Extraction of polarizabilities from Compton scattering

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Kolymbari, Crete,
May 15th 2023

SUMMARY

- Physics motivations

Why is Real Compton Scattering (RCS) worthwhile to be measured ?

Study of the nucleon internal structure

see E. Mornacchi's talk

- The Experimental RCS -proton data base

- The new fit method

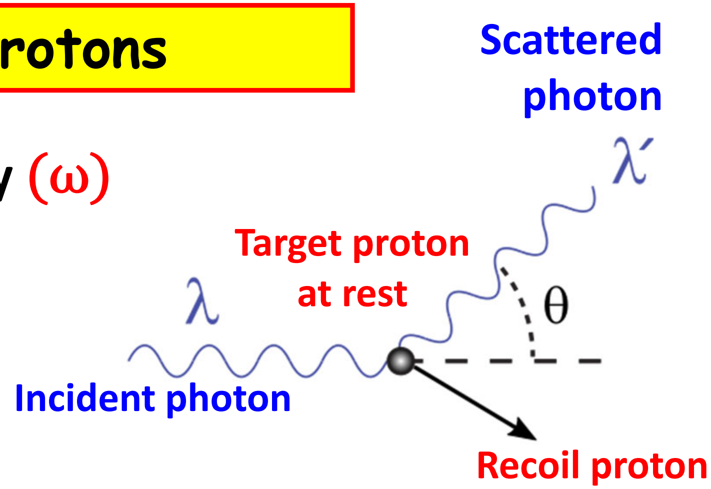
- Selected fit Results

First Concurrent Extraction of all the 6 Leading-Order Proton Polarizabilities

B. Pasquini, P.P., S. Sconfietti PRC 89 015204 (2018)
B. Pasquini, P.P., S. Sconfietti JPG 46 104001 (2019)
P. P., S. Sconfietti, JPG 47, 054001 (2020)
E. Mornacchi, S. Rodini, B. Pasquini, P.P.
PRL 129 102501 (2022)

- Comments and Outlook

Real Compton Scattering off protons



Expansion of the effective Hamiltonian in incident photon energy (ω)

0th order ⟹ **charge, mass**

1st order ⟹ **magnetic moment**

} «point-like» nucleon (Born terms)

2nd order ⟹ **2 scalar polarizabilitites**

Baldin's sum rule (BSR): $(\alpha_{E1}+\beta_{M1}) \approx$ known from other experiments $[= 14.2 \pm 0.5 \; (10^{-4}\ \mathrm{fm}^3)]$

PDG value

$$H_{eff}^{(2)} = -4\pi\left[\frac{1}{2}\alpha_{E1}\vec{E}^2 + \frac{1}{2}\beta_{M1}\vec{H}^2\right]$$

3rd order ⟹ **4 spin (vector) polarizabilitites:**

$$H_{eff}^{(3)} = -4\pi\left[\begin{array}{c}\frac{1}{2}\gamma_{E1E1}\vec{\sigma}\cdot\left(\vec{E}\times\dot{\vec{E}}\right) + \frac{1}{2}\gamma_{M1M1}\vec{\sigma}\cdot\left(\vec{H}\times\dot{\vec{H}}\right) \\ -\gamma_{M1E2}E_{ij}\sigma_i H_j + \gamma_{E1M2}H_{ij}\sigma_i E_j\end{array}\right]$$

$$E_{ij} = \frac{1}{2}\left(\nabla_i E_j + \nabla_j E_i\right)$$
$$H_{ij} = \frac{1}{2}\left(\nabla_i H_j + \nabla_j H_i\right)$$

$$\boxed{\gamma_0} = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$
$$\boxed{\gamma_\pi} = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

(γ_0 value also given by the GGT sum rule $\approx$ known from other experiments) = −0.9 ± 0.1± 0.1 (10^{-4} fm^4); B. Pasquini, P. P. D. Drechsel, PLB 687 160 (2010)

What do we need to determine the polarizability values ?

- (High-quality) experimental data on different observables

- A theoretical model predicting the functional shape of the RCS cross section

- A fit procedure using the two previous ingredients to give an estimate of all different polarizabilities

Up to now, in all existing fits of the RCS data, some of the polarizabilities have been fixed either using theoretical calculations or empirical evaluations from other reactions

⇒ Comparisons between different extractions of polarizabilities and comparisons with theoretical models are often quite difficult

Polarizabilities: how can they be accessed ?

Rough indications !!!

Different observables must be measured

Several different experiments must be performed 13 (!!) possible observabes

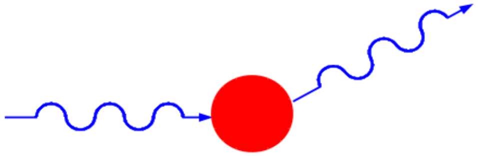
only some points from two measurements

P. Martel et al, PRL 114, 112501 (2015)

D. Paudyal et al PRC 102, 035205 (2020)

- Unpolarized photons, unpolarized protons

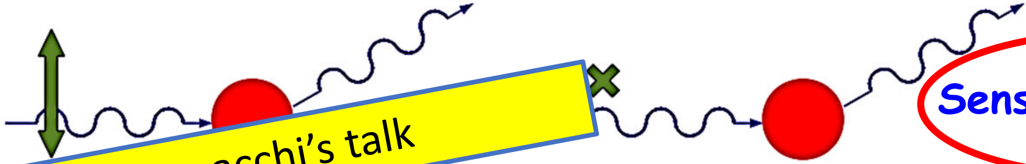
DCS = $\frac{d\sigma}{d\Omega} \propto \frac{N_{tot}}{I_\gamma}$



Sensitive to α_{E1} β_{M1} γ_0 γ_π

- Linearly polarized photons, unpolarized protons.

$$\Sigma_3 = \frac{N_\parallel - N_\perp}{N_\parallel + N_\perp}$$

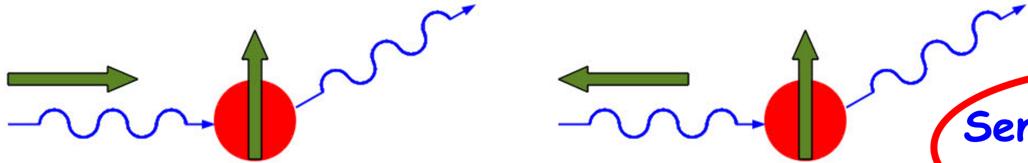


see E. Mornacchi's talk

Sensitive to γ_{M1M1}

- Circularly polarized photons, transversely polarized protons.

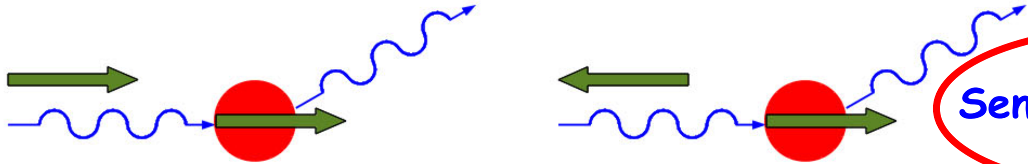
$$\Sigma_{2x} = \frac{N^R_{+x} - N^L_{+x}}{N^R_{+x} + N^L_{+x}}$$



Sensitive to γ_{E1E1}

- Circularly polarized photons, longitudinally polarized protons.

$$\Sigma_{2z} = \frac{N^R_{+z} - N^L_{+z}}{N^R_{+z} + N^L_{+z}}$$

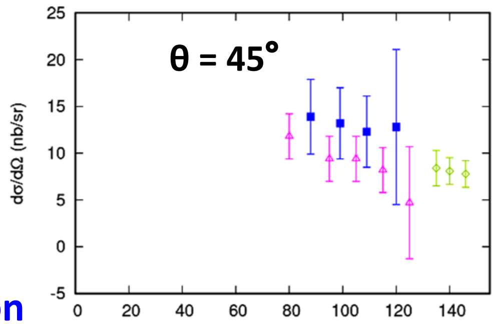


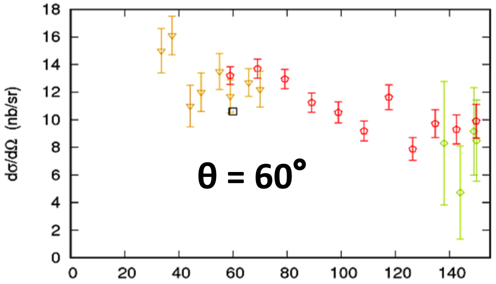
Sensitive to γ_{M1M1}

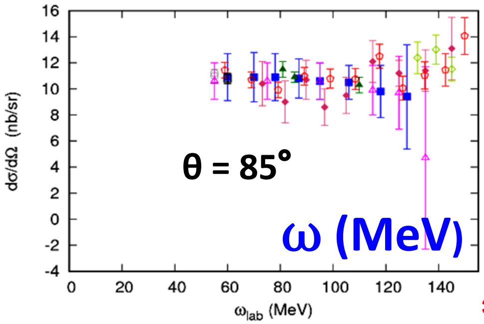
The RCS-proton data base (ω < 150 MeV) – up to 2022

(data mostly sensitive to α_{E1} , β_{M1})

Differential Cross section (nb/sr)





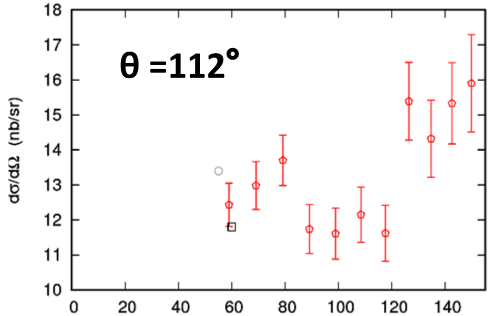


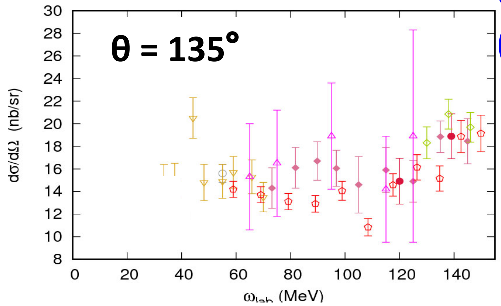
Symbol	Set	Ref	N.points
	GOL 60	Goldansky et al.	5
	OdL 01	Olmos de León	55 *
	HAL 93	Hallin et al.	13
	HYM 59	Hyman et al.	8
	PUG 67	Pugh et al.	16
	FED 91	Federspiel et al.	16
	BER 61	Bernardini et al.	2
	BAR 74	Baranov et al.	7
	OXL 58	Oxley	4
	MAC 95	MacGibbon et al.	18

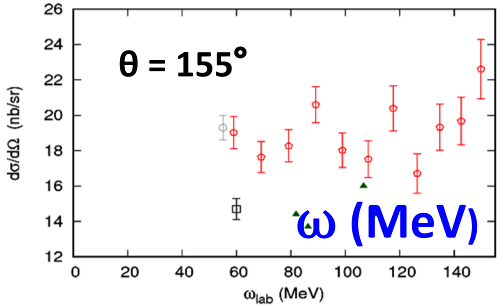
(Only) 150 points

(Half of the Spartans that King Leonidas led to the Battle of Thermopylae in 480 BC)

* Data from TAPS coll.







