

# Prospects in Nucleon Structure at Low $Q^2$

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2nd Workshop on Nucleon Structure at Low  $Q^2$  and  $\mu$ ASTI  
15 - 21 May, 2023, AVRA Imperial, Kolymbari, Crete

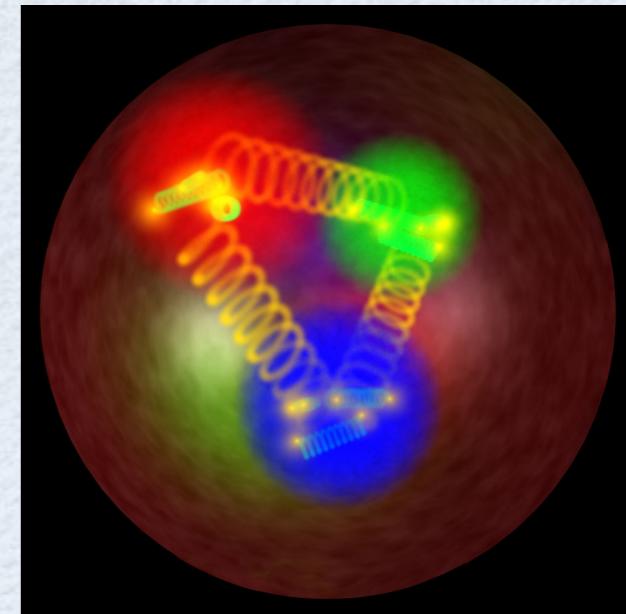
# Nucleon spin structure at low Q : dawn of hadron physics



Otto Stern's measurement of the g-factor of the proton (1932-33):

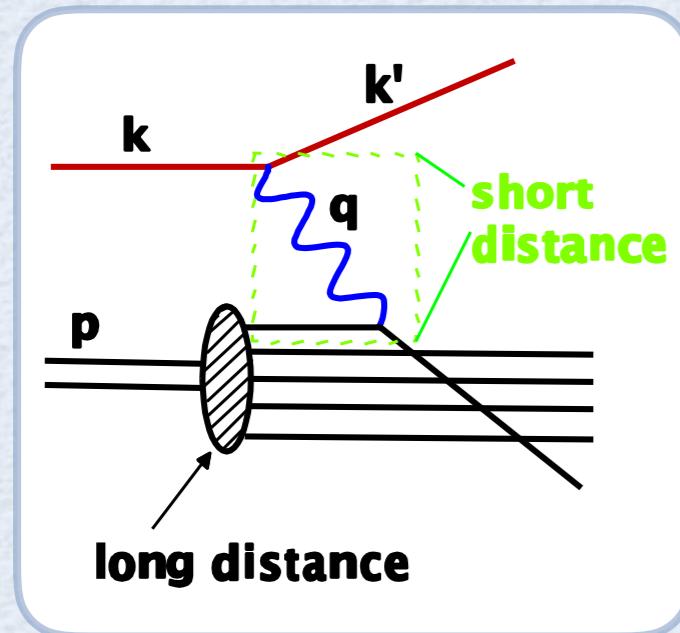
$$g_{\text{proton}} = 5.586$$

Deviation from point particle value ( $g=2$ ) indicates that proton has internal structure



Nobel Prize Physics (1943):  
“for his contribution to the development of the molecular ray method and his discovery of the **magnetic moment of the proton**”

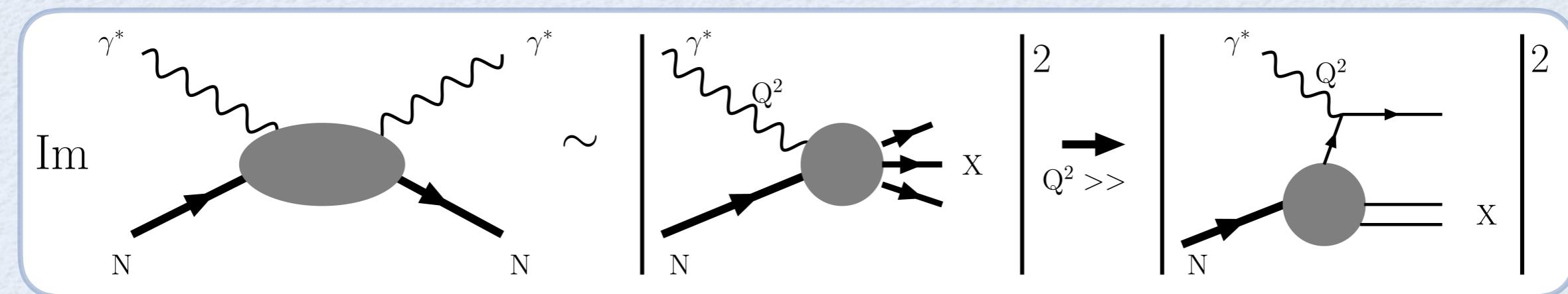
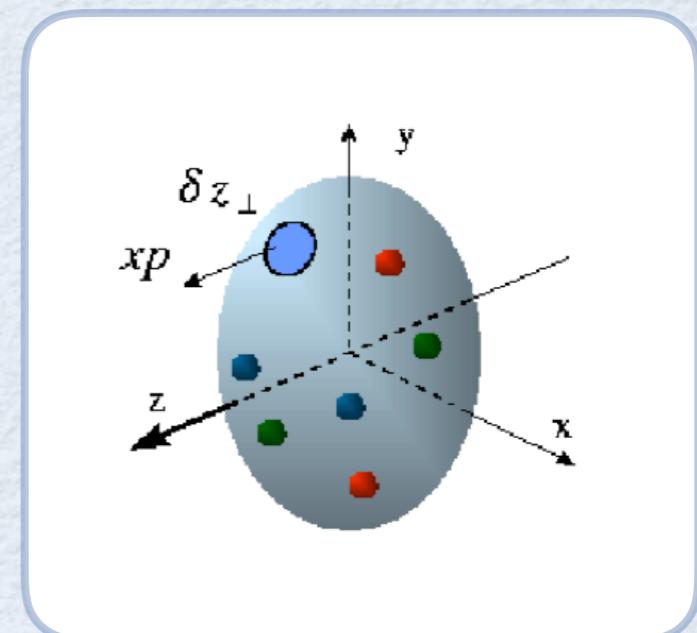
# Nucleon spin structure at large Q : Quark-gluon structure



Deep-inelastic scattering

$$Q^2 \gg 1 \text{ GeV}^2$$

$$\text{At fixed } x_B = \frac{Q^2}{2p \cdot q}$$

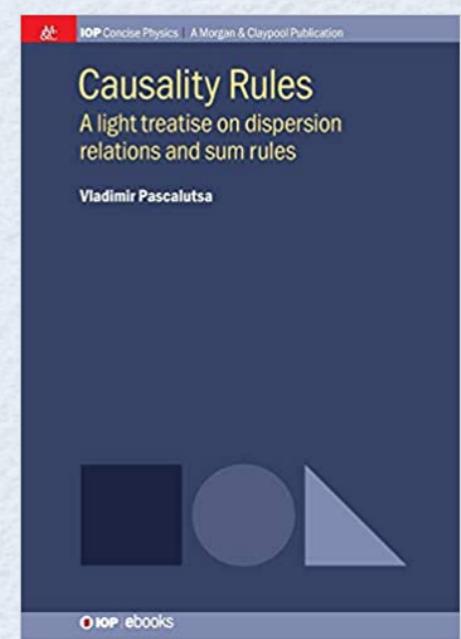
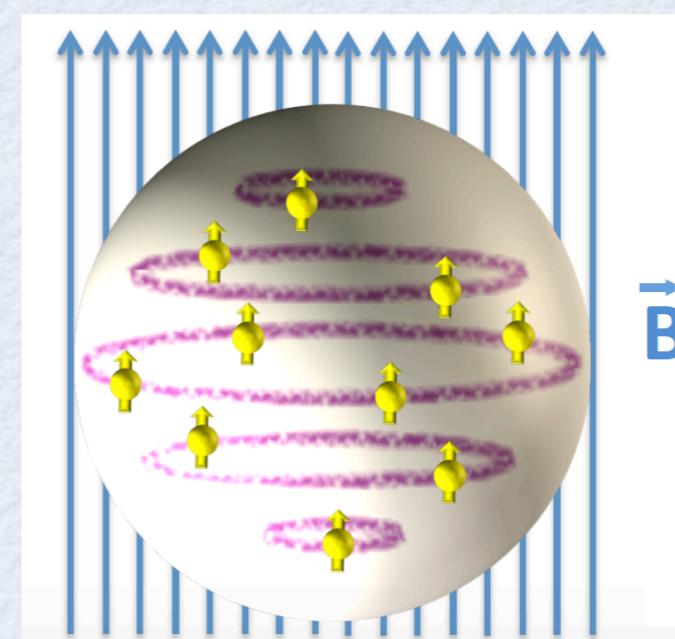
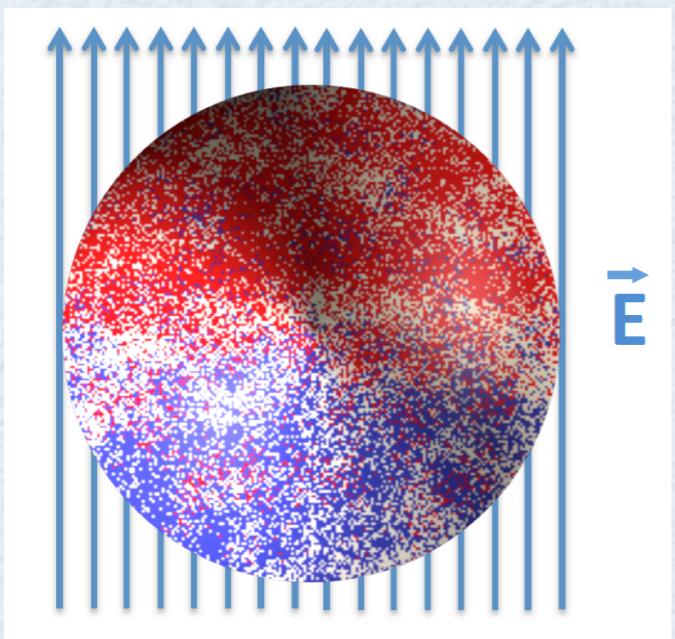


$$d\sigma \sim L_{\mu\nu} W^{\mu\nu}$$

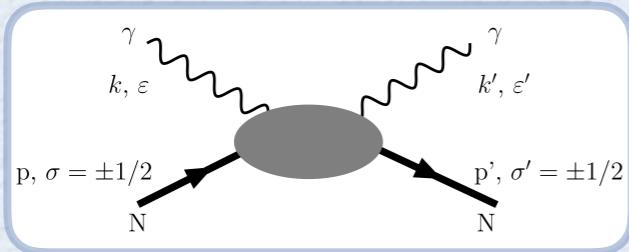
$$W^{\mu\nu} = 2 \left\{ \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x_B, Q^2) + \frac{1}{p \cdot q} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) F_2(x_B, Q^2) \right\} \\ + \frac{2M}{p \cdot q} i \varepsilon^{\mu\nu\alpha\beta} q_\alpha \left\{ S_\beta g_1(x_B, Q^2) + \left[ S_\beta - \frac{S \cdot q}{p \cdot q} p_\beta \right] g_2(x_B, Q^2) \right\}$$

$F_1, F_2, g_1, g_2$  : nucleon structure functions

# Nucleon structure from Compton processes and sum rules



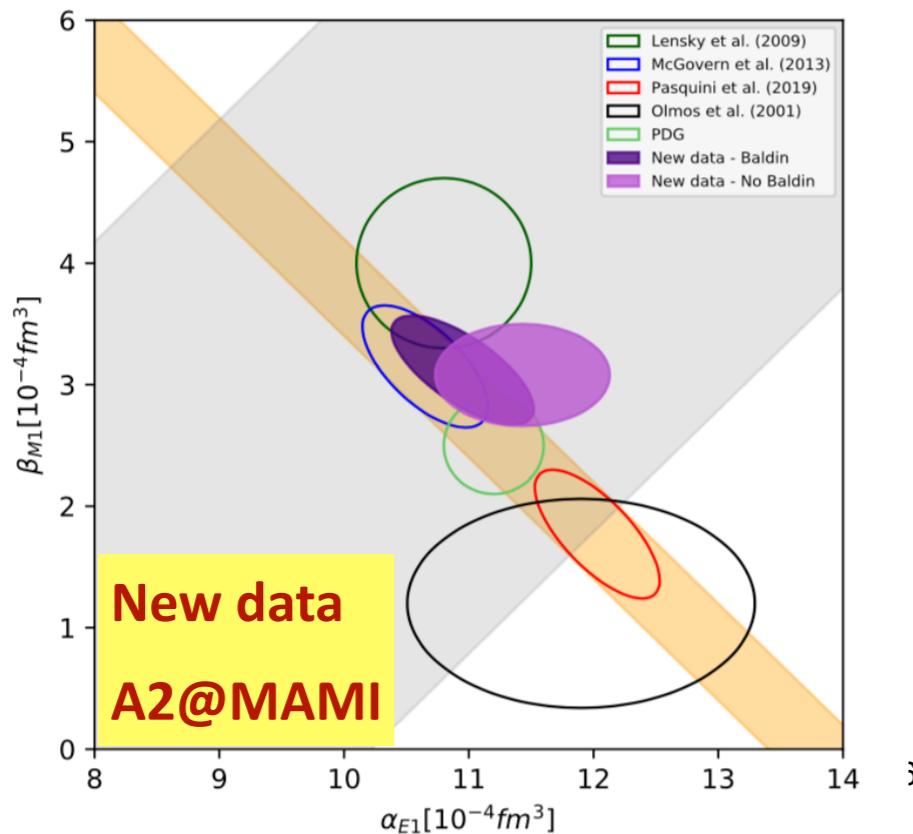
# Nucleon structure at low Q with real photons: RCS



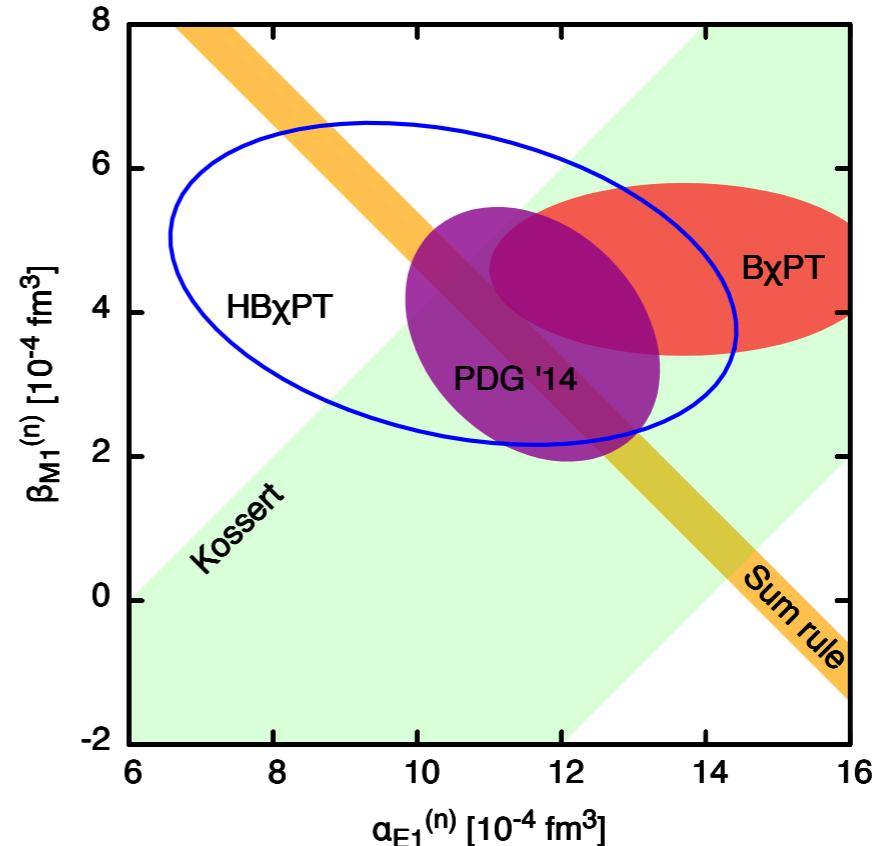
$$H_{\text{eff}}^{(2)} = -4\pi \left[ \frac{1}{2} \alpha_{E1} E^2 + \frac{1}{2} \beta_{M1} H^2 \right]$$

Nucleon static polarizabilities

Proton



Neutron



$$\alpha_E = (11.2 \pm 0.4) \times 10^{-4} \text{ fm}^3$$

PDG '14 values

$$\alpha_E = (11.8 \pm 1.1) \times 10^{-4} \text{ fm}^3$$

$$\beta_M = (2.5 \pm 0.4) \times 10^{-4} \text{ fm}^3$$

$$\beta_M = (3.7 \pm 1.2) \times 10^{-4} \text{ fm}^3$$

New A2@MAMI increase world data set by factor 5

New HIGS data with linear photon pol. at variance

See talks: Mornacchi, Howell, Pedroni (exp);  
McGovern (EFT), Lee (lattice); Larin, Danilkin (pion pol.)

