{14}} \right)$			
Fermion		$-\frac{8\pi \langle 23 \rangle [14]}{M_{Pl}^2} \left(\frac{s_{13}}{s_{12}} + \frac{b}{2} \right)$	$\frac{8\pi \langle 23 \rangle [14] \langle 3 \hat{P}_{12} 4 \rangle}{M_{Pl}^2 s_{12}}$	$-\frac{8\pi \langle 23 \rangle [14] \langle 3 \hat{P}_{12} 4 \rangle^3}{M_{Pl}^2 s_{12}s_{13}s_{14}}$
		$-\frac{8\pi \langle 23 \rangle [14]}{M_{Pl}^2} \left(\frac{s_{13}}{s_{12}} + \frac{s_{12}}{s_{13}} + b \right)$		
Vector			$-\frac{8\pi \langle 23 \rangle^2 [14]^2}{M_{Pl}^2 s_{12}}$	$\frac{8\pi \langle 23 \rangle^2 [14]^2 \langle 3 \hat{P}_{12} 4 \rangle^2}{M_{Pl}^2 s_{12}s_{13}s_{14}}$
			$-\frac{8\pi \langle 23 \rangle^2 [14]^2}{M_{Pl}^2} \left(\frac{1}{s_{12}} + \frac{1}{s_{13}} \right)$	
Graviton				$\frac{8\pi \langle 23 \rangle^4 [14]^4}{M_{Pl}^2 s_{12}s_{13}s_{14}}$

Maximal Helicity (non) Violation

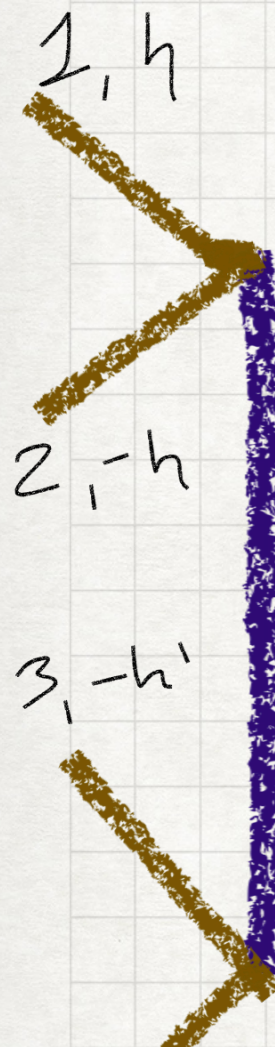
$$\sum h_i = 0$$

If we exchange a massive spin J ? Legendre



$$\begin{aligned}
 & \frac{gM \left(\frac{\langle q_I \hat{P}_{12} q_I \rangle}{M^2} \right)^J}{q^2 - M^2} = \frac{g^2 (2J)!!}{(2J-1)!!} \frac{M^2}{s_{12} - M^2} \\
 & \frac{gM \left(\frac{\langle q_I \hat{P}_{34} q_I \rangle}{M^2} \right)^J}{q^2 - M^2} = \frac{(2J-2m)! (-P_{12}^2 P_{14}^2)^m (P_{12} \cdot P_{34})^{J-2m}}{m!(J-m)!(J-2m)2^J M^{2J}} \\
 & = \frac{P_J(x)}{4^J}, \quad x = 1 + \frac{2s_{13}}{M^2}.
 \end{aligned}$$

If we exchange a massive spin J ? Jacobi



$$\begin{aligned}
 & gM \frac{(\langle q_I \hat{P}_{12} q_I \rangle)^{J-2h} ([1q] \langle 2q \rangle)^{2h}}{M^{2J}} \\
 & \frac{1}{q^2 - M^2} \\
 & gM \frac{(\langle q_I \hat{P}_{12} q_I \rangle)^{J-2h} ([4q] \langle 3q \rangle)^{2h}}{M^{2J}}
 \end{aligned}
 =
 \begin{aligned}
 & \frac{g^2 (2J)!! (J-2h')! (J+2h')!}{(2J-1)!! J!^2} \\
 & \left(\frac{[14] \langle 32 \rangle}{M^2} \right)^{2h} \left(\frac{\langle 3 \hat{P}_{12} 4 \rangle}{M^2} \right)^{2h-2h} \\
 & P_{J-2h'}^{(2h'-2h, 2h'+2h)}(x) \frac{M^2}{s_{12} - M^2}
 \end{aligned}$$

$$P_n^{(a,b)}(x) \equiv \sum \binom{n+a}{n-k} \binom{n+b}{k} \left(\frac{x-1}{2} \right)^k \left(\frac{x+1}{2} \right)^{n-k} \quad x = 1 + \frac{2s_{13}}{M^2}$$

Angular Analysis and unitarity? Wigner



$$s_{12} \rightarrow s, \quad s_{13} \rightarrow -s \sin^2(\theta/2), \quad \langle 13 \rangle \rightarrow \sqrt{s} \sin(\theta/2), \quad \dots$$

$$\left(\frac{[14]\langle 32 \rangle}{M^2} \right)^{2h} \left(\frac{\langle 3\hat{P}_{12}4 \rangle}{M^2} \right)^{2h-2h}$$

$$P_{J-2h'}^{(2h'-2h, 2h'+2h)}(x)$$

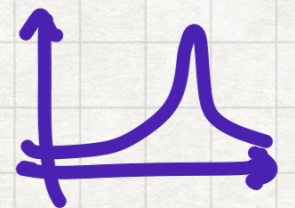
$$\left(\frac{s \cos^2(\theta/2)}{M^2} \right)^{2h} \left(\frac{s \sin(\theta/2) \cos(\theta/2)}{M^2} \right)^{2h-2h}$$