Differential Cross section (nb/sr)

⇒ Poor quality of the data set (... a difficult experiment to perform ➡ atomic e.m. background)
Large statistical -and systematic- errors ; possible inconsistencies between subsets ....

➢ There is not a common agreement on the definition of a "good" data set below pion-production threshold

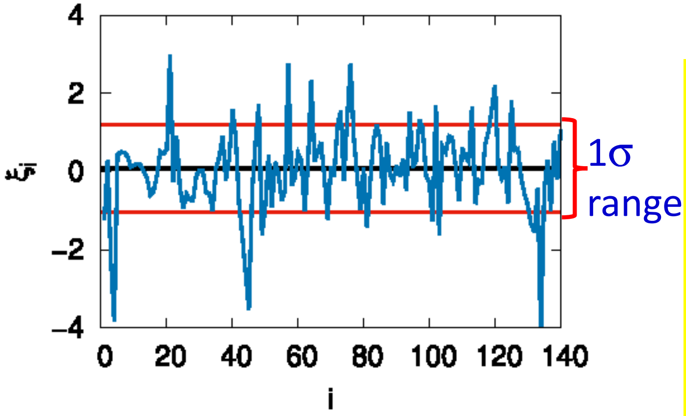
Residual test: 1-parameter fit $(\alpha_{E1} - \beta_{M1})$ without systematic errors, $(\alpha_{E1} + \beta_{M1})$ from Baldin's sum rule and constany $\gamma_i' s$

Residual distribution

Residual QQ-plot

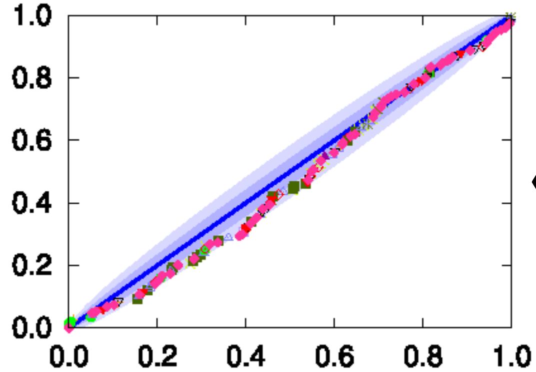
Full data set
(150 points)

3 points outside the 3σ range: probability 1%



data index

Experimental residual quantiles



Theoretical gaussian quantiles

Fit $p-\text{value} \cong 3$*

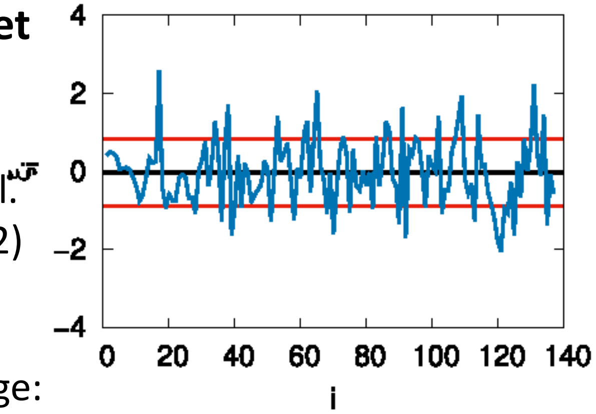
«too slow» convergence

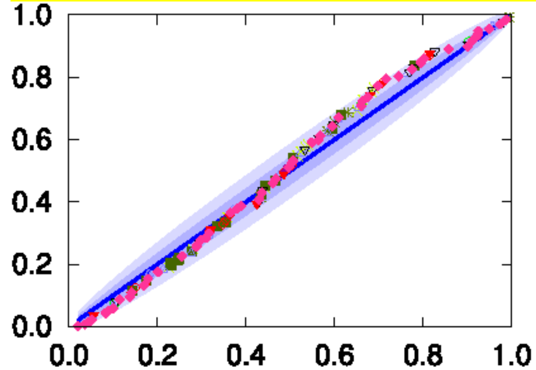
* $p-\text{value} \cong 12\%$
with systematic errors included (see below)

«Pruned» data set
(137 data points)

Griesshammer et al.
PPNP 67, 841 (2012)

(only) 2 points outside the 2σ range: probability 3%





(blue bands: uncertainty due to the finite dimension of the data set)

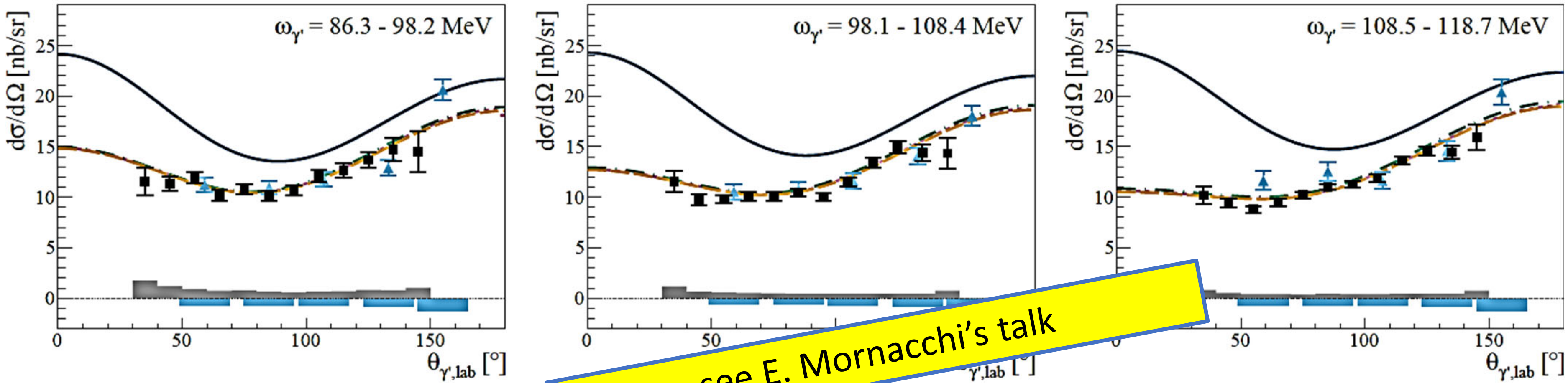
Fit $p-\text{value} \cong 98\%$
(suspiciously very high …)

«too fast» convergence

More details in JPG-2019

➢ Situation drastically improved with the publication of new A2-Mainz data

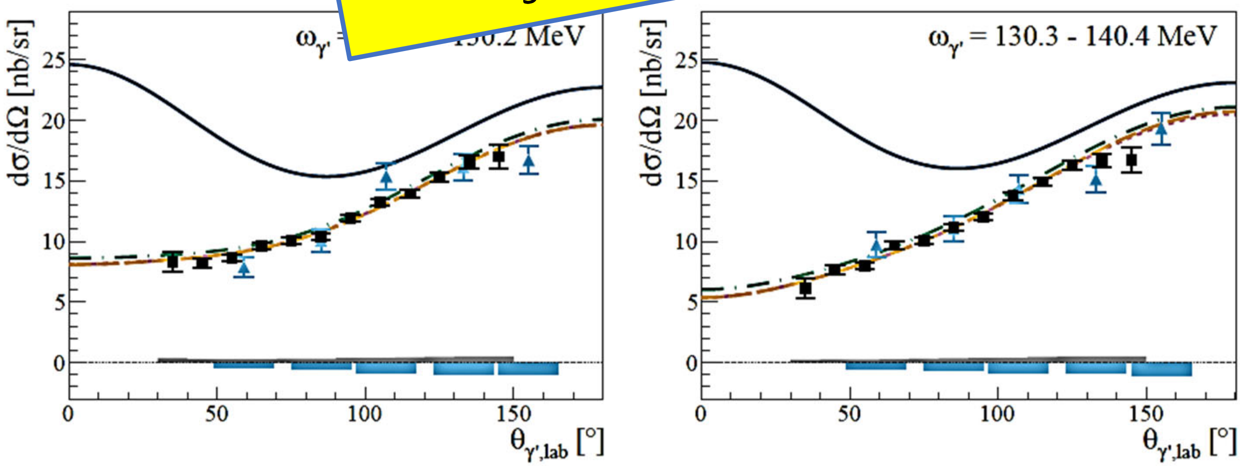
Differential Cross section (nb/sr)



E. Mornacchi et al.,
PRL 128, 132503 (2022)

.... the previous article is ...
X. Li, et al.,
PRL 128, 132502 (2022).

(9 Compton points for $d\sigma_{\perp}$, $d\sigma_{\parallel}$, Σ_3 at ≃83 MeV)



△ O. De Leon (2001) – TAPS collaboration

■ Mornacchi et al.