Plan: reduction of error by factor of 2

→ precision comparable with proton

→ impact light muonic atom program

# Proton spin polarizabilities

→ effective Hamiltonian (3rd order):

$$H_{\text{eff}}^{(3)} = -4\pi \left[ \frac{1}{2} \gamma_{E1E1} \boldsymbol{\sigma} \cdot (\mathbf{E} \times \dot{\mathbf{E}}) + \frac{1}{2} \gamma_{M1M1} \boldsymbol{\sigma} \cdot (\mathbf{H} \times \dot{\mathbf{H}}) \right. \\ \left. - \gamma_{M1E2} E_{ij} \sigma_i H_j + \gamma_{E1M2} H_{ij} \sigma_i E_j \right]$$

Nucleon spin polarizabilities

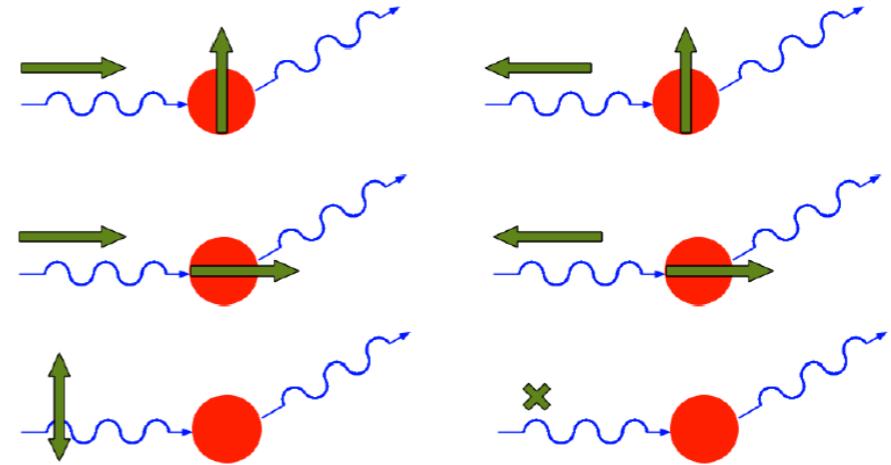
→ polarized Compton scattering

measure angular dependence of asymmetries

Martel et al. (2015); Paudyal et al. (2020)



A2@MAMI  
data



	EXP. (A2@MAMI)	DRs	HBChPT	BChPT	$L_\chi$
$\gamma_{E1E1}$	$-2.87 \pm 0.52$	-4.5	$-1.1 \pm 1.8$	$-3.3 \pm 0.8$	-3.7
$\gamma_{M1M1}$	$2.70 \pm 0.43$	3.0	$2.2 \pm 0.7$	$2.9 \pm 1.5$	2.5
$\gamma_{E1M2}$	$-0.85 \pm 0.72$	-0.08	$-0.4 \pm 0.4$	$0.2 \pm 0.2$	1.2
$\gamma_{M1E2}$	$2.04 \pm 0.43$	2.3	$1.9 \pm 0.4$	$1.1 \pm 0.3$	1.2

→ theory predictions (units  $10^{-4} \times \text{fm}^4$ ): dispersion theory

Drechsel, Pasquini, Vdh (2000)  
Pasquini, Vdh: ARNPS 68 (2018)

HBChPT

McGovern, Phillips, Griesshammer (2013)

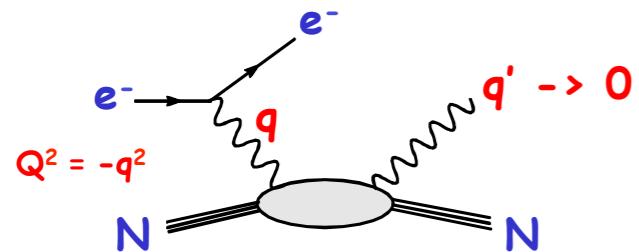
BChPT

Lensky, McGovern, Pascalutsa (2015)

Chiral Lagrangian

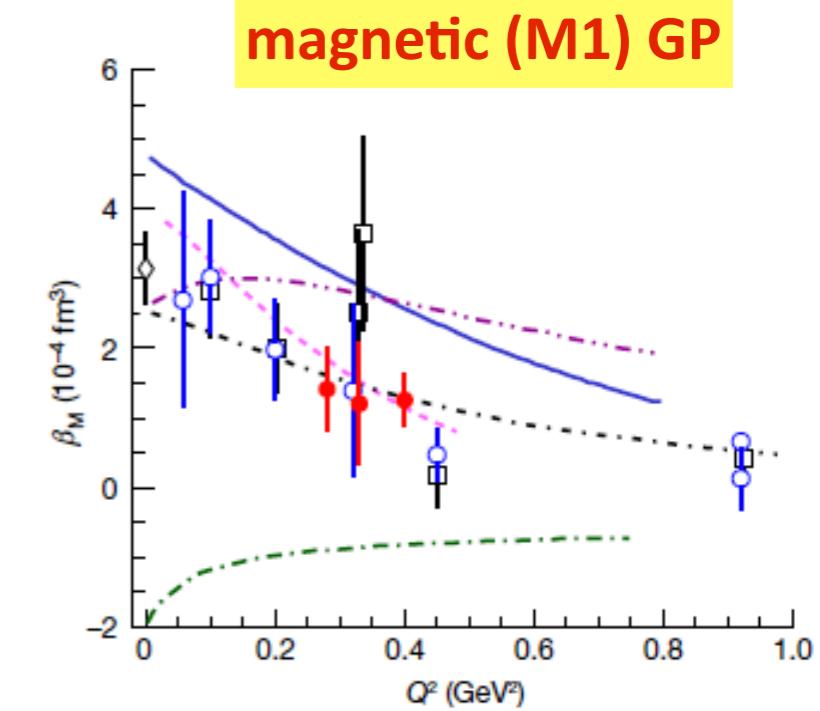
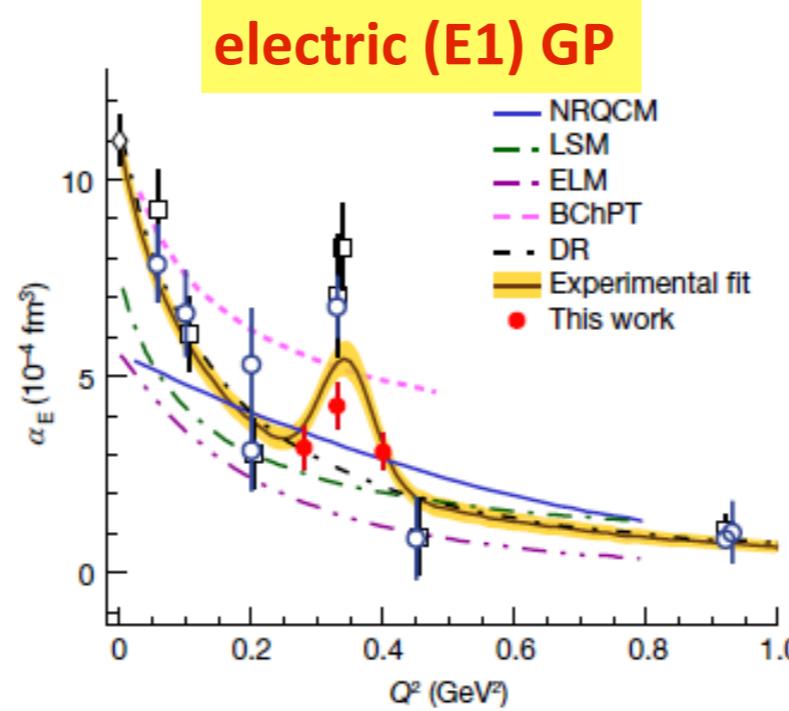
Gasparyan, Lutz, Pasquini (2011)

# Nucleon structure at low Q with virtual photons: Generalized Polarizabilities in VCS



BATES, MAMI,  
JLab data

see talk: Sparveris

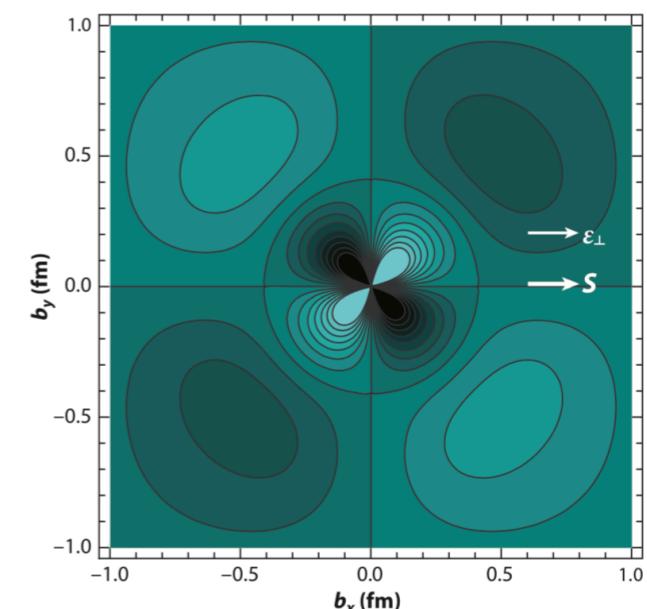
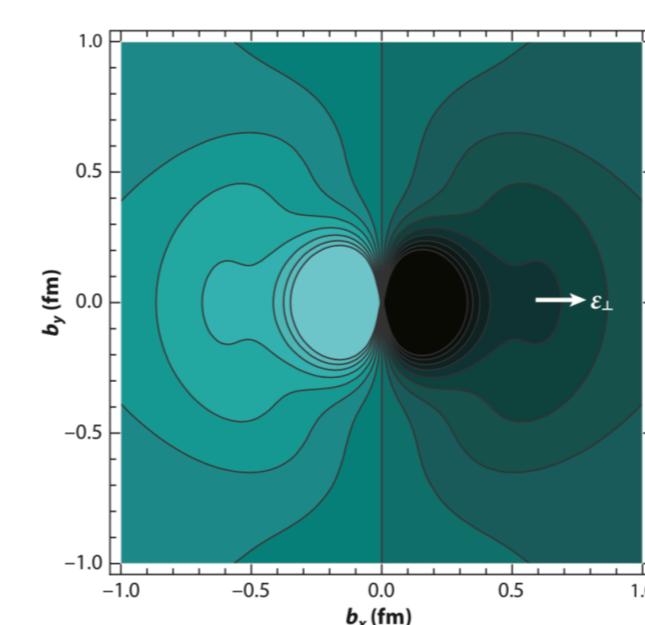


energy shift:  $\delta E = -\vec{E} \cdot \vec{P}_0$

induced polarization

$$\begin{aligned} \vec{P}_0(\vec{b}) &= \int \frac{d^2 \vec{q}_\perp}{(2\pi)^2} e^{-i \vec{q}_\perp \cdot \vec{b}} \vec{P}_0(\vec{q}_\perp) \\ &= \hat{b} \int_0^\infty \frac{dQ}{(2\pi)} Q J_1(bQ) A(Q^2) \end{aligned}$$

combination of nucleon  
generalized polarizabilities

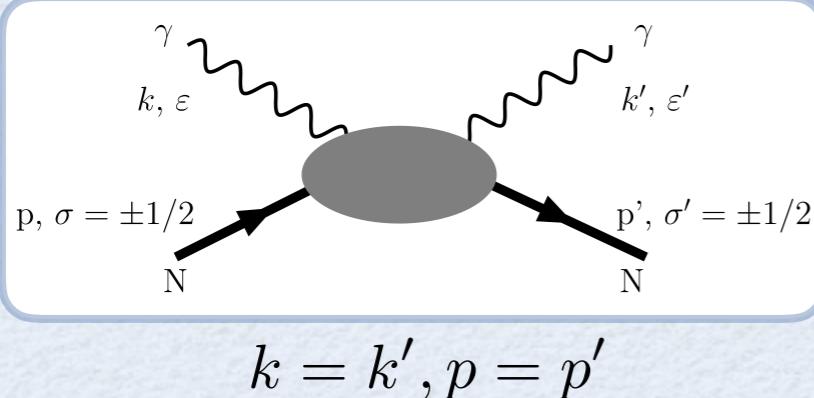


Gorchtein, Lorcé, Pasquini, Vdh (2009) ;

Pasquini, Vdh (2018)

# Nucleon structure with real photons: sum rules

→ forward real Compton scattering



$$T(\nu, \theta = 0) = \vec{\varepsilon}'^* \cdot \vec{\varepsilon} f(\nu) + i \vec{\sigma} \cdot (\vec{\varepsilon}'^* \times \vec{\varepsilon}) g(\nu)$$

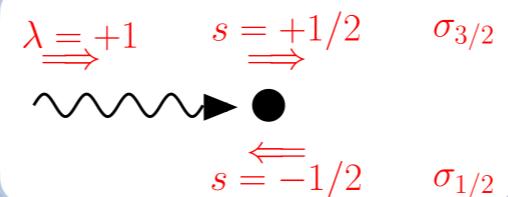
low-energy expansion

$$\begin{aligned} f(\nu) &= \frac{-e^2}{4\pi M} + (\alpha_E + \beta_M) \nu^2 + \mathcal{O}(\nu^4) \\ g(\nu) &= \frac{-e^2 \kappa^2}{8\pi M^2} \nu + \gamma_0 \nu^3 + \mathcal{O}(\nu^5) \end{aligned}$$