$$P_{J-2h'}^{(2h'-2h, 2h'+2h)}(1 - (1 - \cos \theta)s/M^2)$$

On-shell

$$s = M^2$$

$$P_{J-2h'}^{(2h'-2h, 2h'+2h)}(\cos \theta)$$

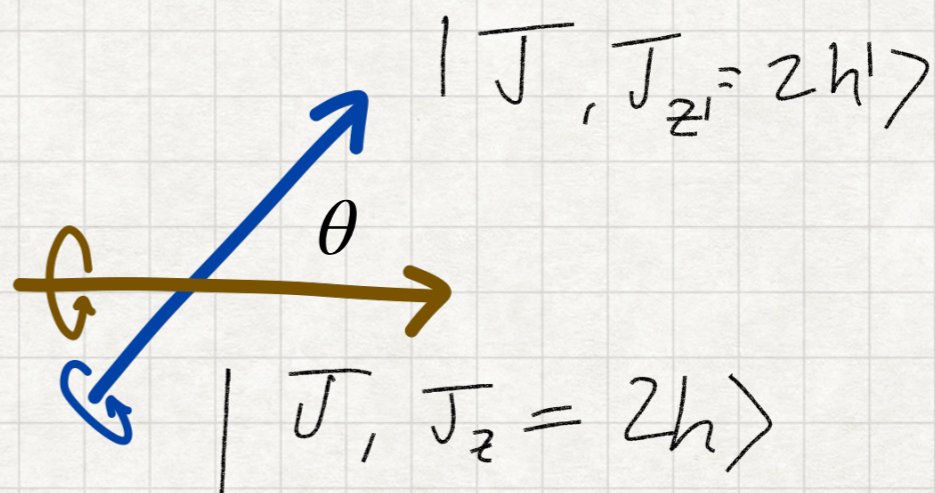
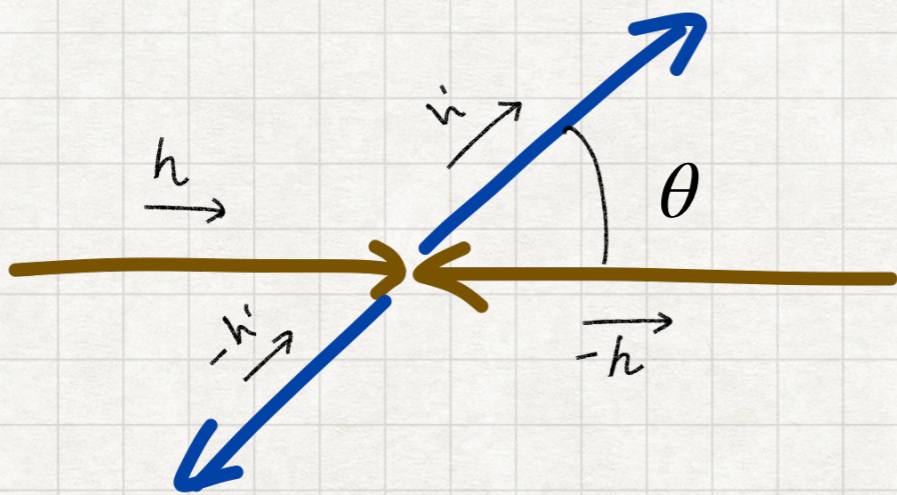


Wigner d-function and angular analysis

$$\sin^{2h'-2h}(\theta/2) \cos^{2h+2h'}(\theta/2) P_{J-2h'}^{(2h'-2h, 2h'+2h)}(\cos \theta)$$

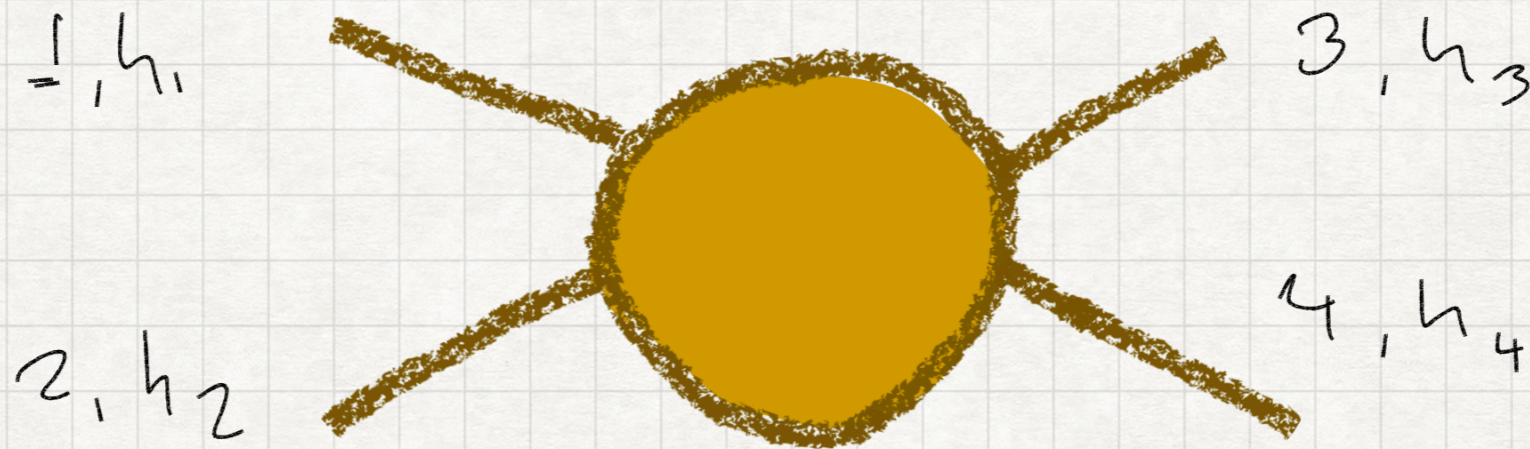
$$\propto d_{2h, 2h'}^J(\theta)$$

$$d_{m', m}^J = \langle J, m' | R(\theta) | J, m \rangle$$



Wigner d-function and angular analysis

$$\mathcal{A} = \sum 16\pi(2J+1)a^J(s)d_{h_1-h_2, h_3-h_4}^J(\theta)$$



Partial wave expansion

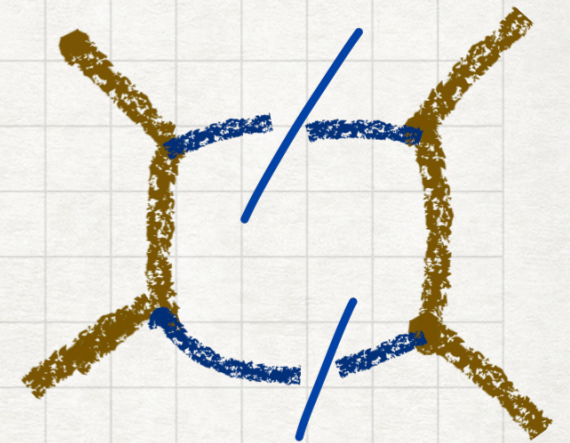
a 's bounded to be less than one to conserve probability

$$a^J = \frac{1}{32\pi} \int d\cos(\theta) d_{h_1-h_2, h_3-h_4}^J(\theta) \mathcal{A}(s, \theta)$$

