

# **Spin sum rules and polarizabilities from the JLab CLAS EG4 experiment.**

**A. Deur** (for Marco Ripani)

Thomas Jefferson National Accelerator Facility

# The EG4 experiment Group

Main goal: measurement of the generalized **Gerasimov-Dreall-Hearn (GDH)** sum for the **proton**, **deuteron** and **neutron** at very low  $Q^2$ .

## E03-006 ( $\text{NH}_3$ ):

Spokespeople: **M. Ripani**, M. Battaglieri, A.D., R. de Vita

Students: H. Kang (Seoul U.), K. Kovacs (UVa)

X. Zheng *et al.* (CLAS Collaboration),  
Nature Physics, vo. 17 736-741 (2021)

## E05-111 ( $\text{ND}_3$ )

Spokespeople: **A.D.**, G. Dodge, M. Ripani, K. Slifer

Students: K. Adhikari (ODU)

K.P. Adhikari *et al.* (CLAS Collaboration),  
PRL 120, 062501 (2018)

EG4 ran at Jefferson Lab (JLab) from Feb. to May 2006.

Focus on inclusive analyses, but exclusive analysis ( $\vec{e} \vec{p} \rightarrow e \pi^+(n)$ ) also available.

X. Zheng *et al.* (CLAS Collaboration), PRC 94, 045206 (2016)

# The EG4 experiment Group

Main goal: measurement of the generalized **Gerasimov-Dreall-Hearn (GDH)** sum for the **proton**, **deuteron** (and **neutron**) at very low  $Q^2$ .

Neutron information from EG4 will be shown with that of the Hall A neutron ( ${}^3\text{He}$ ) E97110 experiment, Thursday, 10:30.

# The GDH and Generalized GDH Sum Rules

**Sum rule:** relation between an integral of a dynamical quantity (cross section, structure function,...) and a global property of the target (mass, spin,...).

Can be used to:

- Test theory (e.g. QCD) and hypotheses with which they are derived. e.g. GDH, Ellis-Jaffe, Bjorken sum rules.
- Access the global property (e.g. spin polarizability sum rules)

**GDH sum rule:** derived for real photons ( $Q^2=0$ ):

$$\int_{v_{\text{thr}}}^{\infty} \frac{\sigma_A(v) - \sigma_P(v)}{v} dv = \frac{-4\pi^2 S \alpha \kappa^2}{M^2}$$

Diagram illustrating the GDH sum rule equation:

- Left side:**  $\int_{v_{\text{thr}}}^{\infty} \frac{\sigma_A(v) - \sigma_P(v)}{v} dv$
- Right side:**  $\frac{-4\pi^2 S \alpha \kappa^2}{M^2}$
- Annotations:**
  - QED coupling constant:**  $\alpha$  (blue arrow)
  - target anomalous magnetic moment:**  $S$  (green arrow)
  - target mass:**  $M$  (purple arrow)
  - target spin:**  $\kappa$  (red arrow)
  - photoprod. cross section with photon spin anti-parallel to S:**  $\sigma_A(v)$  (blue arrow)
  - photoprod. cross section with photon spin parallel to S:**  $\sigma_P(v)$  (red arrow)

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QED coupling constant  
target anomalous magnetic moment  
target mass  
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photoprod. cross section with photon spin anti-parallel to S

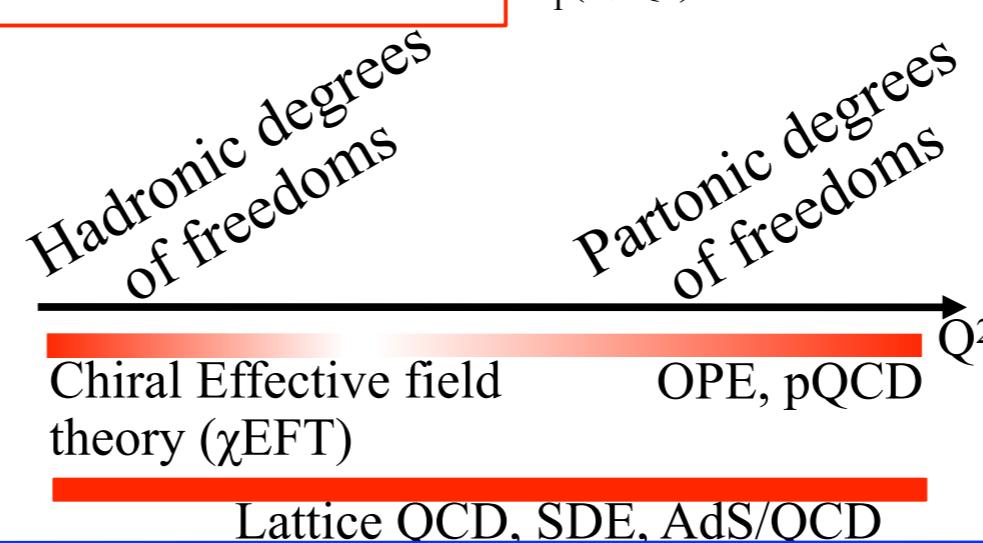
**Generalized GDH sum rule:** valid for any  $Q^2$ . Recover the original GDH sum rule as  $Q^2 \rightarrow 0$

$$\Gamma_1(Q^2) = \int_0^{x_{\text{th}}} g_1(x, Q^2) dx = \frac{Q^2}{2M^2} I_1(0, Q^2)$$

$g_1(v, Q^2)$ : first spin structure function (mostly a longit. target pol. observable)  
 $I_1(v, Q^2)$ : first covariant polarized VVCS amplitude

⇒ Study QCD at any scale

$I_1(0, Q^2) :$



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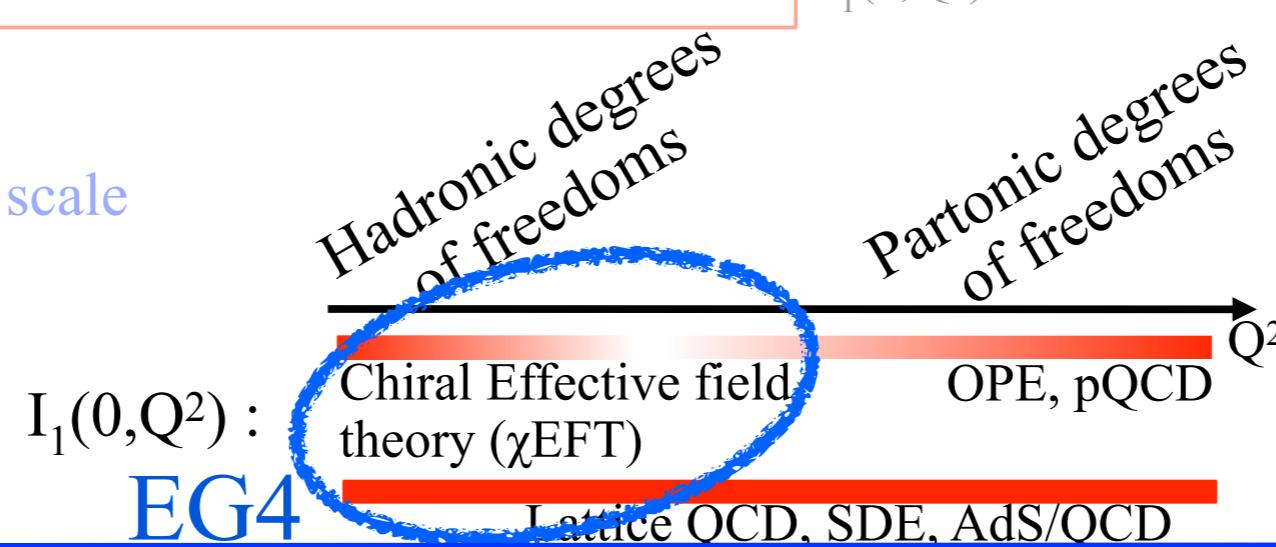
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# Spin polarizabilities sum rules