LET

polarizabilities

$$\frac{e^2 \pi \kappa^2}{2M^2} = \int_{\nu_0}^{\infty} d\nu' \frac{\sigma_{3/2} - \sigma_{1/2}}{\nu'}$$



$$\Delta\sigma = \sigma_{3/2} - \sigma_{1/2} \quad \vec{\gamma} \vec{p} \rightarrow X$$

Gerasimov (1966); Drell, Hearn (1966)

Ihs SR: 204  $\mu\text{b}$     rhs SR:  $(211 \pm 15) \mu\text{b}$     ✓

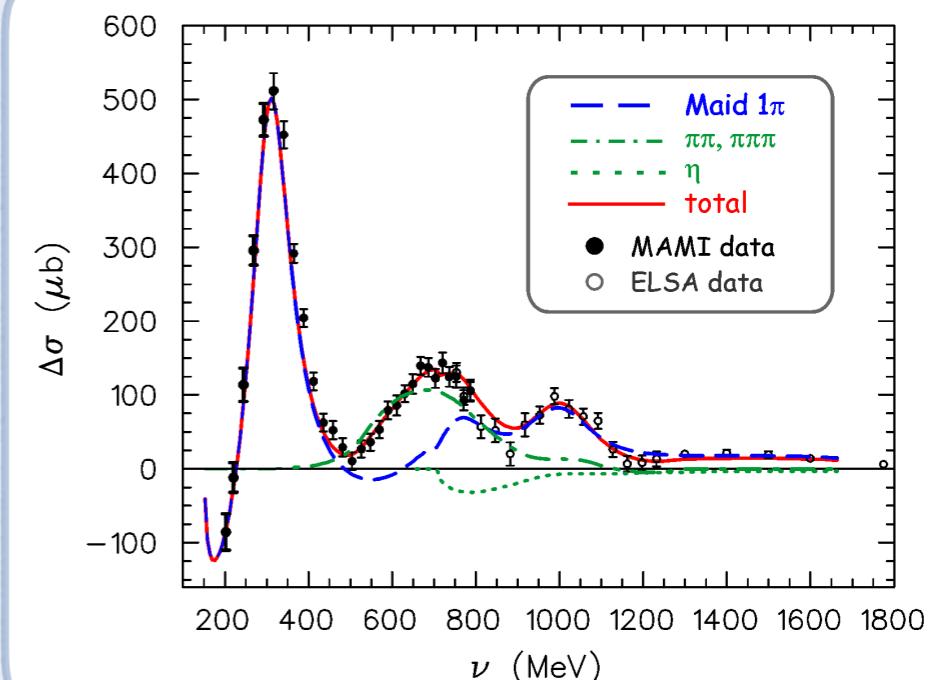
But: region  $\nu > 2.9 \text{ GeV}$  not measured so far (- 7%)

→ term in  $\nu^3$ : forward spin polarizability sum rule

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1E2} - \gamma_{M1M1}$$

$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} d\nu' \frac{\sigma_{3/2} - \sigma_{1/2}}{\nu'^3}$$

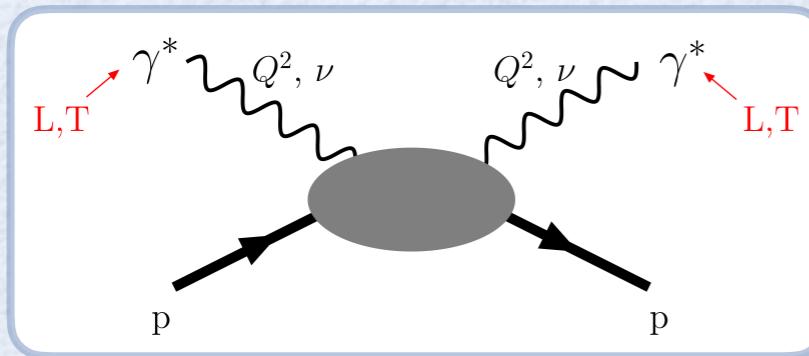
proton:  $\gamma_0 = -(1.01 \pm 0.08 \pm 0.10) \times 10^{-4} \text{ fm}^4$



Drechsel, Tiator (2004)

# Nucleon spin structure with virtual photons: VVCS

→ forward, virtual Compton scattering



$$T_{spin}^{\mu\nu} = \frac{i}{M} \epsilon^{\nu\mu\alpha\beta} q_\alpha s_\beta S_1(\nu, Q^2) + \frac{i}{M^3} \epsilon^{\nu\mu\alpha\beta} q_\alpha (p \cdot q s_\beta - s \cdot q p_\beta) S_2(\nu, Q^2)$$

$$\text{Im } S_1(\nu, Q^2) = \frac{e^2}{4M} \frac{M}{\nu} g_1(x, Q^2), \quad \text{Im } S_2(\nu, Q^2) = \frac{e^2}{4M} \frac{M^2}{\nu^2} g_2(x, Q^2)$$

→ moments of spin structure functions ( $x_0$  : inelastic threshold)

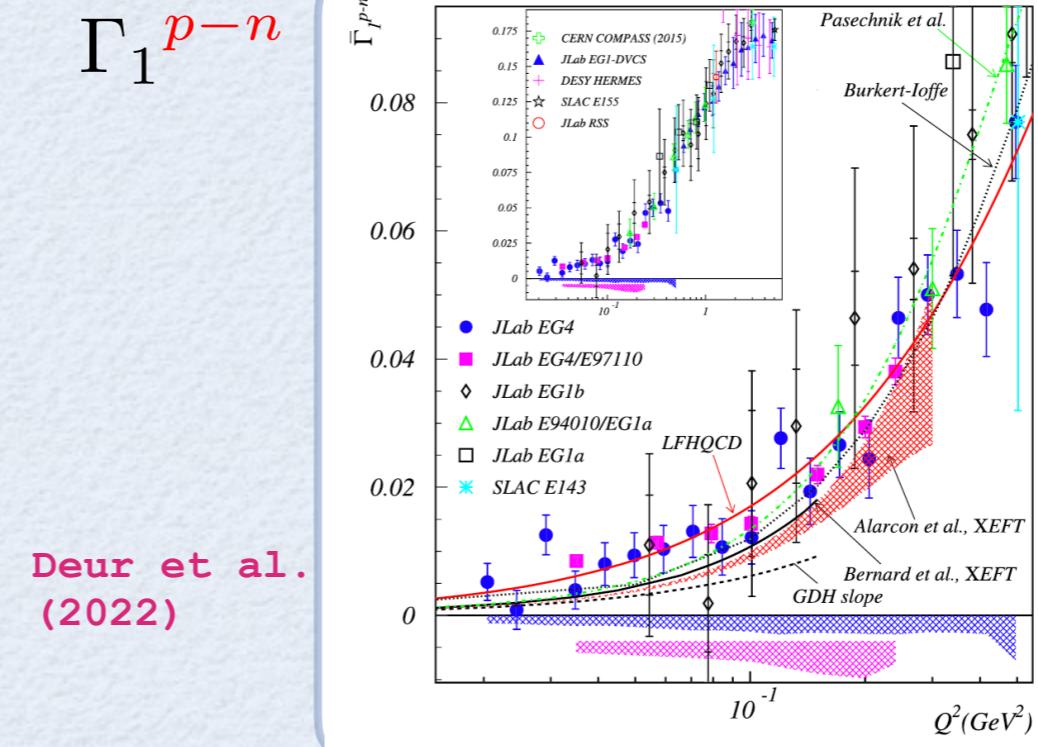
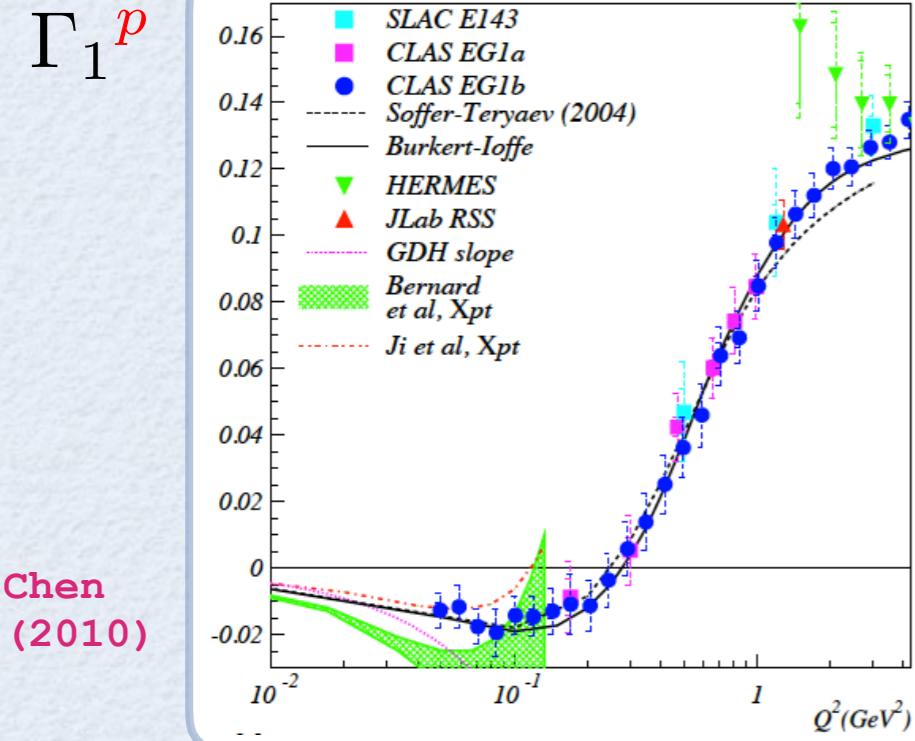
$$\Gamma_1(Q^2) = \int_0^{x_0} dx g_1(x, Q^2)$$

$$\frac{Q^2}{2M^2} \left( -\frac{\kappa^2}{4} \right), \quad Q^2 \rightarrow 0$$

$$\Gamma_1(Q^2)|_{scaling}, \quad Q^2 \rightarrow \infty$$

GDH sum rule

parton helicity structure (DIS)



$$\text{in DIS: } \frac{1}{6} g_A \left\{ 1 - \frac{\alpha_s}{\pi} + \dots \right\}$$

Bjorken sum rule

see talk:  
Chen

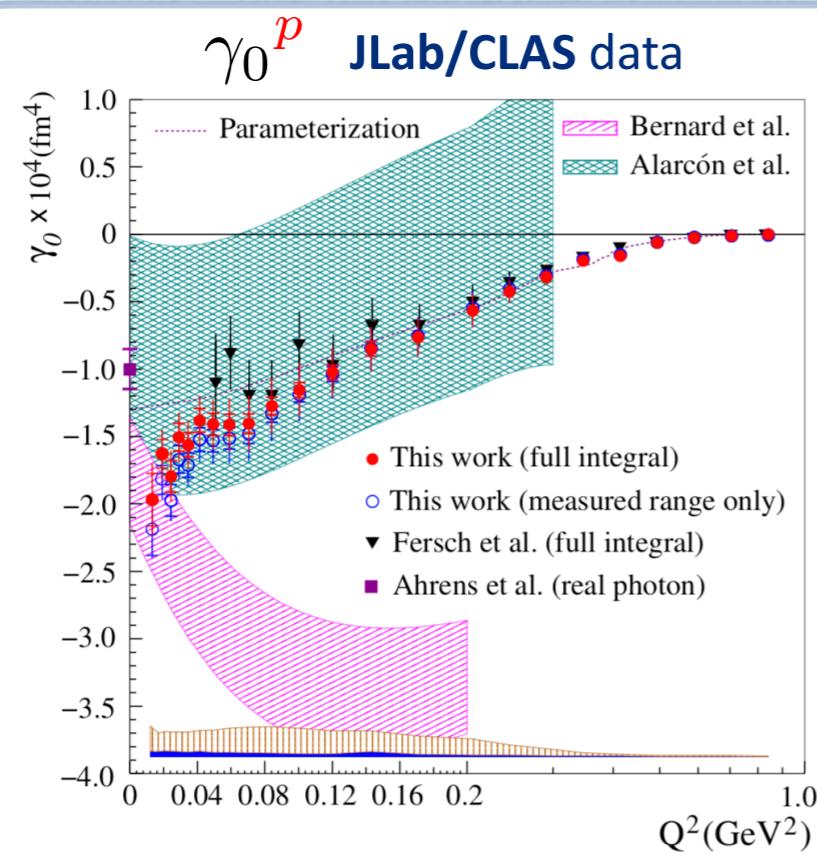
# Nucleon spin polarizabilities with virtual photons: VVCS

→ higher moments of spin structure functions: spin polarizabilities

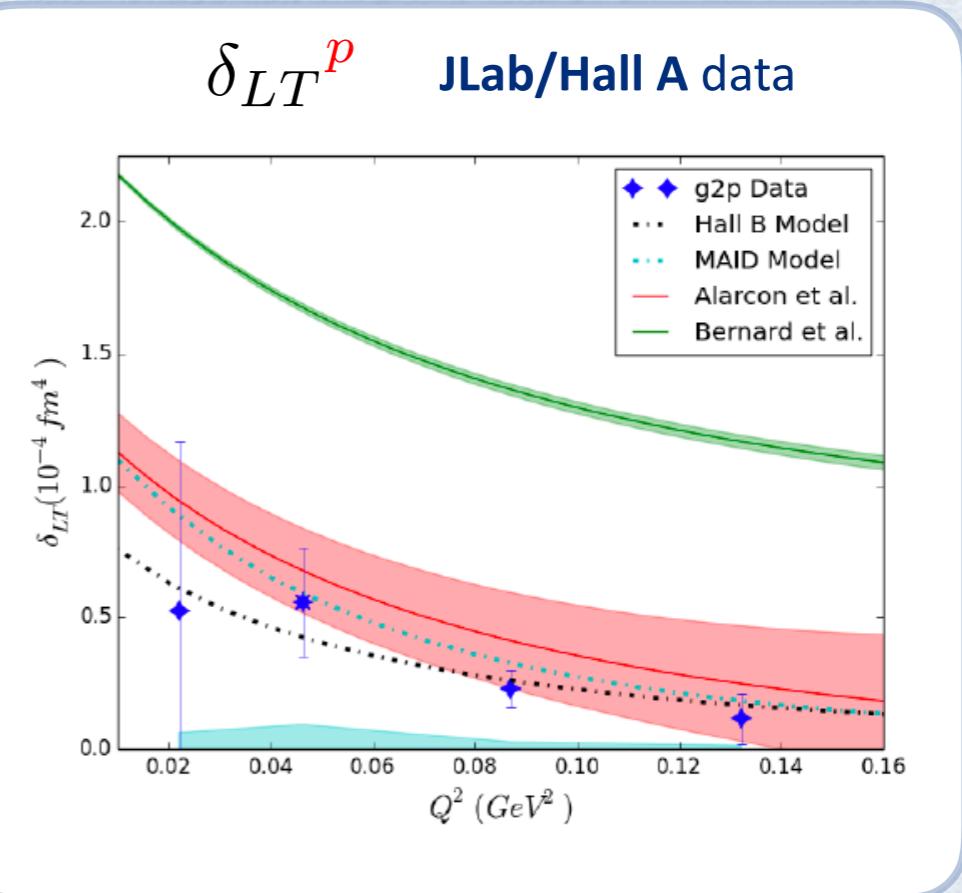
$$\gamma_0(Q^2) = \frac{4M^2e^2}{\pi Q^6} \int_0^{x_0} dx x^2 \left\{ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right\} \rightarrow \gamma_0, \quad Q^2 \rightarrow 0$$

$$\delta_{LT}(Q^2) = \frac{4e^2 M^2}{\pi Q^6} \int_0^{x_0} dx x^2 \{ g_1(x, Q^2) + g_2(x, Q^2) \}$$

see talks: Deur, Peng, Ripani,  
Ruth, Slifer



Zheng et al. (2021)



Ruth et al. (2022)

→ provide strong test for theory: chiral EFT calculations / lattice QCD see talk: Pascalutsa

# Sum rules at low Q relating RCS, VCS, and VVCS (I)

Can we formulate sum rules relating observables to observables for virtual photons as is the case for real photons (GDH, FSP) ?