⇒ Unitarity ⇐

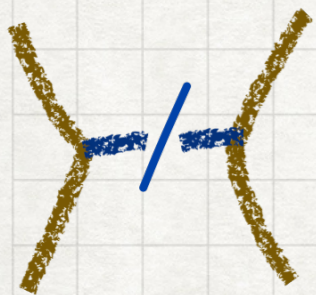
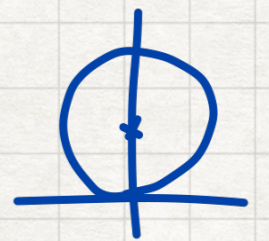
$$(1 + i\mathcal{A})(1 + i\mathcal{A})^\dagger = 1$$

$$2 \operatorname{Im}(\mathcal{A}_{12 \rightarrow 34}) = \int \prod_i \frac{dp_i^3}{(2E_i)(2\pi)^3} \mathcal{A}_{34 \rightarrow n}^* \mathcal{A}_{12 \rightarrow n}$$



If $12=34$ and we do angular dec. $\operatorname{Im}(a^J) \leq 1, \operatorname{Re}(a^J) \leq 1/2.$

For a single particle intermediate state n



$$2 \operatorname{Im}(\mathcal{A}_{12 \rightarrow 34}) = \pi \mathcal{A}_{34 \rightarrow J}^*(\theta) \mathcal{A}_{12 \rightarrow J}(\theta) \delta(s - M^2)$$

$$\frac{1}{s - M^2 + i\epsilon} = \mathcal{PV} \left[\frac{1}{s - M^2} \right] - i\pi \delta(s - M^2)$$

Unitarity

$$(1 + i\mathcal{A})(1 + i\mathcal{A})^\dagger = 1$$

$$2 \operatorname{Im}(\mathcal{A}_{12 \rightarrow 34}) = \pi \mathcal{A}_{34 \rightarrow J}^*(\theta) \mathcal{A}_{12 \rightarrow J}(\theta) \delta(s - M^2)$$

If $12=34$

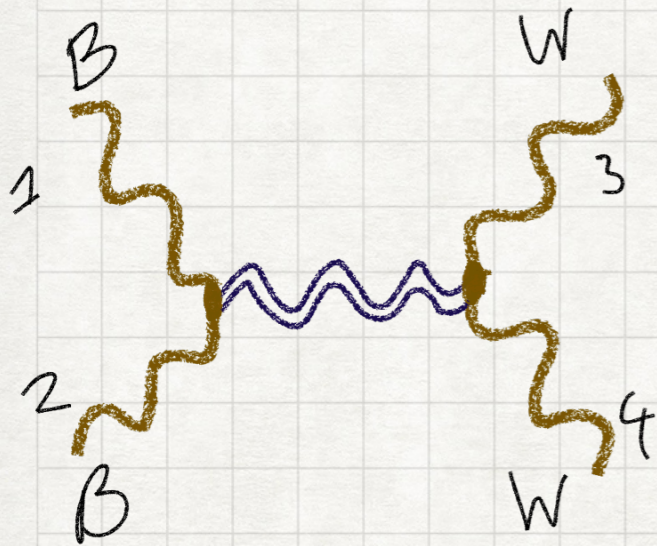
$$2 \operatorname{Im}(\mathcal{A}_{12 \rightarrow 12}) = \pi |\mathcal{A}_{12 \rightarrow J}(\theta)|^2 \delta(s - M^2)$$

Positivity

If in the forward direction

$$2 \operatorname{Im}(\mathcal{A}_{12 \rightarrow 34})(\theta = 0) = 16\pi(2J + 1)M\Gamma_{J \rightarrow 12} \delta(s - M^2)$$

Gravity in the ultraviolet

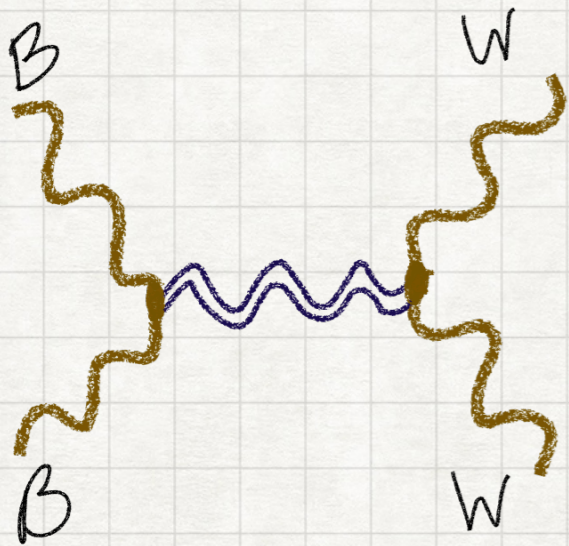


$$\mathcal{A}_{B,W} = - \frac{8\pi \langle 32 \rangle^2 [41]^2}{M_{\text{pl}}^2 s_{12}}$$

Second partial
wave coeff.

$$a_{\text{GR}}^2 = \frac{s}{10M_{\text{pl}}^2}$$

Gravity in the ultraviolet

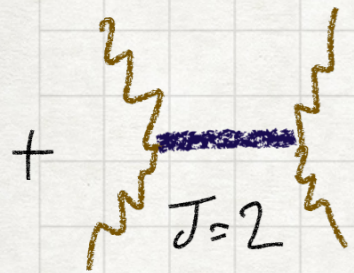


$$\mathcal{A}_{B,W} = - \frac{8\pi \langle 32 \rangle^2 [41]^2}{M_{\text{pl}}^2 s_{12}}$$