Significant improvement with respect to all previous data (~ same number of DCS points of the TAPS experiment -the most comprehensive single data set up to now- with much higher precision. In addition, also $\mathbf{\Sigma_3}$ 36 data points)

The RCS-proton data base ($\omega > 150$ MeV)

… RCS is an even more difficult experiment to perform …. additional (huge) background from the **$p\pi^0$** production process (cross section 2-3 orders of magnitude higher; when one of the two π^0 decay photons is not detected, **$p\gamma$** kinematics can mimic Compton reaction)

Only «new» esperiment using tagged photons were considered (over-determined event-by-event kinematics is essential – together with a precise MC simulation - to subtract this background)

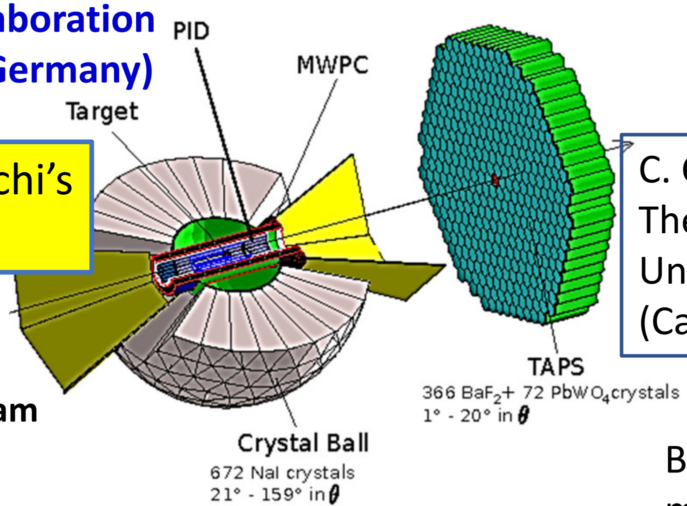
$\gamma p \rightarrow p\gamma'$

Detected both p and γ' – **Measured** E_γ ; $\theta_{\gamma'}\ \varphi_{\gamma'}\ E_{\gamma'}$; $\theta_p\ \varphi_p$ (only E_P is missing)

Applied kinematical cuts on $\Delta\varphi = (\varphi_{\gamma'} - \varphi_p)$ and $\Delta\theta = (\theta_{\gamma'} - \theta_p)$

A2@MAMI collaboration set-up (Mainz, Germany)

see E. Mornacchi's talk

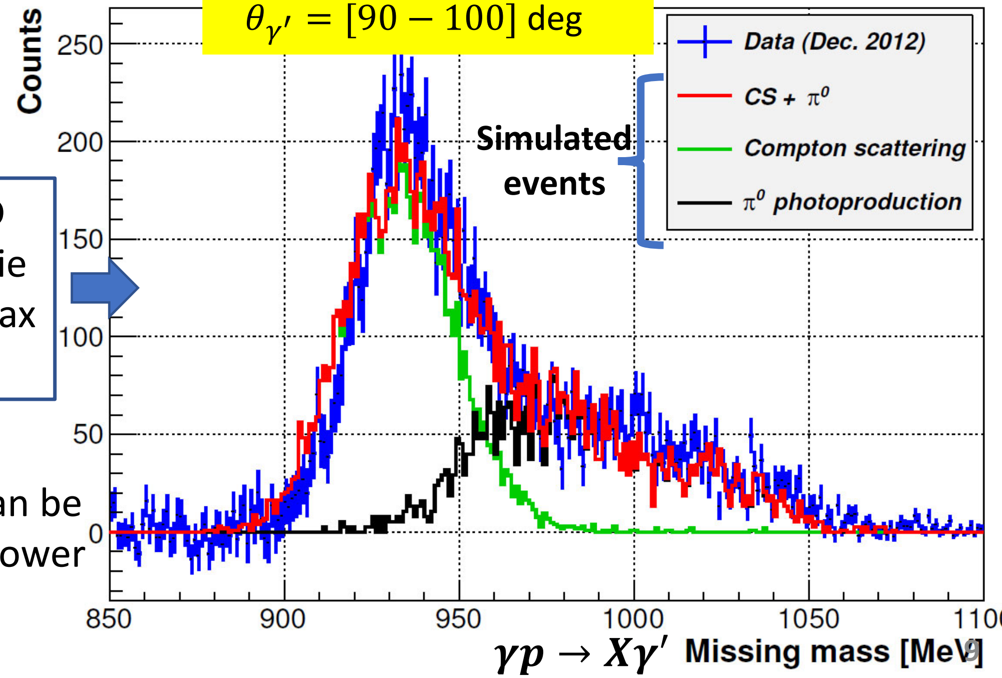


tagged photon beam

Geometrical acceptance $\cong 97\%$ of 4π.
Optimized for the detection of π^0 decay photons

C. Collicott, PhD Thesis, Dalhousie University, Halifax (Canada), 2015

$E_\gamma = [267 - 287]$ MeV
$\theta_{\gamma'} = [90 - 100]$ deg



Background can be much higher/lower depending on E_γ ; $\theta_{\gamma'}$

The RCS-proton data base (150 MeV < ν < 300 MeV)

Upper limit of validity of the used model

Two large data sets have been collected in this energy region (properties of the $\Delta(1232)$ resonance)

LEGS G. Blanpied et al., Phys. Rev. C 64, 025203 (2001).
LARA S. Wolf et al., Eur. Phys. J. A 12, 231 (2001).

LEGS 82 DCS and 82 Σ_3 points (58 below 300 MeV)
LARA 340 DCS points (128 below 300 MeV)

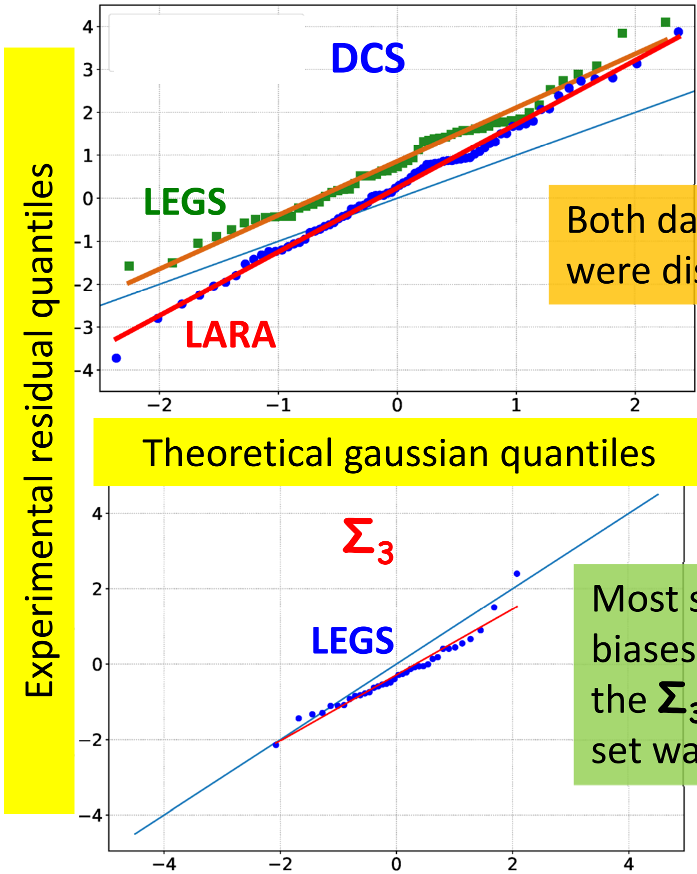
However, these two DCS data sets are known be **inconsistent** between each other (see M. Schumacher, Prog. Part. Nucl. Phys. 55, 567 (2005), H. W. Griesshammer et al, Prog. Part. Nucl. Phys. 67, 841 (2012)). **Which is the «correct» one?**

Used DCS data base above 150 MeV

	N.Points
O. De Leon et al. (TAPS)	10
Camen et al.	5
Peise et al.	8
Wissmann et al.	6
Molinari et al.	4

Residual analysis: a fit test by alternatively including LARA or LEGS data in the database (results with MAID-2021 but nothing changes with the other solutions)

Residual QQ-plot



Both data sets were discarded

Most systematic biases vanish in the Σ_3 ratio. This set was used

The DR model

- RCS Amplitudes described using 6 Lorentz-invariant amplitudes $\boldsymbol{A_i(\nu, t)}$

$\nu \to \omega + t/4M$
$t \to$ transferred momentum

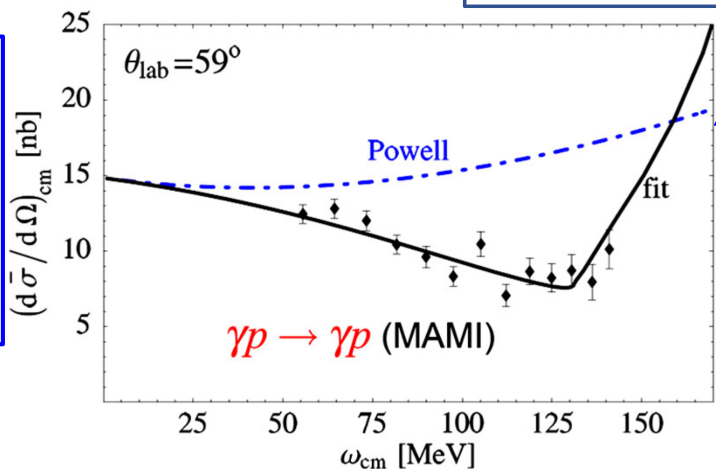
- They are determined using fixed-*t* subtracted dispersion relations

$$\mathbf{Re}\, A_i(\nu,t) = A_i^B(\nu,t) + \boxed{[A_i(0,t) - A_i^B(0,t)]} + \frac{2}{\pi}\nu^2 \mathcal{P}\int_{\nu_{thr}}^{+\infty} d\nu' \frac{\boxed{\mathbf{Im}_s A_i(\nu',t)}}{\nu'(\nu'^2 - \nu^2)}$$