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Spin polarizability sum rules involve higher moments:

**Generalized forward spin polarizability:**

$$\gamma_0 = \frac{4e^2 M^2}{\pi Q^6} \int x^2 \left( g_1 - \frac{4M^2}{Q^2} x^2 g_2 \right) dx$$

$g_2(v, Q^2)$ : second spin structure function (mostly a perp. target pol. observable)

**Longitudinal-Transverse polarizability:**

$$\delta_{LT} = \frac{4e^2 M^2}{\pi Q^6} \int x^2 (g_1 + g_2) dx$$

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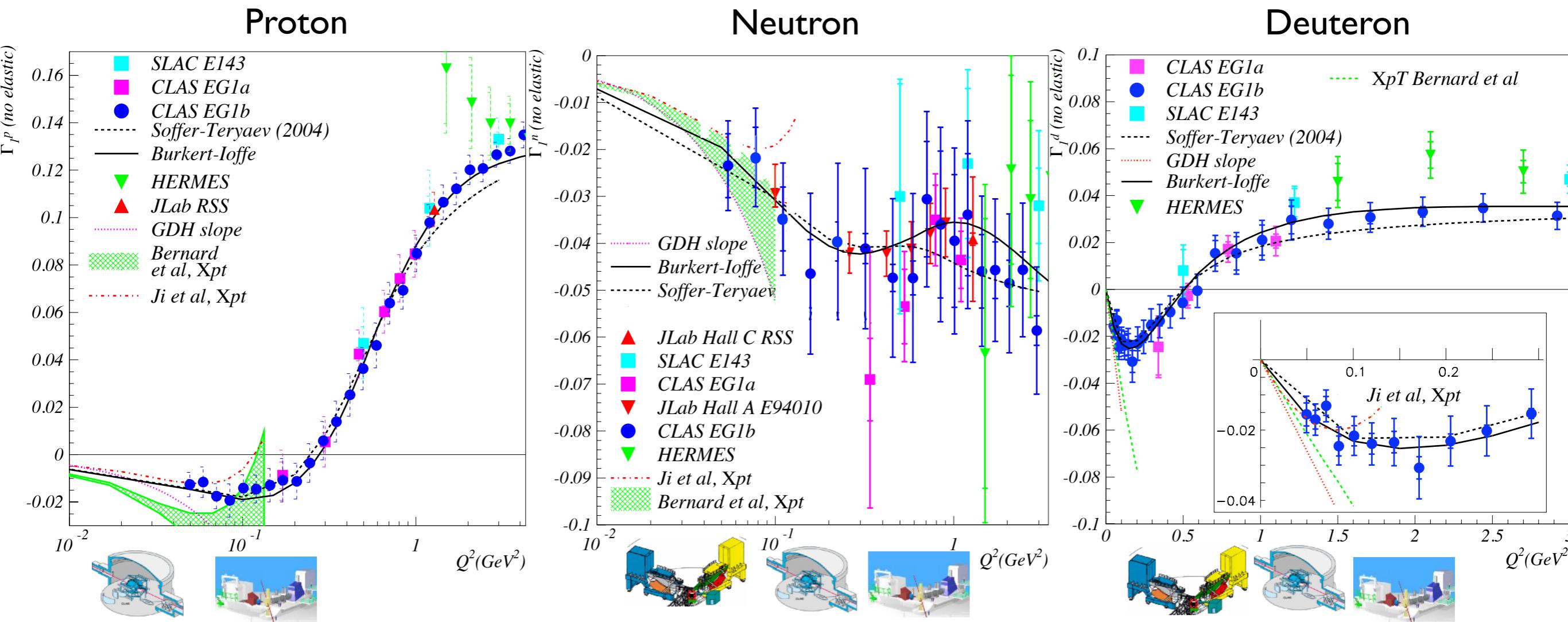
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# Previous JLab data: high to intermediate $Q^2$

$\Gamma_1(Q^2)$  Before EG4 run:



Precise mapping of spin structure function moments in intermediate  $Q^2$  region for p, n and d.

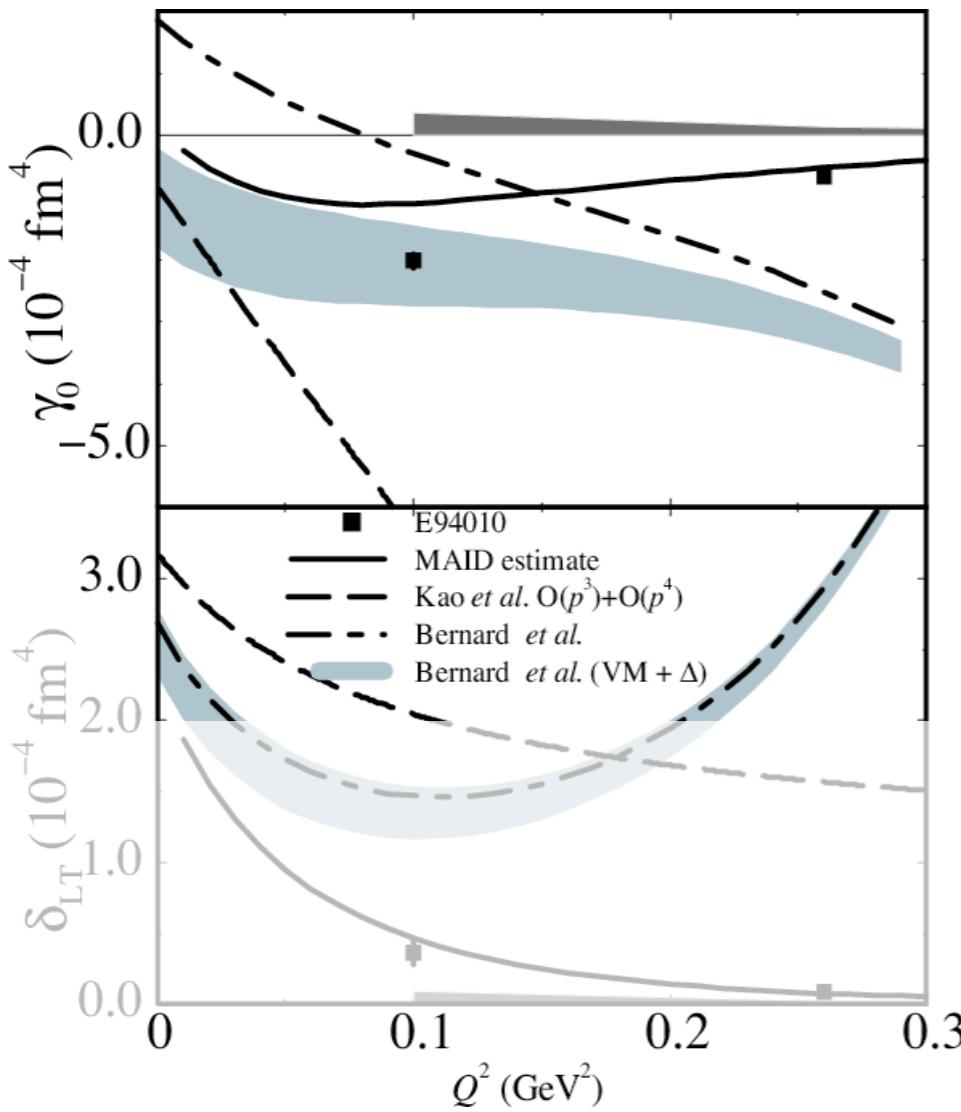
PQCD, models and data agree.