Second partial
wave coeff.

$$a_{\text{GR}}^2 = \frac{s}{10M_{\text{pl}}^2}$$

constant

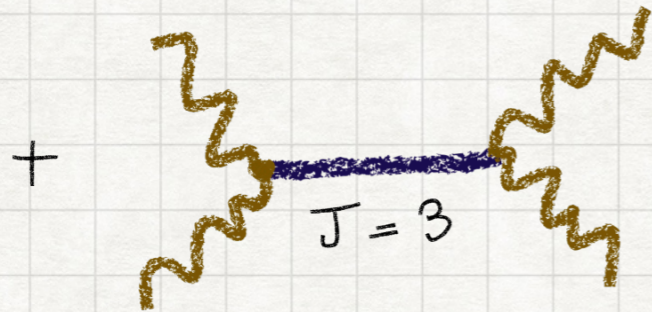


$$\mathcal{A}_{B,W}^{\text{GR}} + \mathcal{A}_{B,W}^{J=2} = \langle 23 \rangle^2 [14]^2 \left(\frac{8\pi}{M_{\text{pl}}^2 s} + \frac{g_B g_W P_0^{(0,4)}(x)}{M_2^2 (s - M_2^2)} \right)$$

$$g_B g_W = -8\pi \frac{M_2^2}{M_{\text{pl}}^2}$$

But opposite sign does not extend
to a third species, e.g. gluon

Gravity in the ultraviolet



$$\mathcal{A}_{B,W}^{\text{GR}} + \mathcal{A}_{B,W}^{J=3} = \langle 23 \rangle^2 [14]^2 \left(\frac{8\pi}{sM_{\text{pl}}^2} + \frac{g_B g_W P_1^{(0,4)}(x)}{M_3^2 (s - M_3^2)} \right)$$

$1 + 6t/M_3^2$
↓

$$a_{\text{GR}}^2 = \frac{s}{10M_{\text{pl}}^2} - \frac{g_B g_W s^2}{80M_3^4}$$


Gravity in the ultraviolet

$$\begin{aligned} \mathcal{A}_{B,W}^{\text{GR}} + \sum_J \mathcal{A}_{B,W}^J &= \langle 23 \rangle^2 [14]^2 \left(\frac{8\pi}{M_{\text{pl}}^2} + \sum_J \frac{P_J^{(0,4)}(1 + 2t/M_J^2)}{s - M_J^2} \right) \\ &= \frac{\langle 23 \rangle^2 [14]^2 \prod_n (t - f_n(s))}{s M_{\text{pl}}^2 \prod_i (s - M_i^2)} \end{aligned}$$

Solving for f_n is not straightforward so we will make a number of assumptions ★

Gravity in the ultraviolet

$$\frac{\langle 23 \rangle^2 [14]^2}{s M_{\text{pl}}^2} \frac{\prod (t - f_n(s))}{\prod (s - M_i^2)} \Big|_{s=M_i} \propto \prod_{i=1}^{\infty} (t - f_n(M_i^2))$$

 f_n analytic around M

We introduce inverse powers of t

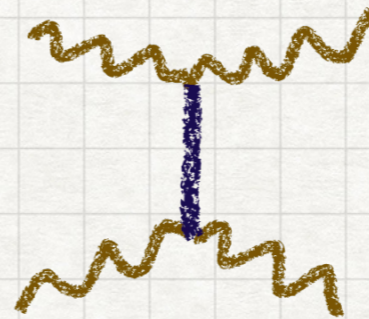
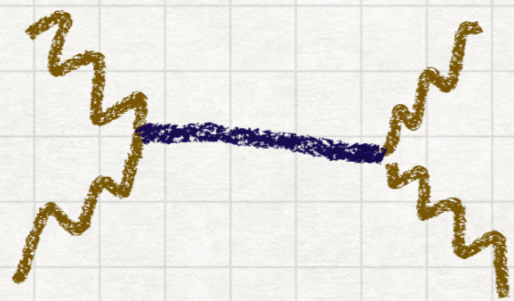
$$\frac{\langle 23 \rangle^2 [14]^2}{s M_{\text{pl}}^2} \frac{\prod (t - f_n(s))}{\prod (s - M_i^2) \prod (t - \hat{M}_j^2)}$$

$$\{f_n(s_i)\} \supset \{\hat{M}_j^2\}, \quad \forall i.$$

The zeroes contain
the poles

Gravity in the ultraviolet

$$\frac{\langle 23 \rangle^2 [14]^2}{s M_{\text{pl}}^2} \frac{\prod (t - f_n(s))}{\prod (s - M_i^2) \prod (t - \hat{M}_j^2)}$$



$$\frac{\langle 23 \rangle^2 [14]^2}{s M_{\text{pl}}^2} \frac{\prod (t - f_n(s))}{\prod (s - M_i^2) \prod (t - \hat{M}_j^2)} \Bigg|_{t=\hat{M}_j^2} \propto \frac{\prod (\hat{M}_j^2 - f_n(s))}{\prod (s - M_i^2)}$$

$$\{f_n^{-1}(t_j)\} \supset \{M_i^2\}, \quad \forall j.$$

The zeroes contain
the poles (also in s)

Gravity in the ultraviolet

$$\frac{\langle 23 \rangle^2 [14]^2}{s M_{\text{pl}}^2} \frac{\prod (t - f_n(s))}{\prod (s - M_i^2) \prod (t - \hat{M}_j^2)} \Bigg|_{t=\hat{M}_j^2} \propto \frac{\prod (\hat{M}_j^2 - f_n(s))}{\prod (s - M_i^2)}$$

$$\{f_n^{-1}(t_j)\} \supset \{M_i^2\}, \quad \forall j.$$

But how many zeroes for the inverse of f_n ?

A degree r will have r zeroes... but there's an infinite f_n 's!