Evaluated using Nπ multipoles from **all** the latest version of the MAID, SAID, BNGA analyses and contributions of Nππ… channels

πN threshold

Determined by additional once subtracted DRs in the *t*-channel with $\mathbf{Im}_t$ calculated using $\gamma\gamma \to \pi\pi$ and $\pi\pi \to N\bar{N}$ processes. The subtraction constants $\equiv \boxed{[A_i(0{,}0) - A_i^B(0{,}0)]}$ are directly related to the 6 polarizabilities



A^B => Powell (Born) cross section: photon scattering off a point-like nucleon with anomalous magnetic moment

Upper limit of validity ~300 MeV

D. Drechsel, M. Gorchtein, B. Pasquini, M. Vanderhaeghen, Phys.Rev. C61 (1999) 015204
B. Pasquini , D. Drechsel M. Vanderhaeghen, Phys.Rev. C76 (2007) 015203
B. Pasquini, M. Vanderhaeghen, Ann.Rev.Nucl.Part.Sci. 68 (2018) 75-103

The Fit procedure

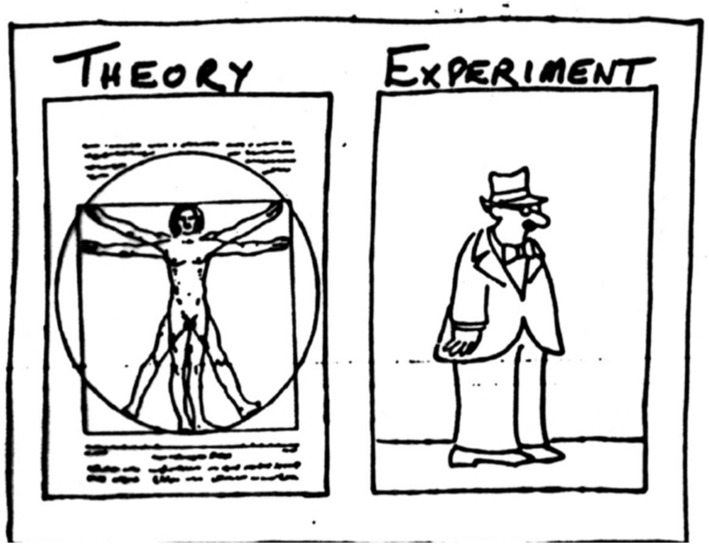
- **RCS data base**: **388 points from 25 different data sets and 6 different observables** $(d\sigma_{\text{unpol}}, d\sigma_{\parallel}, d\sigma_{\perp}, \Sigma_3, \Sigma_{2x}, \Sigma_{2z})$

- **For each data set, systematic uncertainties have to be taken into account in the fit procedure, to perform the correct error propagation.**

  - ✓ «point to point» systematic uncertainties (when present) are quadratically combined with the statistical uncertainties

  - ✓ the remaining systematic uncertainites are are **common scale factors** They are assumed to be uniformly distributed – if non otherwise specified- («maximum ignorance» principle)

Best fit of fully correlated data



- **Fit parameters:** $(\alpha_{E1}+\beta_{M1})$ $(\alpha_{E1}-\beta_{M1})$ γ_{E1E1} γ_{M1M1} γ_0 γ_π

Fit with systematic uncertainties-I

G .D'Agostini,
NIM A 346, 306 (1994)

➢ **Standard method (a single subset and a single common scale factor):**

$$\chi^2_{mod}(\theta,\lambda) = \sum_{i=1}^{N}\left(\frac{(\lambda\cdot y_i - \mu_i(\theta,x_i))}{\lambda\cdot\sigma_i}\right)^2 + \left(\frac{\lambda-1}{\sigma_{sys}}\right)^2 \equiv \chi^2_{mod}(\theta,\alpha) = \sum_{i=1}^{N}\left(\frac{(y_i - \alpha\cdot\mu_i(\theta,x_i))}{\sigma_i}\right)^2 + \left(\frac{\alpha-1}{\sigma_{sys}}\right)^2$$

with $\boldsymbol{\lambda = 1/\alpha}$

Drawbacks:

- Valid only for gaussian systematic uncertainties
- When different data subsets are fitted, one additional normalization factor per subset is needed (problem with large data bases) ⟹ **[in our case this would mean to have 25 additional fit parameters (!!)]**
- In general, systematic errors may also vary within a given subset (i.e. for angular –dependent errors) and statistical errors may also not be gaussian (there could, for instance, be asymmetric errors).
- In general, the minimum value $\hat{\chi}^2_{mod}$ is not distributed according to the chi-squared density (sum in quadrature of variables which are neither independent nor gaussian). What is the correct goodness-of-fit distribution ?
- are errors on the final $\hat{\theta}$ values Gaussian-distributed (product/ratio of 2 gaussians is not *a priori* gaussian) ?

Fit with systematic uncertainties-II

P. Pedroni, S. Sconfietti, JPG 47, 054001 (2020)

A new boostrap-based method (a single subset and a single common scale factor):

known (kinematical variables, in our case E_γ ; $\theta_{\gamma'}$)

$$Y_i \sim p(x_i, \mu_i) \quad \Rightarrow \quad Y_i^* \sim p^*(x, \mu_i = y_i)$$

A model for the distribution of the experimental data

known (is the experimental resolution) unknown true value of $d\sigma/d\Omega(x_i)$ known measured value

- ✓ Generate a bootstrapped data sample :$y_1^{*\prime}$, $y_2^{*\prime}, \dots, :y_N^{*\prime}$
- ✓ Fit this virtual data sample to get $\hat{\theta}^{*\prime}$
- ✓ Repeat the previous steps to get a sample $\hat{\theta}^{*\prime}, \hat{\theta}^{*\prime\prime}, \dots, \hat{\theta}^{*\prime\prime\prime\dots\prime\prime\prime}$ from which to obtain the final results

$$y_i^* = (1 + \xi_{sys})(y_i + \tau \cdot \sigma_{stat})$$

with

$\xi_{sys} \sim p_{sys}$ $\quad$ $\tau \sim Gauss(0, 1)$

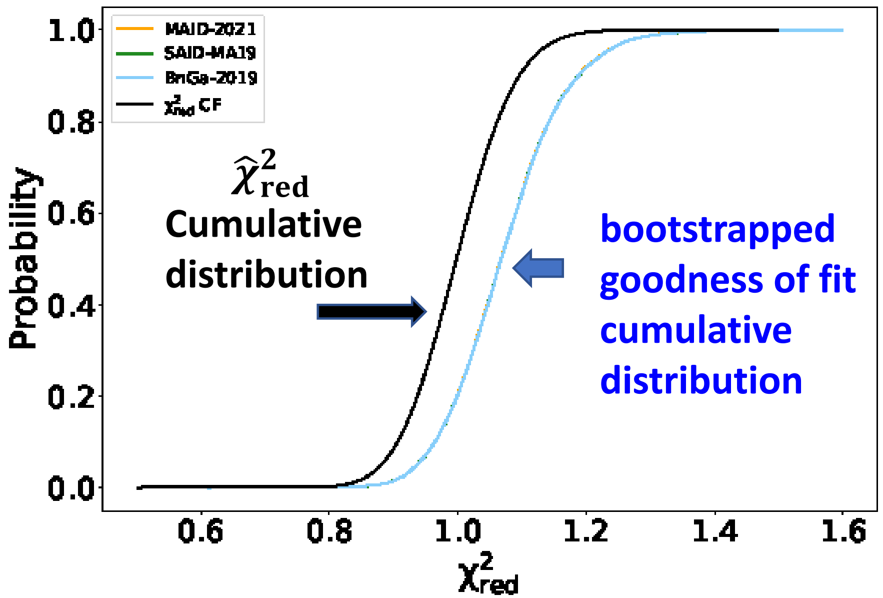
same value for all points of a given subset

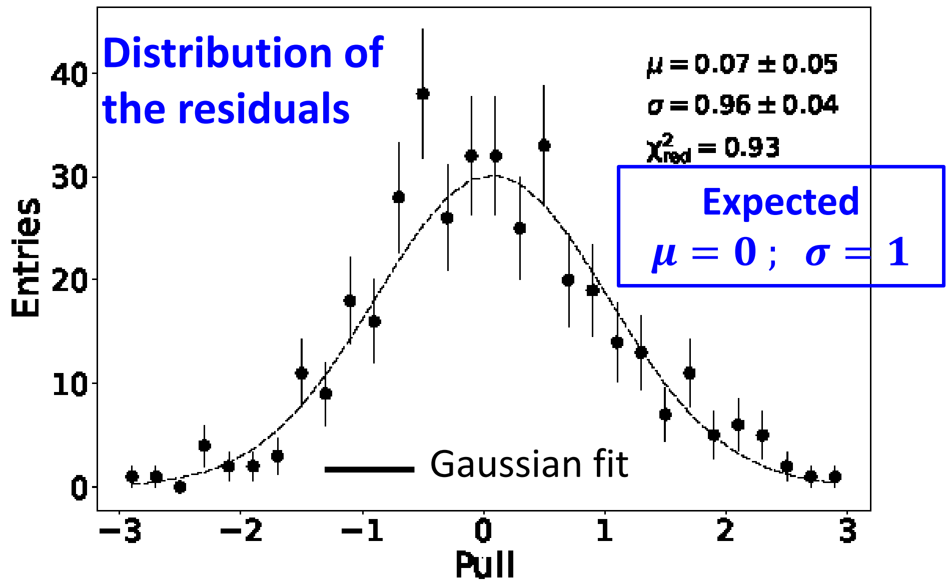
Advantages:

- No a priori assumption needed for both the systematic and the statistical errors (any type of distribution can be easily simulated); the correct p.d.f. of the fit parameters is always provided automatically by the procedure
- No additional fit parameter is needed
- The correct goodness-of-fit distribution and the correct *p*-value of the "χ^2 test" are also always provided by this procedure