Not so clear for the  $\chi$ EFT predictions available at that time.

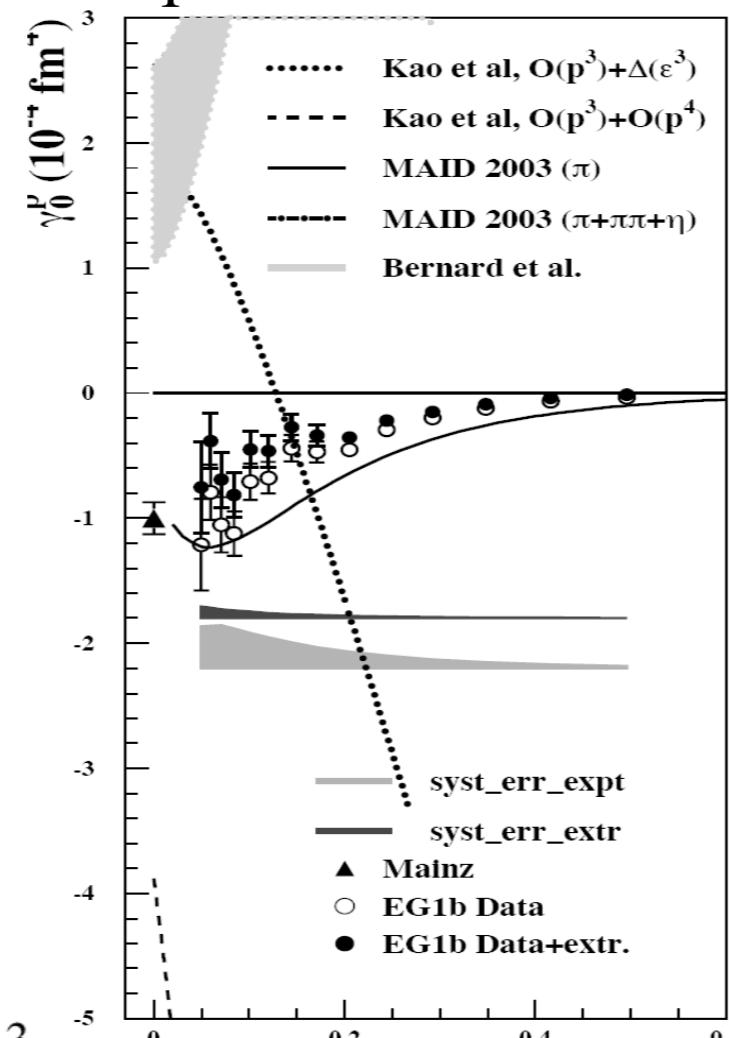
# Previous JLab data: high to intermediate $Q^2$

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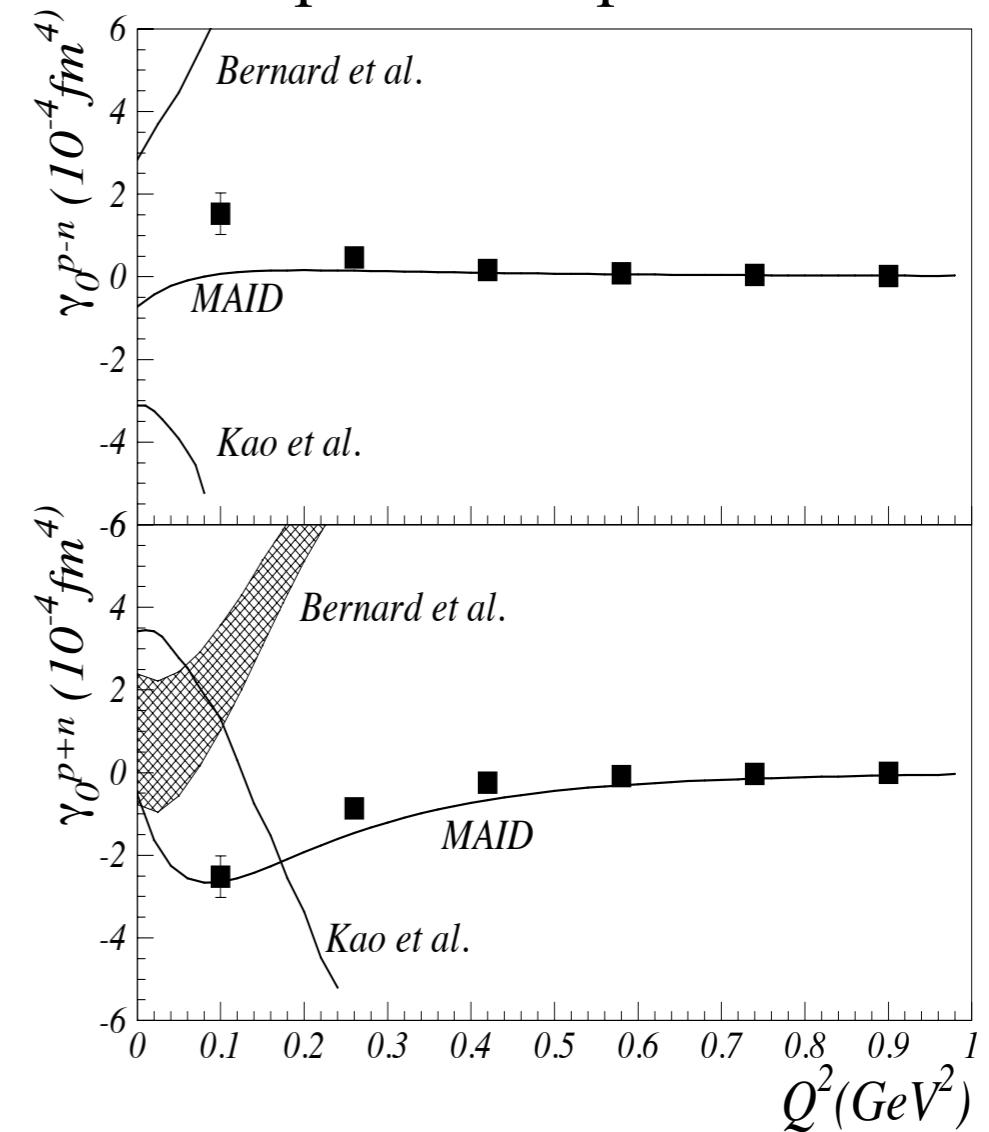
neutron JLab E94-010