★ Assume f_n^{-1} have one solution

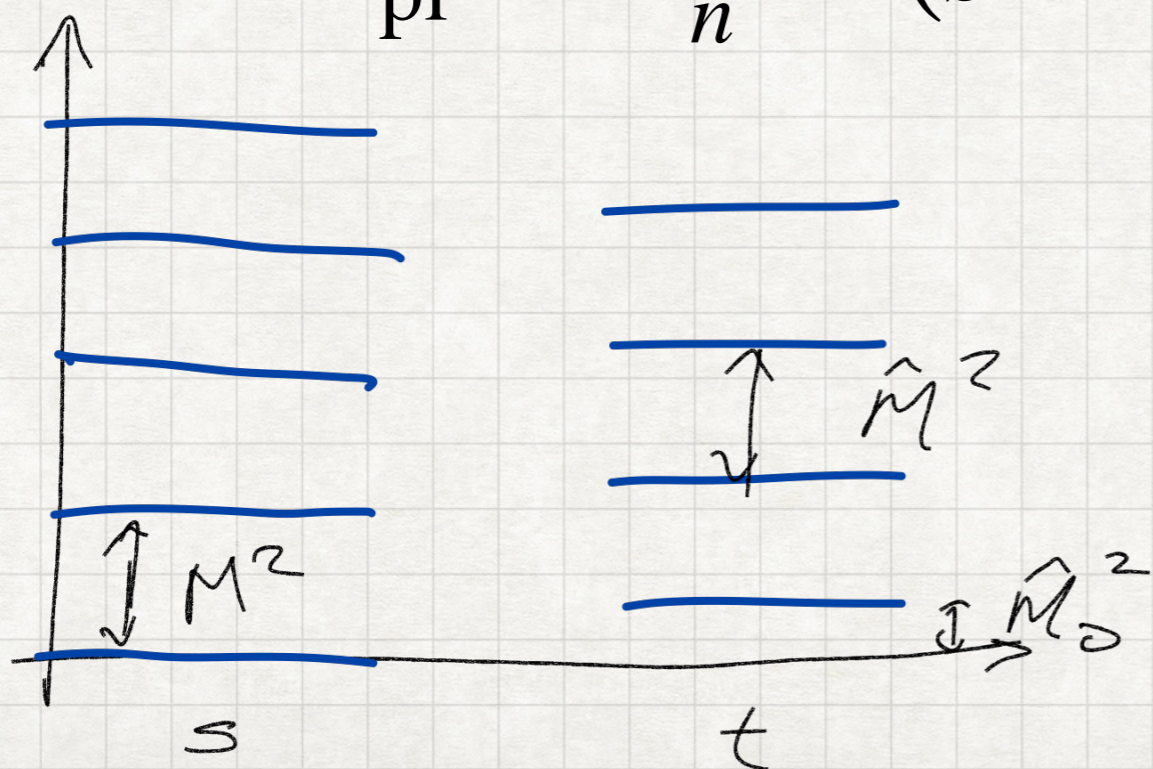
$$\rightarrow \text{linear} \quad f_n = f_n' s + f_n^0$$

Gravity in the ultraviolet



Finally we assume the spin (degree of polynomial in t) of resonances increases with n

$$\frac{\langle 23 \rangle^2 [14]^2}{s M_{\text{pl}}^2} \prod_n \frac{M^2 t + \hat{M}^2 s - M^2 (n \hat{M}^2 + M_0^2)}{(s - n M^2)(t - \hat{M}_0^2 - n \hat{M}^2)},$$



Recalling the following definition of the Gamma function

$$\Gamma(z) = \frac{1}{z} \prod_n \frac{(1 + 1/n)^z}{(1 + z/n)}$$

Gravity in the ultraviolet

$$\mathcal{A} = \frac{8\pi \langle 23 \rangle^2 [14]^2 \Gamma(1 - \tilde{s}) \Gamma(1 - \hat{t})}{M_{\text{pl}}^2 s \Gamma(1 - \tilde{s} - \hat{t})}$$

$$\tilde{s} = s/M^2, \quad \hat{t} = (t - M_0^2)/\hat{M}^2$$

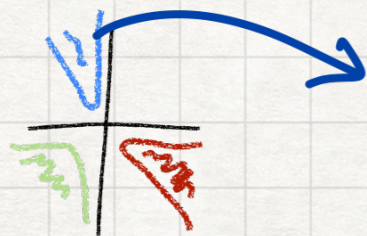
This looks familiar but... did we solve the problem we set out to?

$$\mathcal{A} \rightarrow s e^{R\tilde{s}} \quad R = \log \left((1 - \eta s_{\theta/2}^2)^{\eta s_{\theta/2}^2} (\eta s_{\theta/2}^2)^{\eta s_{\theta/2}^2} \right)$$

$\eta \equiv M^2/\hat{M}^2$ for η less (or =) than one it decays exponentially!

Positivity from unitarity

The t resonances we found are accessible in the crossed process which furthermore is subject to positivity



$$s_{12} \rightarrow t, \quad [14] \rightarrow \sqrt{s} \cos(\theta/2) \quad \text{etc}$$

$$\mathcal{A} = \frac{8\pi \langle 23 \rangle^2 [14]^2 \Gamma(1 - \eta \tilde{s}_{13}) \Gamma(1 - \tilde{s}_{12})}{s_{12} M_{\text{pl}}^2 \Gamma(1 - \tilde{s}_{12} - \eta \tilde{s}_{13})}$$

Make (blue) substitutions & evaluate @ poles

$$\eta \tilde{s}_{13} = n$$

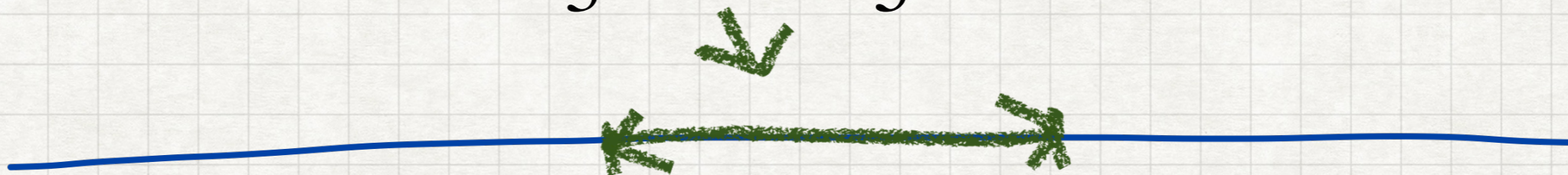
$$\alpha_{4,4}^J = \frac{1}{32\pi} \int d\cos\theta d_{4,4}^J(\theta) \text{Res}(\mathcal{A}(s, \theta))_{s=M_n^2}$$

$$\alpha^J \geq 0 \quad \Rightarrow \quad (\eta^{-1} - 1) \leq \frac{3}{2n} \quad \Rightarrow \quad M = \hat{M} \quad \text{!}$$