Drawback (as any other MC-based method): a relevant computational time may be needed to get precise results

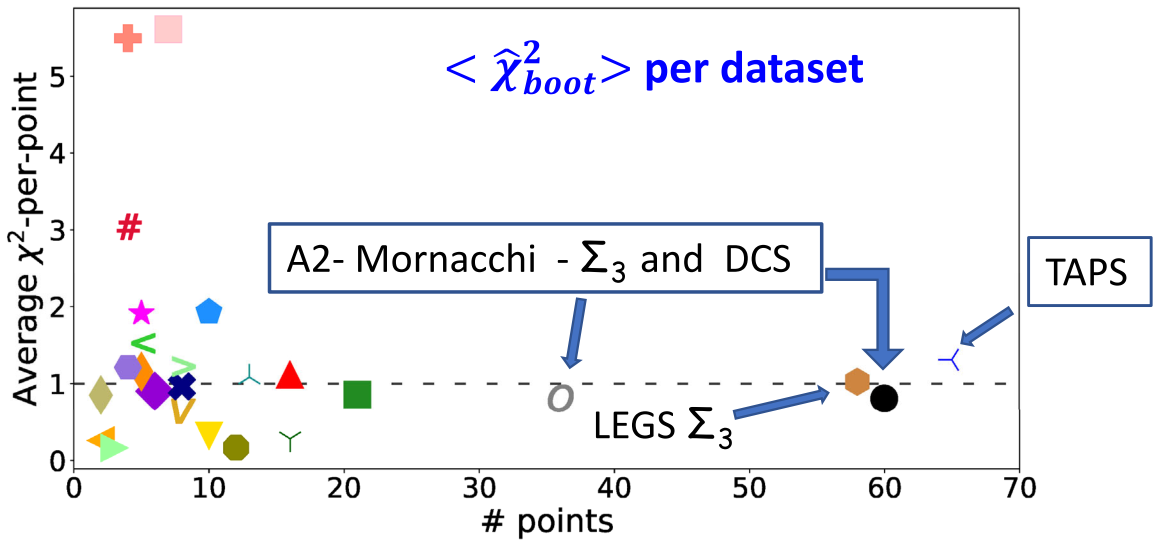
Fit Results





Basically the same values/distributions for each multipole solution

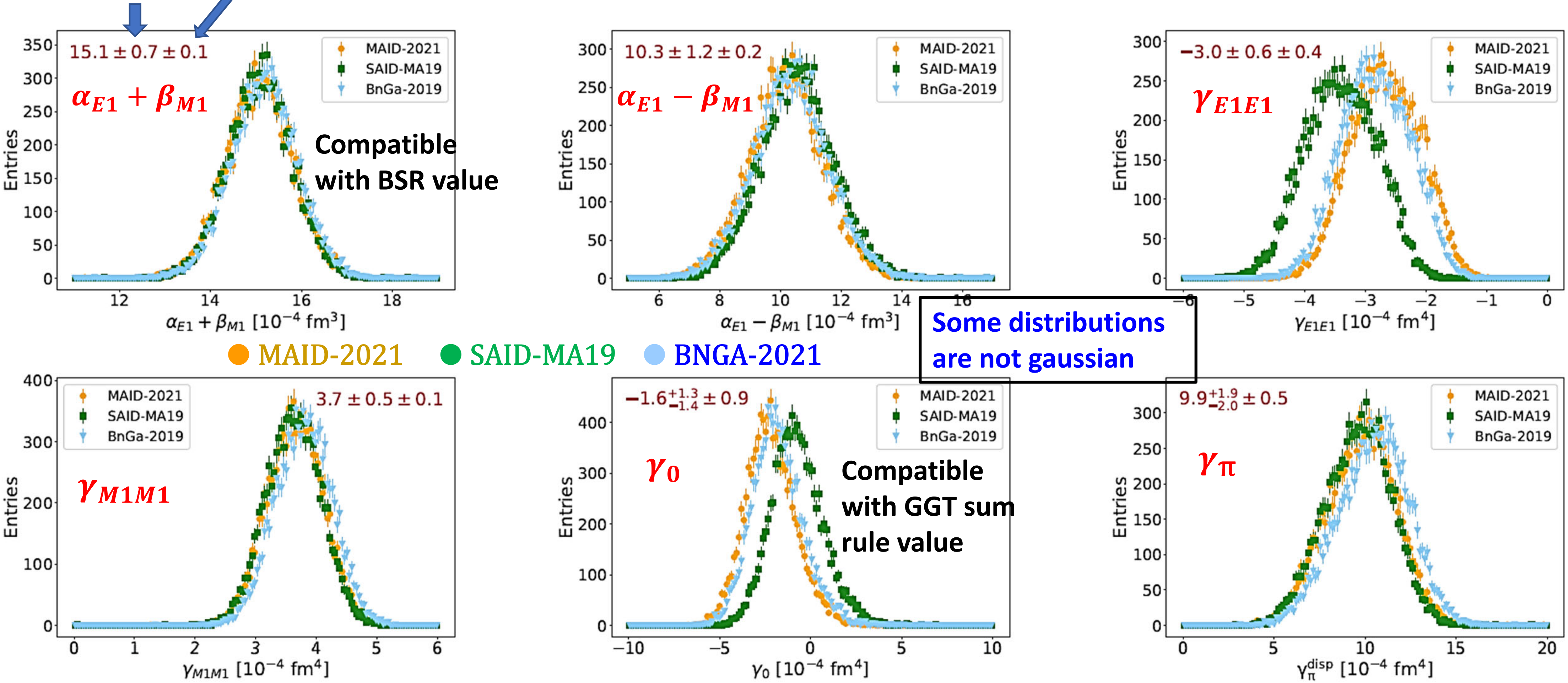
$\widehat{\chi}^2_{boot}$ = 1.13
p-value = 24%



Fit Results- parameter distributions

68% CL (stat+sys)

model error



Different multipole solutions give (very) similar results.

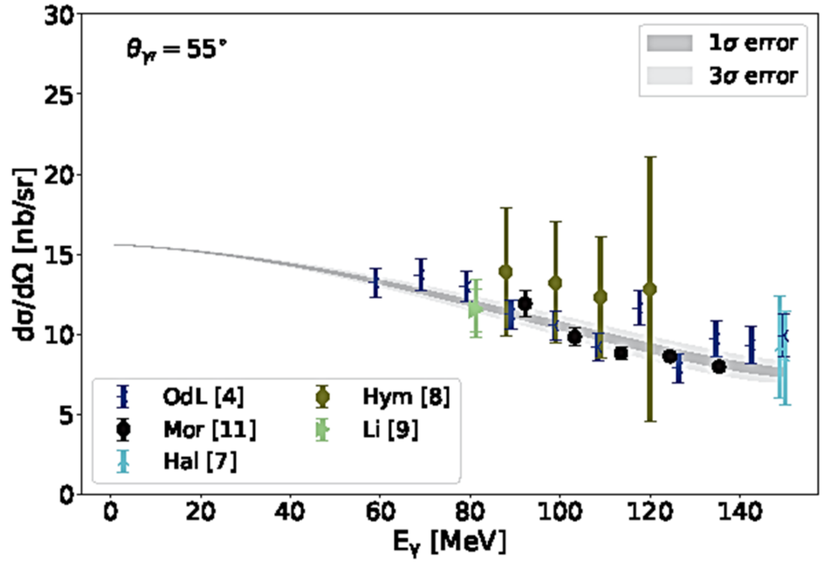
Estimate of the model error: largest of the differences between each set of fit values and the average was used to estimate an additional model error (conservatively considered as a standard deviation)

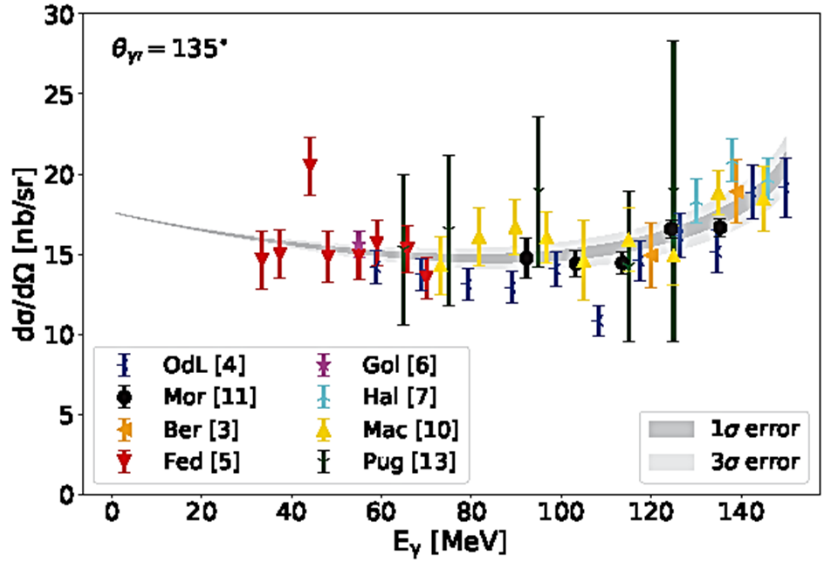
Fit Results –correlation matrix

In agreement with Baldin's sum rule

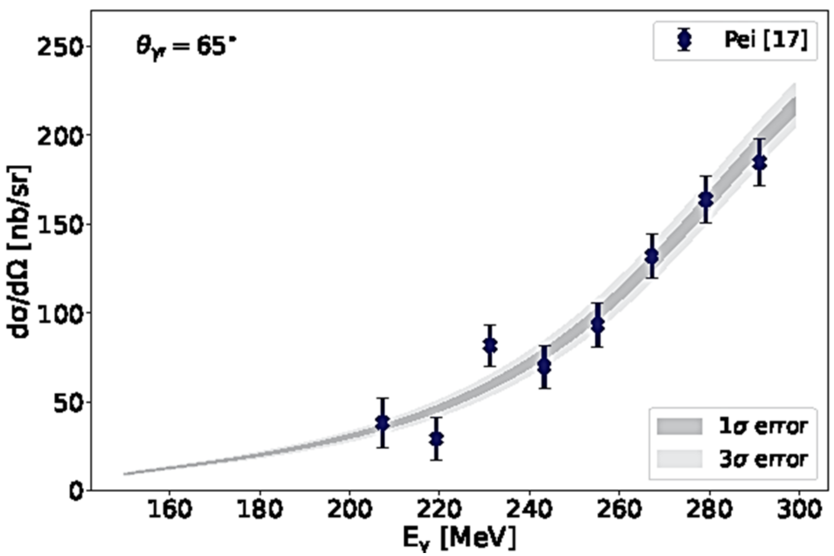
	$\alpha+\beta$	$\alpha-\beta$	γ_{E1E1}	γ_{M1M1}	γ_0	γ_π
$\alpha+\beta$	1.0	0.34	0.04	-0.2	-0.17	-0.32
$\alpha-\beta$		1.0	-0.51	0.34	0.36	0.62
γ_{E1E1}			1.0	-0.35	-0.21	-0.67
γ_{M1M1}				1.0	-0.39	0.81
γ_0					1.0	0.13
γ_π						1.0