proton JLab EG1b



Isospin decomposition



No agreement with the  $\chi$ EFT predictions available at that time.

# Previous data: high to intermediate $Q^2$

State of  $\chi$ EFT affairs before EG4 run:

A: ~agree

X: ~disagree

- : No prediction available

Ref.	$\Gamma_1^p$	$\Gamma_1^n$	$\Gamma_{\mathbf{1}}^{\mathbf{p}-\mathbf{n}}$	$\Gamma_1^{p+n}$	$\gamma_0^p$	$\gamma_0^n$	$\gamma_{\mathbf{0}}^{\mathbf{p}-\mathbf{n}}$	$\gamma_0^{p+n}$	$\delta_{LT}^p$	$\delta_{LT}^n$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X

1990s-2000s  $\chi$ EFT predictions in tension with spin observable data more often than not.

# Testing $\chi$ EFT

Ref.	$\Gamma_1^p$	$\Gamma_1^n$	$\Gamma_1^{p-n}$	$\Gamma_1^{p+n}$	$\gamma_0^p$	$\gamma_0^n$	$\gamma_0^{p-n}$	$\gamma_0^{p+n}$	$\delta_{LT}^p$	$\delta_{LT}^n$
			😊		😊	😊	😊 😊	😊	😊 😊	😊 😊

More robust measurements (no significant missing low-x contribution).

Nucleon resonance  $\Delta_{1232}$  contribution suppressed  
(More robust  $\chi$ EFT calculations)

# Testing $\chi$ EFT

State of  $\chi$ EFT affairs before EG4 run:

A: ~agree  
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- : No prediction available

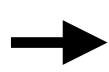
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Kao 2002	-	-	-	-	X	X	X	X		X

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The discrepancies for  $\delta_{LT}^n$  was particularly puzzling:

- Expected to be a robust  $\chi$ EFT prediction;
- Expected to be a robust measurement.

$\chi$ EFT calculation problem? Or were the experiments not reaching well enough into the  $\chi$ EFT applicability domain, i.e., reaching low enough  $Q^2$ ?



- Refined  $\chi$ EFT calculations, with improved expansion schemes & including the  $\Delta_{1232}$ .
- New experimental program at JLab reaching well into the  $\chi$ EFT applicability domain & with improved precision.

# Testing $\chi$ EFT

State of  $\chi$ EFT affairs before EG4 run:

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 X: ~disagree  
 - : No prediction available

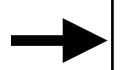
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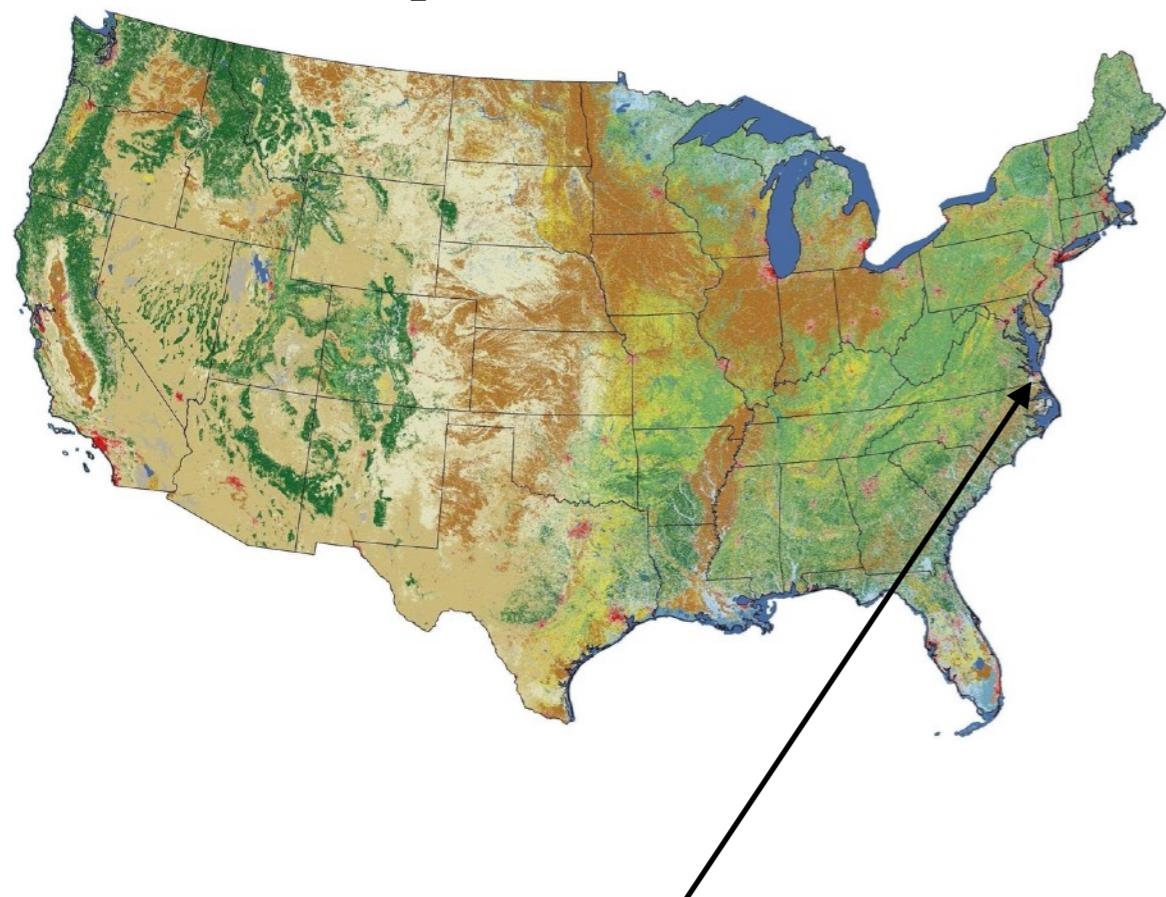


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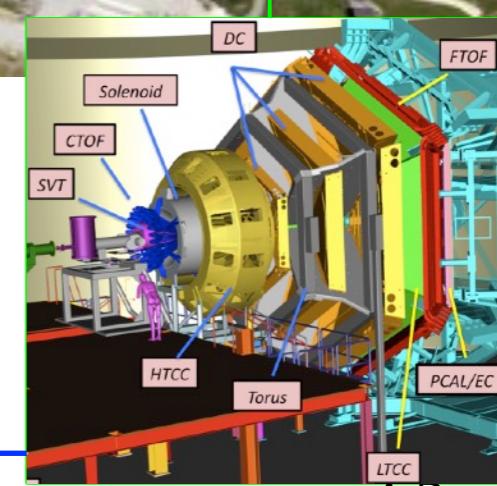
Continuous e<sup>-</sup> beam.  
up to 12 GeV.  
Polarization: ~90%  
Up to 200  $\mu$ A.  
4 experimental halls.