Fermions are special

$$\frac{8\pi\langle 23 \rangle [14]}{M_{\text{pl}}^2 s_{12}} \left(\frac{s_{13}}{s_{12}} + \frac{s_{12}}{s_{13}} + b \right) \frac{\Gamma(1 - \tilde{s}_{12})\Gamma(1 - \tilde{s}_{13})}{\Gamma(1 - \tilde{s}_{12} - \tilde{s}_{13})}$$

$$\frac{2}{3} \leq b \leq \frac{22}{5}$$

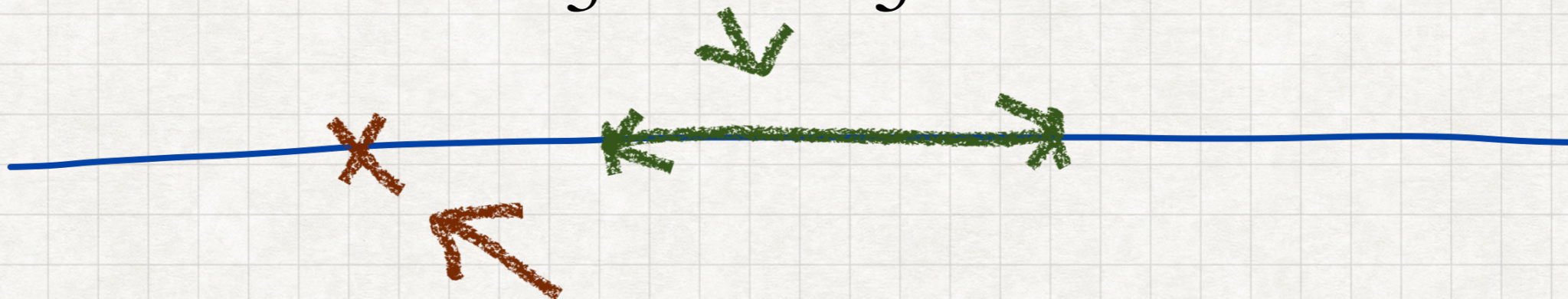


But b we can compute from Feynman rules of GR!

Fermions are special

$$\frac{8\pi\langle 23 \rangle [14]}{M_{\text{pl}}^2 s_{12}} \left(\frac{s_{13}}{s_{12}} + \frac{s_{12}}{s_{13}} + b \right) \frac{\Gamma(1 - \tilde{s}_{12})\Gamma(1 - \tilde{s}_{13})}{\Gamma(1 - \tilde{s}_{12} - \tilde{s}_{13})}$$

$$\frac{2}{3} \leq b \leq \frac{22}{5}$$



But b we can compute from Feynman rules of GR!

$$b = 1/2 !$$

Modify the low energy content of gravity

We can fix this introducing a 3-form H

$$-\frac{g}{M} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} \psi^\dagger \sigma_\mu \psi + (H^2)$$