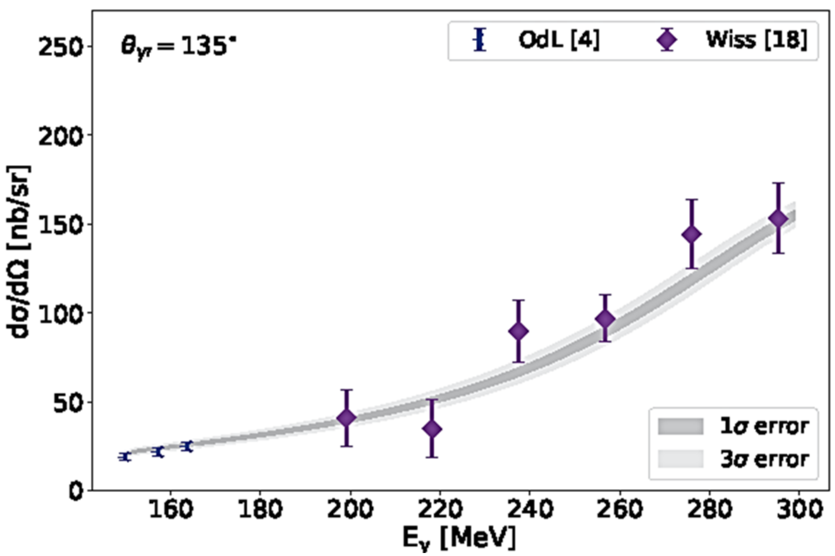
Compton data can hardly disentangle γ_π from γ_{M1M1}





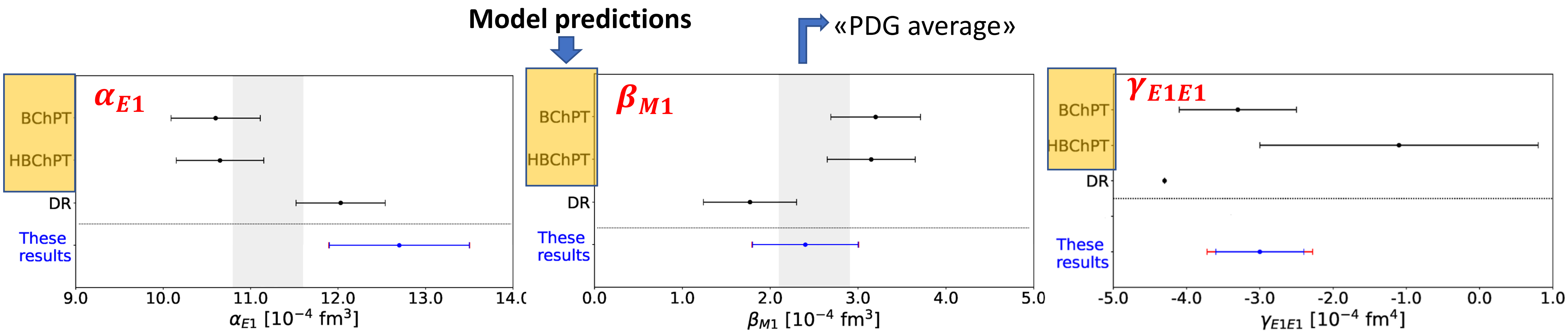
Fit Results – comparison with data



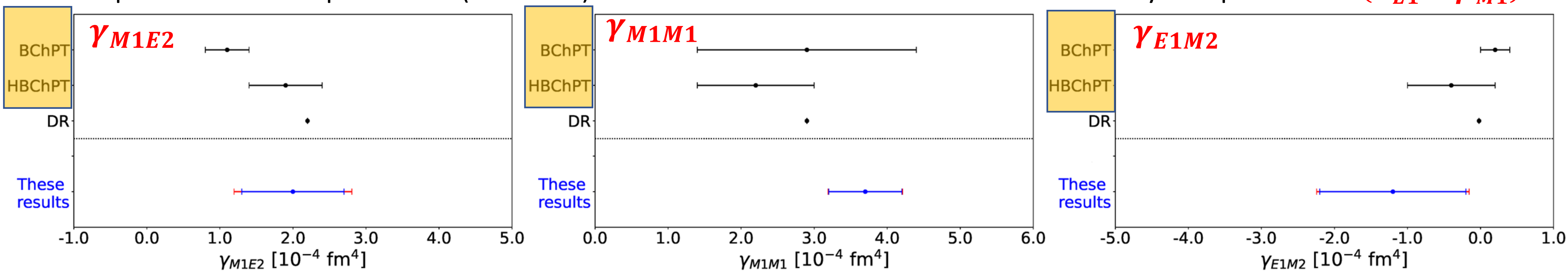


➢ First consistent comparison with existing theoretical models

BChPT: V. Lensky et al, Phys. Rev. C 89, 032202 (2014) **HBChPT**: J. McGovern et al, Eur. Phys. J. A 49, 12 (2013)



DR: our previous bootstrap-based fit (JPG-2020): "old" 150 DCS data below threshold with only 1 fit parameter $(\alpha_{E1} - \beta_{M1})$



$\alpha_{E1} = 12.7 \pm 0.8 \pm 0.1$ $\beta_{M1} = 2.4 \pm 0.6 \pm 0.1$ $\gamma_{E1E1} = -3.0 \pm 0.6 \pm 0.4$

$\gamma_{M1M1} = 3.7 \pm 0.5 \pm 0.1$ $\gamma_{E1M2} = -1.2 \pm 1.0 \pm 0.3$ $\gamma_{M1E2} = 2.0 \pm 0.7 \pm 0.4$

Conclusions and Outlook

- **Last A2 proton-Compton DCS data have reached a significant precision and accuracy (stat.errors ≃ sys.errors $\approx$ 2-5%) over a wide angular and energy range.**

- **This critical experimental improvement and a new boostrap-based fitting method have allowed to obtain the first concurrent extraction of all 6 leading-order proton polarizabilities**

> We have already performed the first extraction of the **scalar dynamical polarizabilities** values – See PRC-2018

- **However this important milestone is only the first step of a long run …**
  - ✓ relative uncertainties (in std.dev. units) on α_{E1} (7%) and β_{M1} (30%) are quite large
  - ✓ relative uncertainties on all γ_i are even larger (from 15% to 100%)

- **… since the overall quality of the proton-Compton DB is still quite poor**
  - ✓ **Many More** DCS data, especially at backward angles and below/**above** pion threshold are needed (and with very small stat. and systematic uncertainties) to increase precision on α_{E1} and β_{M1}
  - ✓ **Many more** data on polarization observables (especially for $\Sigma_{2x,z}$) are mandatory to increase precsision on all γ_i. Also additional polarization (beam-recoil) observables must be measured

Backup

Multipole Expansion for RCS

R.Hildebtrandt et al., EPJA 20, 293 (2004)

$$T_{fi} = \frac{4\pi W}{M} \sum_{i=1}^{6} \rho_i R_i(\omega, \cos\theta)$$

$R_i \Rightarrow$ 6 Independent amplitudes

$$R_1 = \sum_{l\geq 1} \{[(l+1) f_{EE}^{l+} + l f_{EE}^{l-}](l P_l' + P_{l-1}'') - [(l+1) f_{MM}^{l+} + l f_{MM}^{l-}] P_l''\}$$

$$R_2 = \sum_{l\geq 1} \left\{[(l+1) f_{MM}^{l+} + l f_{MM}^{l-}](l P_l' + P_{l-1}'') - [(l+1) f_{EE}^{l+} + l f_{EE}^{l-}] P_l''\right\}$$

2 spin-independent amplitudes

$f_{TT'}^{l\pm}$ Corresponds to the transition $Tl \rightarrow T'l'$ with $T, T' = E, M \quad ; \quad l' = l \pm \{0,1\}$

Multipole expansion + nucleon polarizabilities $\Rightarrow$ **Dynamical polarizabilities**

$$\alpha_{E1-DYN}(\omega) = \frac{2 f_{EE}^{1+}(\omega) + f_{EE}^{1-}(\omega)}{\omega^2} \quad ; \quad \beta_{M1-DYN}(\omega) = \frac{2 f_{MM}^{1+}(\omega) + f_{MM}^{1-}(\omega)}{\omega^2}$$

$$\alpha_{E1} = \lim_{\omega\to 0} \alpha_{E1-DYN}(\omega) \quad ; \quad \beta_{M1} = \lim_{\omega\to 0} \beta_{M1-DYN}(\omega)$$

How to extract dynamical polarizabilites ?