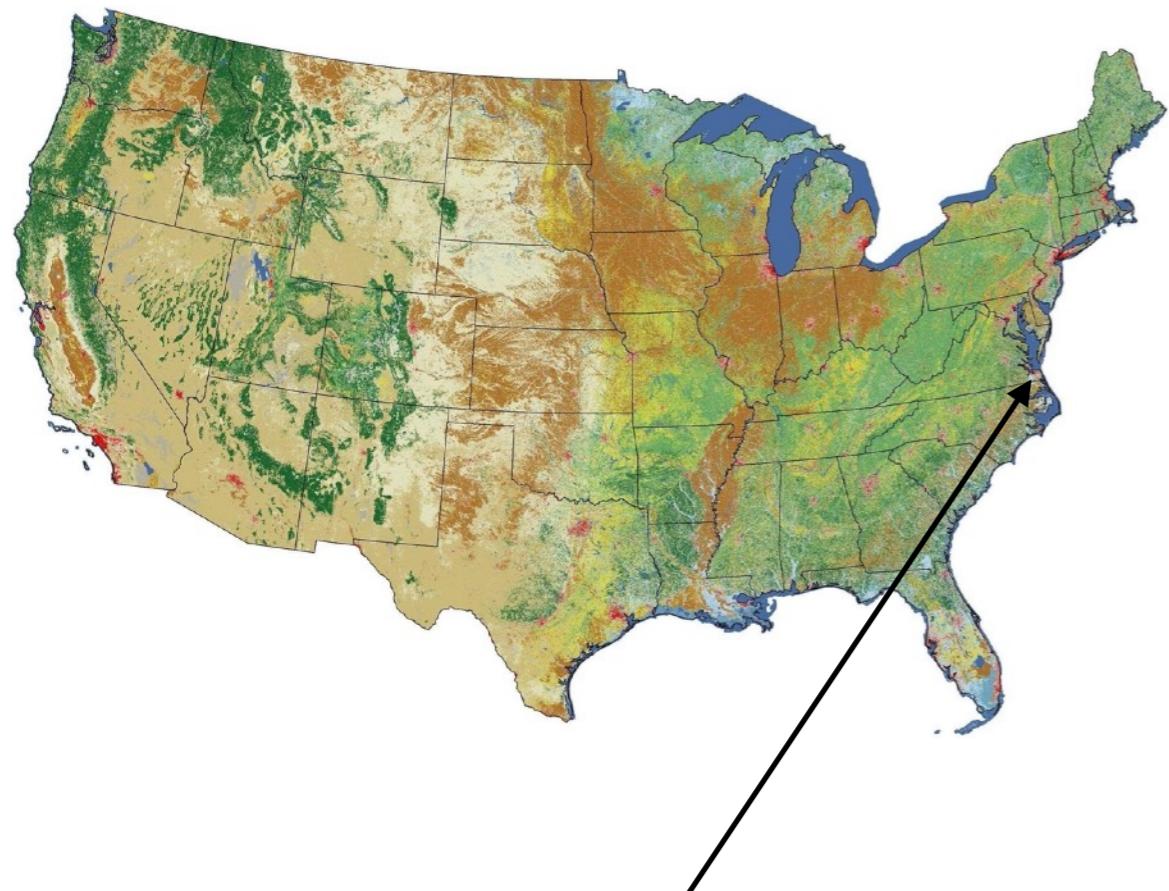
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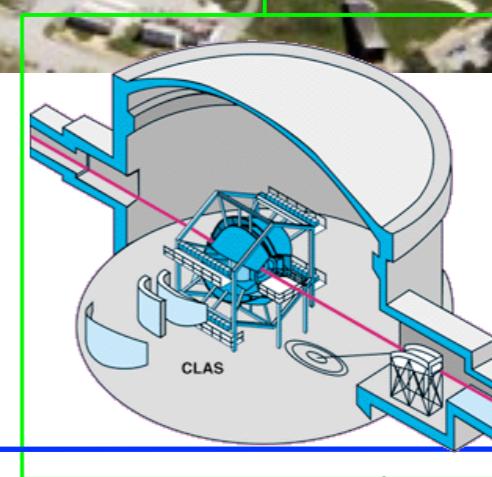
Newportnews, VA, USA