→ low-energy expansion for virtual photons

$$(S_1 - S_1^{\text{pole}})(\nu, Q^2) = -\frac{e^2}{8\pi M} \kappa^2 + M\gamma_0 \nu^2 + M\gamma_L Q^2 + \mathcal{O}(Q^4, \nu^4, Q^2\nu^2)$$

$$\gamma_L = \frac{e^2}{24\pi M^2} \kappa^2 \langle r_2^2 \rangle + \gamma_{E1M2} - 3M \frac{e^2}{4\pi} [P'^{(M1,M1)1}(0) + P'^{(L1,L1)1}(0)]$$

$P^{(M1,M1)1}(Q^2), \quad P^{(L1,L1)1}(Q^2)$  Generalized Polarizabilities

-> measured in Virtual Compton Scattering

→ sum rule for  $I_1$

$$(S_1 - S_1^{\text{pole}})(\nu = 0, Q^2) = \frac{e^2}{2\pi M} I_1(Q^2) \quad \text{with}$$

$$I_1(Q^2) = \frac{2M^2}{Q^2} \int_0^{x_0} dx g_1(x, Q^2)$$

$$I'_1(0) = \frac{\kappa^2}{12} \langle r_2^2 \rangle + \frac{2\pi M^2}{e^2} \gamma_{E1M2} - \frac{3M^3}{2} [P'^{(M1,M1)1}(0) + P'^{(L1,L1)1}(0)]$$

Sum rules relating RCS, VCS, VVCS  
Can be tested by experiment !

# Sum rules at low Q relating RCS, VCS, and VVCS (II)

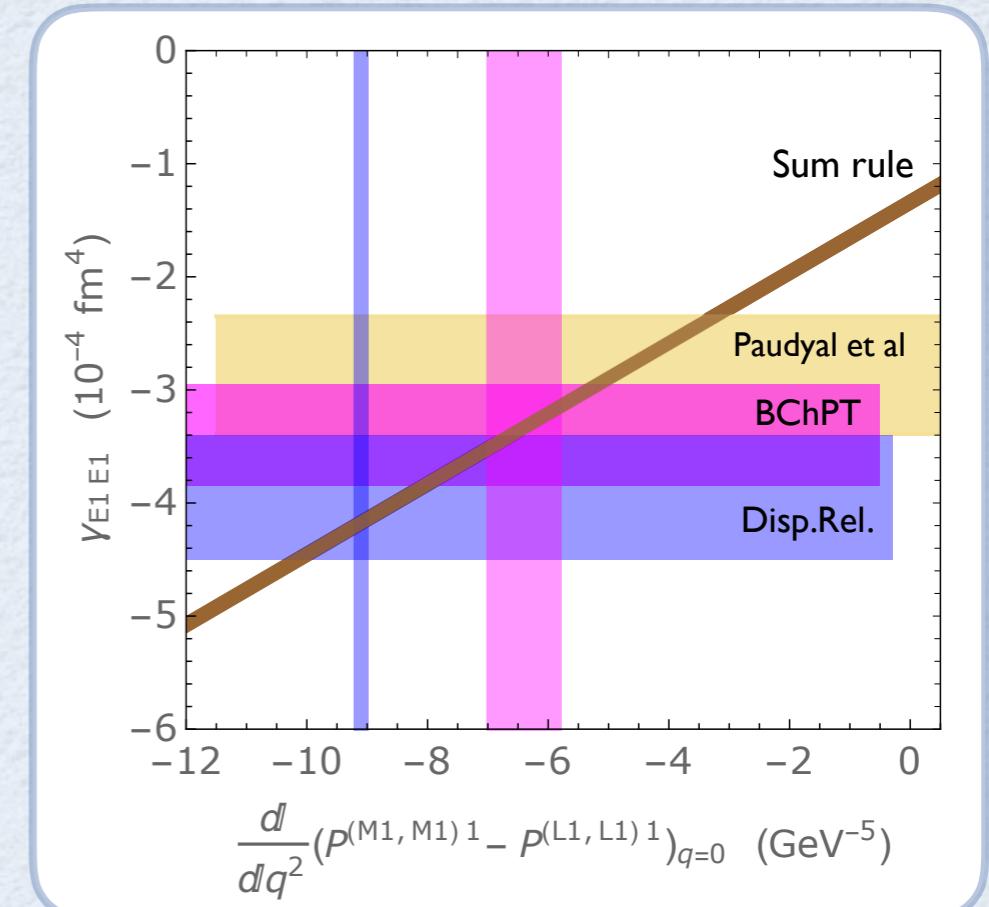
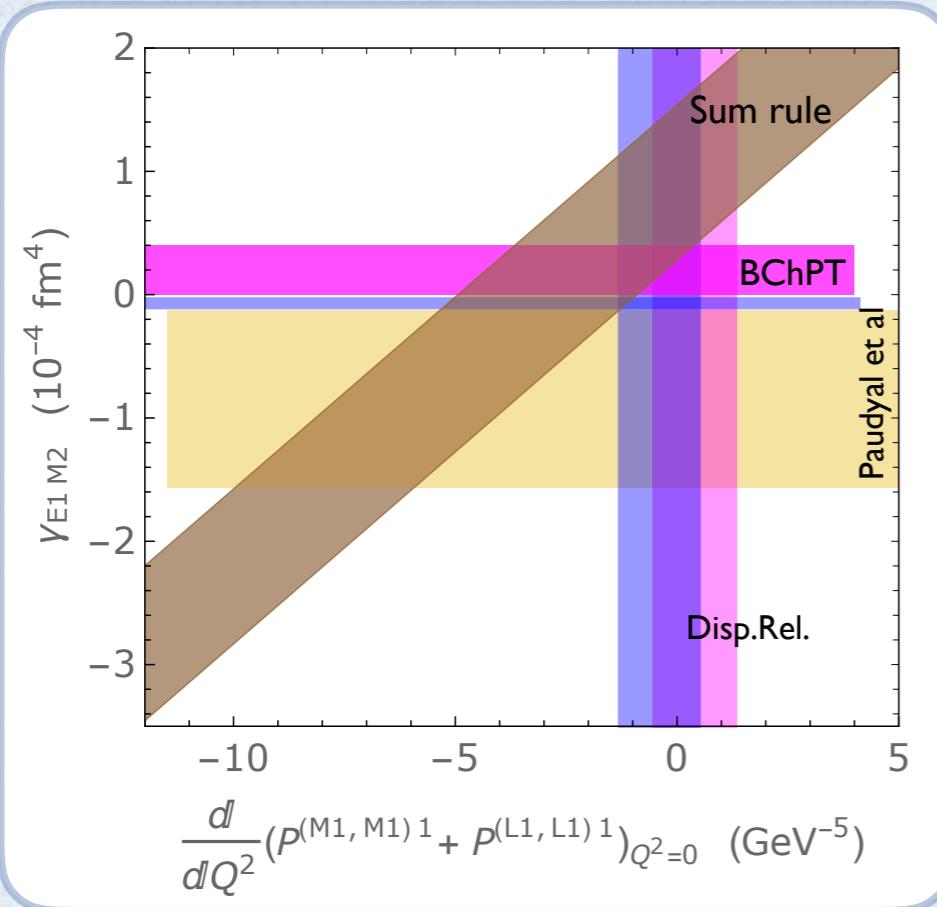
$$I_1(Q^2) = \frac{2M^2}{Q^2} \int_0^{x_0} dx g_1(x, Q^2)$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} dx x^2 [g_1 + g_2](x, Q^2)$$

$$I'_1(0) = \frac{\kappa^2}{12} \langle r_2^2 \rangle + \frac{2\pi M^2}{e^2} \gamma_{E1M2} - \frac{3M^3}{2} [P'^{(M1,M1)1}(0) + P'^{(L1,L1)1}(0)]$$

$$\delta_{LT}(0) = -\gamma_{E1E1} + 3M \frac{e^2}{4\pi} [P'^{(M1,M1)1}(0) - P'^{(L1,L1)1}(0)]$$

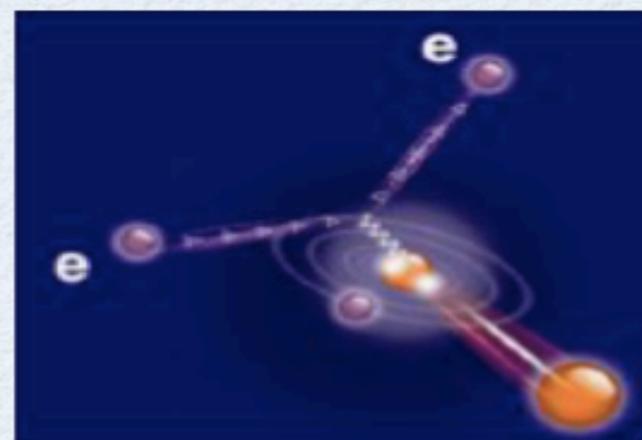
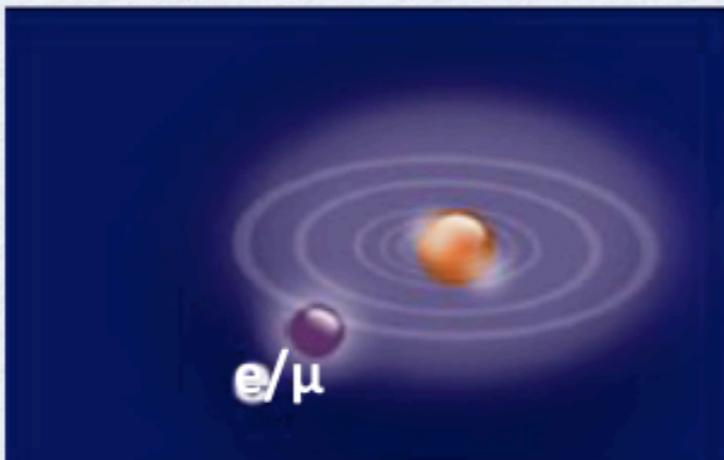
Pascalutsa, Vdh (2015) ; Lensky, Pascalutsa, Vdh (2017)



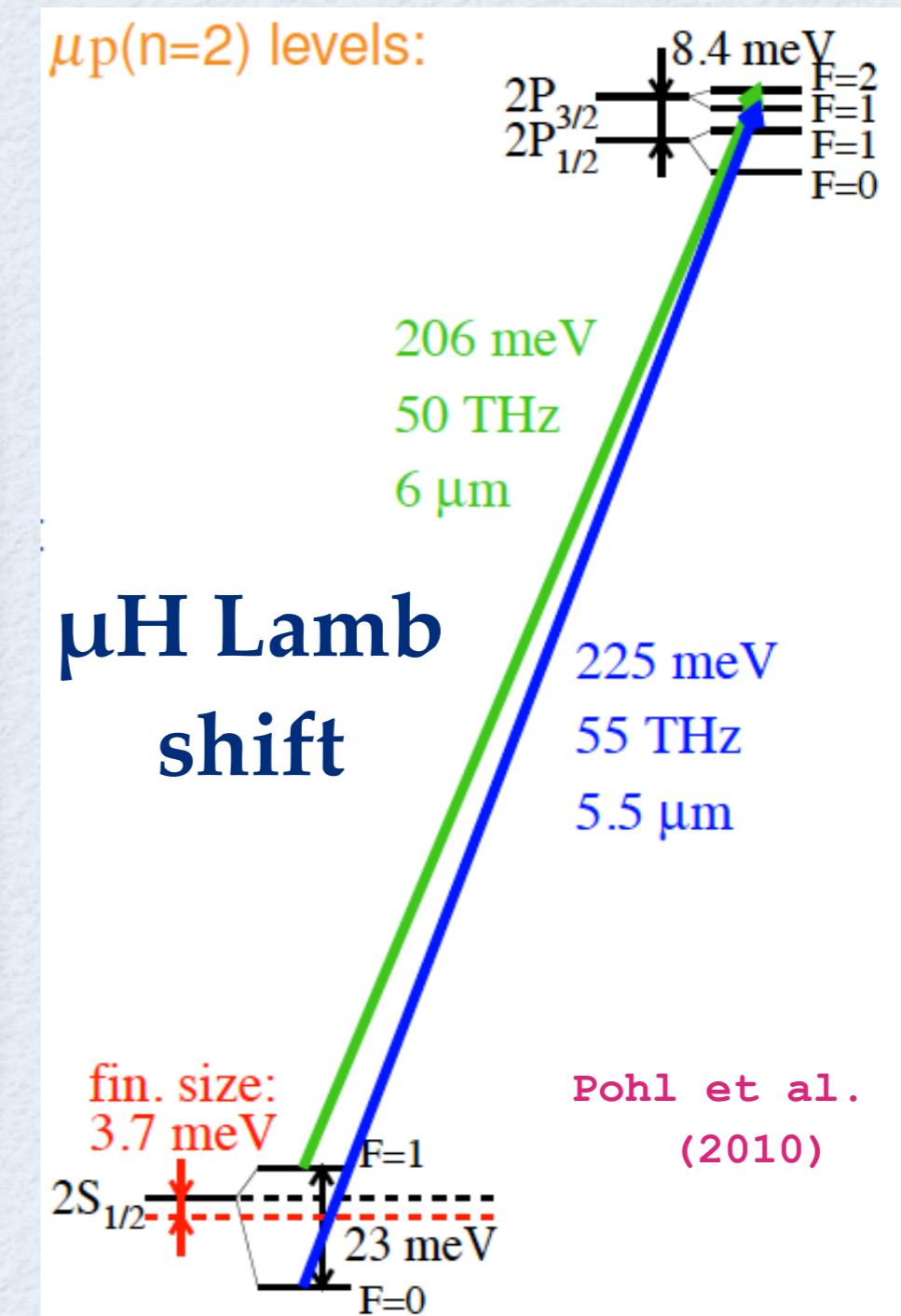
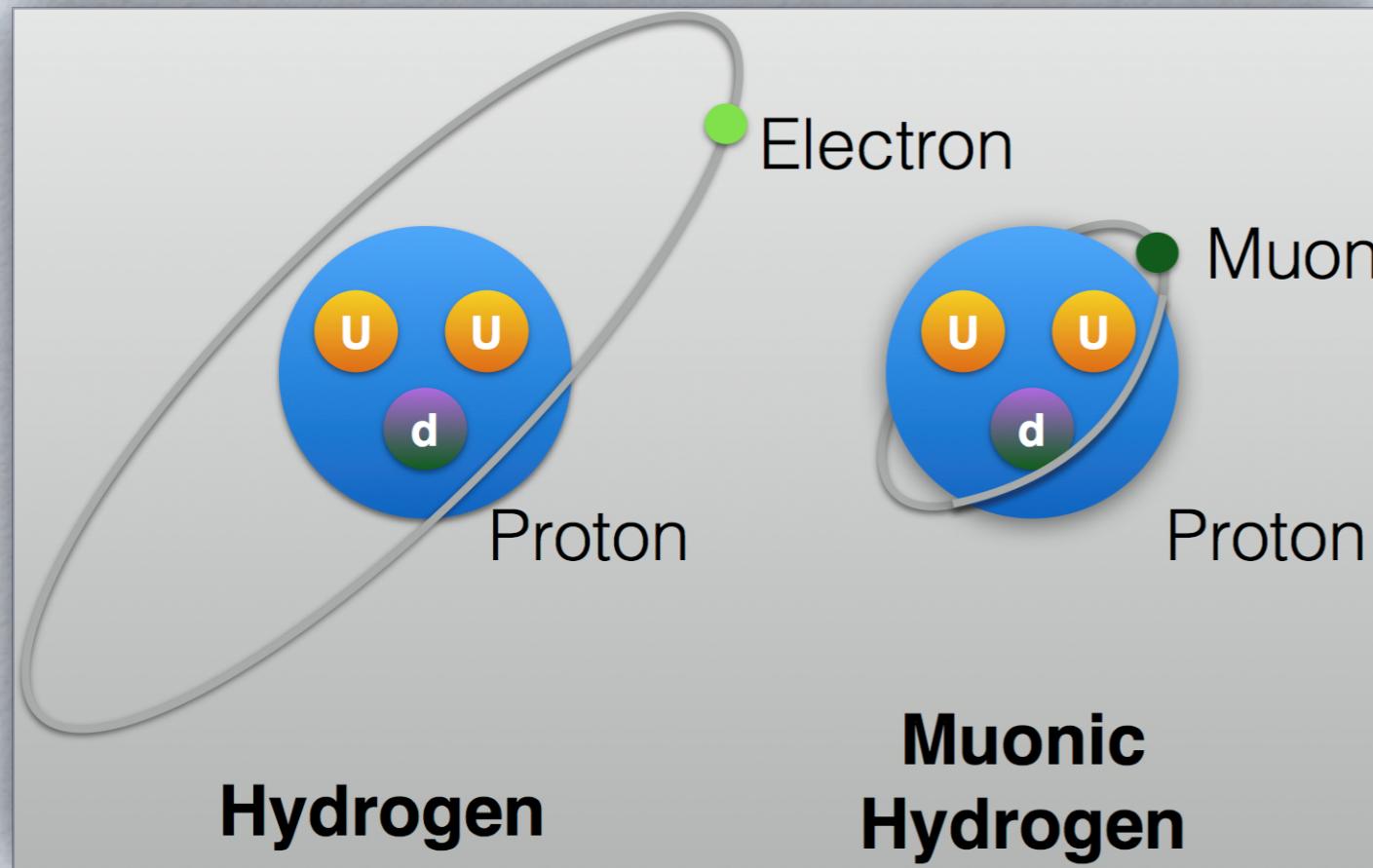
Further sum rules: see talks: Pascalutsa, Biloshytskyi