Which nonetheless is not dynamical but it integrates out to

$$\frac{g^2}{M^2} \left(\psi^\dagger \sigma_\mu \psi \right)^2$$

and the amplitude we obtain is reconciled with the low energy EFT for the range

$$\frac{1}{108} \leq \frac{g^2 M_{\text{pl}}^2}{\pi M^2} \leq \frac{13}{60}$$

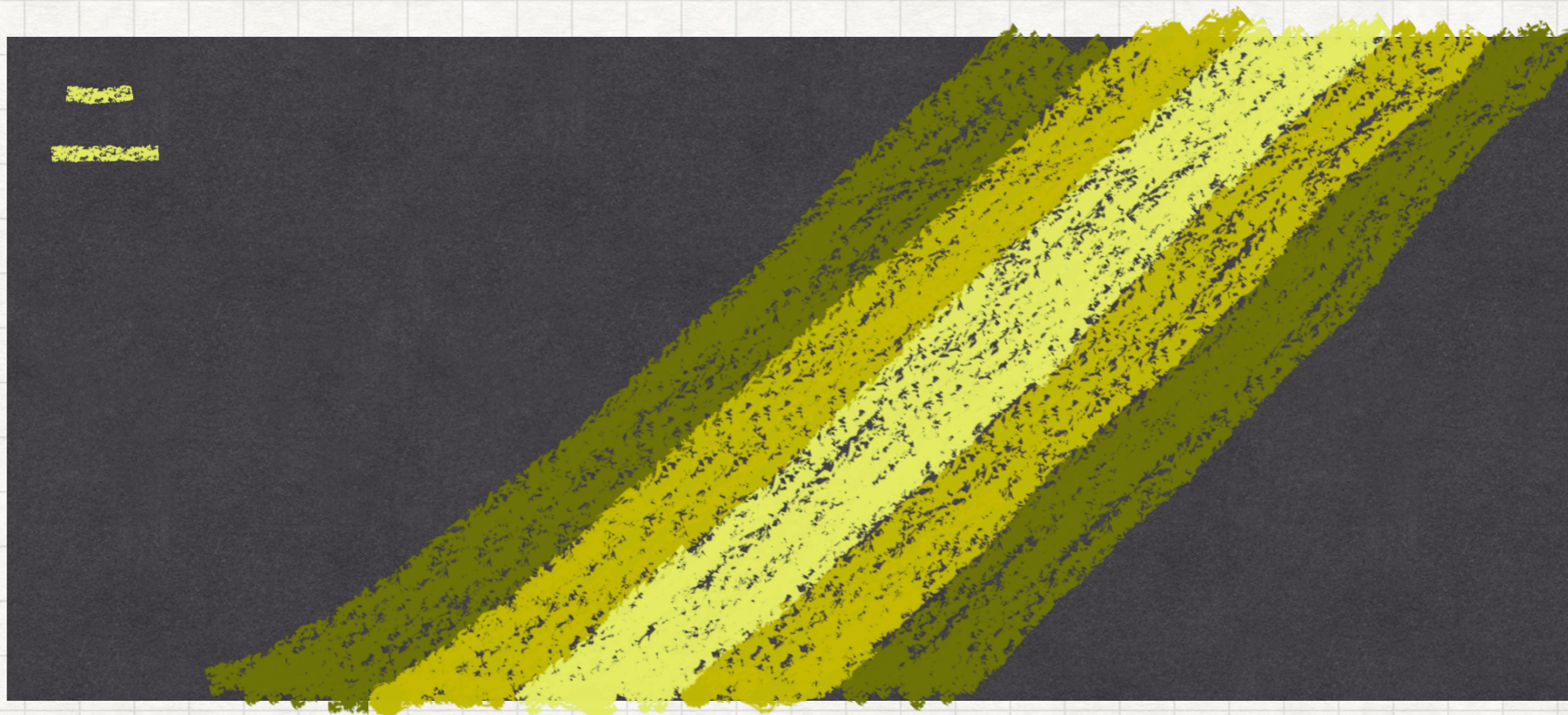
Summary

A sample of how amplitude methods
Provide a new angle to approach gravity

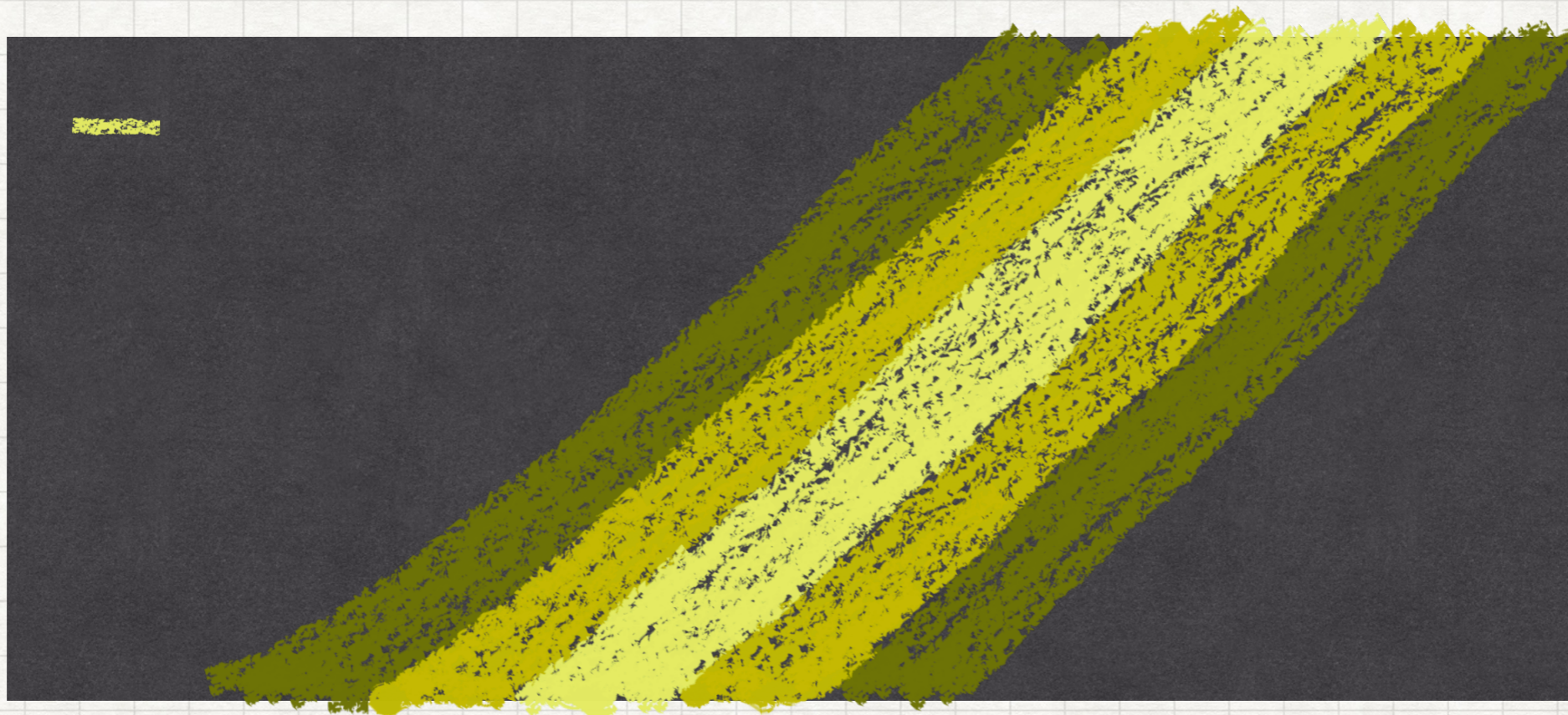
... and quite remarkably give predictions $\hat{-}\hat{}$

