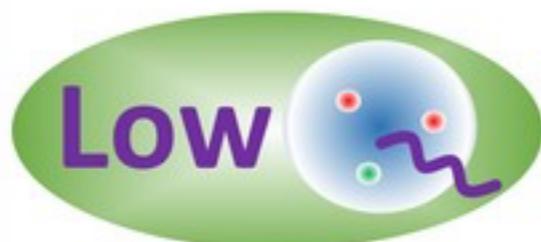
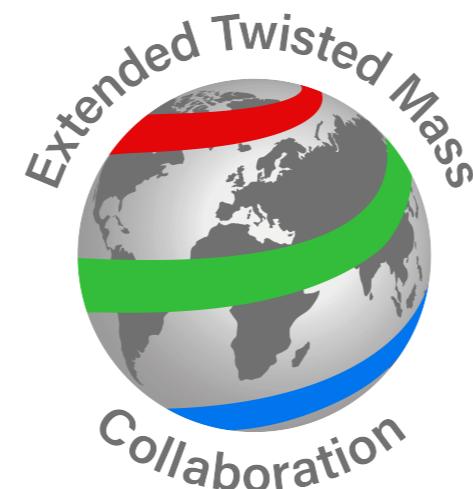


Recent highlights from lattice QCD



Constantia Alexandrou



Nucleon Structure at Low Q
AVRA IMPERIAL HOTEL, CRETE, GREECE, 15 MAY – 22 MAY 2023

Outline

*Introduction

- State-of-the-art lattice QCD simulations

* 3D structure of the nucleon

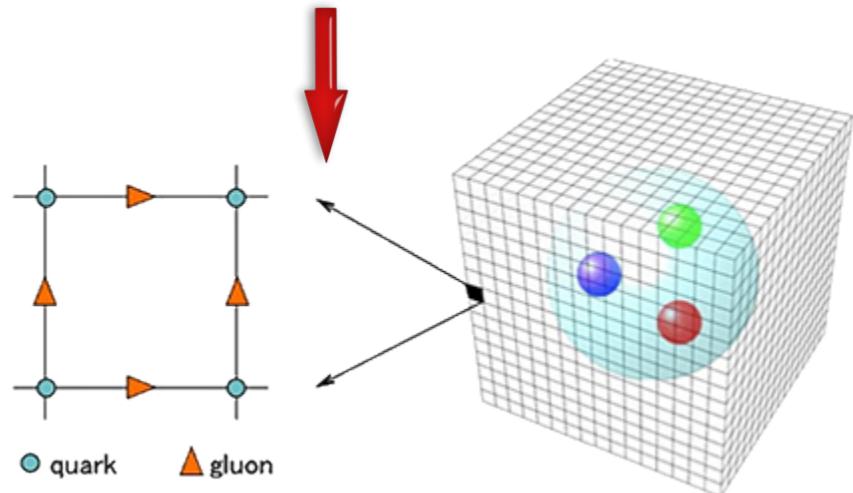
- Mellin moments (for direct computation of GPDs see talks by K. Orgas and M. Constantinou)
 - Scalar and tensor charges
 - Electromagnetic form factors and strangeness of the nucleon
 - Spin content of the nucleon

*Hadronic vacuum polarisation contribution to the muon g-2

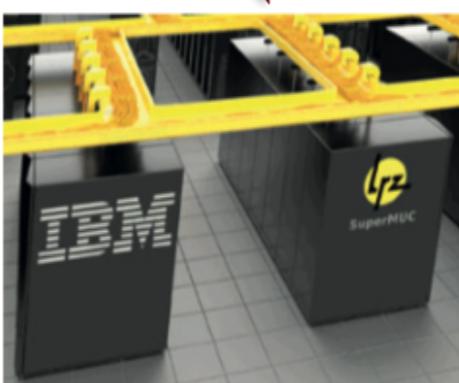
*Conclusions

Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \text{Det}(D_f[U]) \right) e^{-S_{\text{QCD}}[U]}$$



Simulation of gauge
ensembles $\{U\}$

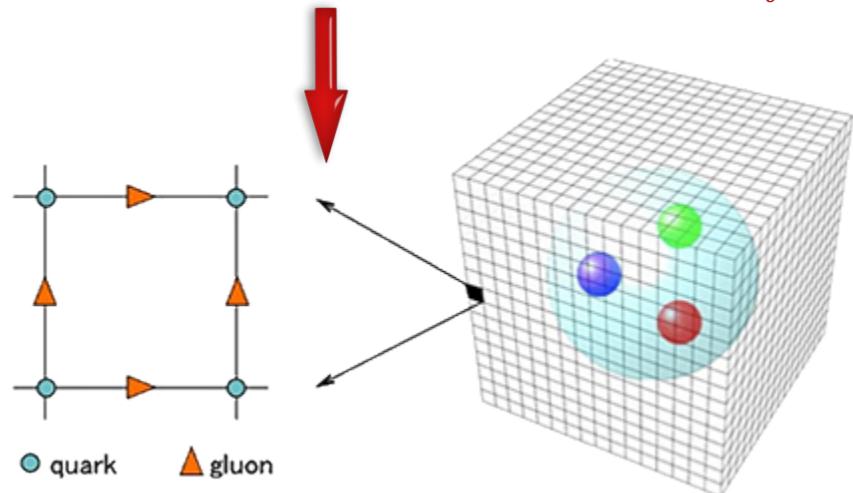


Quark & gluon
propagators

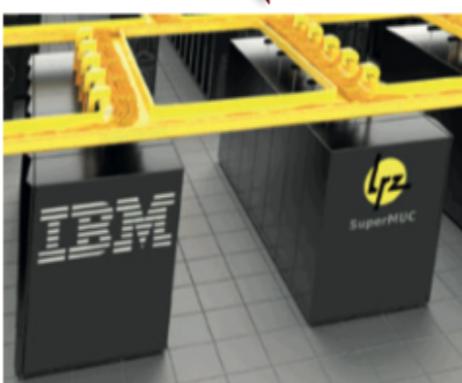


Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] O(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \text{Det}(D_f[U]) \right) e^{-S_{\text{QCD}}[U]}$$



Simulation of gauge ensembles $\{U\}$

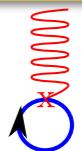
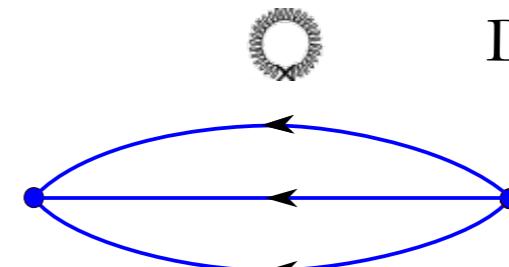


Quark & gluon propagators

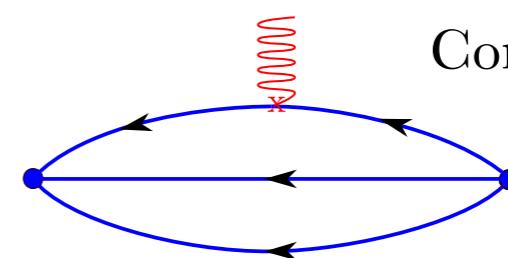


contractions

Disconnected

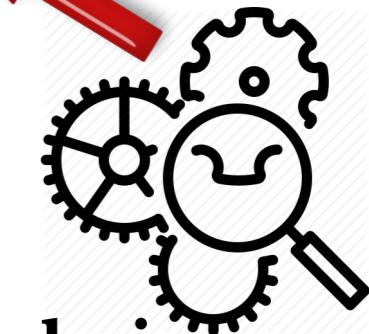
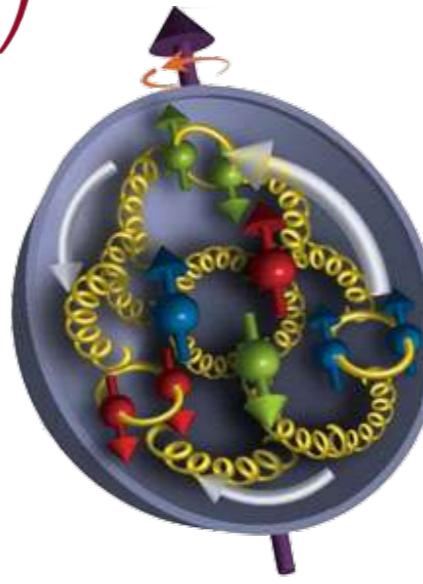
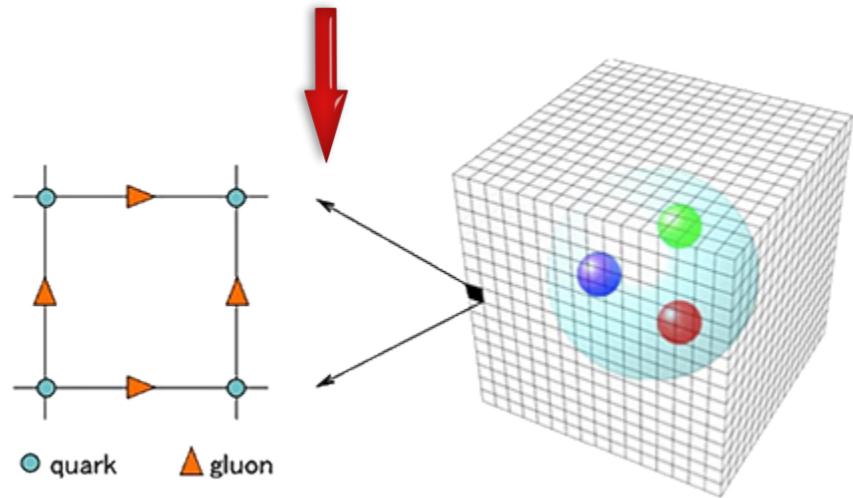


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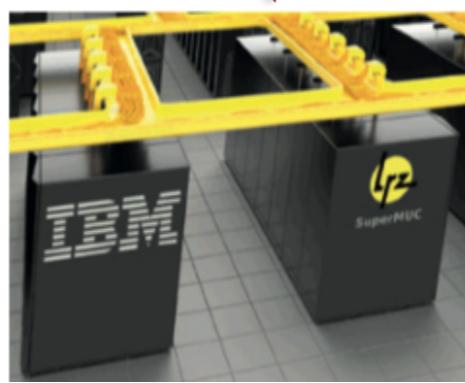


Lattice QCD

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D_f^{-1}[U], U) \left(\prod_{f=u,d,s,c} \text{Det}(D_f[U]) \right) e^{-S_{\text{QCD}}[U]}$$



Simulation of gauge ensembles $\{U\}$



Quark & gluon propagators



contractions

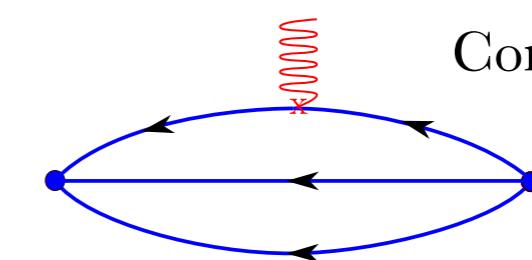
Data Analysis



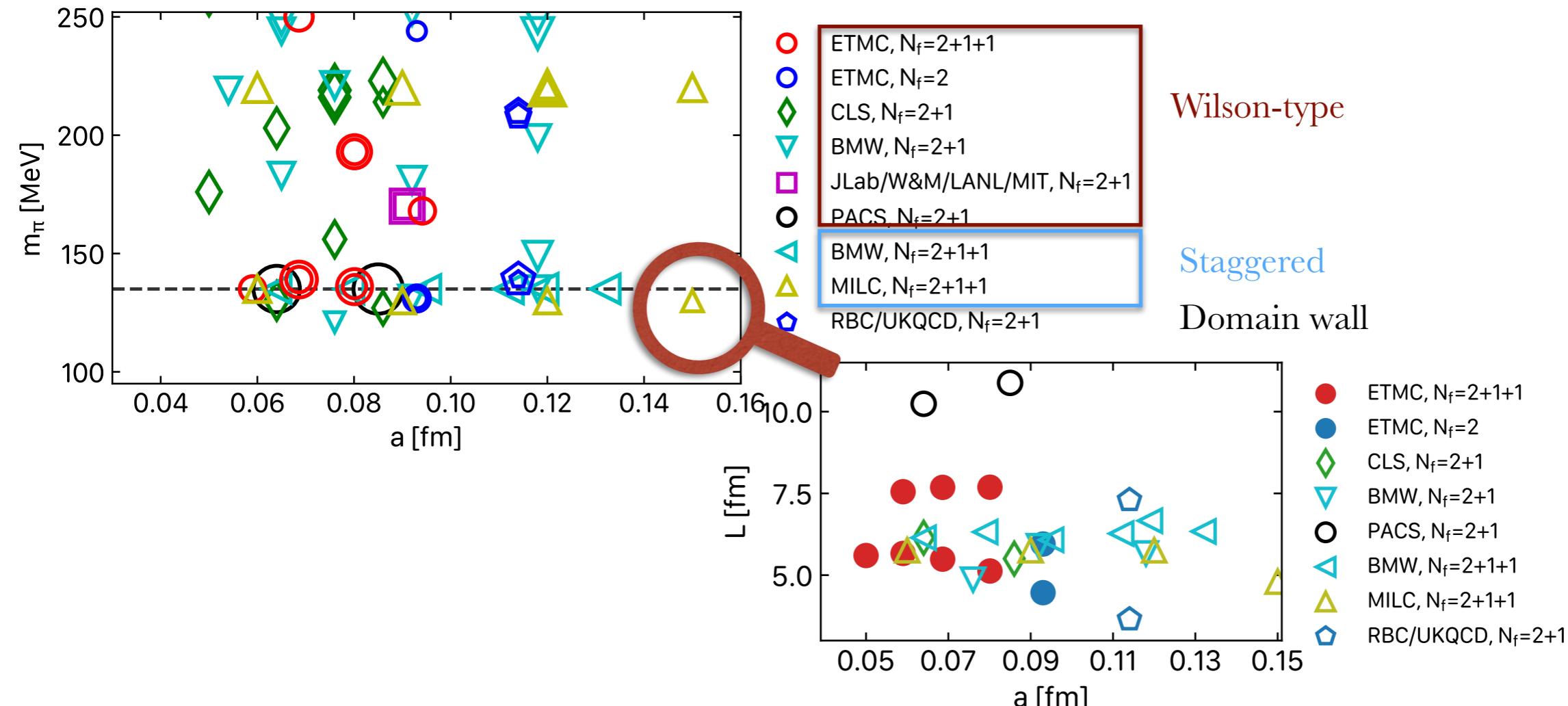
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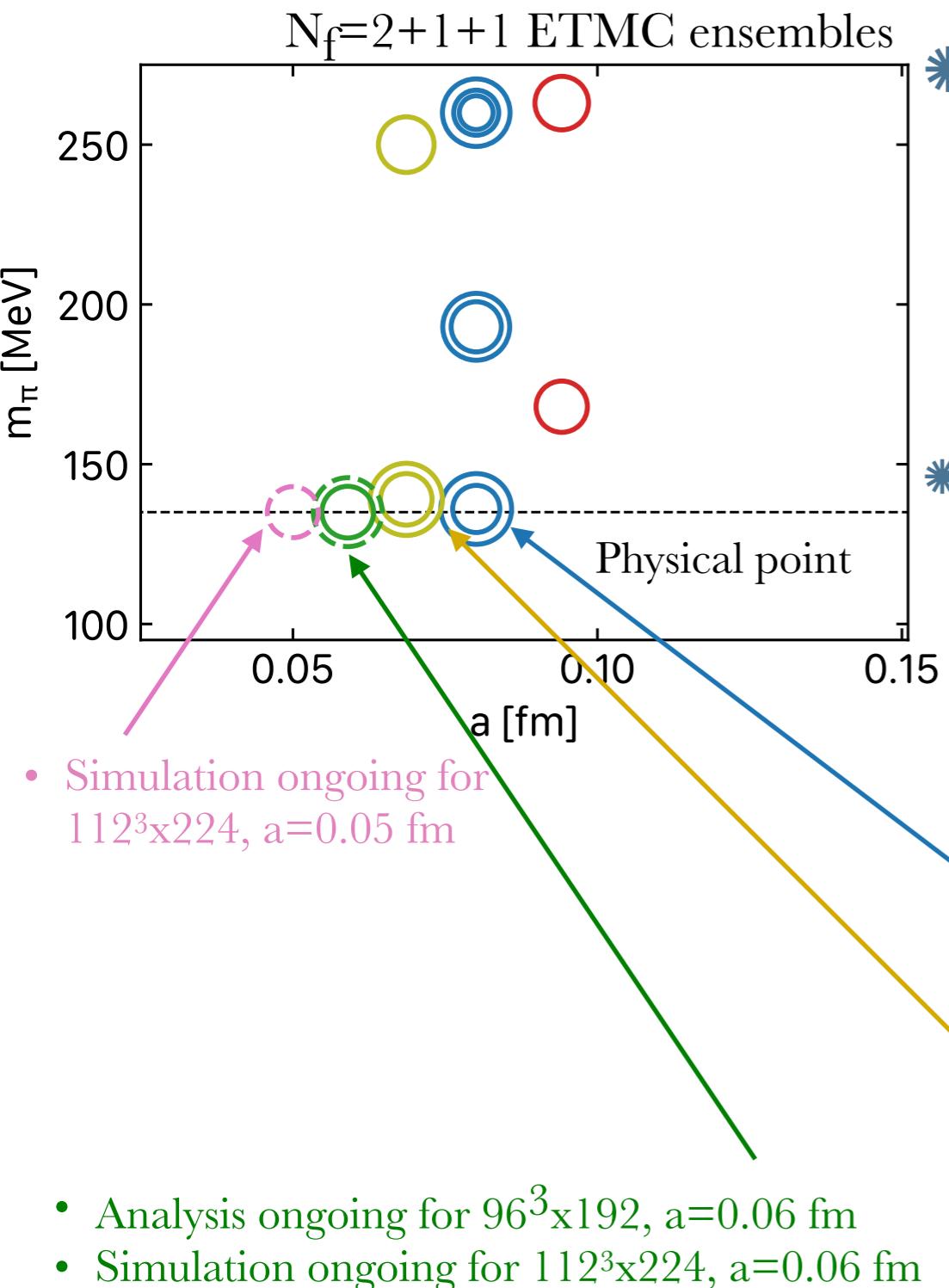
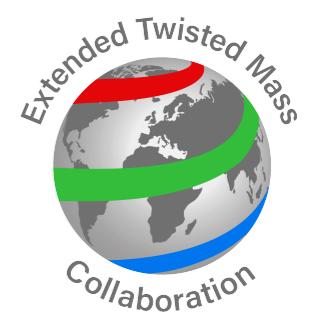
Status of current simulations