**measurement of  $P_{TT}$  in VCS  
(separation in  $\varepsilon$ ) at low  $Q^2$  can verify !**

# Proton size and electromagnetic structure



# Proton radius from Hydrogen spectroscopy



$$\Delta E_{LS} = 206.0336(15) - 5.2275(10) R_E^2 + \Delta E_{TPE} \text{ meV}$$

Antognini et al. (2013)

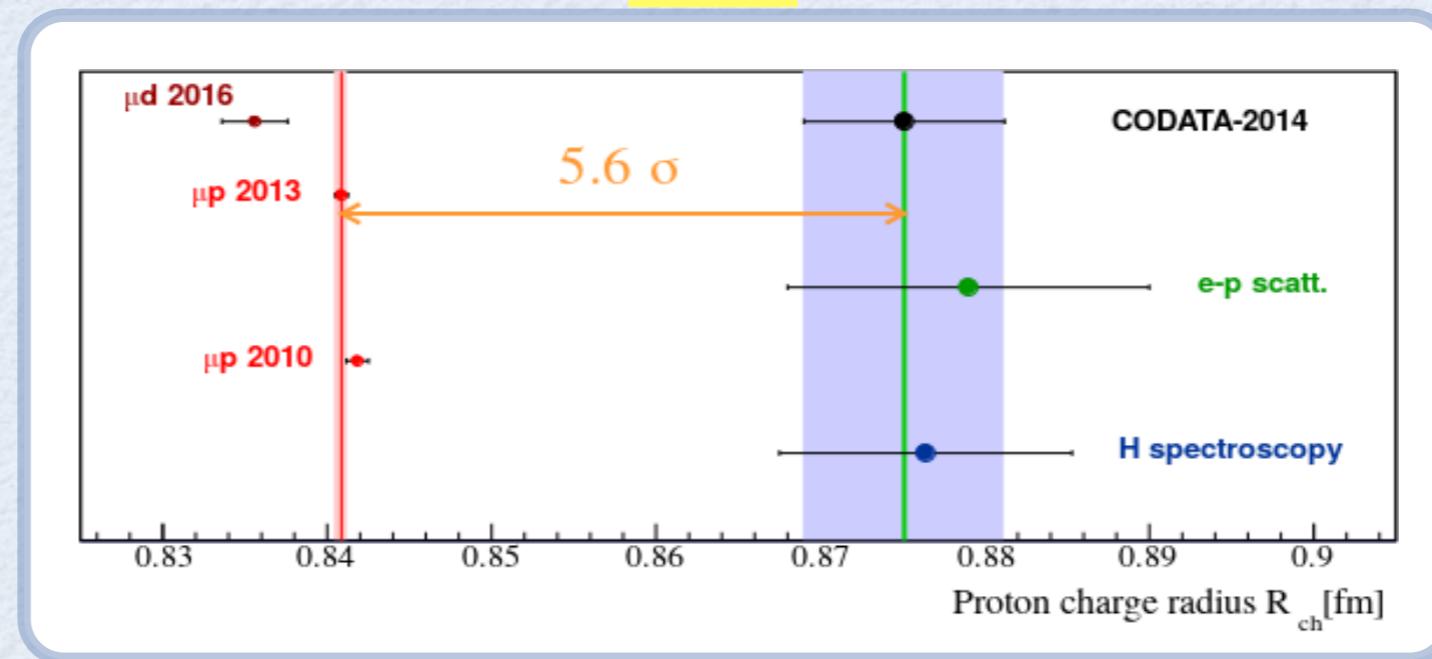
3.70 meV

0.033 (2) meV

$O(\alpha^5)$  correction

# Proton charge radius: from puzzle to precision

2016



## → μ atom Lamb shift:

- μ atom Lamb shift:  
μ D, μ <sup>3</sup>He<sup>+</sup>, μ <sup>4</sup>He<sup>+</sup> performed
- 2γ-exchange correction scrutinized,  
still limiting uncertainty to date

## → electron scattering analysis:

- radius extraction fits
- radiative corrections,  
two-photon exchange corrections

## → electronic H spectroscopy:

- new measurements,  
new Lamb shift experiment!

## → electron scattering experiments:

- new G<sub>Ep</sub> expts. down to  $Q^2 \approx 2 \times 10^{-4}$  GeV<sup>2</sup>
- MAMI/A1: Initial State Radiation (pilot expt.)
- PRad@JLab: HyCal, magnetic spectrometer-free exp.

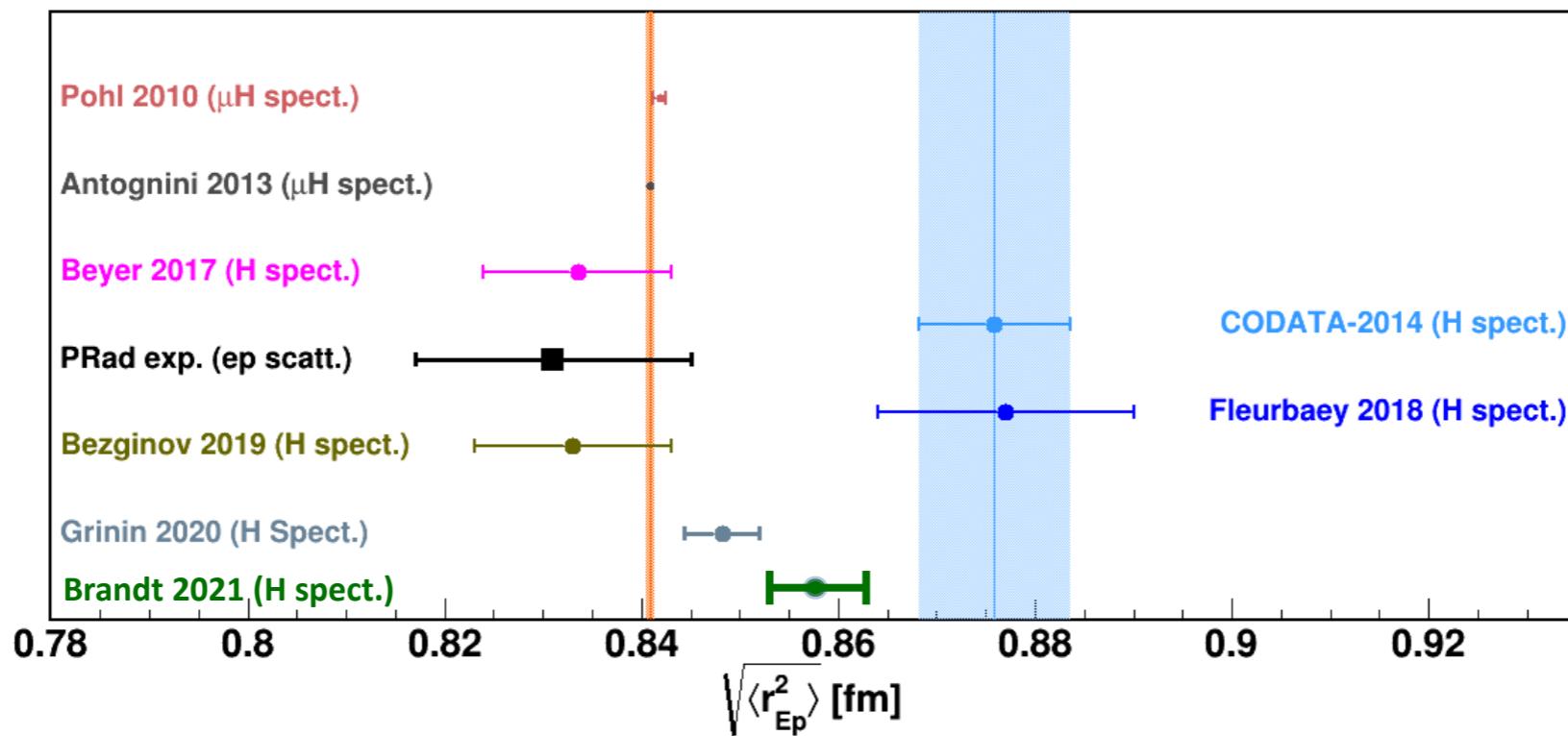
see talks: Bernauer, Higinbotham

# Proton charge radius: present experimental status

2021

Hydrogen 2S-4P  
Hydrogen 2S-2P  
Hydrogen 1S-3S  
Hydrogen 2S-8D

Hydrogen 1S-3S



from recent compilation

Rev. Mod. Phys. 94 (2022) 015002

H. Gao, M. Vdh

- 3 out of 6 new results are fully consistent with muonic hydrogen result
- inconsistency between Fleurbaey et al. (Paris) and Grinin et al. (Garching) results for 1S-3S H : Grinin et al.: factor 2 more precise,  $\sim 2\sigma$  smaller than Fleurbaey et al.,  $\sim 2\sigma$  larger than  $\mu$ H result
- Brandt et al. (Colorado) result is  $\sim 3\sigma$  larger than CODATA 2018 / muonic atom spect.

# Muonic atom spectroscopy needs nucleon/nuclear input

## 2S-2P Lamb Shift:

### THEORY

### EXPERIMENT

	$\Delta E_{TPE} \pm \delta_{theo} (\Delta E_{TPE})$	Ref.	$\delta_{exp}(\Delta_{LS})$	Ref.
$\mu\text{H}$	$33 \text{ }\mu\text{eV} \pm 2 \text{ }\mu\text{eV}$	Antognini et al. (2013)	$2.3 \text{ }\mu\text{eV}$	Antognini et al. (2013)
$\mu\text{D}$	$1710 \text{ }\mu\text{eV} \pm 15 \text{ }\mu\text{eV}$	Krauth et al. (2015)	$3.4 \text{ }\mu\text{eV}$	Pohl et al. (2016)
$\mu^3\text{He}^+$	$15.30 \text{ meV} \pm 0.52 \text{ meV}$	Franke et al. (2017)	$0.05 \text{ meV}$	
$\mu^4\text{He}^+$	$9.34 \text{ meV} \pm 0.25 \text{ meV}$ $-0.15 \text{ meV} \pm 0.15 \text{ meV (3PE)}$	Diepold et al. (2018) Pachucki et al. (2018)	$0.05 \text{ meV}$	Krauth et al. (2020)

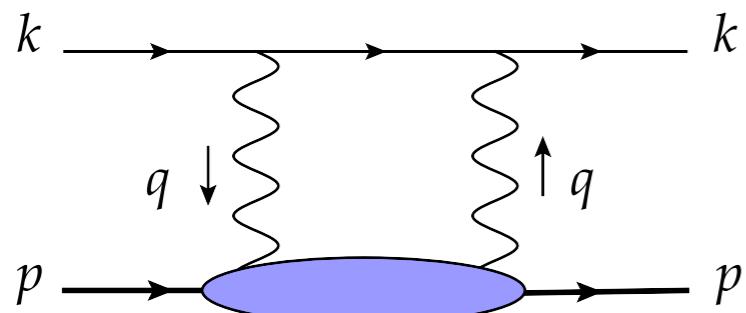
$\mu\text{H}$ :

present accuracy comparable with experimental precision  
Future: factor 5 improvement planned @PSI

$\mu\text{D}, \mu^3\text{He}^+, \mu^4\text{He}^+:$

present accuracy factor 5-10 worse than experimental precision

# Two-photon exchange: hadronic corrections



$$T^{\mu\nu}(p, q) = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\nu, Q^2) + \frac{1}{M^2} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right) T_2(\nu, Q^2)$$