Our method $\Rightarrow$ Dispersion relations (DRs) + Low Energy Expansion (LEX) $(\omega < 140$ **MeV)**

B.Holstein et al., PRC61, 034316 (2000)

RCS differential cross section → 6 amplitudes $A_i(\nu,t)$ $\nu \rightarrow \omega + t/4M$

$t \rightarrow$ transferred momentum

$A_i(\nu,t)$ are connected to the multipoles $f_{TT'}^{l\pm}$

$\left[A_i(0,0) - A_i^B(0,0)\right]$ are connected to the 6 static polarizabilites

$$\mathrm{Re}\,A_i(\nu,t) = A_i^B(\nu,t) + \left[A_i(0,t) - A_i^B(0,t)\right] + \frac{2}{\pi}\nu^2 P\int_{\nu_{thr}}^{+\infty} d\nu' \frac{\mathrm{Im}_s\, A_i(\nu',t)}{\nu'(\nu'^2-\nu^2)}$$

Dispersion relations

Born terms (can be exactly calculated)

Can be evaluated from $\gamma\gamma \rightarrow \pi\pi$, $\pi\pi \rightarrow N\overline{N}$, $\gamma N \rightarrow N\pi(\pi)$ data

$$\alpha_{E1-DYN}^{DR}(\omega) = f_\alpha(\alpha_{E1}, \beta_{M1}, \alpha_{E1\nu}, \beta_{M1\nu}) + g_\alpha(\gamma_i) + h_\alpha(\text{any other term})$$

$$\beta_{M1-DYN}^{DR}(\omega) = f_\beta(\alpha_{E1}, \beta_{M1}, \alpha_{E1\nu}, \beta_{M1\nu}) + g_\beta(\gamma_i) + h_\beta(\text{any other term})$$

(up to ω^5)

2 additional parameters to be fitted

Calculated using measured γ_i values

evaluated with DRs

Complications

3-parameter fit $(\alpha_{E1}-\beta_{M1})$; $\alpha_{E1\nu}$; $\beta_{M1\nu}$ $\rightarrow$ $(\alpha_{E1}+\beta_{M1})$ from Baldin's sum rule

Standard gradient (Newton) method to find the minimum of the "χ^2 function" using first and second derivatives

MINUIT WARNING IN HESSE
============== MATRIX FORCED POS-DEF BY ADDING 0.13727E-01 TO DIAGONAL.

Too high correlations between fitted parameters!

VERY low sensitivity of the data to dynamical polarizabilities

NO WAY to find the "right" minimum and to define "right" errors on fit parameters

Combination of SIMPLEX method and BOOTSTRAP technique
(purely geometrical search) (Monte Carlo)

Bootstrap and dynamical polarizabilities

3-parameter fit $(\alpha_{E1}-\beta_{M1})$; $\alpha_{E1\nu}$; $\beta_{M1\nu}$

✓ Baldin's sum rule

✓ Systematical errors ON

✓ FULL data set (150 data)

✓ TAPS data set (55 data) (O. De Leon et al., 2001)

✓ Errors on Baldin's sum rule and γ_π included in the procedure

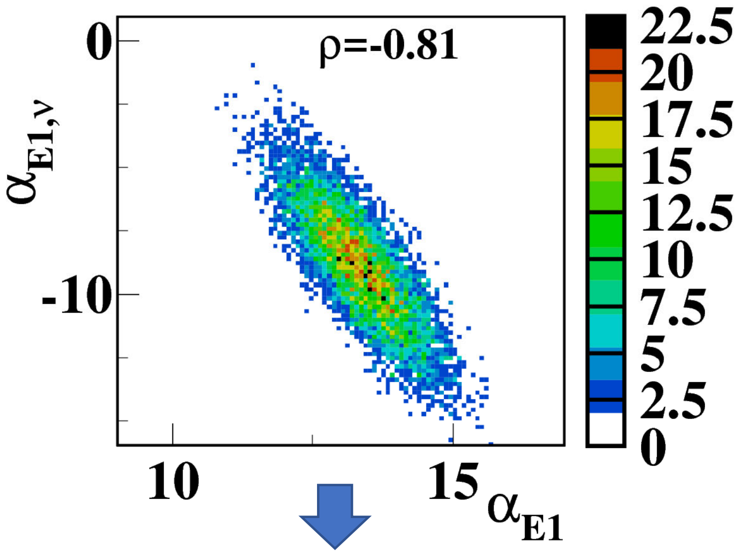
Dynamical polarizabilities: fit results

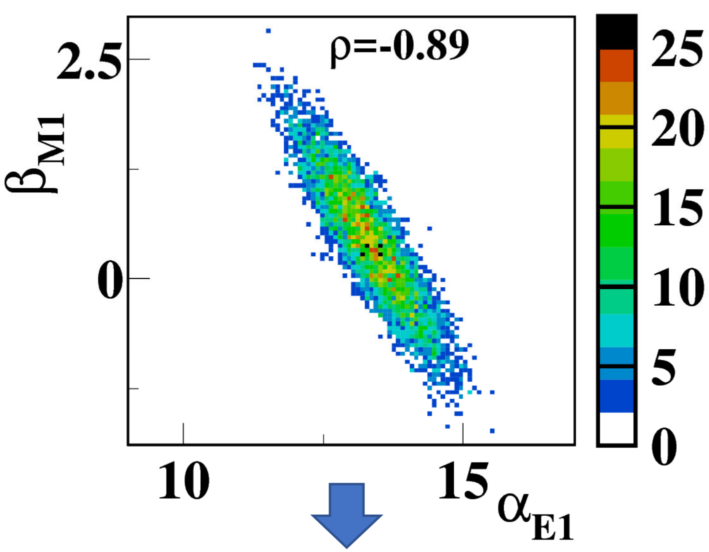
PRC-2019

		FULL	TAPS	BChPT model
α_{E1}	$(10^{-4}\mathrm{fm}^3)$	13.3 ± 0.8	11.6 ± 1.1	11.2±0.7
$\alpha_{E1,\nu}$	$(10^{-4}\mathrm{fm}^5)$	-8.8 ± 2.5	-3.2 ± 3.1	1.3±1.0
β_{M1}	$(10^{-4}\mathrm{fm}^3)$	0.4 ∓ 0.9	2.2 ∓ 1.1	3.9±0.7
$\beta_{M1,\nu}$	$(10^{-4}\mathrm{fm}^5)$	10.8 ± 2.8	5.1 ± 3.7	7.1±2.5

V. Lensky et al., EPJC 75, 604 (2015)

Quite strong dependence on data set (maybe due to different covered angular regions ?)



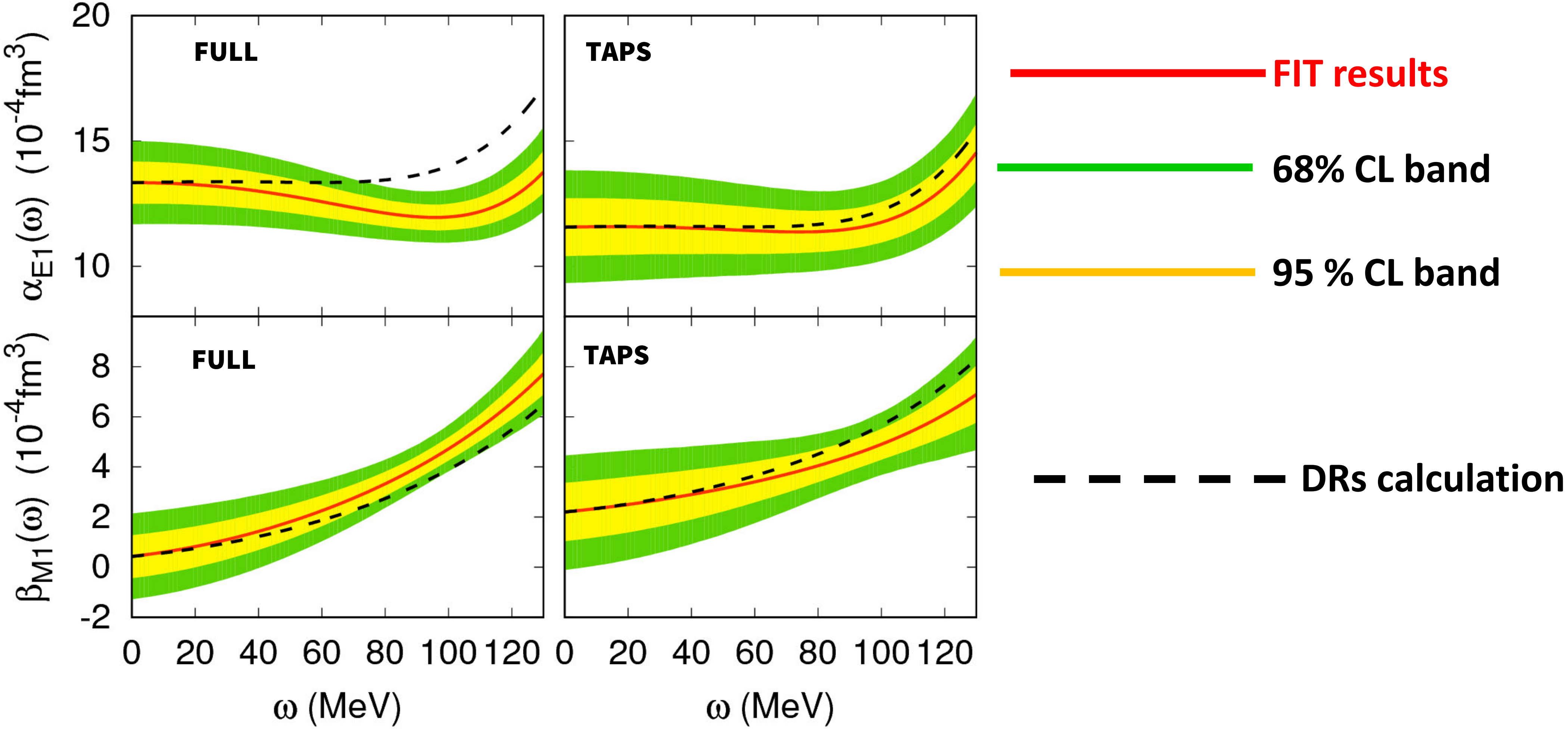


Very strong correlations between the fit parameters

Very low sensitivity of the data to $\alpha_{E1\nu}$

$(\alpha_{E1}+\beta_{M1})$ Constrained by the Baldin's rum rule

Dynamical polarizabilities: fit results



Parametric bootstrap

A.C.Davidson, D.V.Hinkley **Bootstrap Methods and Their Applications**
Cambridge University Press, 1997