* A number of collaborations has physical point ensembles:

- ▶ Wilson-type: **BMW**, **ETMC**, **CLS**, **PACS**
 - **BMW**, **CLS** and **ETMC** have multiple lattice spacings $0.05 < a < 0.1$ fm
 - PACS has two large volume ensembles at two lattice spacings
- ▶ Staggered at physical point: **MILC** with 4 and **BMW** with 6 lattice spacings $0.05 < a < 0.15$ fm
- ▶ Domain wall at physical point: **RBC/UKQCD** with 2 lattice spacings

Gauge ensembles generated by ETMC



- * 5 ensembles completed and 2 under production at physical pion mass
 - 4 lattice spacings $0.05 < a < 0.1$ fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
 - Two volumes at $a=0.08$ fm, 0.07 fm and 0.06 fm of $Lm_\pi \sim 3.6$ (5.1 fm) and $Lm_\pi \sim 5.4$ (7.7 fm)
- * Algorithmic improvements needed to go to $a < 0.05$ fm due to critical slow down in HMC (long autocorrelations) —> new approaches e.g. Machine learning approaches using equivariant flows
 - G. Kanwar, et al., Phys. Rev. Lett. 125 (2020) 121601, arXiv:2003.06413; D. Boyda, et al., Phys. Rev. D 103 (2021) 074504, arXiv: 2008.05456; M. S. Albergo *et al.*, Phys. Rev. D 104 (2021) 114507, arXiv:2106.05934
 - J. Finkenrath arXiv:2201.02216; S. Bacchio *et al.*, Phys. Rev. D 107 (2023) 5, L051504, arXiv:2212.08469
- Analysis completed for $64^3 \times 128$ $a=0.08$ fm
- Analysis ongoing for $96^3 \times 192$ $a=0.08$ fm
- Analysis completed for $80^3 \times 160$, $a=0.07$ fm
- Analysis ongoing for $112^3 \times 224$, $a=0.07$ fm

Systematics & Challenges

- **Simulations directly at the physical point**



Systematic effects from chiral extrapolation are eliminated

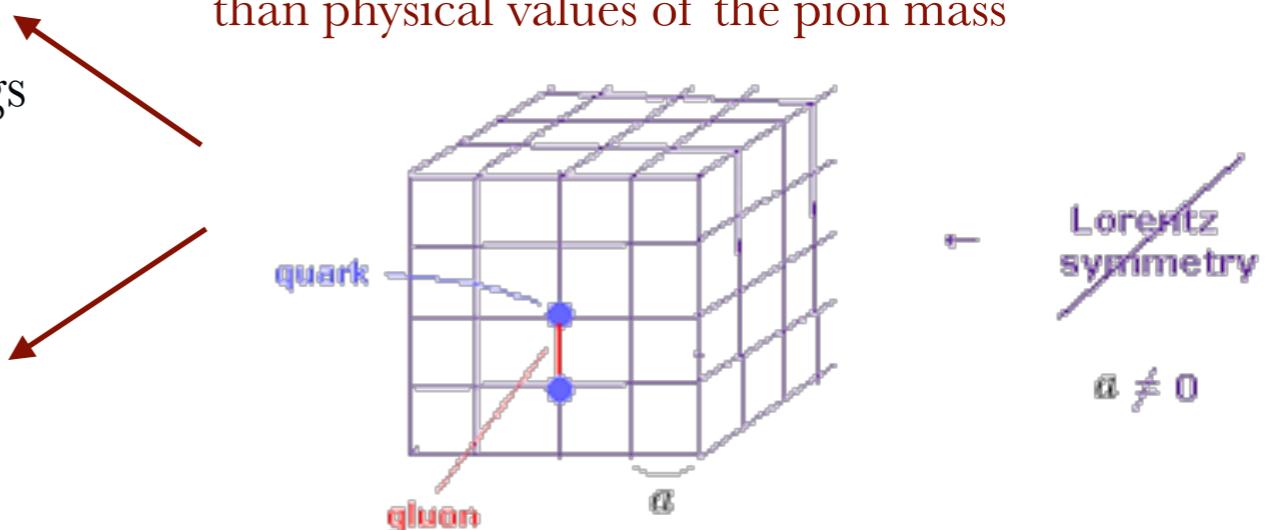
- **Discretisation effect:** Continuum limit

→ need simulations for at least 3 lattice spacings

Typically done using simulations for heavier than physical values of the pion mass

- **Finite volume effects:** Infinite volume limit

→ need simulations for at least 3 volumes



- **Ground-state identification**

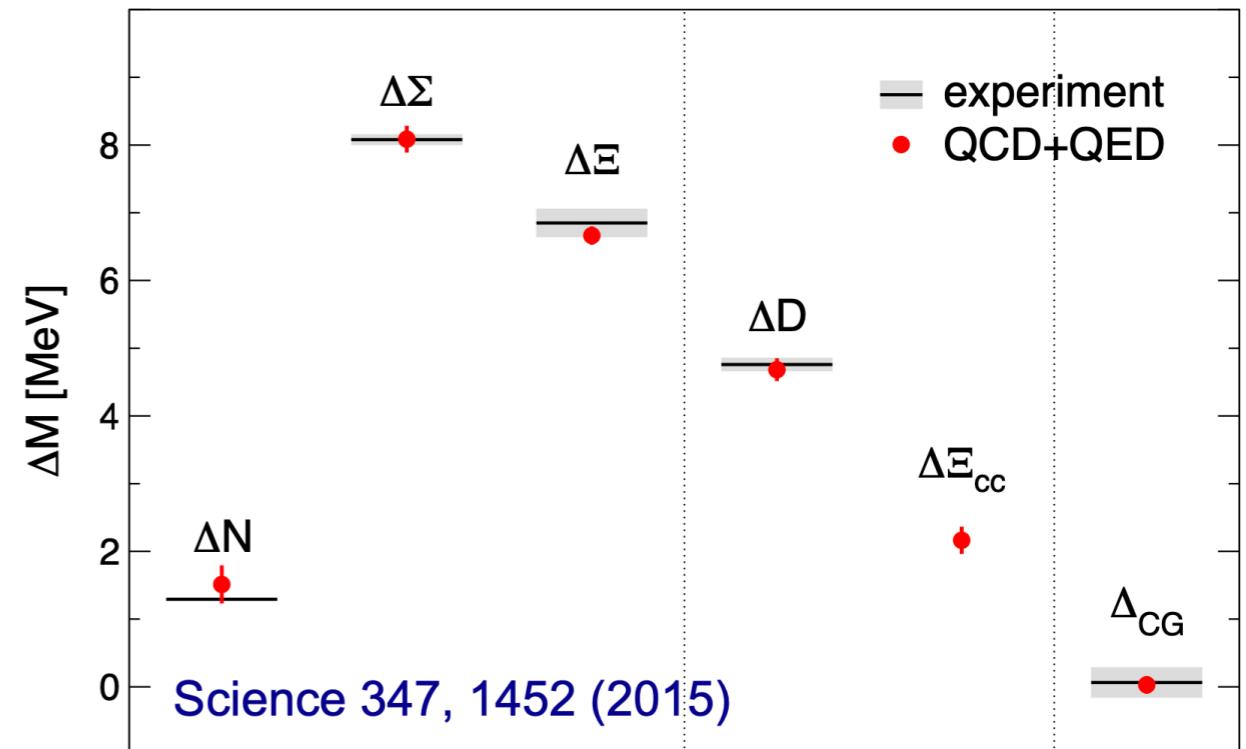
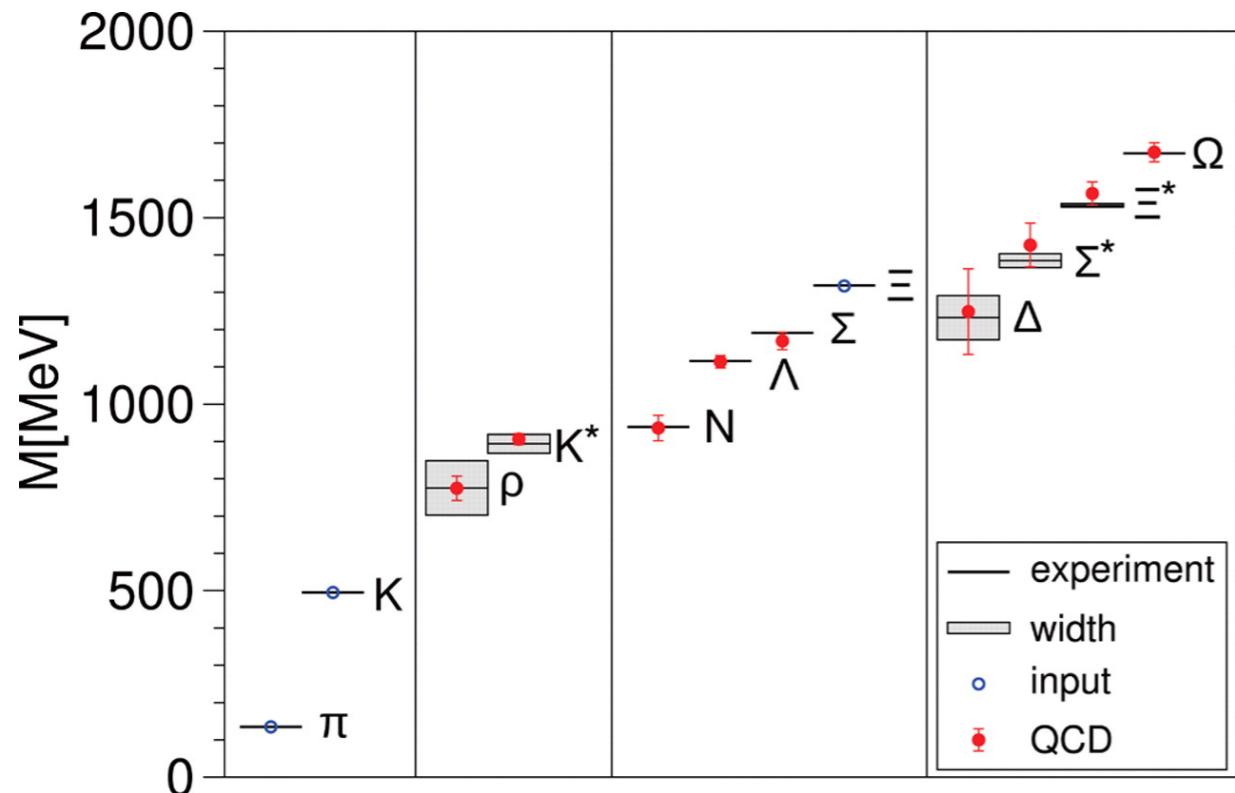
Cross-check (one-, two- and three-state fits, summation), but two or more particle states create difficulties

Low-lying hadron spectrum

BMW collaboration determined the low-lying hadron masses,
as well as the mass splittings

S. Durr *et al.*, Science 322 (2008) 1224

Sz. Borsanyi *et al.*, Science 347 (2015) 1452



Science 347, 1452 (2015)

	mass splitting [MeV]	QCD [MeV]	QED [MeV]
$\Delta N = n - p$	1.51(16)(23)	2.52(17)(24)	-1.00(07)(14)
$\Delta \Sigma = \Sigma^- - \Sigma^+$	8.09(16)(11)	8.09(16)(11)	0
$\Delta \Xi = \Xi^- - \Xi^0$	6.66(11)(09)	5.53(17)(17)	1.14(16)(09)
$\Delta D = D^\pm - D^0$	4.68(10)(13)	2.54(08)(10)	2.14(11)(07)
$\Delta \Xi_{cc} = \Xi_{cc}^{++} - \Xi_{cc}^+$	2.16(11)(17)	-2.53(11)(06)	4.69(10)(17)
$\Delta_{CG} = \Delta N - \Delta \Sigma + \Delta \Xi$	0.00(11)(06)	-0.00(13)(05)	0.00(06)(02)

Lattice QCD reproduces the low-lying hadron masses and mass splittings

Systematics & Challenges

- **Simulations directly at the physical point**



Systematic effects from chiral extrapolation are eliminated

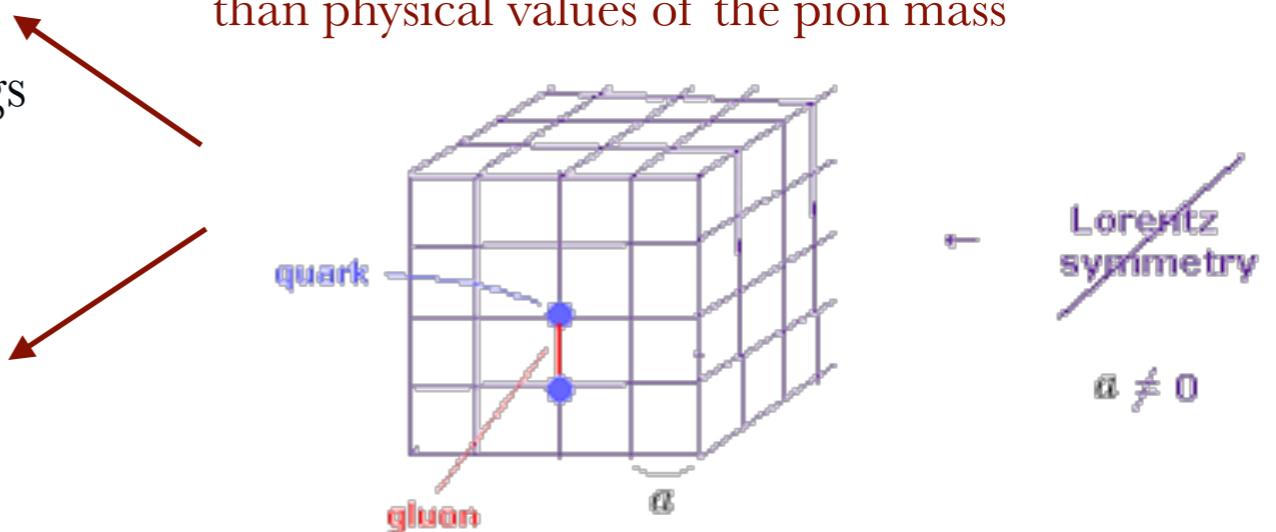
- **Discretisation effect:** Continuum limit

→ need simulations for at least 3 lattice spacings

Typically done using simulations for heavier than physical values of the pion mass

- **Finite volume effects:** Infinite volume limit

→ need simulations for at least 3 volumes



- **Ground-state identification**

Cross-check (one-, two- and three-state fits, summation), but two or more particle states create difficulties