- Two-photon exchange (TPE): lower blob contains both elastic (nucleon) and inelastic states
- Lamb shift: described by unpolarized amplitudes  $T_1, T_2$ : functions of energy  $\nu$  and  $Q^2$
- Hyperfine splitting: described by polarized amplitudes  $S_1, S_2$
- Imaginary parts: directly proportional to nucleon structure functions  $F_1, F_2$  resp.  $g_1, g_2$
- Real parts: obtained as dispersion integral over the imaginary parts modulo a subtraction function in case of  $T_1$

$$\begin{aligned}\Delta E &= \Delta E^{el} \\ &+ \Delta E^{subtr} \\ &+ \Delta E^{inel}\end{aligned}$$

- Elastic state: involves **nucleon form factors**
- Subtraction: involves **nucleon polarizabilities**
- Inelastic state: involves **nucleon structure functions**

**Hadron/Nuclear physics input needed !**

# Two-Photon Exchange (TPE) in Lamb shift

wave function at  
the origin

$$\Delta E(nS) = 8\pi\alpha m \phi_n^2 \frac{1}{i} \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{(Q^2 - 2\nu^2) T_1(\nu, Q^2) - (Q^2 + \nu^2) T_2(\nu, Q^2)}{Q^4(Q^4 - 4m^2\nu^2)}$$

dispersion relation  
& optical theorem

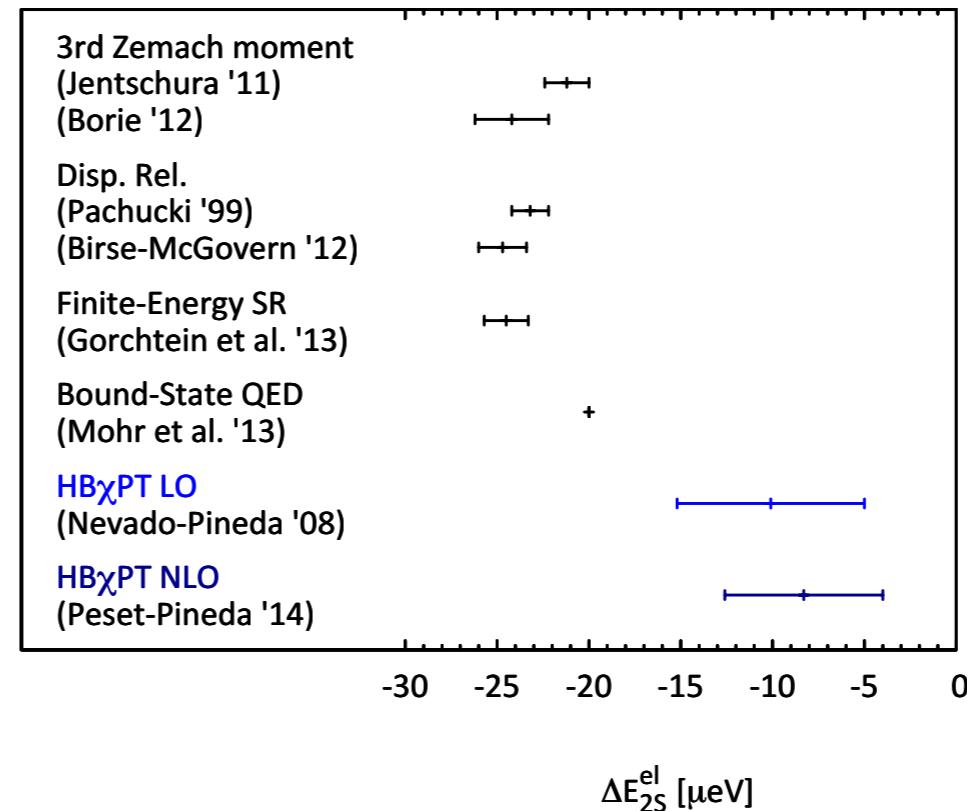
$$T_1(\nu, Q^2) = T_1(0, Q^2) + \frac{32\pi Z^2 \alpha M \nu^2}{Q^4} \int_0^1 dx \frac{x F_1(x, Q^2)}{1 - x^2(\nu/\nu_{\text{el}})^2 - i0^+}$$
$$T_2(\nu, Q^2) = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 dx \frac{F_2(x, Q^2)}{1 - x^2(\nu/\nu_{\text{el}})^2 - i0^+}$$

low-energy expansion:

$$\lim_{Q^2 \rightarrow 0} \overline{T}_1(0, Q^2)/Q^2 = 4\pi\beta_{M1}$$

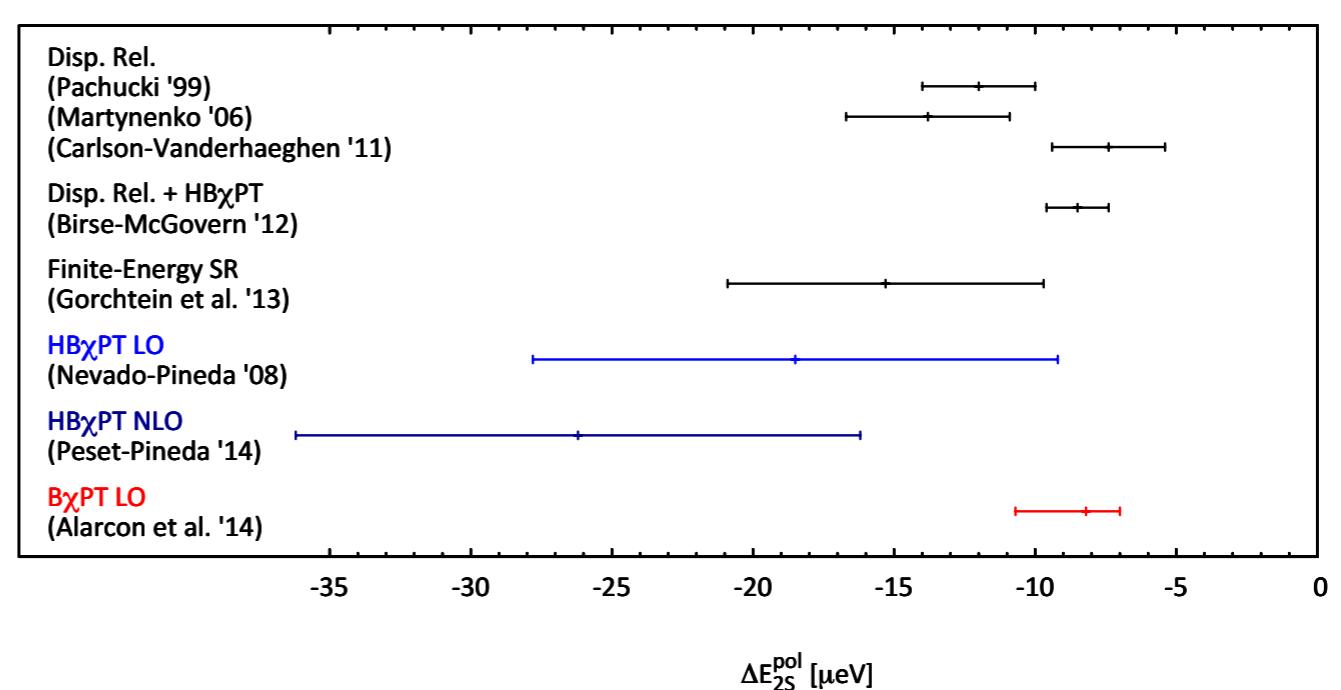
**Caution:**  
in the dispersive approach  
the  $T_1(0, Q^2)$  subtraction function  
is model-dependent!

## TPE elastic correction:



## TPE polarizability correction:

Hagelstein, Miskimen,  
Pascalutsa (2016)



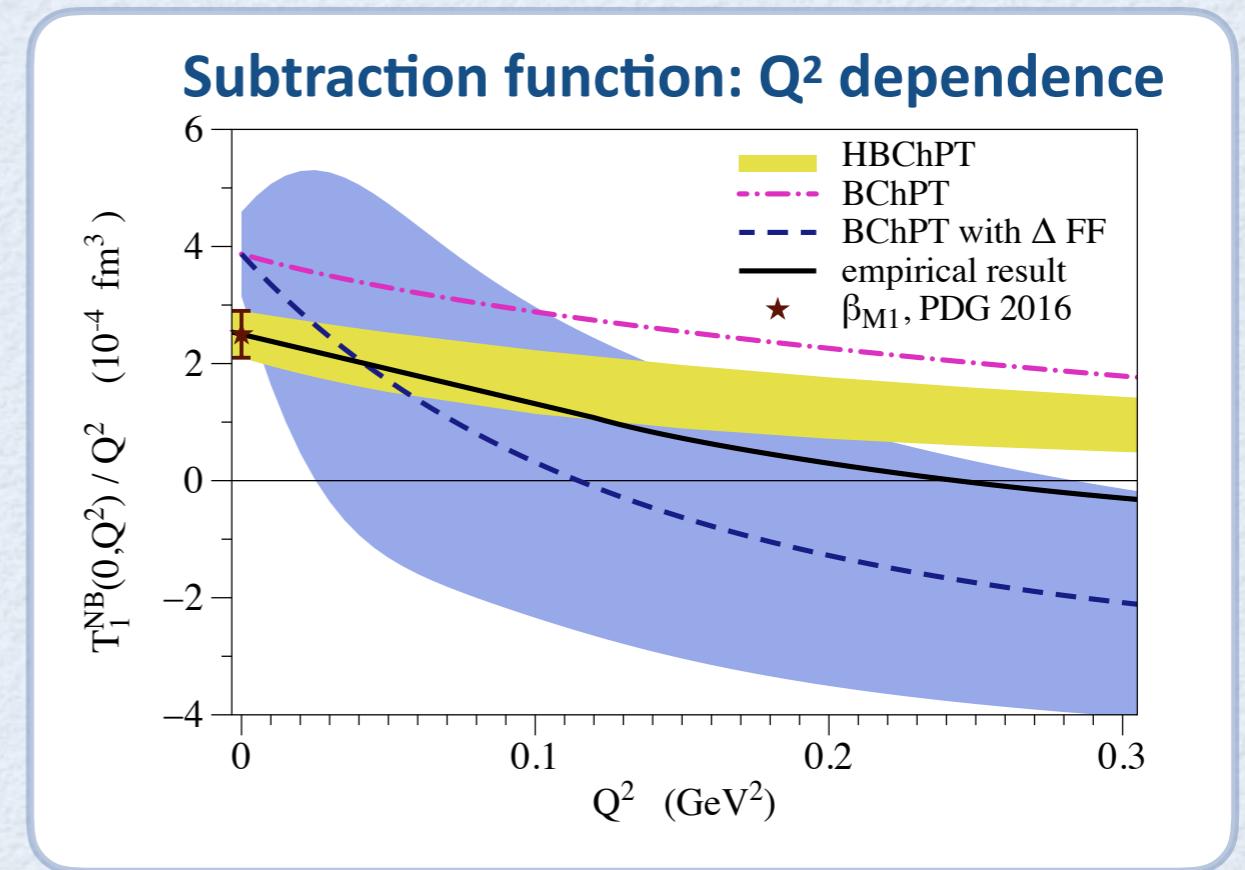
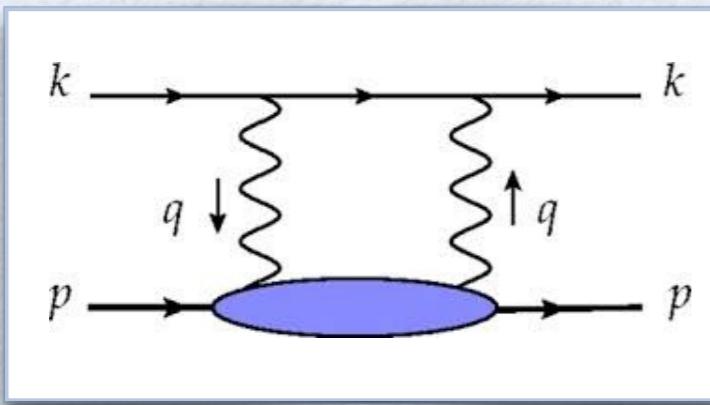
recent review: Antognini, Hagelstein, Pascalutsa Ann.Rev.Nucl.Part.Sci. 72 (2022) 389

see talk: Hagelstein, Feng, and session on Saturday

# Improved determination of subtraction function (Lamb shift)

**Future plan @PSI:  
factor 5 improvement  
on LS for muonic H !**

**Antognini, Pohl**

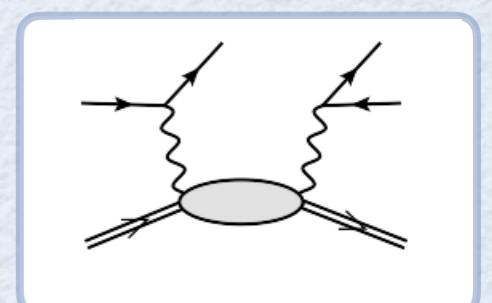


**Lensky, Hagelstein, Pascalutsa, Vdh (2018)**

**To improve on uncertainty due to subtraction function: 3 avenues**

- Full NLO calculation in Baryon ChPT    **Pascalutsa et al.**
- New prospect for lattice determination of subtraction function    **Hagelstein, Pascalutsa (2020)**
- Empirical determination of  $Q^4$  term using dilepton production process

**Pauk, Carlson, Vdh (2020)**



# Two-Photon Exchange (TPE) in Hyperfine Splitting

$$\Delta_{\text{pol}} = \frac{\alpha m}{2\pi(1+\kappa)M} [\Delta_1 + \Delta_2]$$

with  $v = \sqrt{1 + 1/\tau}$ ,  $v_l = \sqrt{1 + 1/\tau_l}$ ,  $\tau_l = Q^2/4m^2$  and  $\tau = Q^2/4M^2$

$$\begin{aligned} \Delta_1 &= 2 \int_0^\infty \frac{dQ}{Q} \left( \frac{5 + 4v_l}{(v_l + 1)^2} [4I_1(Q^2) + F_2^2(Q^2)] - \frac{32M^4}{Q^4} \int_0^{x_0} dx x^2 g_1(x, Q^2) \right. \\ &\quad \times \left. \left\{ \frac{1}{(v_l + \sqrt{1 + x^2\tau^{-1}})(1 + \sqrt{1 + x^2\tau^{-1}})(1 + v_l)} \left[ 4 + \frac{1}{1 + \sqrt{1 + x^2\tau^{-1}}} + \frac{1}{v_l + 1} \right] \right\} \right) \\ \Delta_2 &= 96M^2 \int_0^\infty \frac{dQ}{Q^3} \int_0^{x_0} dx g_2(x, Q^2) \left\{ \frac{1}{v_l + \sqrt{1 + x^2\tau^{-1}}} - \frac{1}{v_l + 1} \right\} \end{aligned}$$

No subtraction function !

$$I_1(Q^2) = \frac{2M^2}{Q^2} \int_0^{x_0} dx g_1(x, Q^2)$$

$$I_1^{\text{non-pol}}(Q^2) = I_A^{\text{non-pol}}(Q^2) = -\frac{1}{4}F_2^2(Q^2)$$

No subtraction hypothesis underlies the **validity of Gerasimov-Drell-Hearn sum rule**

For proton: integrand tested up to photon energies 2.9 GeV (MAMI/ELSA)