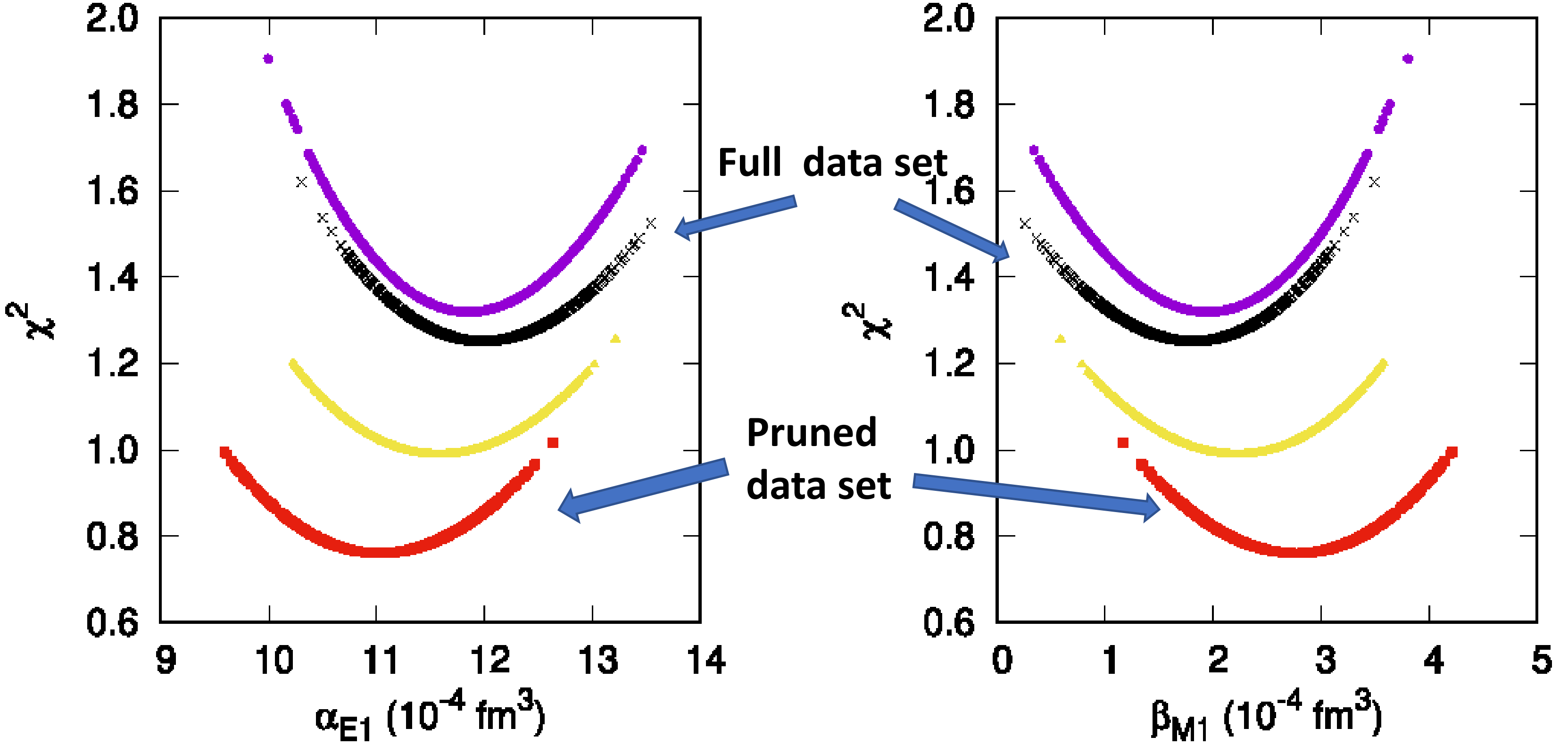
2.2 Parametric Simulation

In the previous section we pointed out that theoretical properties of T might be hard to determine with sufficient accuracy. We now describe the sound practical alternative of repeated simulation of data sets from a fitted parametric model, and empirical calculation of relevant properties of T.

Suppose that we have a particular parametric model for the distribution of the data $y_1, \ldots, y_n$. We shall use $F_\psi(y)$ and $f_\psi(y)$ to denote the CDF and PDF respectively. When ψ is estimated by $\hat{\psi}$ — often but not invariably its maximum likelihood estimate — its substitution in the model gives the *fitted model*, with CDF $\hat{F}(y) = F_{\hat{\psi}}(y)$, which can be used to calculate properties of T, sometimes exactly. We shall use Y^* to denote the random variable distributed according to the fitted model $\hat{F}$, and the superscript * will be used with E, var and so forth when these moments are calculated according to the fitted distribution. Occasionally it will also be useful to write $\hat{\psi} = \psi^*$ to emphasise that this is the parameter value for the simulation model.

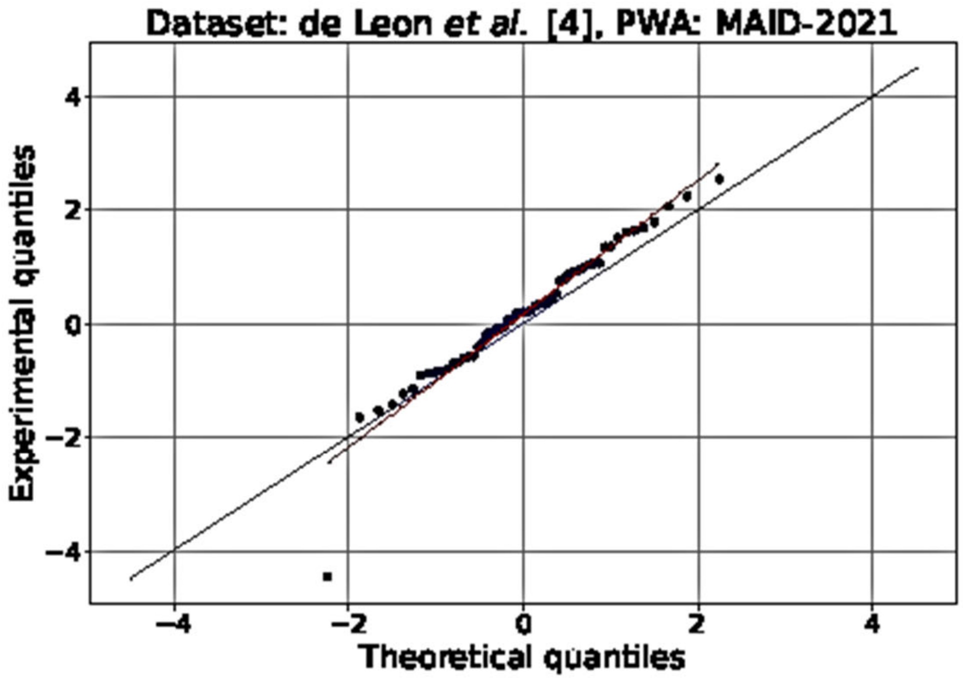
profile fo the χ^2 function of α_{E1} and β_{M1}

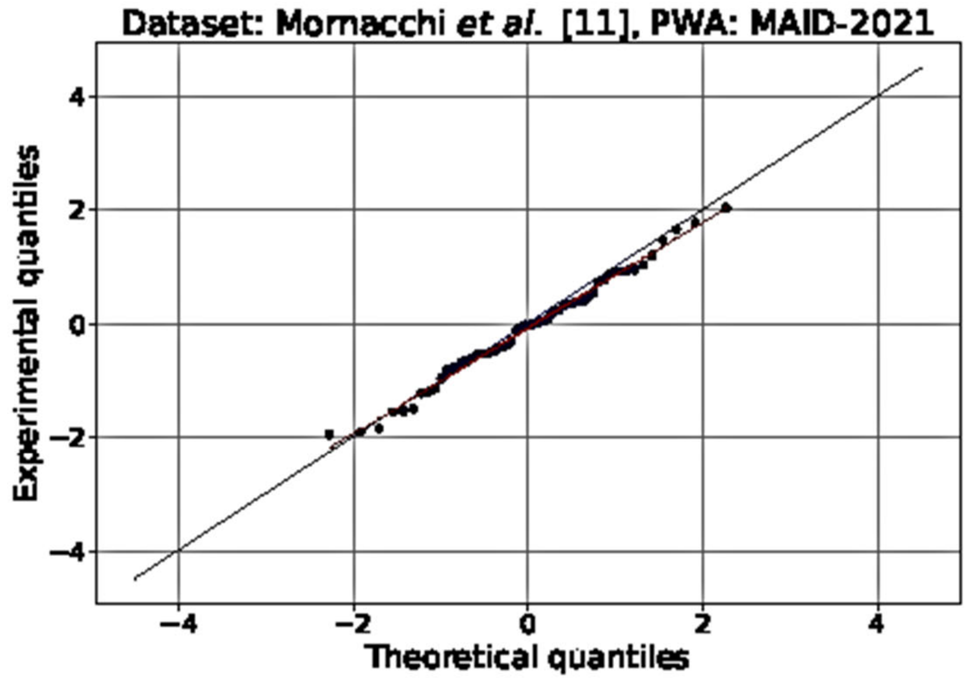
Full data set



«Width» of the parabola is the same

=> same final errors for the pruned and full data sets


Dataset: de Leon et al. [4], PWA: MAID-2021
Experimental quantiles
Theoretical quantiles
4
2
0
−2
−4
−4
−2
0
2
4


Dataset: Mornacchi et al. [11], PWA: MAID-2021
Experimental quantiles
Theoretical quantiles
4
2
0
−2
−4
−4
−2
0
2
4