- **Renormalisation**

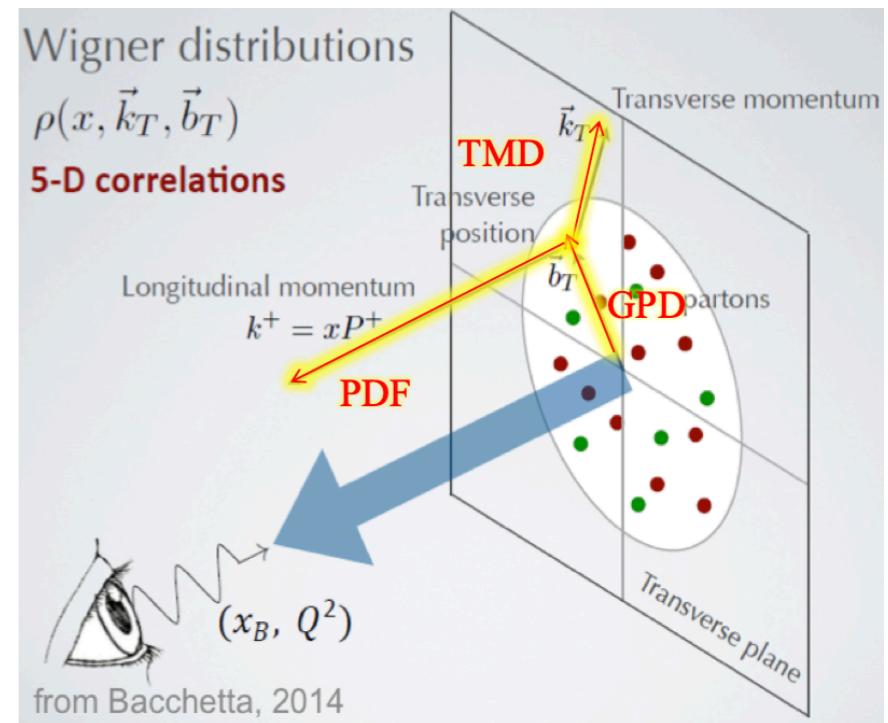
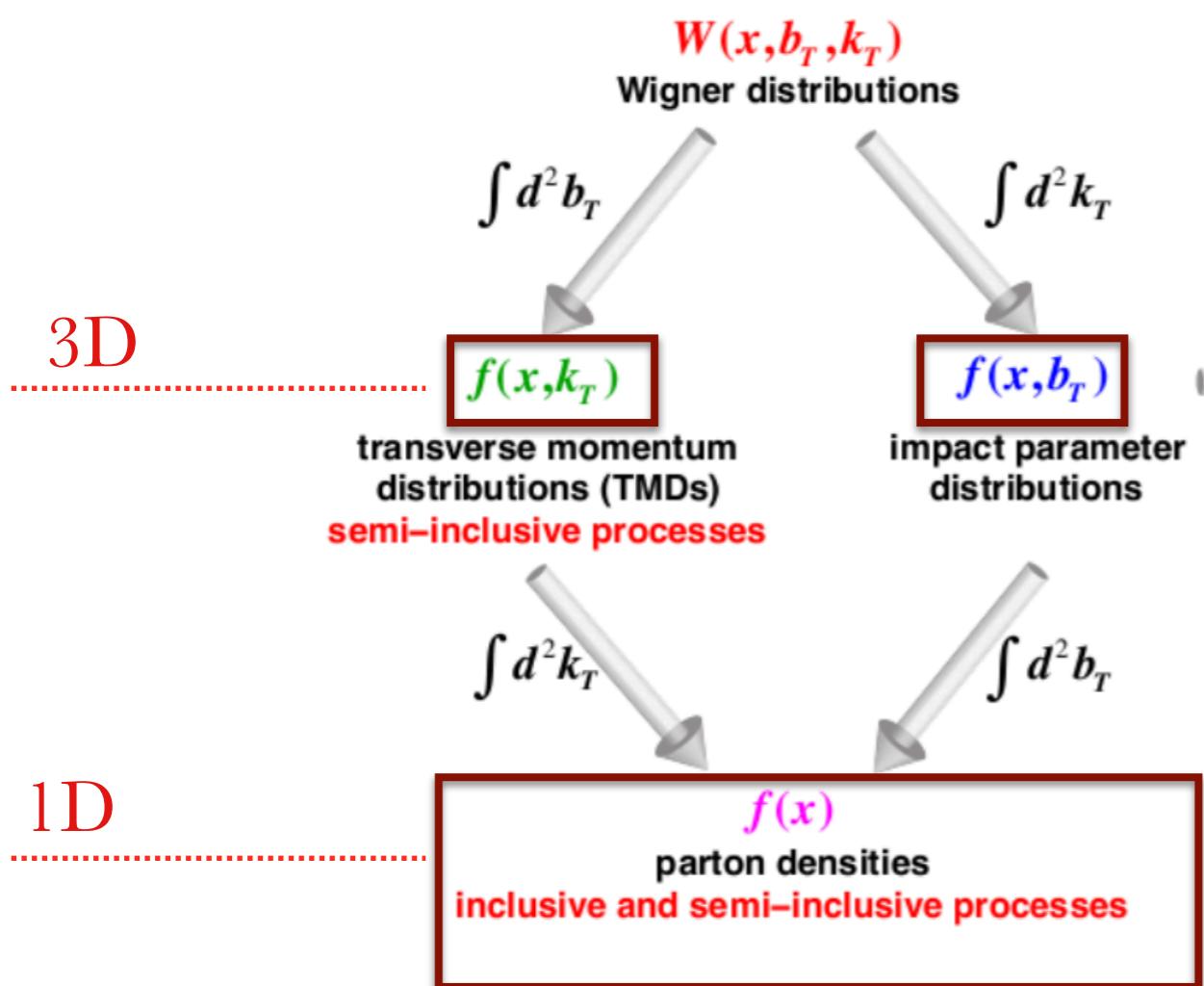
Non-perturbatively with improvements e.g using perturbative subtraction of lattice artefacts - more complicated for extended operators

- In what follows we assume **isospin symmetry** i.e. up and down quarks have equal mass, and **neglect EM effects for all except the muon g-2**

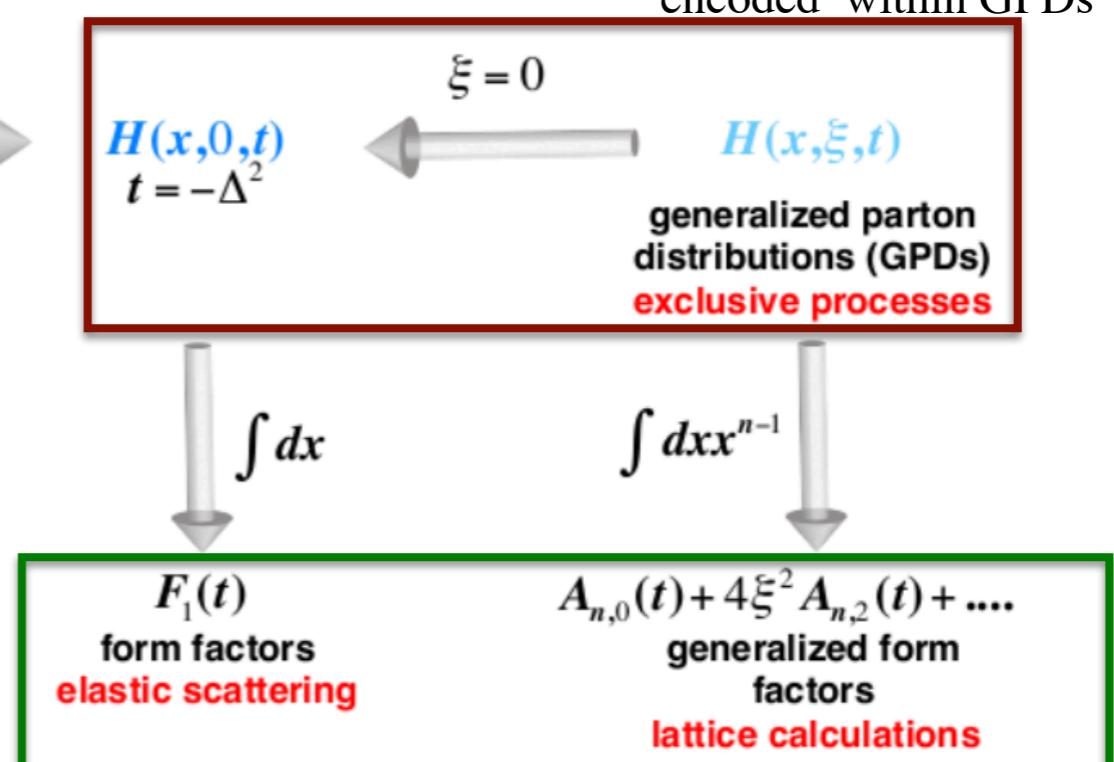
3D structure of the nucleon

⌘ Understanding the 3D-structure of the nucleon from its fundamental constituents, the quarks and the gluons, is major goal of nuclear physics and a key aim of on-going experiments and the future EIC

⌘ Lattice QCD can contribute towards this goal - many recent developments to compute Mellin moments but also directly parton distributions



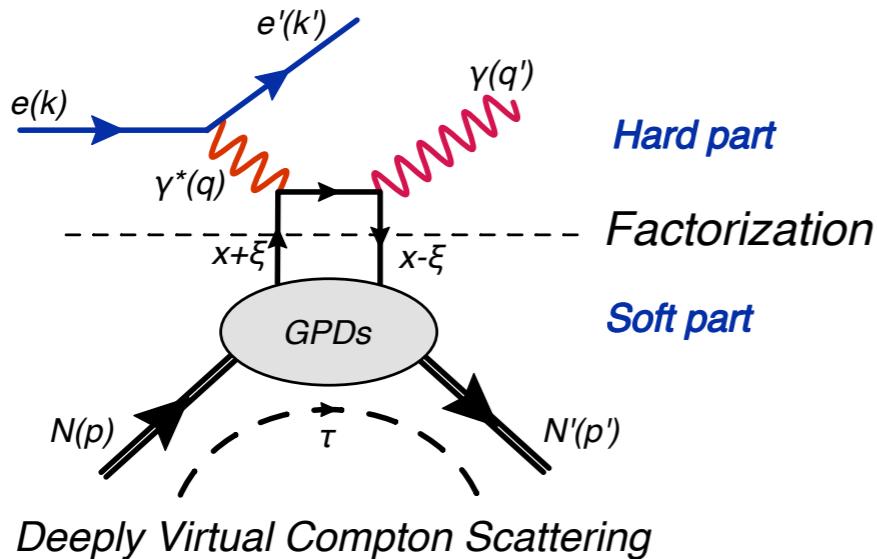
both the form factors and the PDFs are fully encoded within GPDs



Studies in lattice QCD since the 1980s

Generalised Parton Distributions (GPDs)

- * High energy scattering processes: Factorization into a hard partonic subprocess, calculable in perturbation theory, and a universal non-perturbative parton distribution



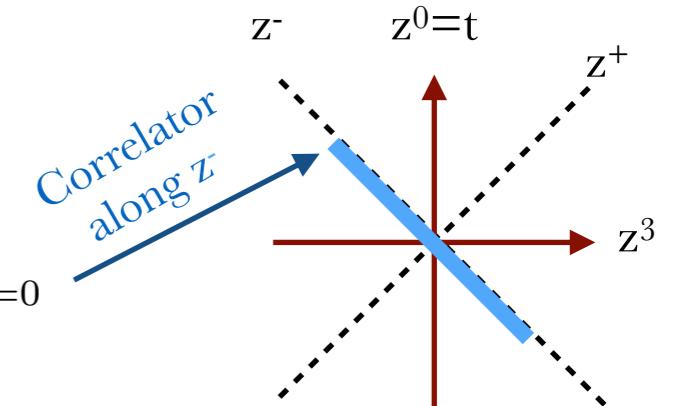
- D. Mueller *et al.*, Fortschr. Phys. 42, 101 (1994)
- A. V. Radyushkin, Phys. Lett. B380, 417 (1996), hep-ph/9604317
- A. V. Radyushkin, Phys. Lett. B385, 333 (1996), hep-ph/9605431
- A. V. Radyushkin, Phys. Rev. D56, 5524 (1997), hep-ph/9704207
- X. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249.
- X. Ji, Phys. Rev. D55, 7114 (1997), hep-ph/9609381
- X. Ji, J. Phys. G24, 1181 (1998), hep-ph/9807358

- * GPDs are light cone matrix elements

$$F_\Gamma(x, \xi, \tau) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle N(p') | \bar{\psi}(-z/2) \Gamma W(-z/2, z/2) \psi(z/2) | N(p) \rangle|_{z^+=0, \bar{z}=0}$$

- $P^+ = \frac{p'^+ + p}{2}$
- $\tau = -Q^2 = (p' - p)^2$
- $\xi = \frac{p^+ - p'^+}{2P^+}$: skewness

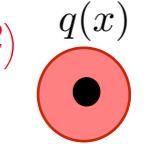
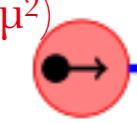
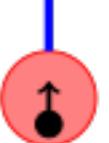
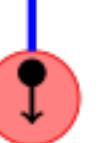
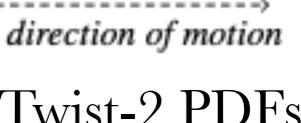
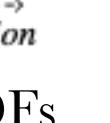
Γ structure defines 3 different types of GPDs



Computation of Mellin moments of GPDs

- * Light-cone matrix elements cannot be computed using a Euclidean lattice formulation of QCD
- * Expansion of light-cone operator leads to a tower of local twist-2 operators —> connected to moments that can be computed in lattice QCD

Forward matrix elements give moments of PDFs

$\mathcal{O}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi$	<i>unpolarized</i>	$\langle x^n \rangle_q = \int_0^1 dx x^n [q(x) - (-1)^n \bar{q}(x)]$	 $f_1(x, \mu^2)$ $q(x)$	
$\tilde{\mathcal{O}}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma_5 \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi$	<i>helicity</i>	$\langle x^n \rangle_{\Delta q} = \int_0^1 dx x^n [\Delta q(x) + (-1)^n \Delta \bar{q}(x)]$		$\Delta q(x) = q^\rightarrow - q^\leftarrow$
$\mathcal{O}_T^{\rho \mu_1 \dots \mu_n} = \bar{\psi} \sigma^\rho \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi$	<i>transversity</i>	$\langle x^n \rangle_{\delta q} = \int_0^1 dx x^n [\delta q(x) - (-1)^n \delta \bar{q}(x)]$		$\delta q(x) = q_\perp + q_\top$
$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$				
 $-$  $-$				
 $-$  $-$				
 $-$  $-$				
direction of motion				
Twist-2 PDFs				

Computation of Mellin moments of GPDs

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$$\mathcal{O}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi \quad \xrightarrow{unpolarized} \quad \langle x^n \rangle_q = \int_0^1 dx x^n [q(x) - (-1)^n \bar{q}(x)]$$

$$\tilde{\mathcal{O}}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma_5 \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi \quad \xrightarrow{helicity} \quad \langle x^n \rangle_{\Delta q} = \int_0^1 dx x^n [\Delta q(x) + (-1)^n \Delta \bar{q}(x)]$$

$$\mathcal{O}_T^{\rho \mu_1 \dots \mu_n} = \bar{\psi} \sigma^\rho \{\mu_1 i D^{\mu_2} \dots i D^{\mu_n\}} \psi \quad \xrightarrow{transversity} \quad \langle x^n \rangle_{\delta q} = \int_0^1 dx x^n [\delta q(x) - (-1)^n \delta \bar{q}(x)]$$

$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_T + q_\perp$

- * Off-diagonal matrix elements yield moments of GPDs or the generalised form factors (GFFs)

$$\int_{-1}^1 dx x^{n-1} H(x, \xi, \tau) = \sum_{i=0,2,\dots}^{n-1} [(2\xi)^i A_{ni}(\tau) + \text{mod}(n,2)(2\xi)^n C_{n0}(\tau)]$$

$$\int_{-1}^1 dx x^{n-1} E(x, \xi, \tau) = \sum_{i=0,2,\dots}^{n-1} [(2\xi)^i B_{ni}(\tau) - \text{mod}(n,2)(2\xi)^n C_{n0}(\tau)]$$

Computation of Mellin moments of GPDs

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$$\tilde{\mathcal{O}}^{\mu_1 \dots \mu_n} = \bar{\psi} \gamma_5 \gamma^{\{\mu_1} i D^{\mu_2} \dots i D^{\mu_n\}} \psi \quad \xrightarrow{\text{helicity}}$$

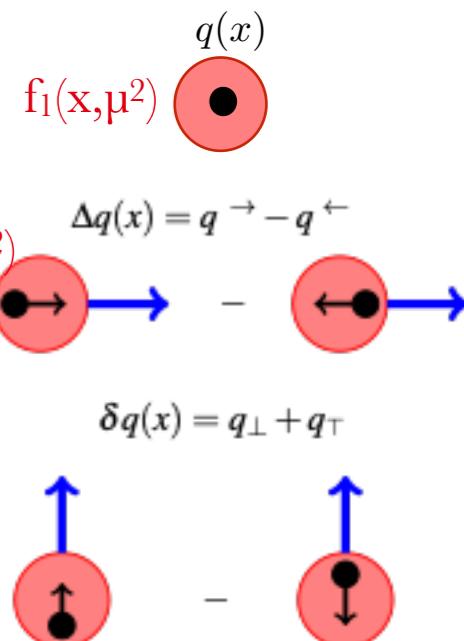
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$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_T + q_\perp$$

$$\langle x^n \rangle_q = \int_0^1 dx x^n [q(x) - (-1)^n \bar{q}(x)]$$

$$\langle x^n \rangle_{\Delta q} = \int_0^1 dx x^n [\Delta q(x) + (-1)^n \Delta \bar{q}(x)]$$

$$\langle x^n \rangle_{\delta q} = \int_0^1 dx x^n [\delta q(x) - (-1)^n \delta \bar{q}(x)]$$