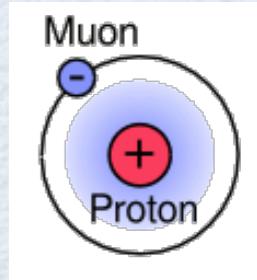
→ will be extended up to 12 GeV at JLab (E12-20-011)

- \* proton-polarizability effect on the HFS is completely *constrained by empirical information*
- \* a ChPT calculation puts the reliability of dispersive calculations to the test, or vice versa

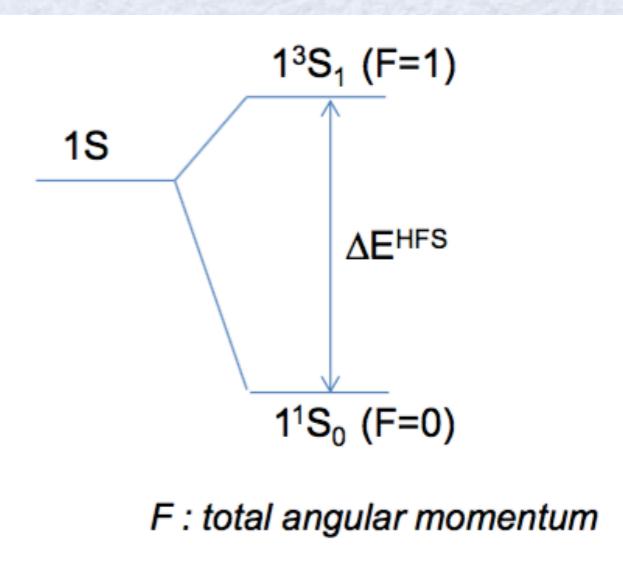
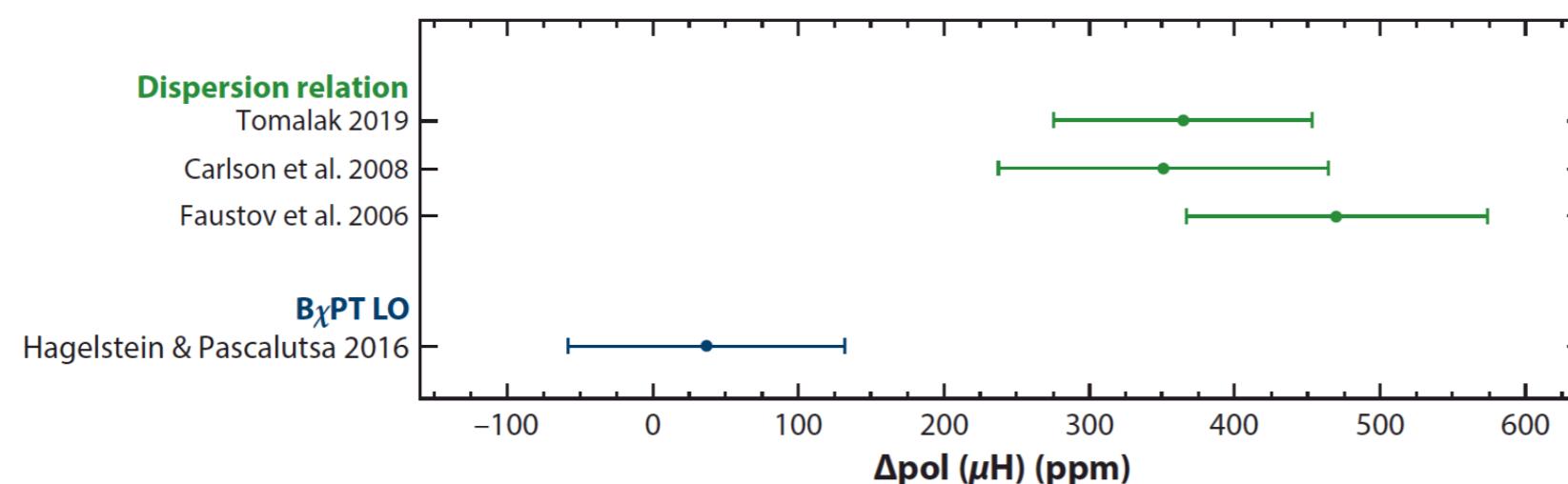
# Hyperfine Splitting in muonic Hydrogen



Measurements of the  $\mu\text{H}$  ground-state HFS planned by  
CREMA, FAMU, J-PARC collaborations **precision goal: 1ppm !**



**Currently: disagreement between data-driven evaluations  
and chiral perturbation theory**

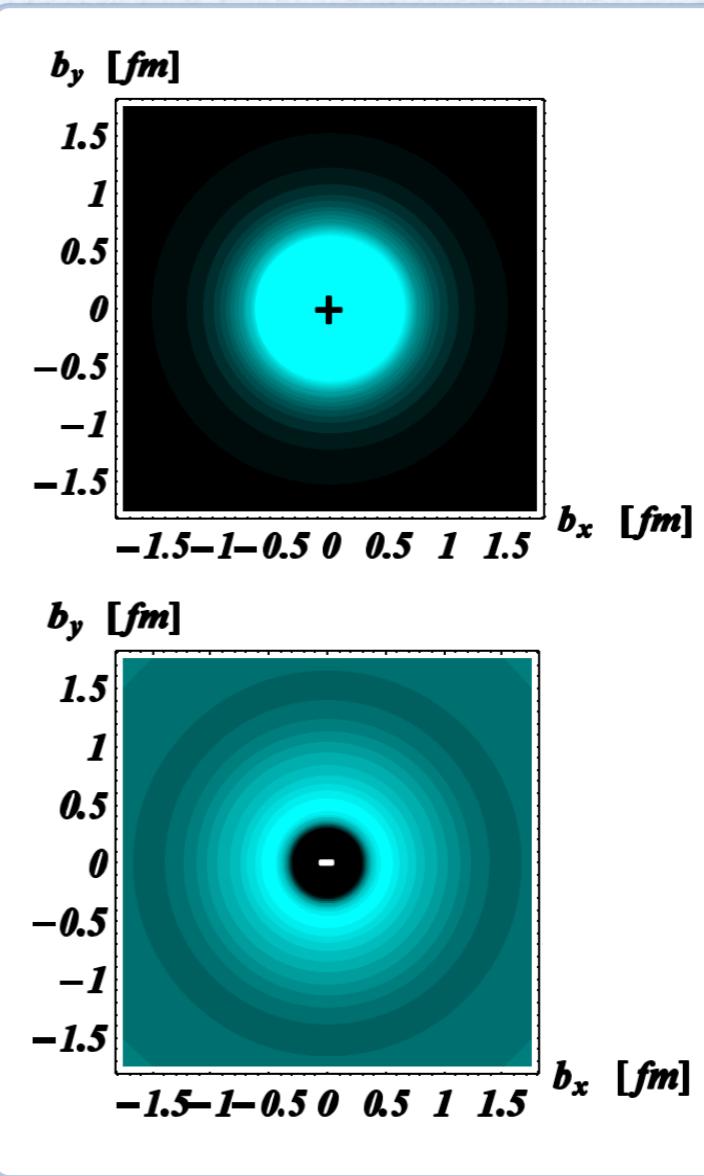
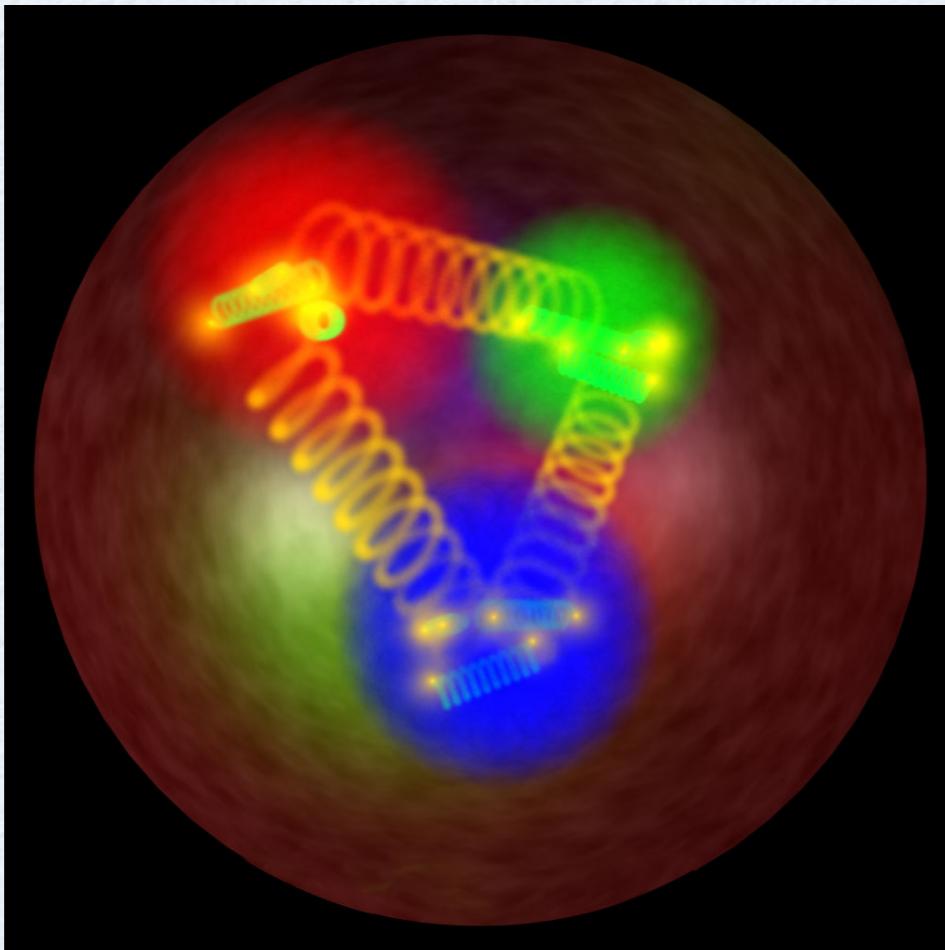


Calls for re-evaluation of empirical parametrizations of nucleon structure functions

Antognini, Hagelstein, Pascalutsa (2022)

see talks: Carlson, Hagelstein (theory) and Ruth (exp)

# Spatial distribution of quarks in nucleons



# Radii of charge distribution for nucleons

→ Radius of 2-dim transverse distribution of quarks of flavor q in proton:

$$\langle b^2 \rangle^q = \frac{\int d^2 \mathbf{b} \mathbf{b}^2 \rho^q(b)}{\int d^2 \mathbf{b} \rho^q(b)} = -4 \frac{F_1'^q(0)}{F_1^q(0)}$$

with:  $F_1'^q(0) \equiv \frac{dF_1^q}{dQ^2} \Big|_{Q^2=0}$

→ Isospin symmetry:  $F_{1p} = e_u F_1^u + e_d F_1^d$   
 $F_{1n} = e_u F_1^d + e_d F_1^u$

$$\begin{aligned}\langle b^2 \rangle_p &= \frac{4}{3} \langle b^2 \rangle^u - \frac{1}{3} \langle b^2 \rangle^d = -4 F_{1p}'(0) \\ \langle b^2 \rangle_n &= \frac{2}{3} \langle b^2 \rangle^d - \frac{2}{3} \langle b^2 \rangle^u = -4 F_{1n}'(0)\end{aligned}$$

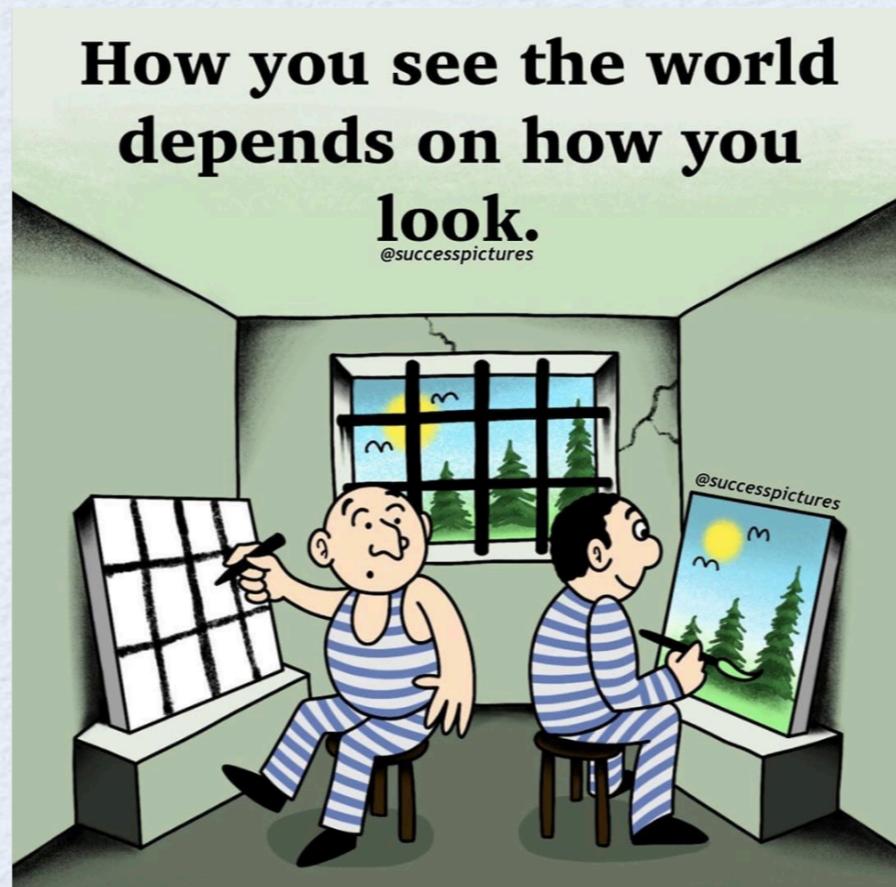
→ Charge radii for proton and neutron:

	$r_E^2$ (fm <sup>2</sup> )	$-\frac{3\kappa_N}{2M^2}$ (fm <sup>2</sup> )	$-6F_1'(0)$ (fm <sup>2</sup> )	$\langle b^2 \rangle$ (fm <sup>2</sup> )
Cui et al. (2021)	proton (e-p) 0.717 ± 0.014		0.598 ± 0.014	0.399 ± 0.009
Antognini et al. (2013)	proton ( $\mu$ H) 0.7071 ± 0.0007	-0.1189	0.5882 ± 0.0007	0.3921 ± 0.0005
	neutron (PDG) -0.1161 ± 0.0022	0.1266	0.0105 ± 0.0022	0.0070 ± 0.0015

H.Gao, M.Vdh  
(2021)

see talks: Atac  
 $N \rightarrow \Delta$  FFs:  
 Papanicolas,  
 Paolone

# Proton gluonic radius



# Nucleon Energy-Momentum Tensor (EMT)

→  $\langle P | T^{\mu\nu} | P \rangle = 2P^\mu P^\nu$

$$2M^2 = \langle P | \frac{\tilde{\beta}(g)}{2g} G_{\alpha\beta}^a G^{a\alpha\beta} | P \rangle + \langle P | \sum_{l=u,d,s} m_l (1 + \gamma_{m_l}) \bar{q}_l \bar{q}_l | P \rangle$$