중문

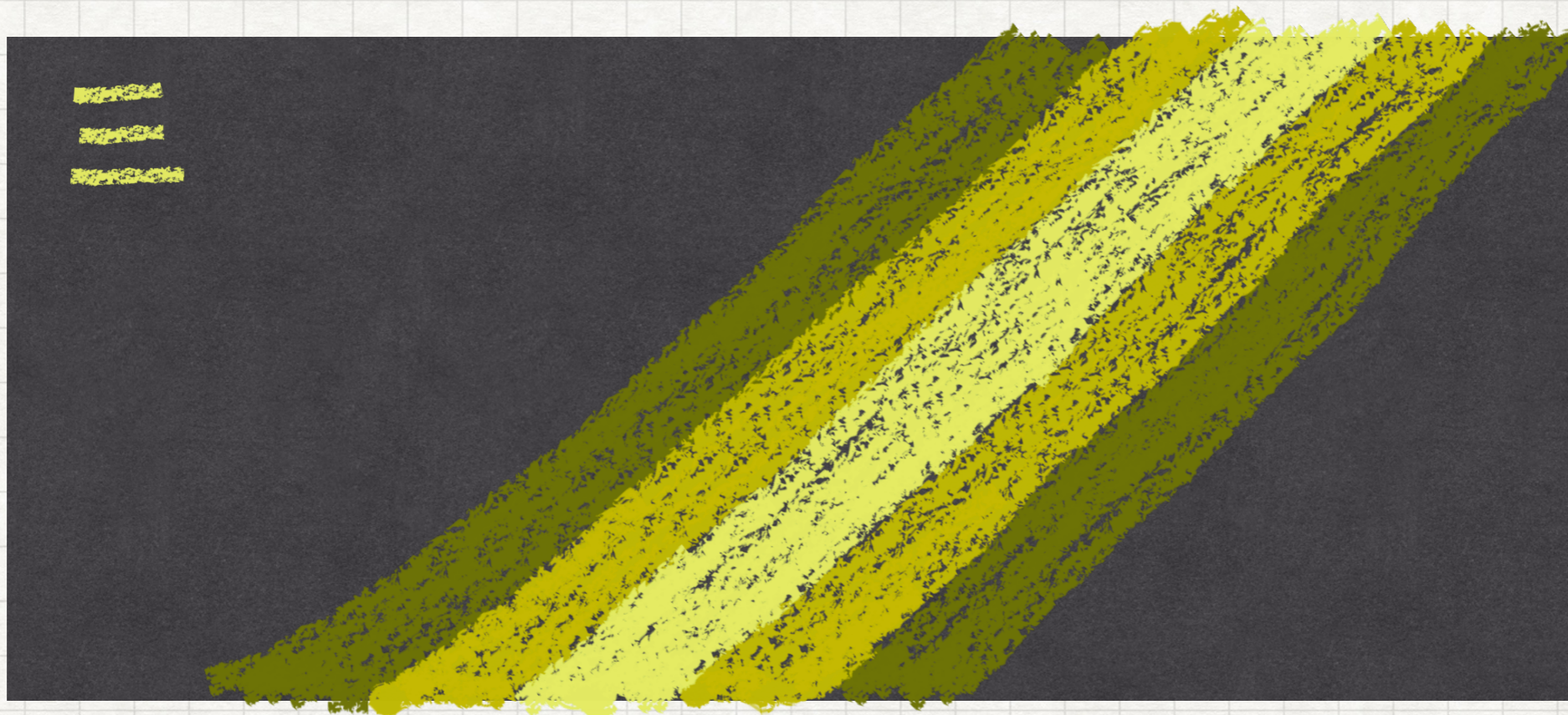




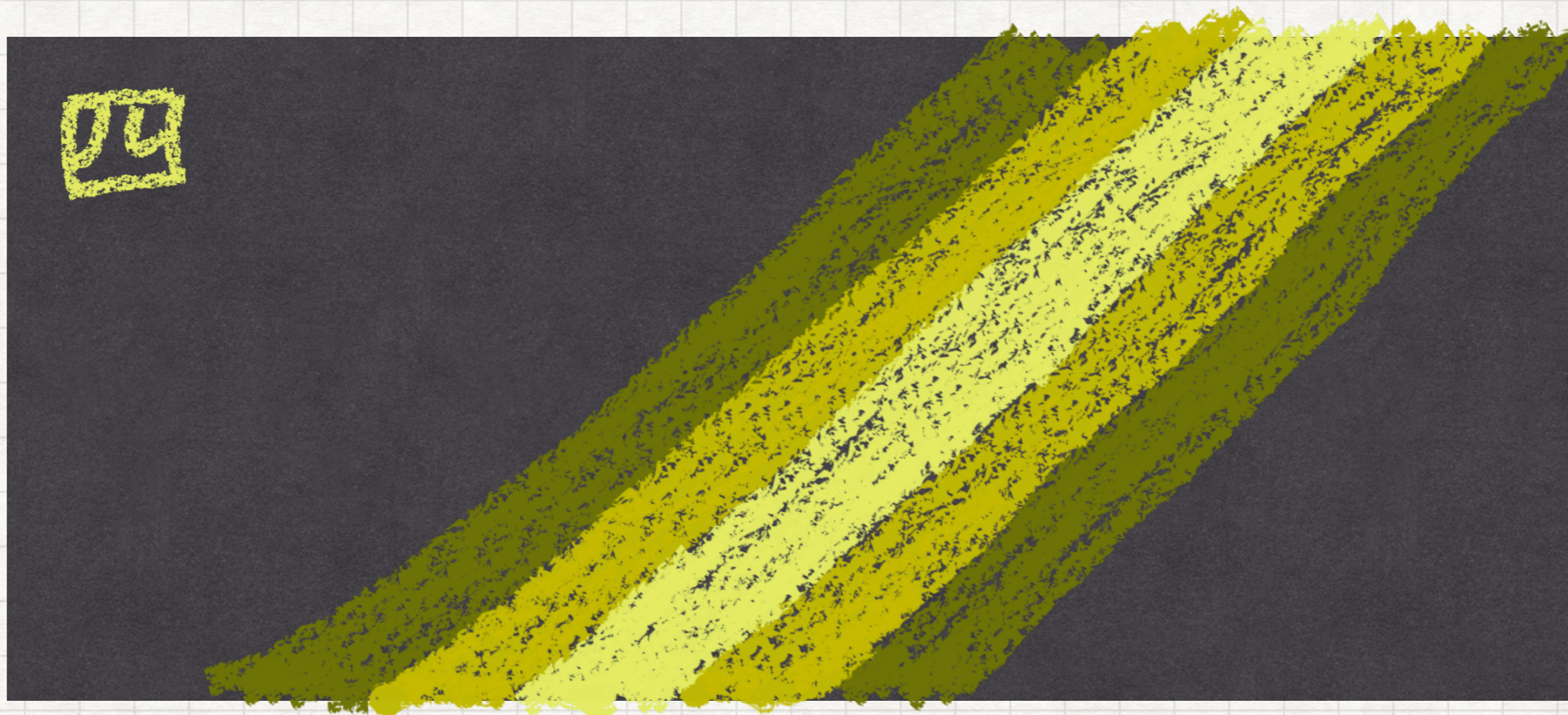
Angular Analysis and Unitarity



On-shell amplitude methods



Gravity in the ultraviolet



Analysis of results