- * For off-diagonal matrix elements we obtain moments of GPDs or the generalised form factors (GFFs)

$$\int_{-1}^1 dx x^{n-1} H(x, \xi, \tau) = \sum_{i=0,2,\dots}^{n-1} [(2\xi)^i A_{ni}(\tau) + \text{mod}(n,2)(2\xi)^n C_{n0}(\tau)]$$

$$\int_{-1}^1 dx x^{n-1} E(x, \xi, \tau) = \sum_{i=0,2,\dots}^{n-1} [(2\xi)^i B_{ni}(\tau) - \text{mod}(n,2)(2\xi)^n C_{n0}(\tau)]$$

Twist-2 PDFs

Special cases: n=1,2 for the nucleon

- n=1: $\tau=0$ —> charges g_V , g_A , g_T

$$\tau \neq 0 \longrightarrow \text{form factors: } A_{10}(\tau) = F_1(\tau), \quad B_{10}(\tau) = F_2(\tau), \quad \tilde{A}_{10}(\tau) = G_A(\tau), \quad \tilde{B}_{10}(\tau) = G_p(\tau)$$

- n=2: generalised form factors: $A_{20}(\tau)$, $B_{20}(\tau)$, $C_{20}(\tau)$, $\tilde{A}_{20}(\tau)$, $\tilde{B}_{20}(\tau)$

$$\langle x \rangle_q = A_{20}(0), \quad \langle x \rangle_{\Delta q} = \tilde{A}_{20}(0), \quad \langle x \rangle_{\delta q} = A_{20}^T(0) \quad \text{and} \quad J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$$

- * Spin and momentum sums: $\sum_q [\frac{1}{2}\Delta\Sigma_q + L_q] + J_g = \frac{1}{2}, \quad \sum_q \langle x \rangle_q + \langle x \rangle_g = 1$

First Mellin moments

- Moments for $n=1,2$ are readily accessible in lattice QCD
- Computation of the low Mellin moments has a long history G. Martinelli and Ch. Sachradja Phys. Lett. B217 (1989) 319
- Only recently we have results directly at the physical point (i.e. simulations with $m_\pi \sim 135 \pm 10$ MeV)

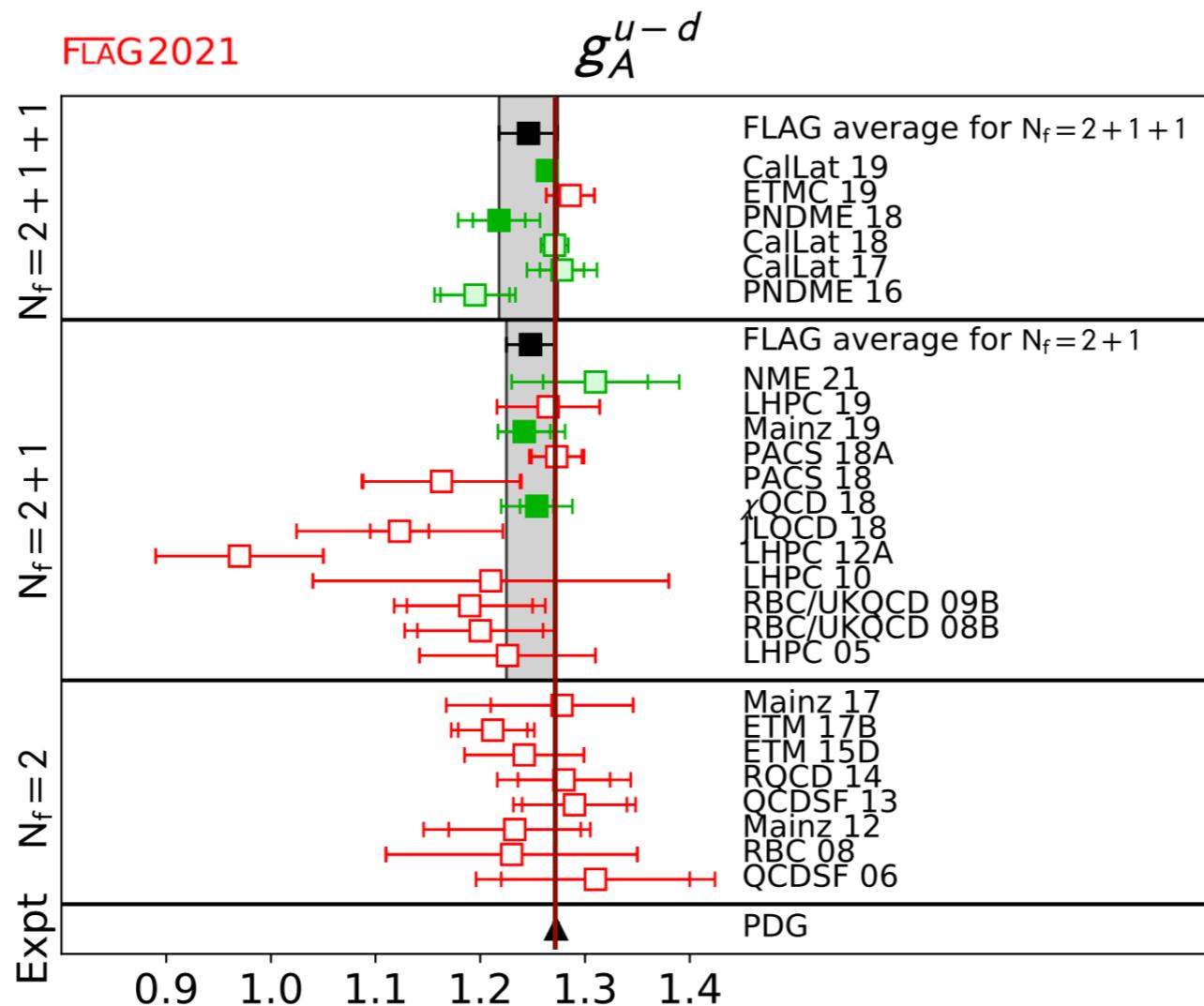
Nucleon isovector charges

$$g_V = \langle 1 \rangle_{u-d}$$

$$g_A = \langle 1 \rangle_{\Delta u - \Delta d}$$

$$g_T = \langle 1 \rangle_{\delta u - \delta d}$$

- $g_V = 1$
- $g_A = 1.2723 \pm 0.0023$ reproduce
- $g_T = 0.53 \pm 0.25$ M. Radici and A. Bacchetta. PRL 120 (2018) 192001



(1) Lattice QCD results on g_A consistent with experimental value

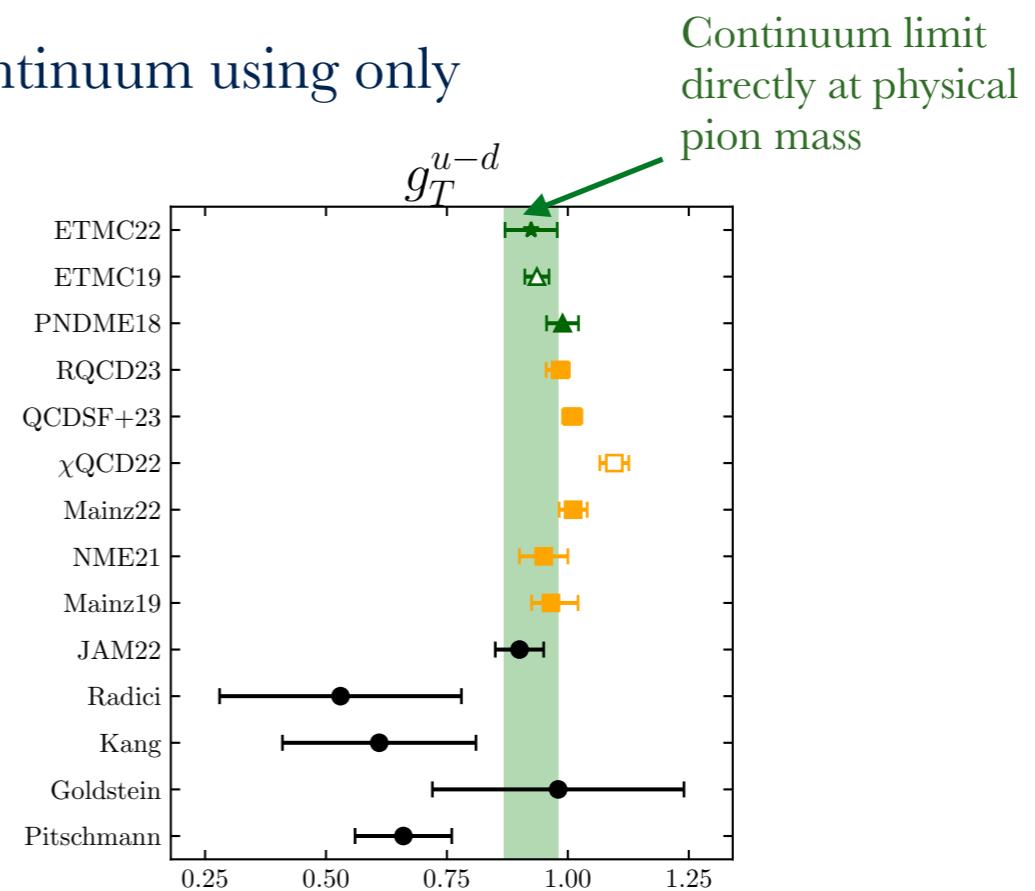
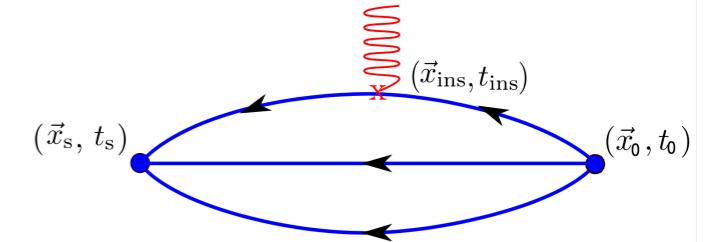
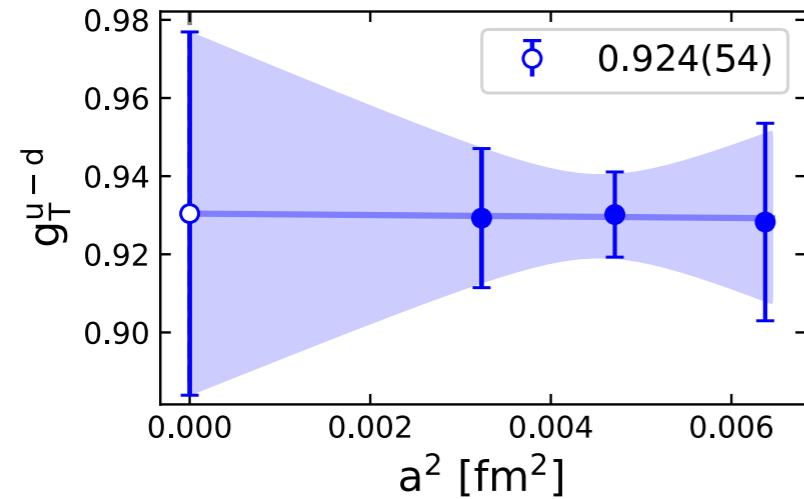
Nucleon isovector (u-d) tensor charge

* Only connected contributions

* Use three gauge ensembles generated using physical values of the light, strange and charm quarks:

- B-ensemble: $64^3 \times 128$, $a \sim 0.08$ fm
- C-ensemble: $80^3 \times 160$, $a \sim 0.07$ fm
- D-ensemble: $96^3 \times 192$, $a \sim 0.06$ fm

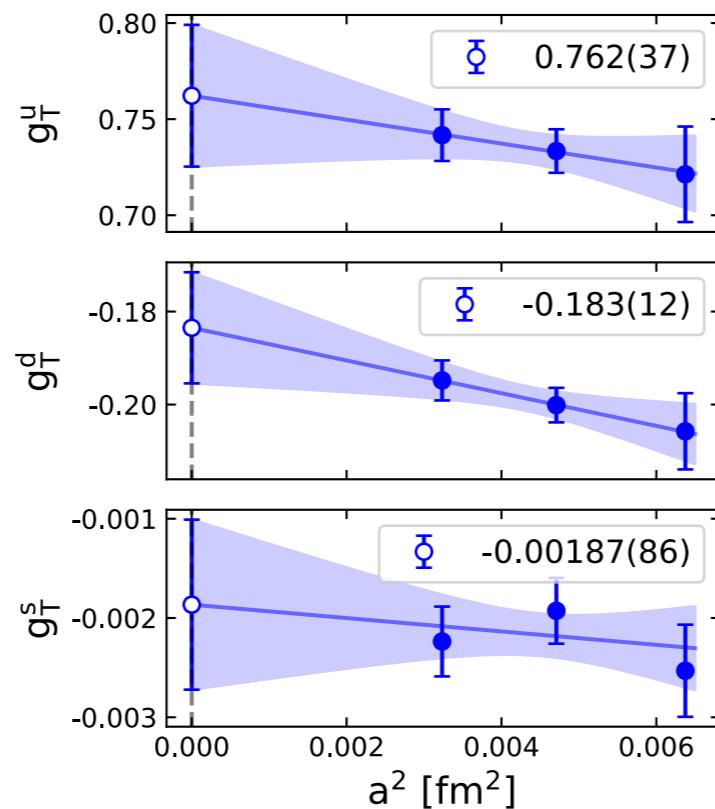
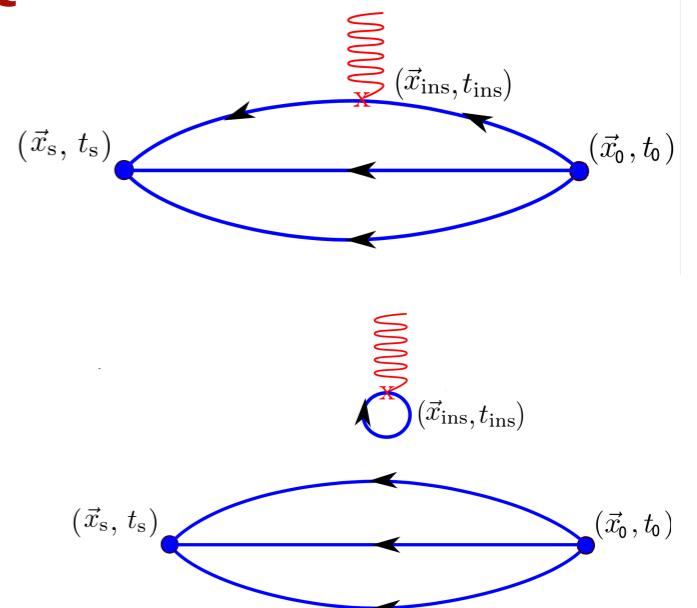
* Obtain the tensor charge for the first time in the continuum using only physical point ensembles



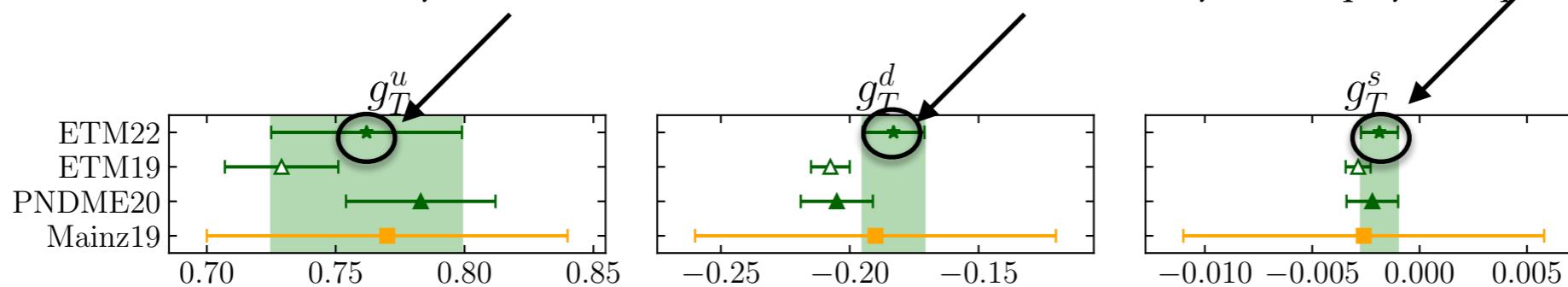
* Precision results on the isovector tensor charge - input for phenomenology e.g. JAM3D-22 analysis