In chiral limit all of hadron mass  
is due to the trace anomaly

Gluonic contribution

Quark contributions to hadron  
mass: sigma-terms

~ 10% (u, d, s) Lattice QCD

→ Matrix element of full nucleon EMT: parametrised by **3 form factors (FFs)**

$$\langle P + \frac{q}{2} | \textcolor{red}{T^{\mu\nu}}(0) | P - \frac{q}{2} \rangle = \bar{u}(P + \frac{q}{2}) \left\{ \textcolor{red}{A(Q^2)} \gamma^{(\mu} P^{\nu)} + \textcolor{red}{B(Q^2)} P^{(\mu} i\sigma^{\nu)\alpha} \frac{q_\alpha}{2M} \right. \\ \left. + \textcolor{red}{C(Q^2)} (q^\mu q^\nu - q^2 g^{\mu\nu}) \frac{1}{M} \right\} u(P - \frac{q}{2})$$

Pagels  
(1966)

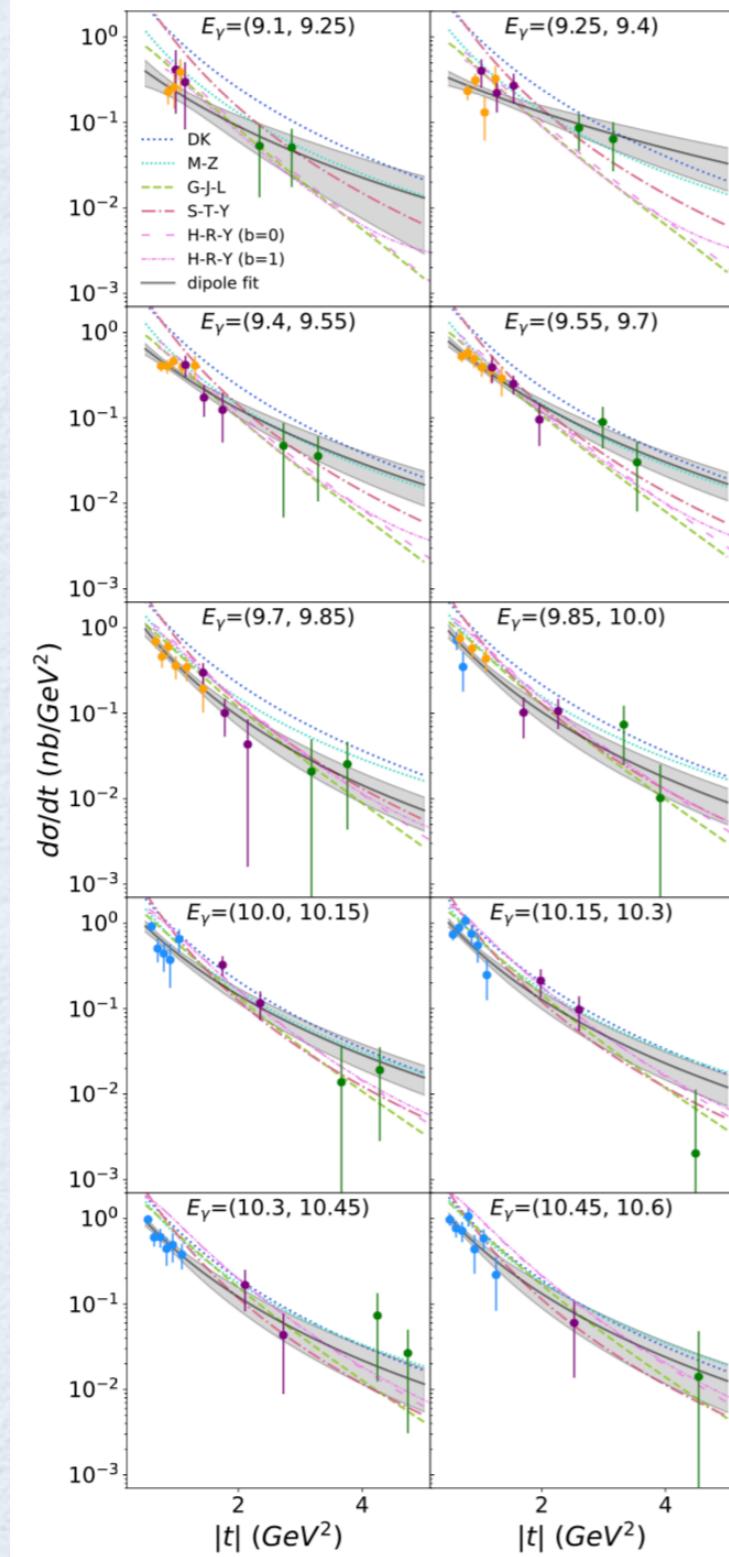
**A** → momentum distribution

**A + B** → angular momentum distribution

**C** → pressure distribution

# Threshold J/ $\psi$ photo-production

JLab/Hall C data

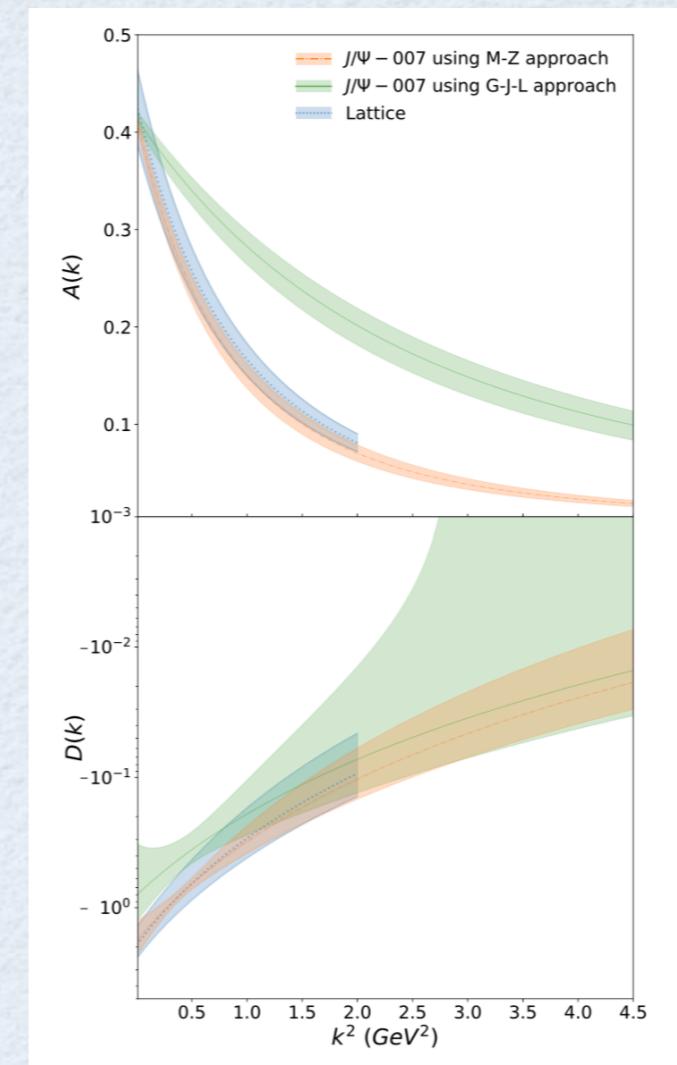


Duran, Meziani, Joosten, et al. (2022)

- Analyzed in 2 model approaches + lattice QCD:
- Holographic QCD model (**M-Z**) Mamo, Zahed (2021)
  - GPD model (**G-J-L**) Guo, Ji, Liu (2021)
  - Lattice QCD Pefkou, Hackett, Shanahan (2022)

$A_g$

$$D_g \equiv 4 C_g$$



see talks:  
Meziani,  
Alharazin

# Correlations in transverse position/longitudinal momentum

elastic  
scattering

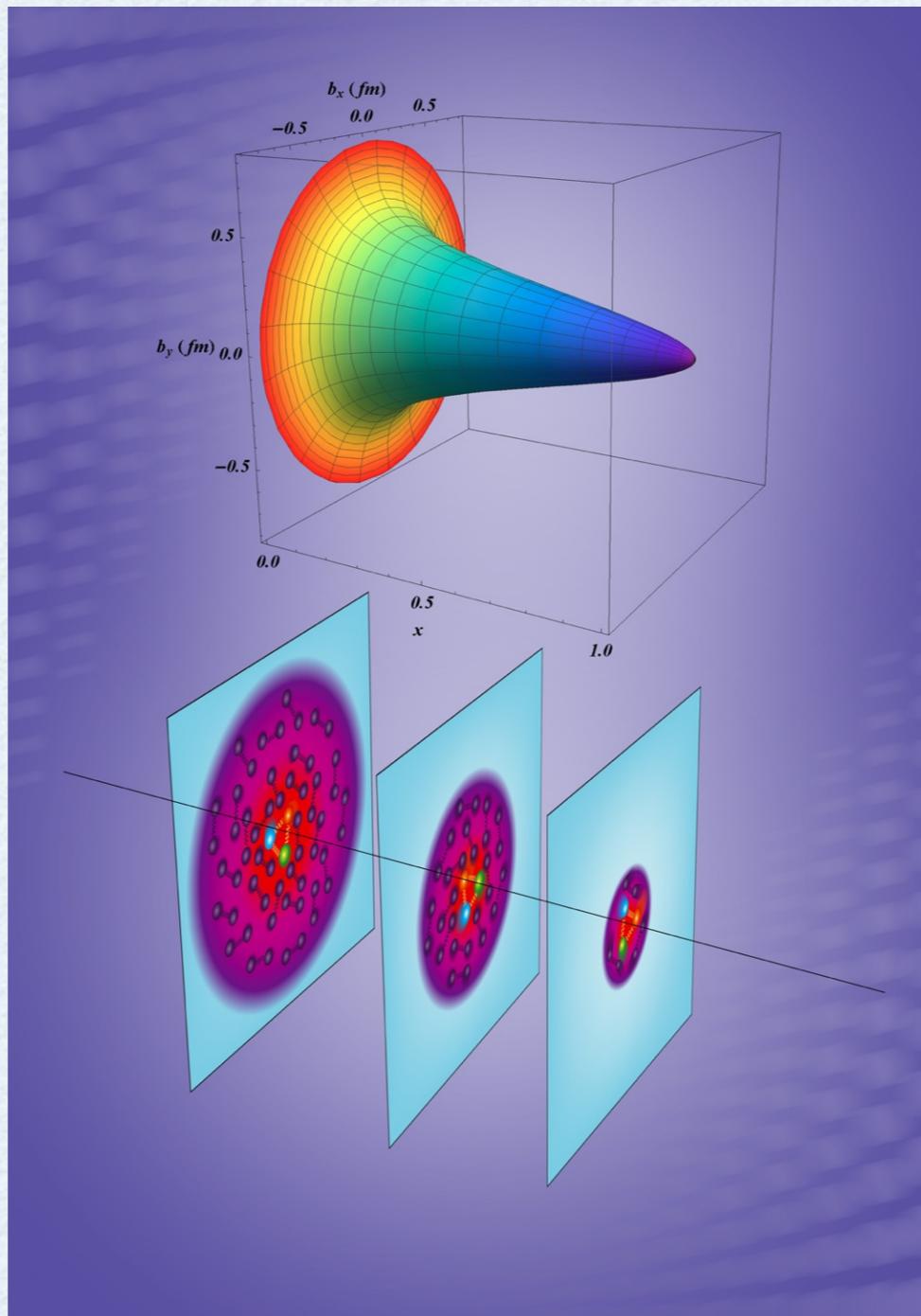


quark  
distributions in  
transverse  
position space

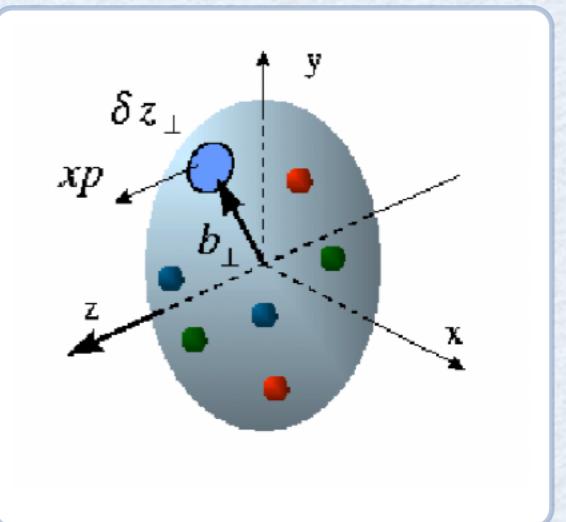
proton  
3D imaging

Burkardt (2000, 2003)

Belitsky, Ji, Yuan  
(2004)



quark  
distributions in  
longitudinal  
momentum

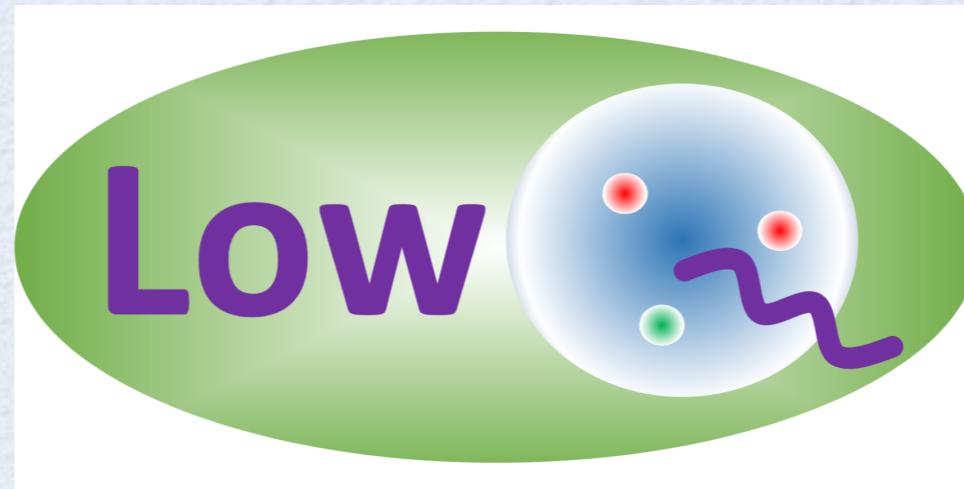


see talks: Alexandrou,  
Constantinou, Orginos



## Near future perspectives at low Q

- **hadronic corrections** to Lamb shift in **muonic atoms**: shift from puzzle to **precision** !
  - **$\mu H$  LS**: CREMA coll. : factor 5 improvement planned
  - **$\mu H$  1S HFS**: next frontier 1ppm precision !
- **muon scattering plans**:
  - MUSE@PSI see talks: Gilman, Lin, Patel
  - AMBER@COMPASS++
- **electron/positron scattering plans**:
  - PRad-II@JLab see talk: Higinbotham
  - ULQ<sup>2</sup>@Tohoku see talk: Suda
  - MAGIX@MESA see talk: Bernauer
  - JLab, e<sup>+</sup> @JLab see talks: Jones, Voutier
- **Close synergy experiment <-> theory to move field forward**



*Wishing us a stimulating workshop*



A F Ḥ + Ā

e-u-po-ro-wo