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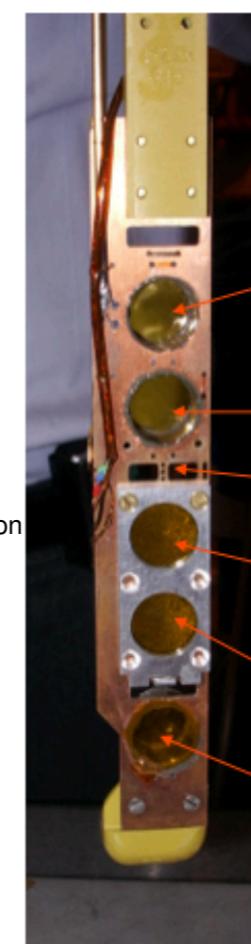
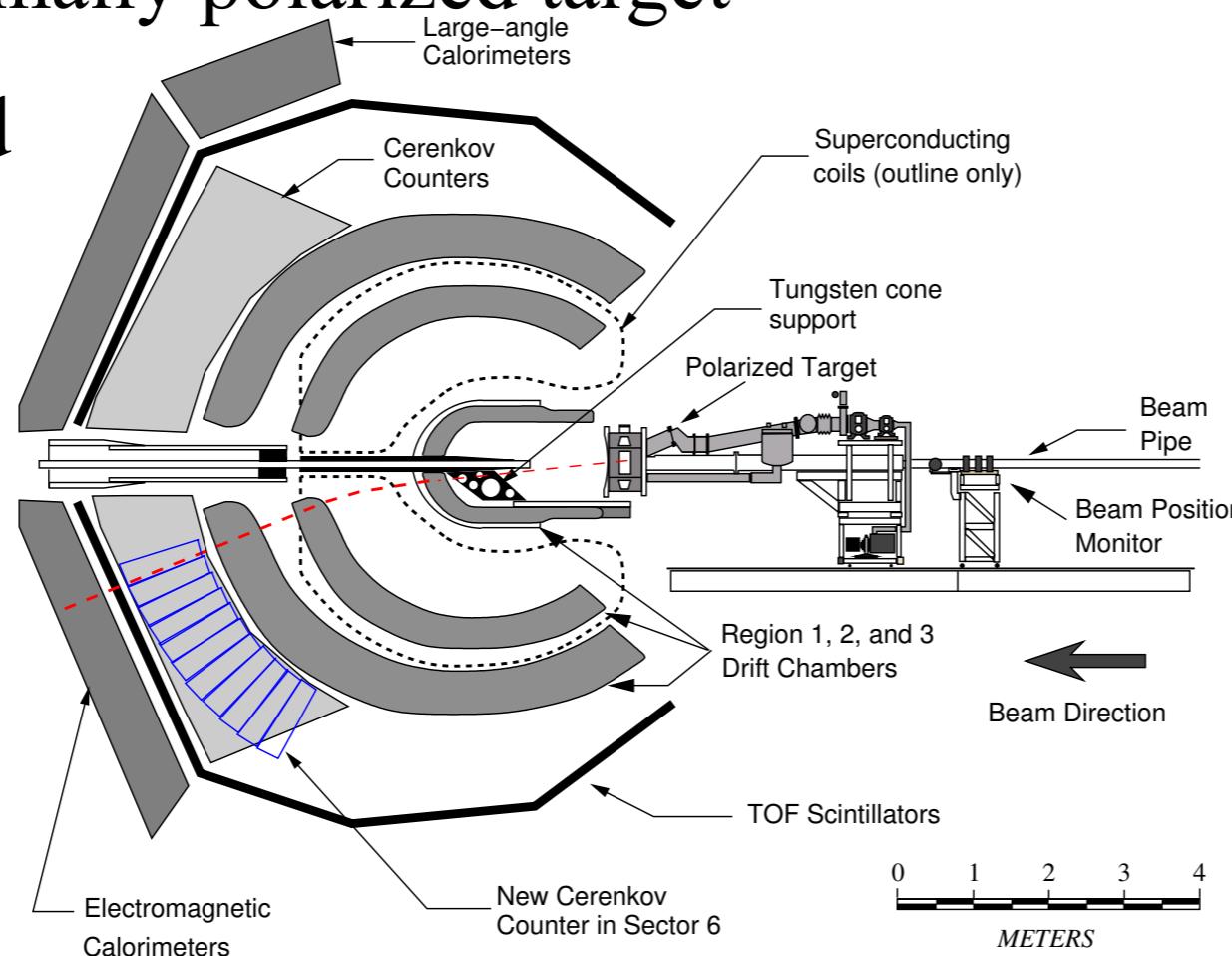
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# EG4 setup

- $Q^2 > 0$ : electron beam (polarized). Energies: 3.0, 2.3, 2.0, 1.3 & 1.0 GeV
- $g_1^{p,n}$ : ~longitudinally polarized target

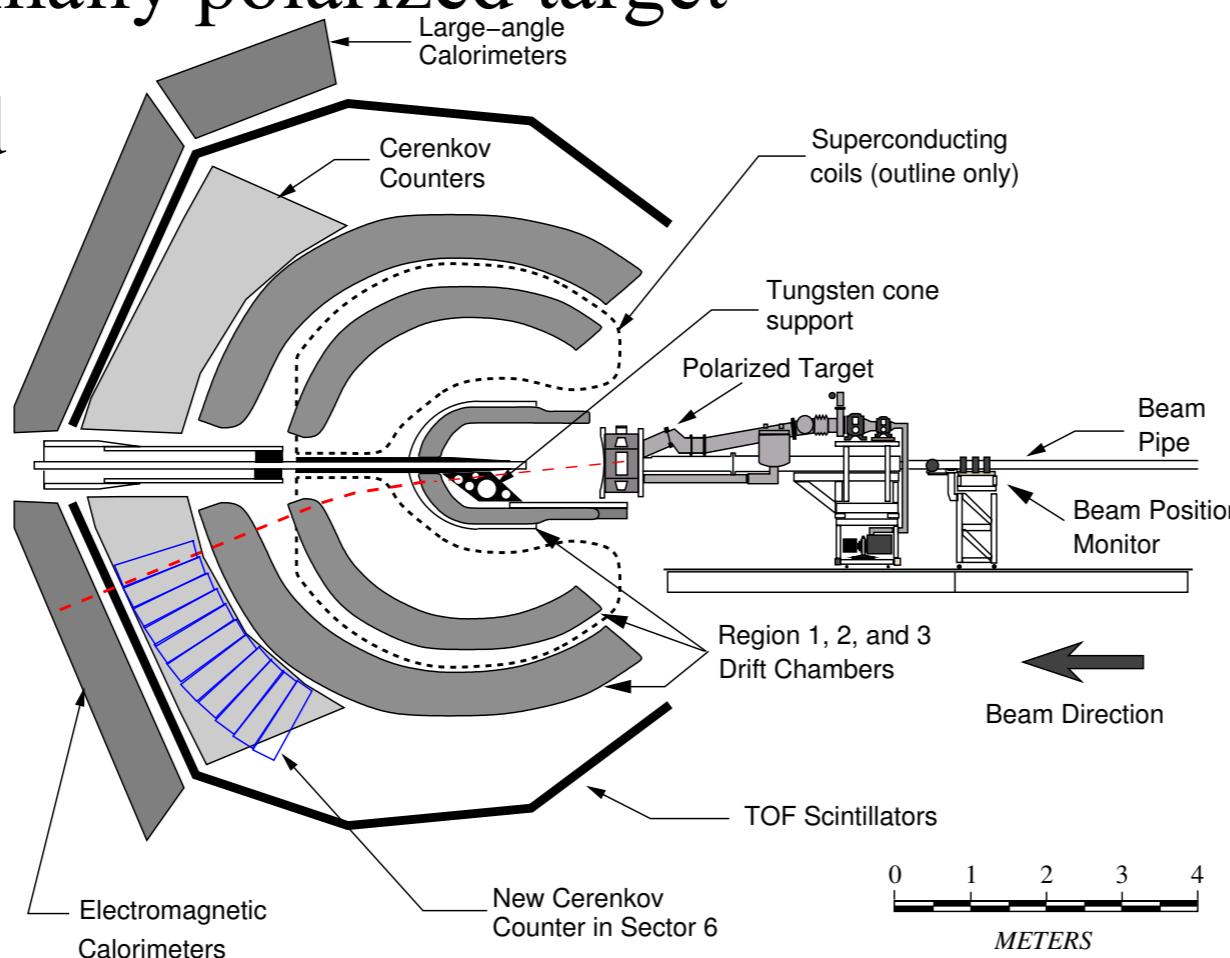
DNP NH<sub>3</sub> and ND<sub>3</sub> target:



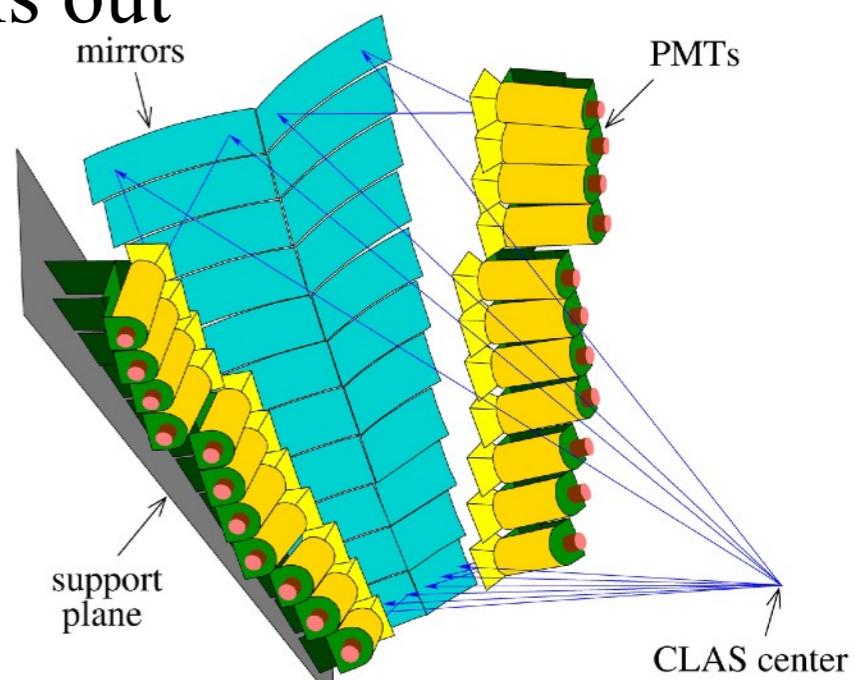
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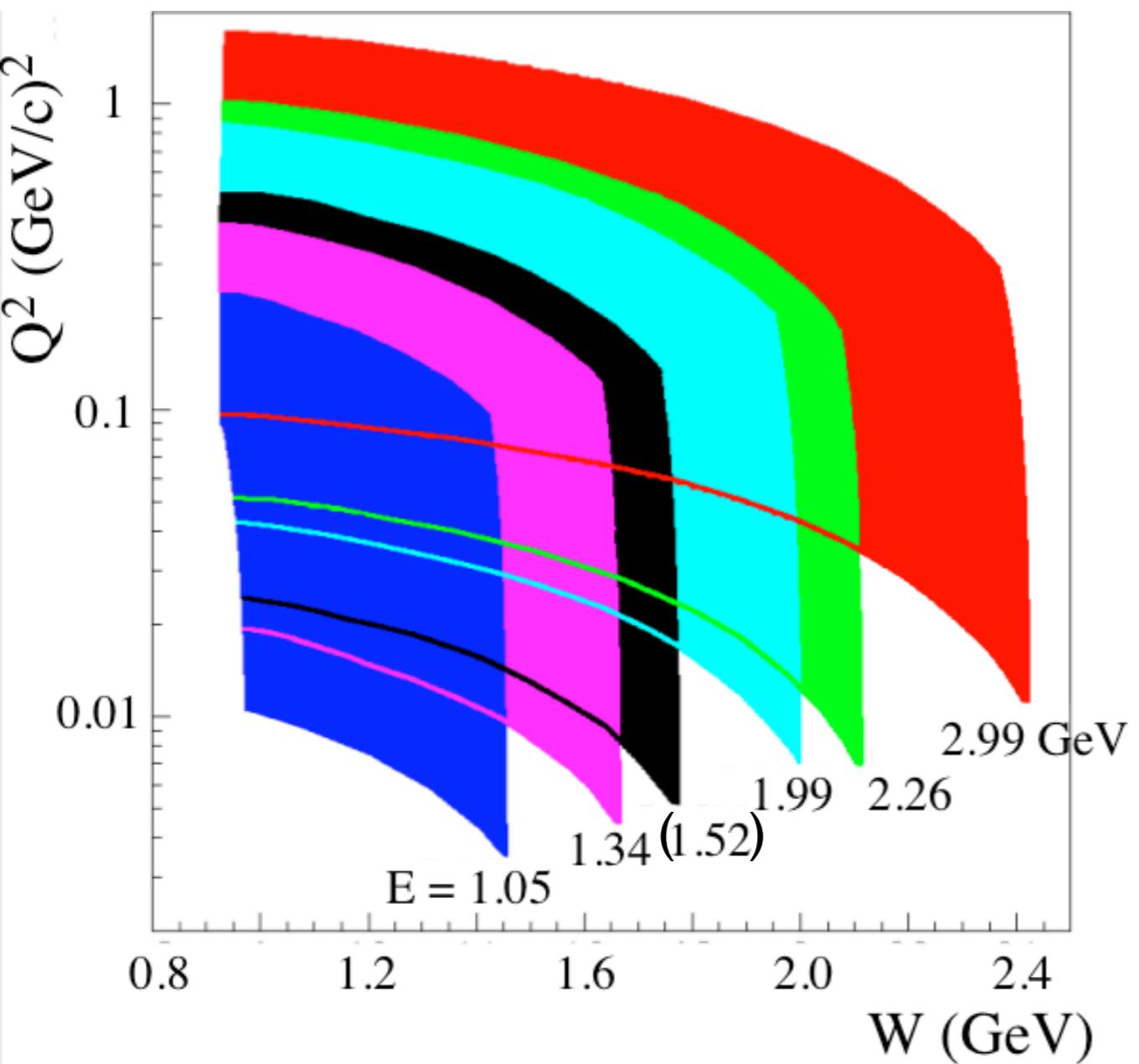


- $g_1$  from inclusive  $eN \rightarrow e'X$  pol. cross-section differences.  
 Advantage: dilution from unpol. target material cancels out
- Small angles: outbending torus field, target at -1m,  
 new Möller shield
- Cross-sections  $\Rightarrow$  controlled (i.e. high) efficiency  
 at small angles. New Cerenkov detector (INFN).  
 Installed in sector 6. Covered down to  $6^\circ$ .