Flavor diagonal tensor charge

- * Evaluate both connected and disconnected contributions
- * Obtain flavor diagonal tensor charge for the first time in the continuum using only physical point ensembles - input for phenomenology

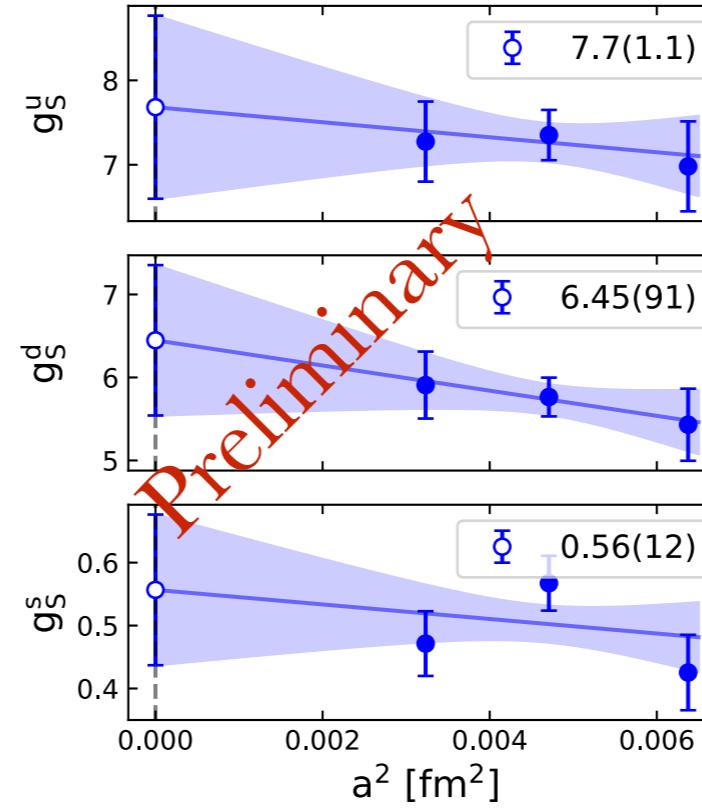
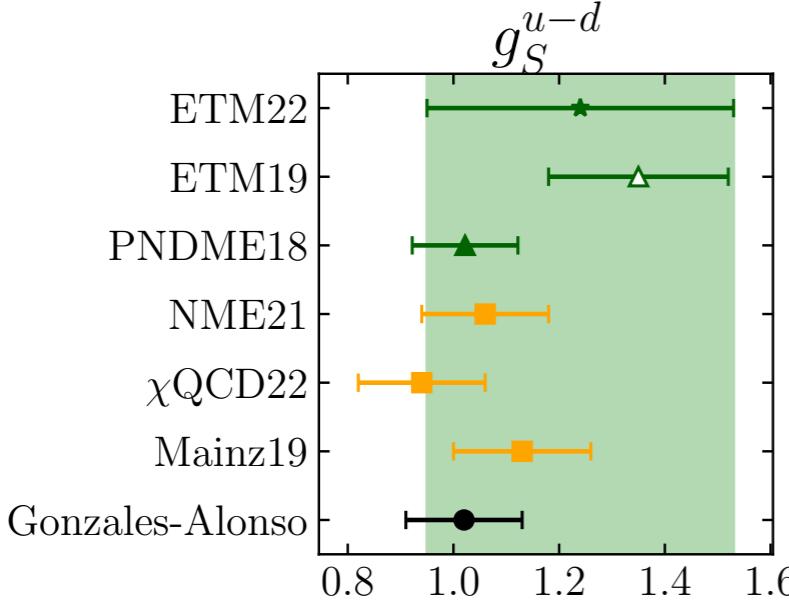


Only calculation in the continuum limit directly at the physical point

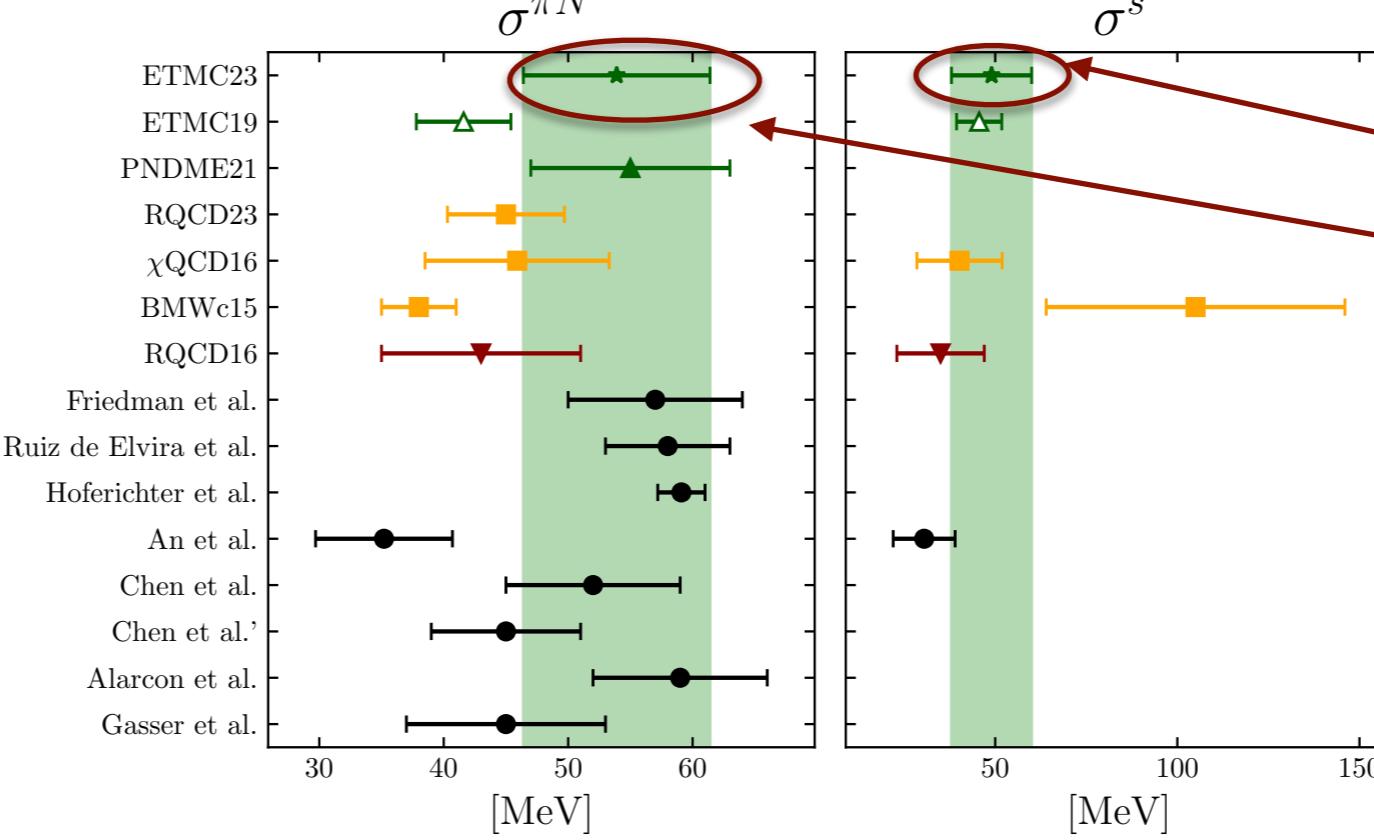
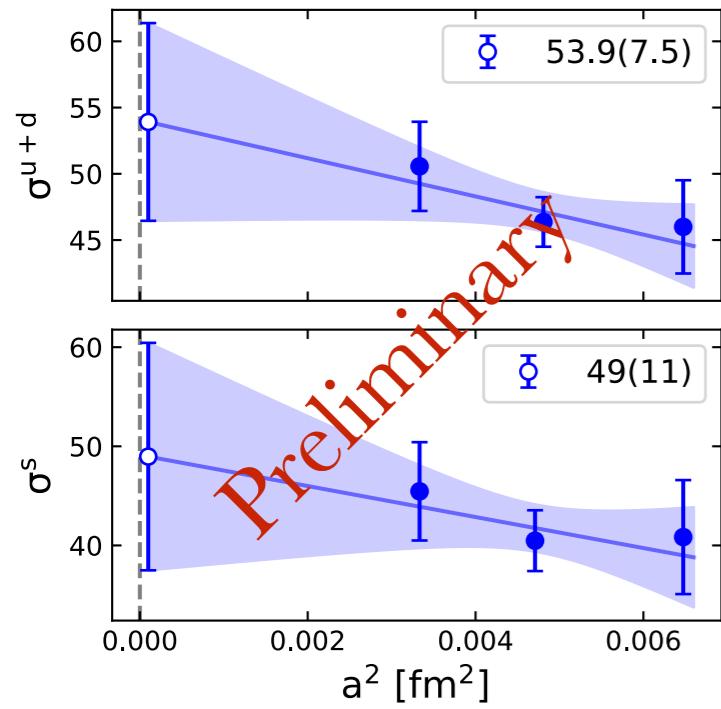


Nucleon scalar charges and σ -terms

* Perform a similar analysis for the scalar charge - important input for direct dark matter searches



* Scalar charge is also directly related to the nucleon σ -terms or quark content $\sigma_q = m_q \langle N | \bar{q}q | N \rangle$

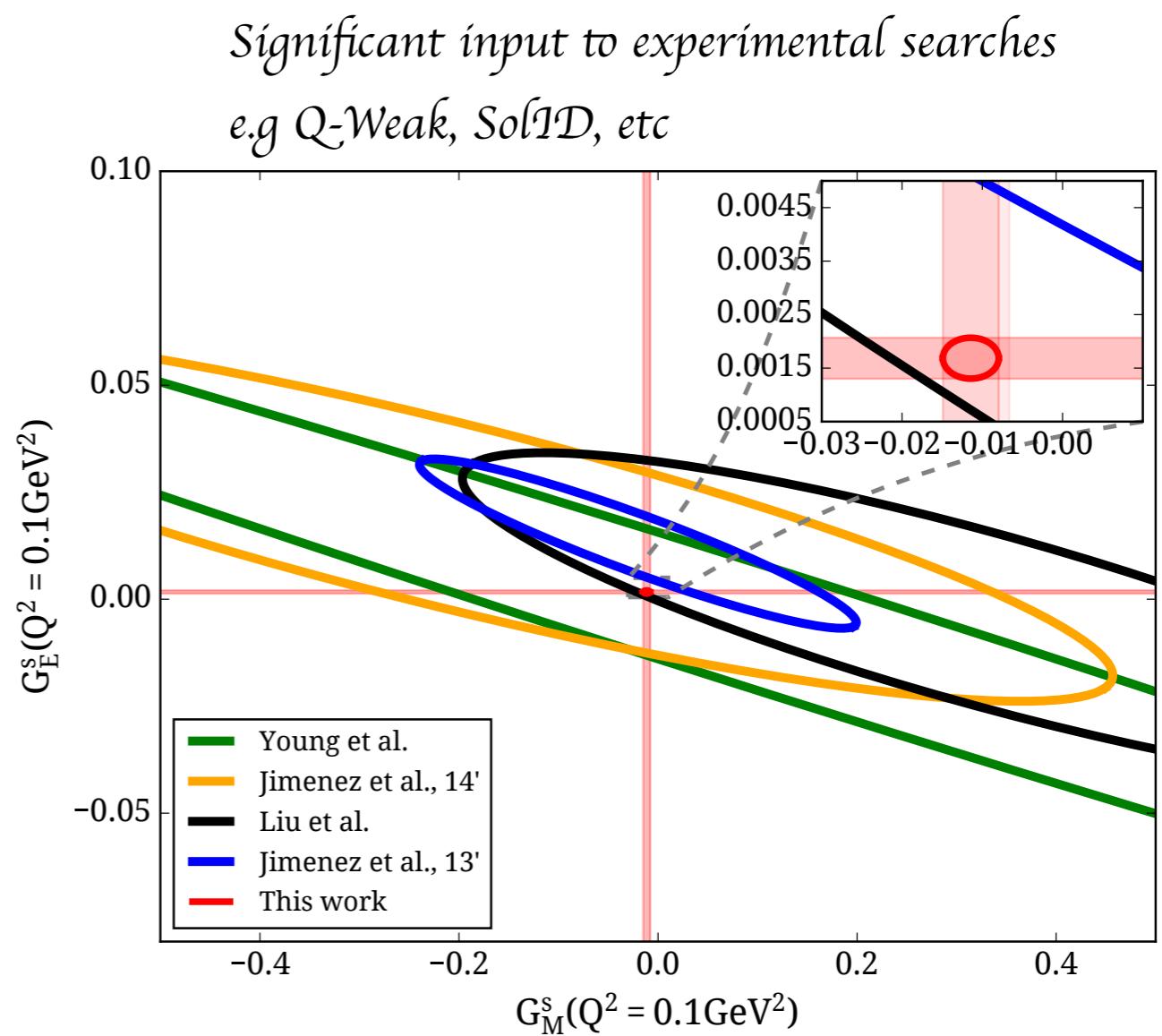
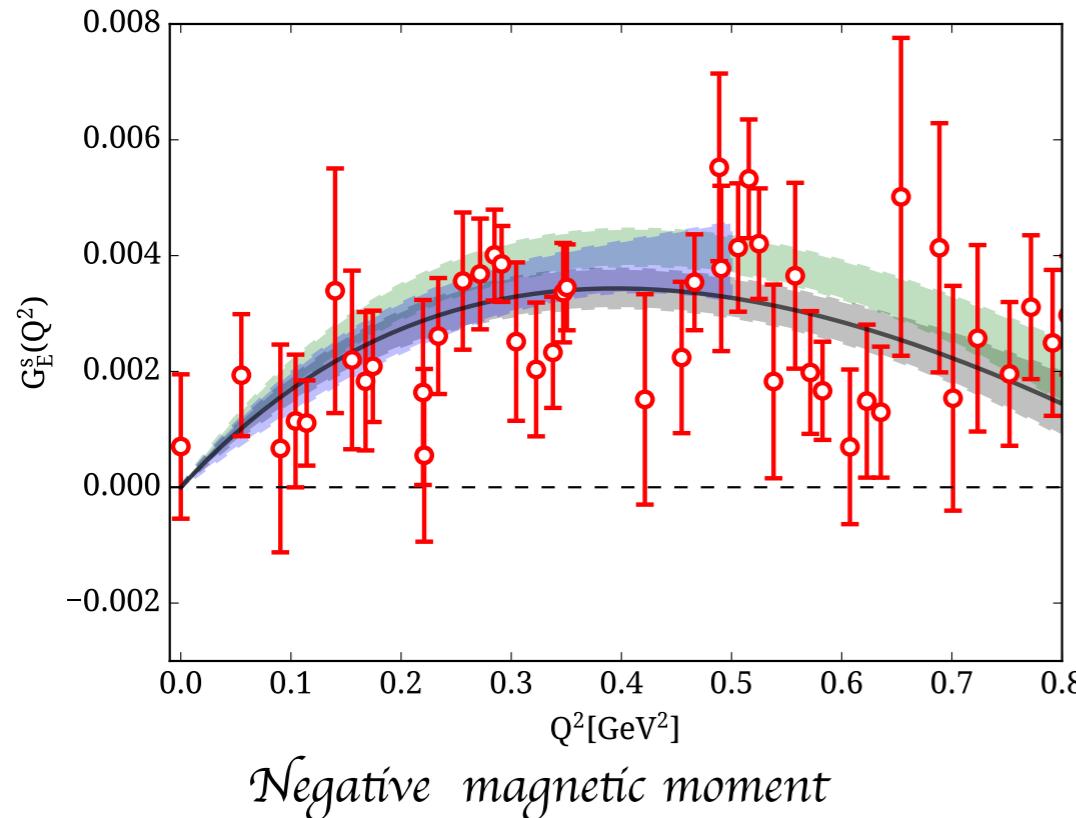


Only calculation in the continuum limit directly at the physical point

Strangeness of the nucleon

- * Sea quark effects can be accurately determined for EM form factors —> provide precise input to experiments

B-ensemble: $64^3 \times 128$, $a \sim 0.08$ fm



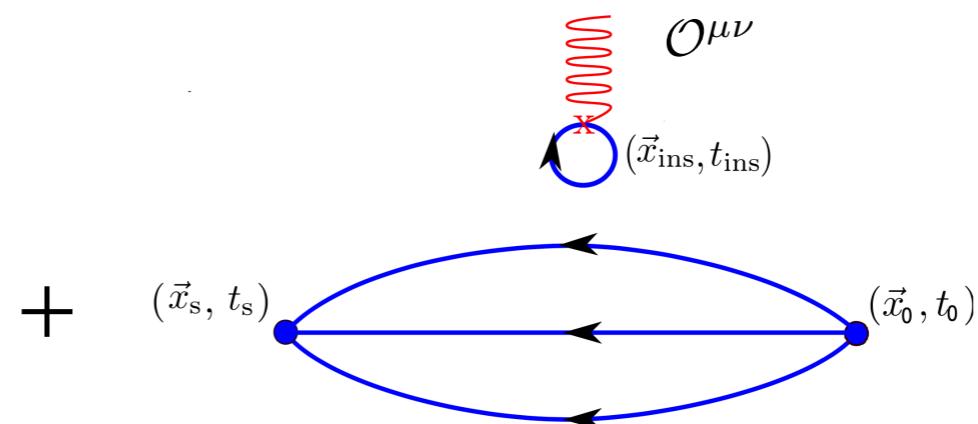
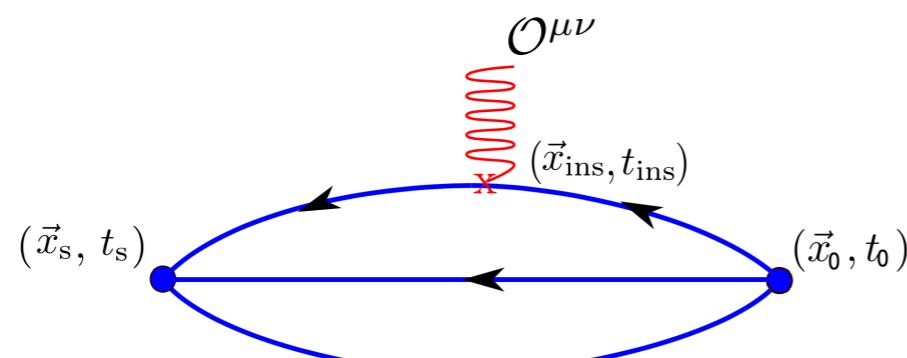
Second Mellin moments

* Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

$$\langle N(p', s') | \mathcal{O}^{\mu\nu,q} | N(p, s) \rangle = \bar{u}_N(p', s') \left[A_{20}^q(q^2) \gamma^{\{\mu} P^{\nu\}} + B_{20}^q(q^2) \frac{i\sigma^{\{\mu\alpha} q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2) \frac{q^{\{\mu} q^{\nu\}}}{m} \right] u_N(p, s)$$

$$\begin{aligned} \langle x \rangle_q &= A_{20}^q(0) \\ J_q &= \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)] \end{aligned}$$

Momentum fraction carried by quark -
best measured



Second Mellin moments

* Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

* Gluon unpolarised moment $\mathcal{O}^{\mu\nu,g} = F^{\{\mu\rho}F_{\rho}^{\nu\}}$ Field strength tensor

$$\langle N(p', s') | \mathcal{O}^{\mu\nu,q} | N(p, s) \rangle = \bar{u}_N(p', s') \left[A_{20}^q(q^2) \gamma^{\{\mu P^\nu\}} + B_{20}^q(q^2) \frac{i\sigma^{\{\mu\alpha} q_\alpha P^{\nu\}}}{2m} + C_{20}^q(q^2) \frac{q^{\{\mu} q^{\nu\}}}{m} \right] u_N(p, s)$$

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↑
Momentum fraction carried by quark -
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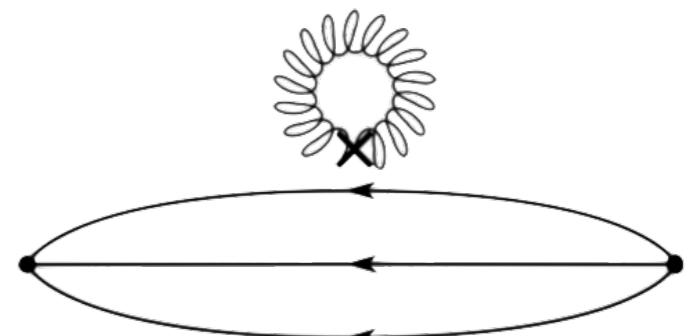
* Equivalent expression for gluon

$$\langle x \rangle_g = A_{20}^g(0) \quad J_g = \frac{1}{2} [A_{20}^g(0) + B_{20}^g(0)]$$

→ Momentum sum: $\sum_q \langle x \rangle_q + \langle x \rangle_g = 1$

→ Spin sum: $\sum_q \left[\frac{1}{2} \Delta \Sigma_q + L_q \right] + J_g = \frac{1}{2}$

J_q



* Matrix elements of helicity and transversity one derivative operators yield $\langle x \rangle_{\Delta q}$, $\langle x \rangle_{\delta q}$

Second Mellin moments

* Quark unpolarised moment $\mathcal{O}^{\mu\nu,q} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$

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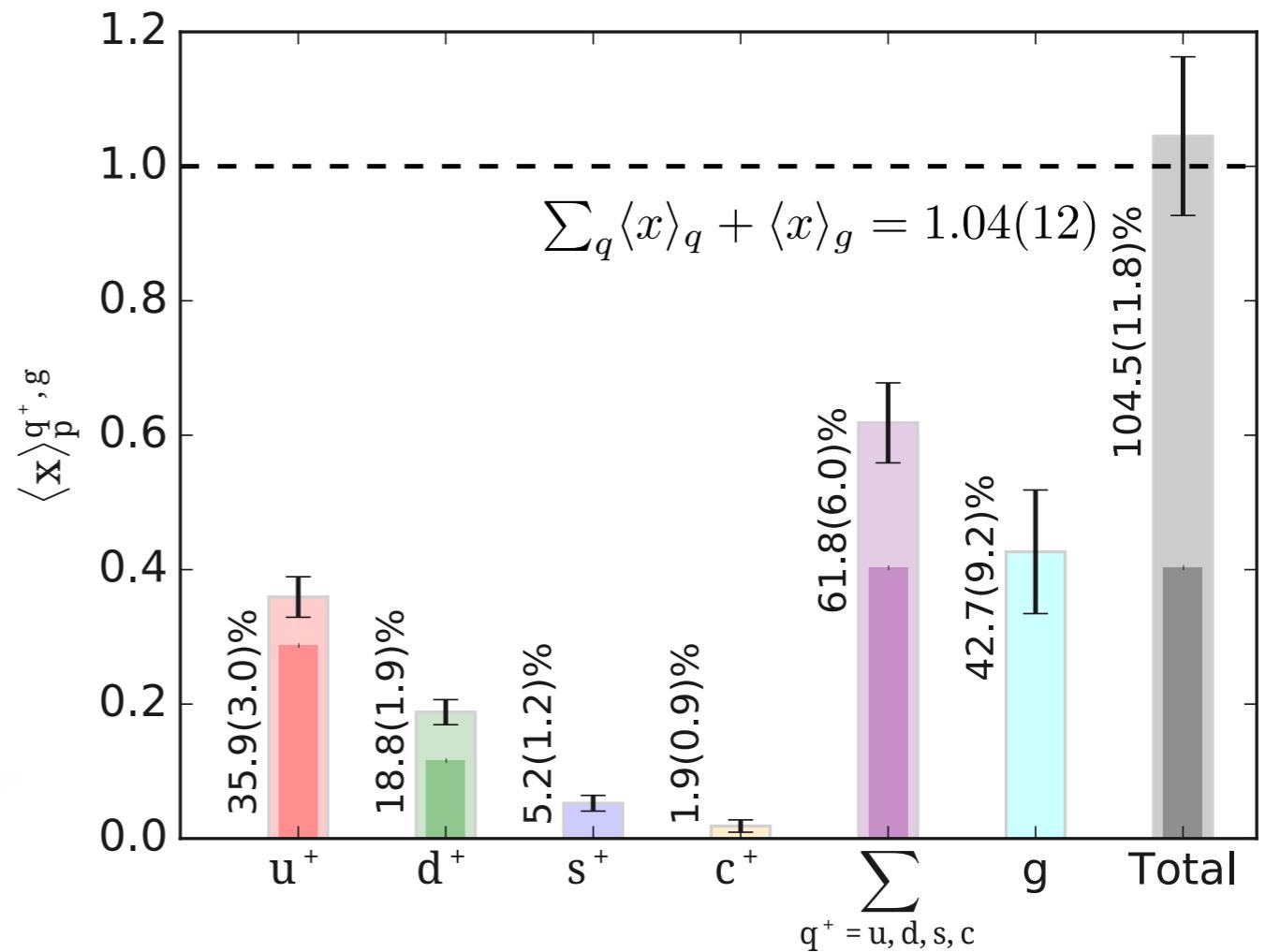
Momentum fraction carried by quark - best measured

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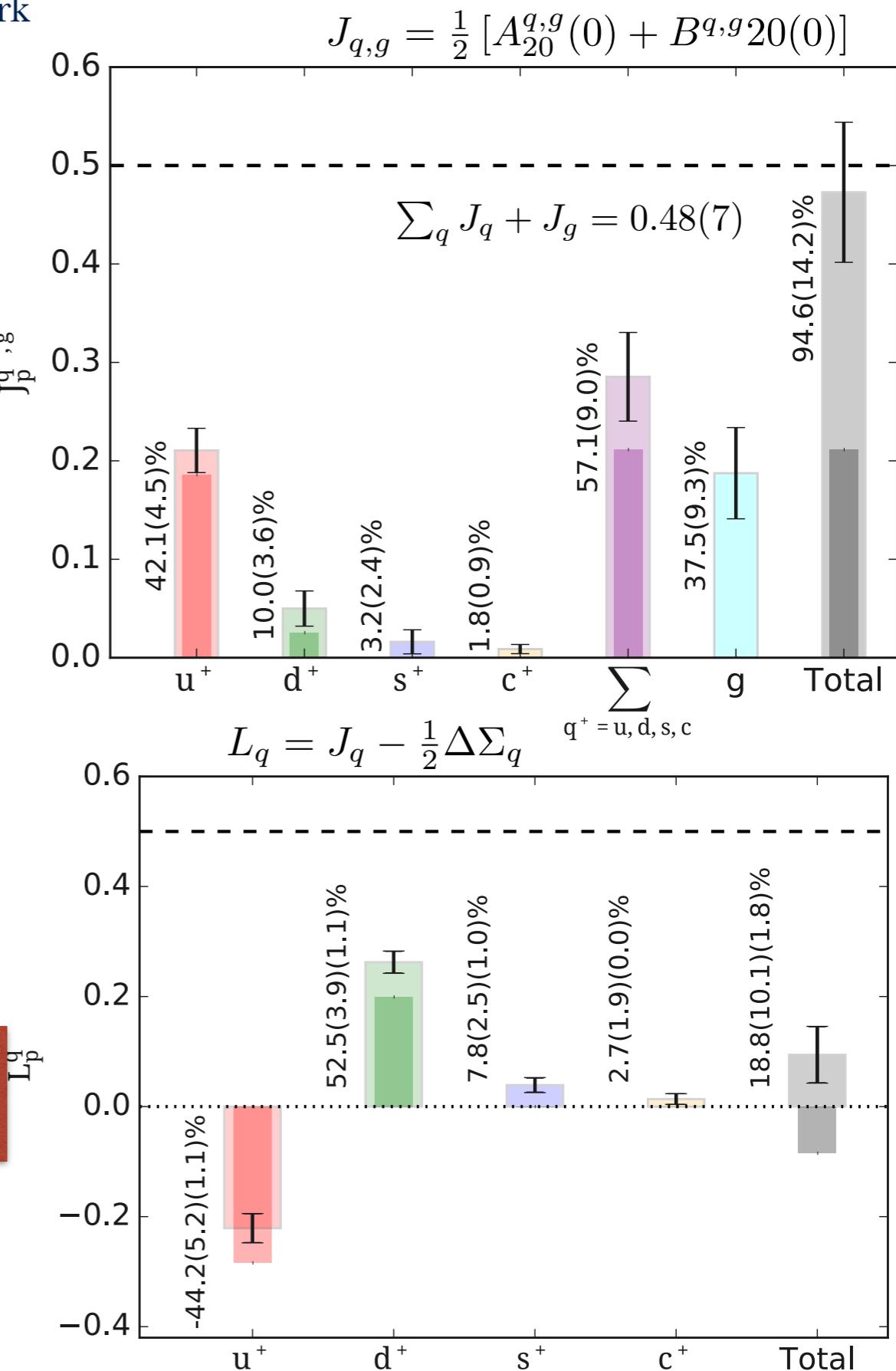
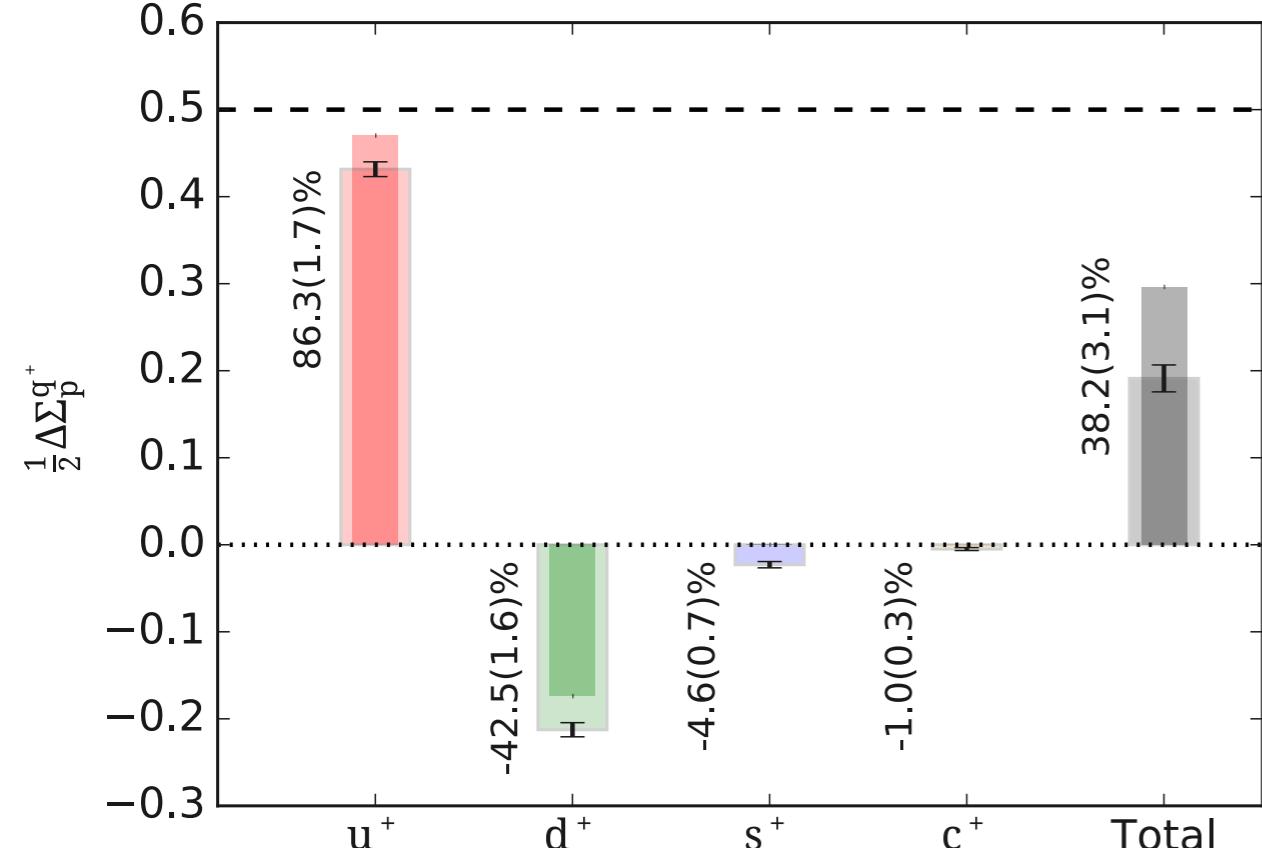
Nucleon momentum sum verified



Momentum and spin sums

* Axial charge determines intrinsic spin carried by each quark

$$\Delta\Sigma_{q+}(\mu^2) = \int_0^1 dx [\Delta q(x, \mu^2) + \Delta \bar{q}(x, \mu^2)] = g_A^q$$



(3) Nucleon spin sum verified - lattice QCD solves a 30 year puzzle

C. A. et al. (ETMC) Phys. Rev. Lett. **119**, 142002, 1909.00485

C. A. et al. (ETMC) Phys. Rev. D **101** (2020) 9, 094513, 2003.08486

Direct computation of PDFs (and GPDs)

See talk by M. Constantinou

- Compute space-like matrix elements for boosted nucleon states and take the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z} \langle P_3 | \bar{\psi}(0) \Gamma W(0, z) \psi(z) | P_3 \rangle|_\mu$$

Renormalise non-perturbatively, $Z(z, \mu)$
 Need to eliminate both UV and exponential divergences

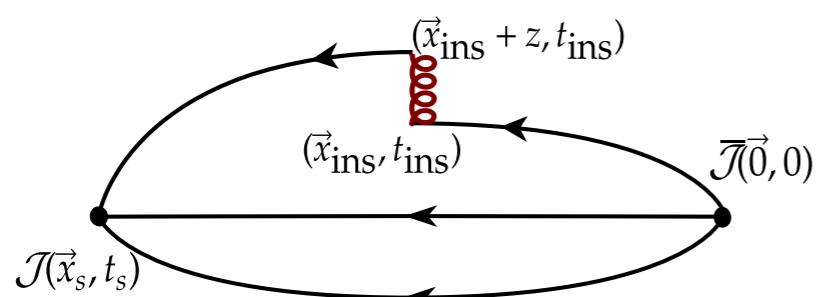
- Match using LaMET

Perturbative kernel

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^1 \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y, \mu) + \mathcal{O}\left(\frac{m_N^2}{P_3^2}, \frac{\Lambda_{\text{QCD}}^2}{P_3^2}\right)$$

X. Ji, Phys. Rev. Lett. 110 (2013) 262002, arXiv:1305.1539

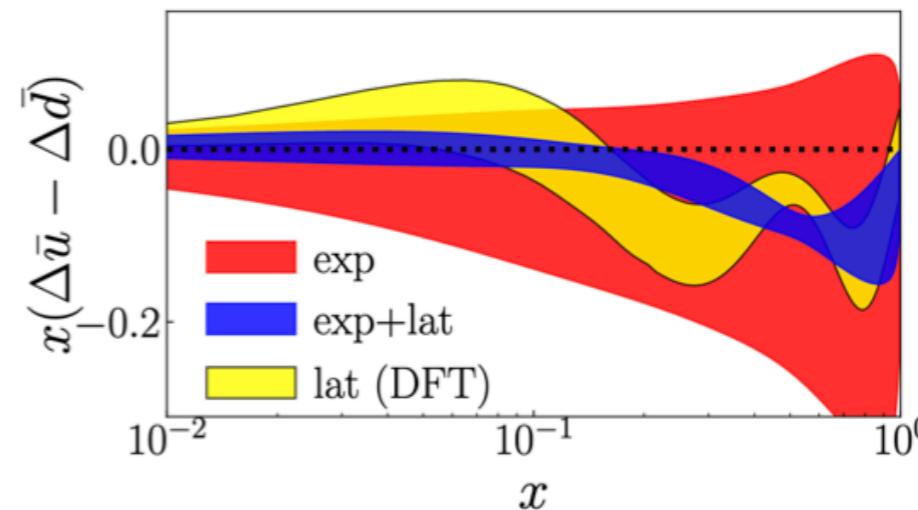
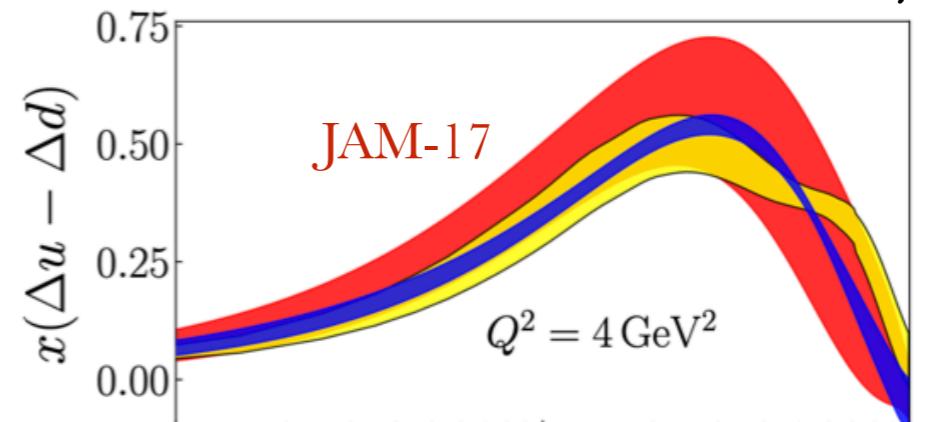
Isovector ($\mathbf{u-d}$)



$\Gamma =$	γ_0	unpolarised
	$\gamma_5 \gamma_3$	helicity
	$\sigma_{3i}, i = 1, 2$	transversity

C.A. et al. (ETMC) Phys. Rev. Lett. **121**, 112001 (2018)

State-of-the-art results on helicity



(4) Parton distribution functions can be computed directly in lattice QCD

Muon anomalous magnetic moment

* Magnetic moment of e, μ , τ : $\vec{\mu} = g \frac{e}{2m} \vec{S}$

* Dirac equation predicts $g=2$

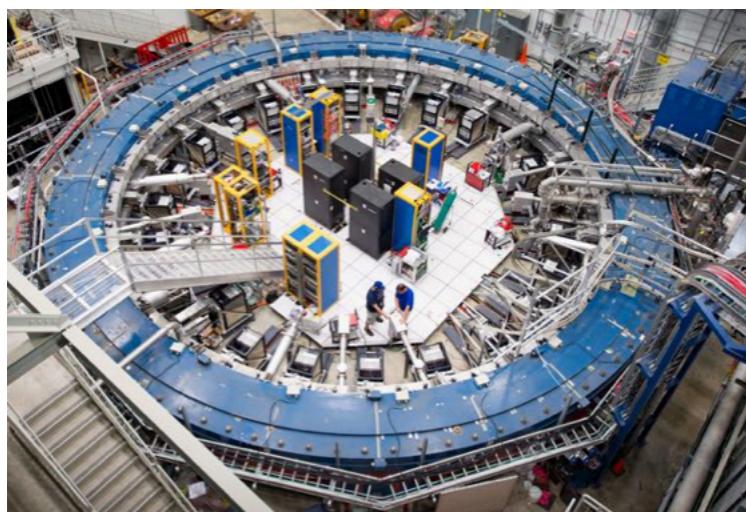
$$\begin{array}{c} \text{Diagram: } \gamma \text{ (photon) } \rightarrow \mu^+ \text{ (muon)} \rightarrow \mu^- \text{ (muon)} \\ = -ie \bar{u}(p') \gamma_\mu u(p) \end{array}$$

* Quantum fluctuations change this value:

$$\begin{array}{c} \text{Diagram: } \gamma \text{ (photon) } \rightarrow \text{muon loop} \rightarrow \mu^+ \text{ (muon)} \rightarrow \mu^- \text{ (muon)} \\ = -ie \bar{u}(p') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p) \\ F_1(0) = 1, \quad g = 2 + 2F_2(0) \end{array}$$

* Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$

FermiLab



* First measurement at CERN in the 60s and 70s

* 1997-2001 measurement at BNL

* Fermilab measurement 2021 (first run)

* J-PARC 2024+

Muon anomalous magnetic moment

* Magnetic moment of e, μ , τ : $\vec{\mu} = g \frac{e}{2m} \vec{S}$

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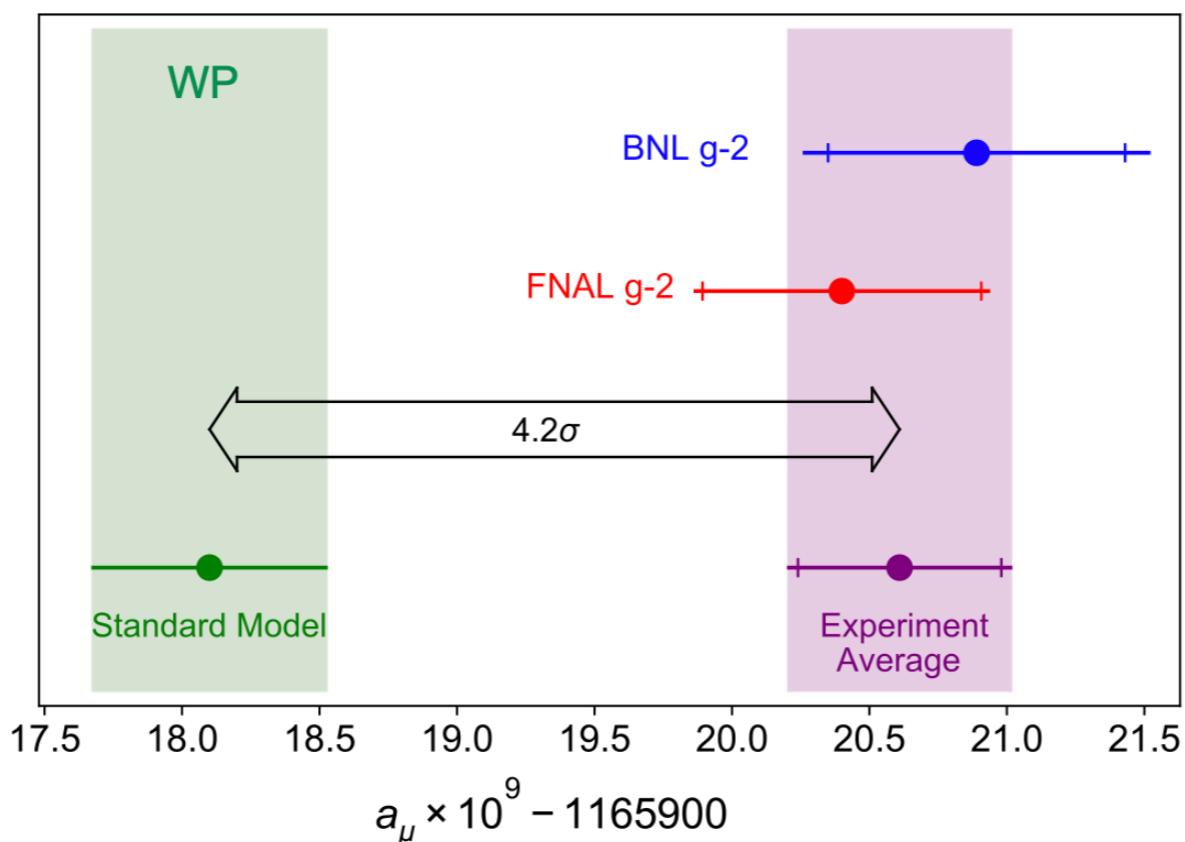
$$\begin{array}{c} \text{Diagram: } \gamma \text{ (photon) } \rightarrow \mu^+ \text{ (muon)} \rightarrow \mu^- \text{ (muon)} \\ = -ie \bar{u}(p') \gamma_\mu u(p) \end{array}$$

* Quantum fluctuations change this value:

$$\begin{array}{c} \text{Diagram: } \gamma \text{ (photon) } \rightarrow \text{Blue Sphere (muon)} \rightarrow \mu^- \text{ (muon)} \\ = -ie \bar{u}(p') \left[\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p) \\ F_1(0) = 1, \quad g = 2 + 2F_2(0) \end{array}$$

* Anomalous magnetic moment: $a \equiv \frac{g - 2}{2} = F_2(0)$

B. Abi *et al.*, Phys. Rev. Lett. 124, 141801 (2021)



SM contributions to muon g-2

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

* QED contribution: computed to α^5

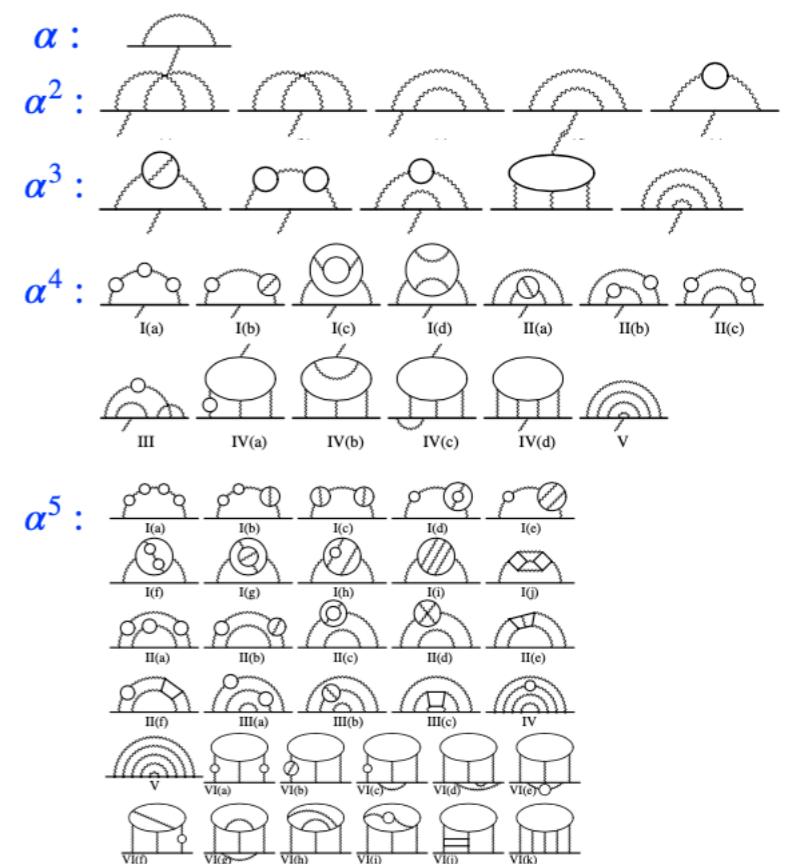
$$a_\mu(\text{QED}) = 116584718.9(1) \times 10^{-11}$$

T. Aoyama et al, arXiv:1205.5370, PRL;
 T. Aoyama, T. Kinoshita, M. Nio, Atoms 7 (1) (2019) 28

* EW contribution: computed to two-loops

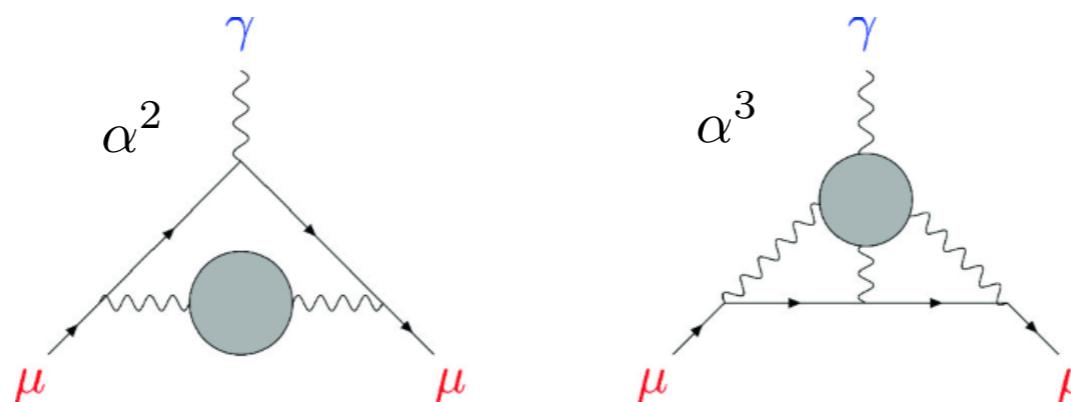
$$a_\mu(\text{EW}) = 153.6(1.0) \times 10^{-11}$$

A. Czarnecki et al, hep-ph/0212229, PRD;
 C. Gnendlinger et al, arXiv:1306.5546, PRD



A. El-Khadra, Lattice 2021

* Leading hadronic contributions: HVP and HLbL

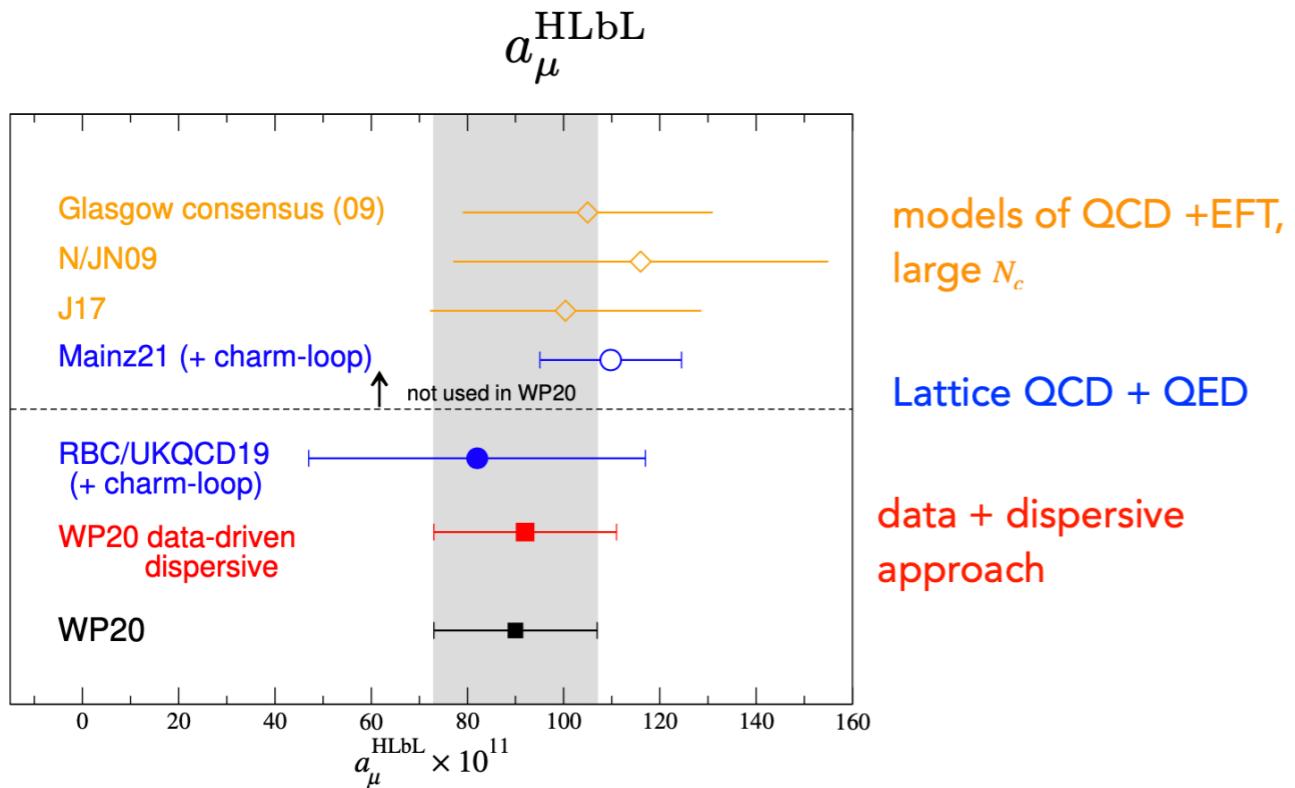


$$a_\mu^{\text{HVP,LO}} = 6931(40) \times 10^{-11} \quad a_\mu^{\text{HLbL}} = 92(19) \times 10^{-11}$$

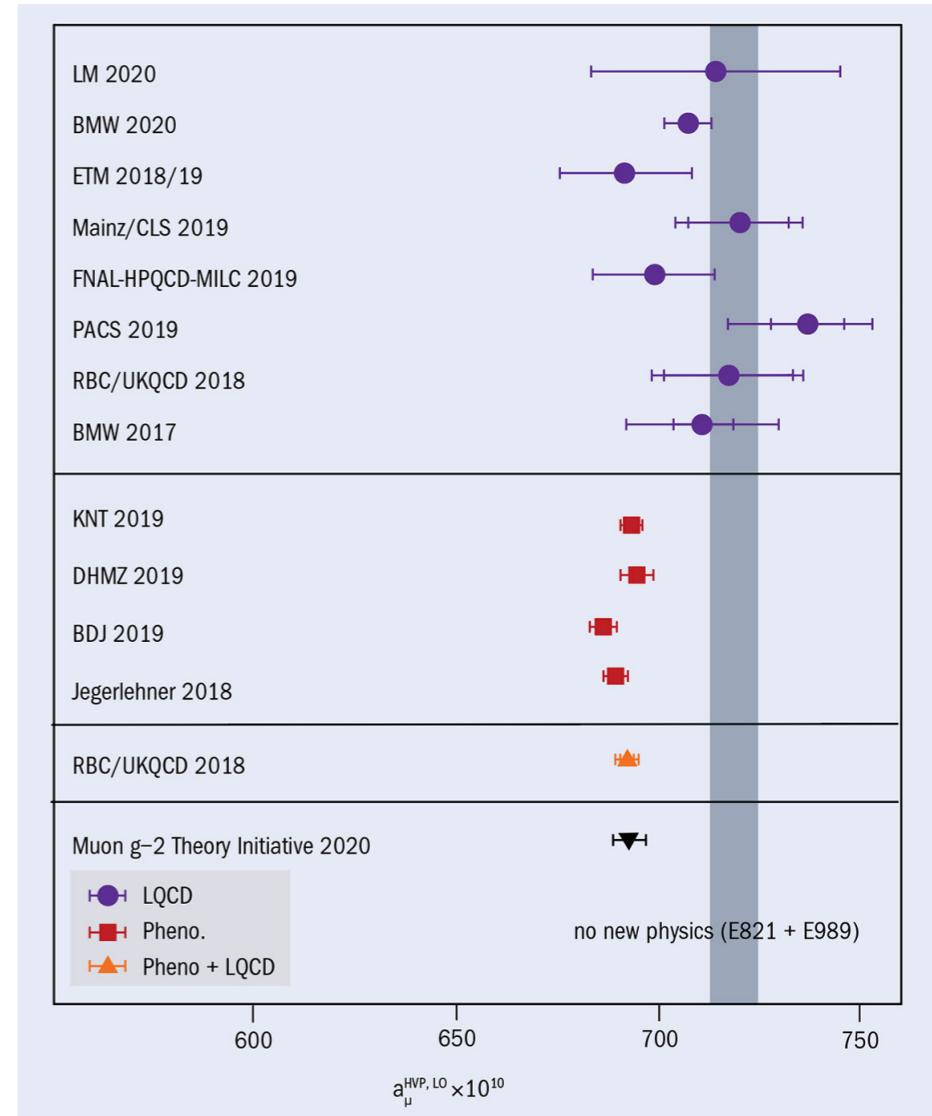
White paper, Theory g-2 initiative, Physics Reports 887 (2020)166, arXiv:2006.04822

Hadronic contributions using lattice QCD

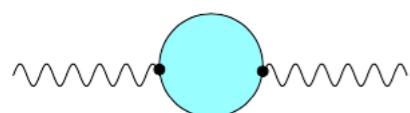
*HLbL computed by two lattice groups



A. El-Khadra, Lattice 2021



*HVP by several lattice groups



$$\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$$

$$\Pi_{\mu\nu}(q^2) = \int d^4x e^{iqx} \langle j_\mu(x) j_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

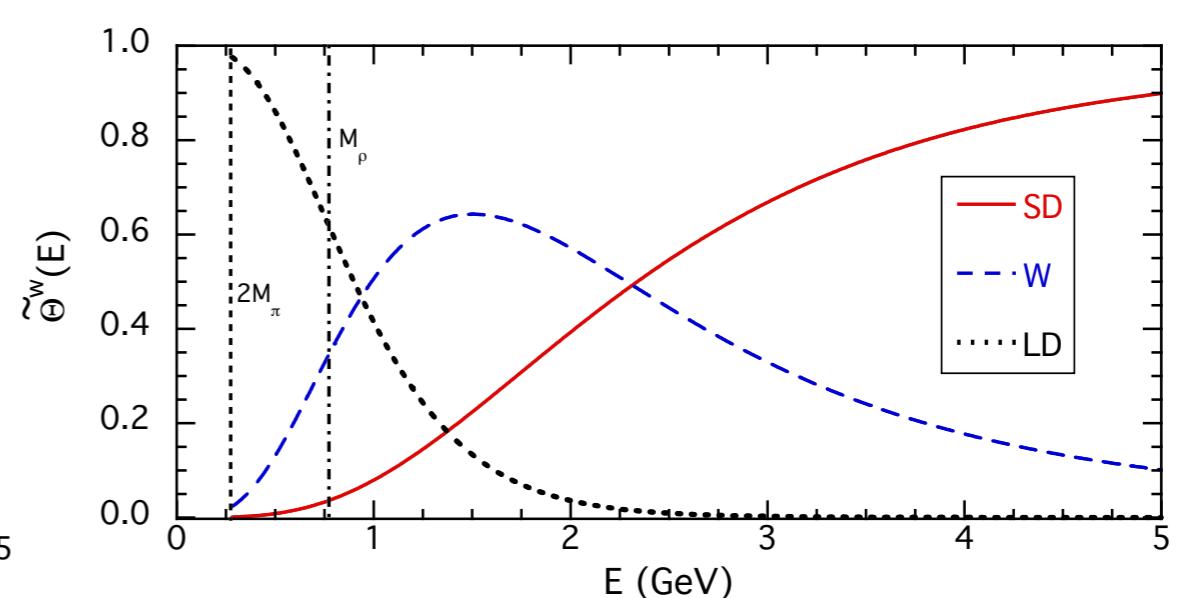
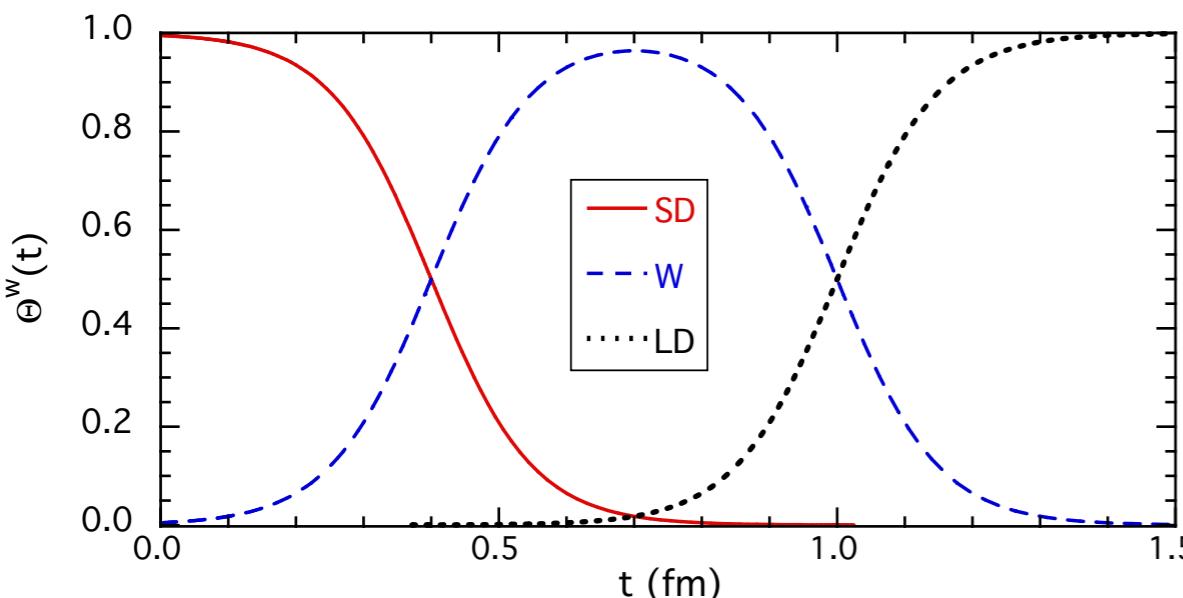
Compute two-point corrector $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i(\vec{x}, t) j_i(\vec{0}, 0) \rangle$ and obtain $a_\mu^{\text{HVP,LO}}$ by doing the Euclidean integral

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi} \right)^2 \int_0^\infty dt t^2 K(m_\mu t) C(t)$$

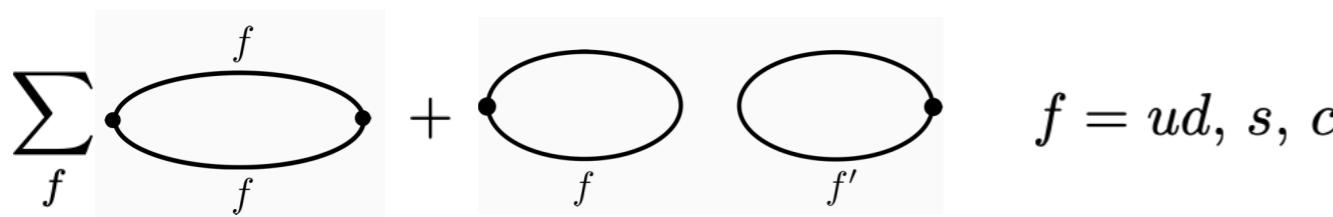
New lattice QCD results on the short- and intermediate-distance windows

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt t^2 K(m_\mu t) C(t)$$

*Consider three-windows as defined by RBC/UKQCD: $a_\mu^{\text{HVP}} = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$



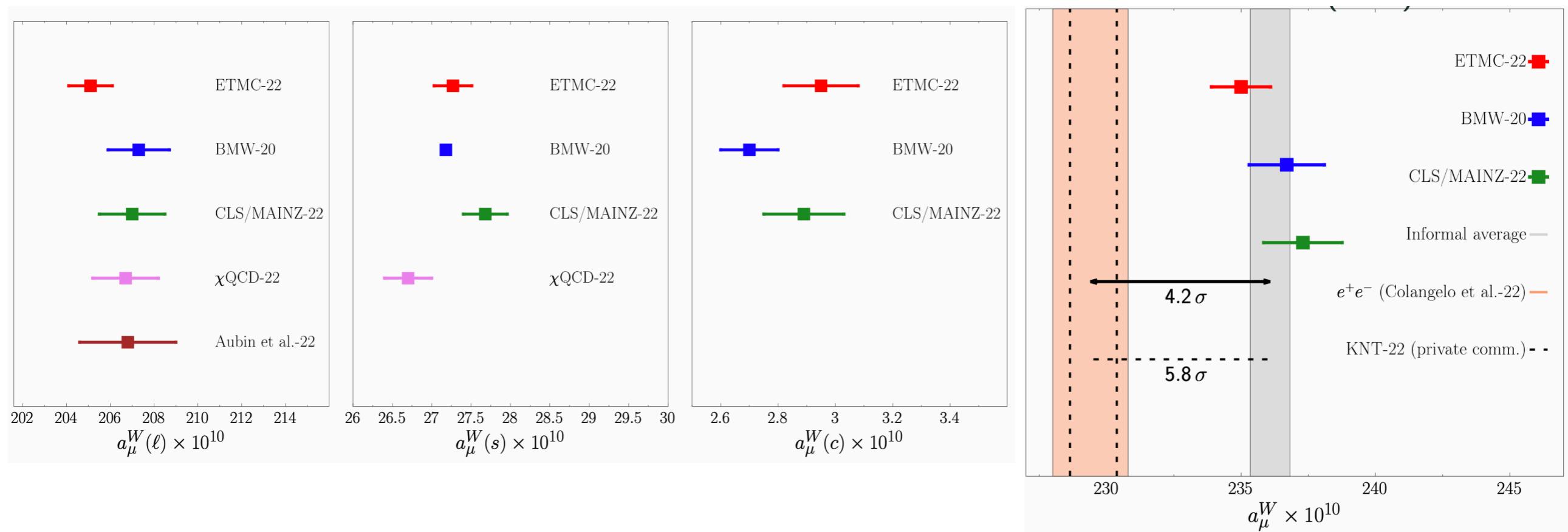
$$a_\mu^w = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt t^2 K(m_\mu t) C(t) \Theta_w(t)$$



Lattice QCD results in the intermediate-distance window

- * ETMC result for a_μ^{SD} in agreement with dispersive analyses
- * All lattice results for a_μ^W in very good agreement, but in tension with dispersive analysis to 4.2σ if we average ETMC, BMW and CLS/Mainz results

Giuseppe Gagliardi, Lattice 2022



→ Deviation of $e^+e^- \rightarrow \text{hadrons}$ data w.r.t. the SM **in the low and (possibly) intermediate energy regions, but not in the high energy region**

C. Alexandrou *et al.* (ETMC) *Phys.Rev.D* 107 (2023) 7, 074506, arXiv: 2206.15084

(5) Lattice QCD input is essential in probing physics beyond the Standard Model

Conclusions

- (1) Lattice QCD yields precision results on e.g. nucleon axial charge, form factors, etc - reproduces benchmark quantities
 - > Precision era of lattice QCD: A number of accurate results with controlled systematics on less known quantities provide valuable input for searches of new physics, e.g nucleon scalar and tensor charges including flavor diagonal, strangeness, ...
- (2) Lattice QCD provides insights on the distribution of spin among the quarks and the gluons in hadrons
 - > Direct computation of PDFs, GPDs and TMDs probing the 3D structure of hadrons is a very active field (see talk by M. Constantinou)
- (3) Lattice QCD provides essential input to searchers beyond the standard model
 - > produced results on muon $g_\mu - 2$ to the same accuracy as experiment
- (4) Many other results are emerging, such as properties of resonances and exotics, phase diagram of QCD and nuclear equation of state, polarizabilities (see talk by X. Feng)

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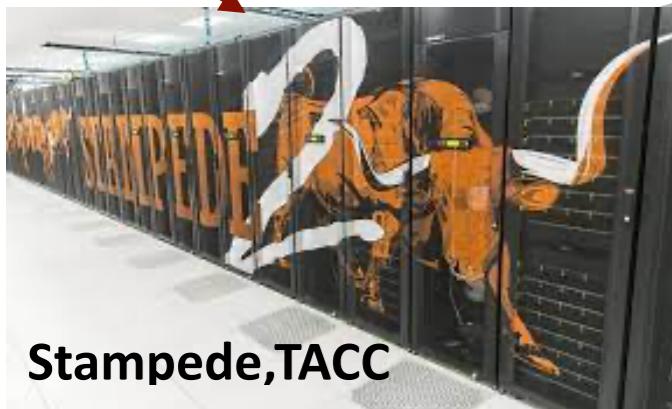




Computational resources



USA



Piz Daint, CSCS



JSC



HAWK, HLRS



SuperMUC, LRZ



THE CYPRUS
INSTITUTE

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