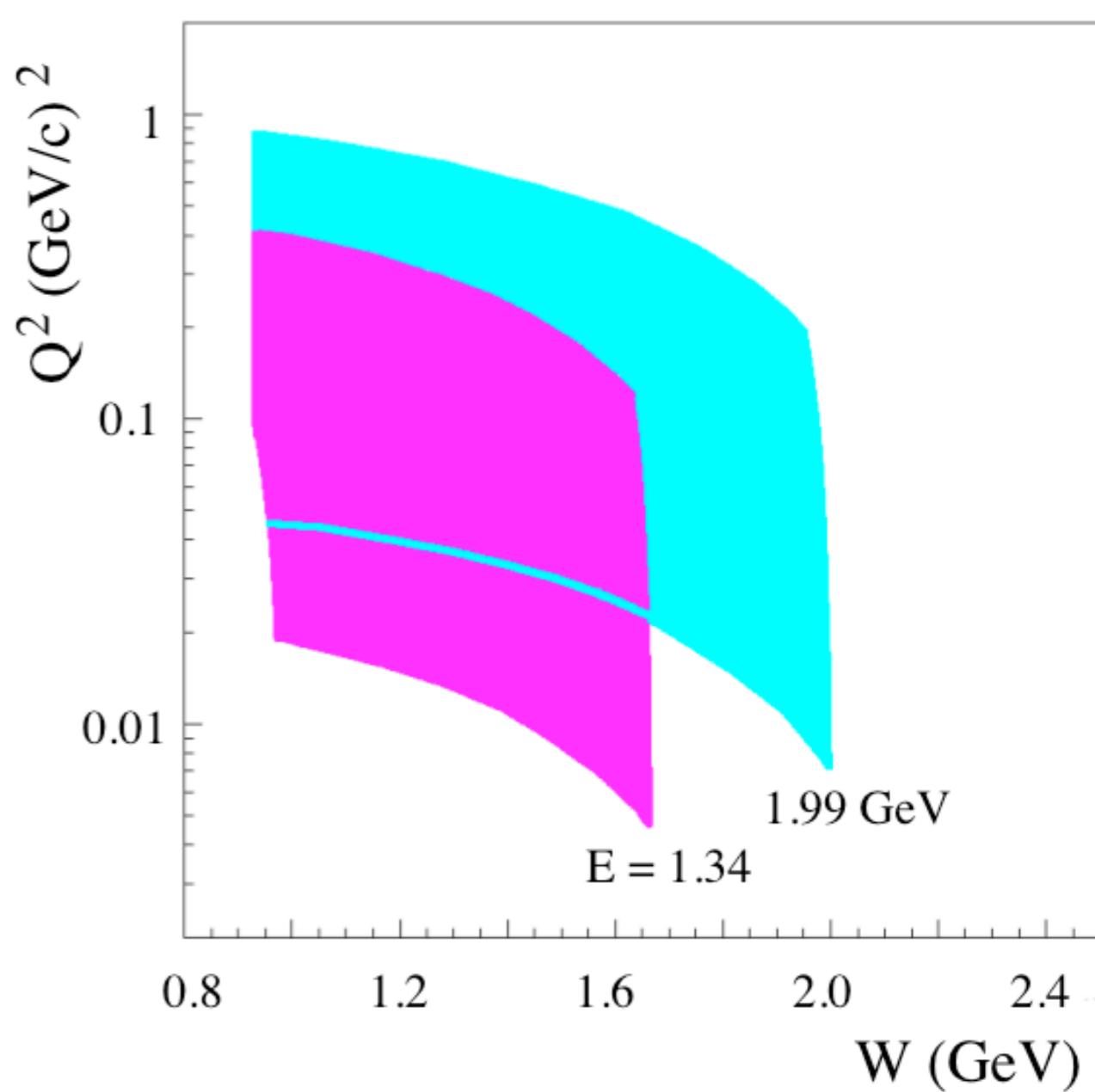


# EG4 kinematic coverage

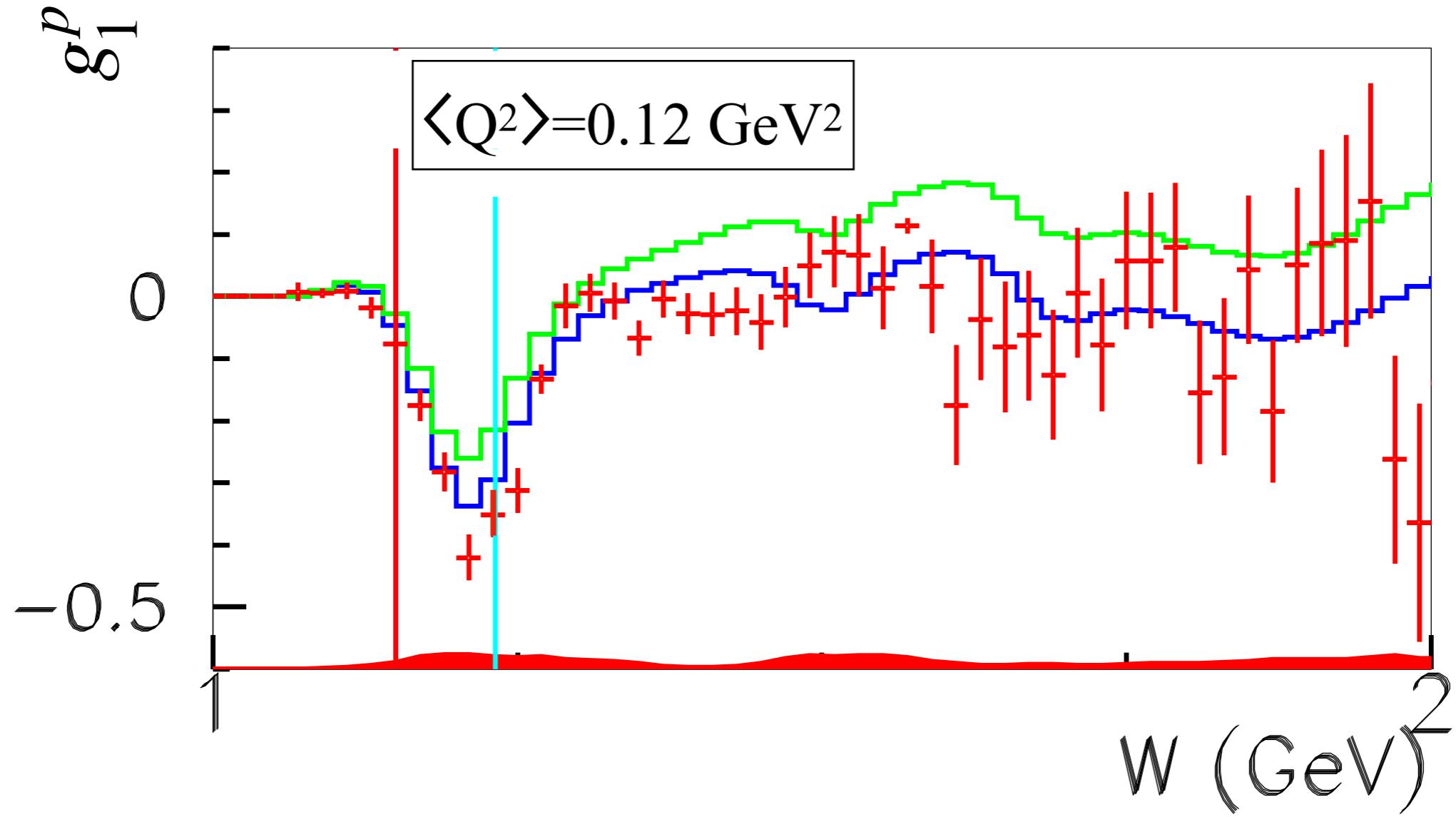
Proton



Deuteron



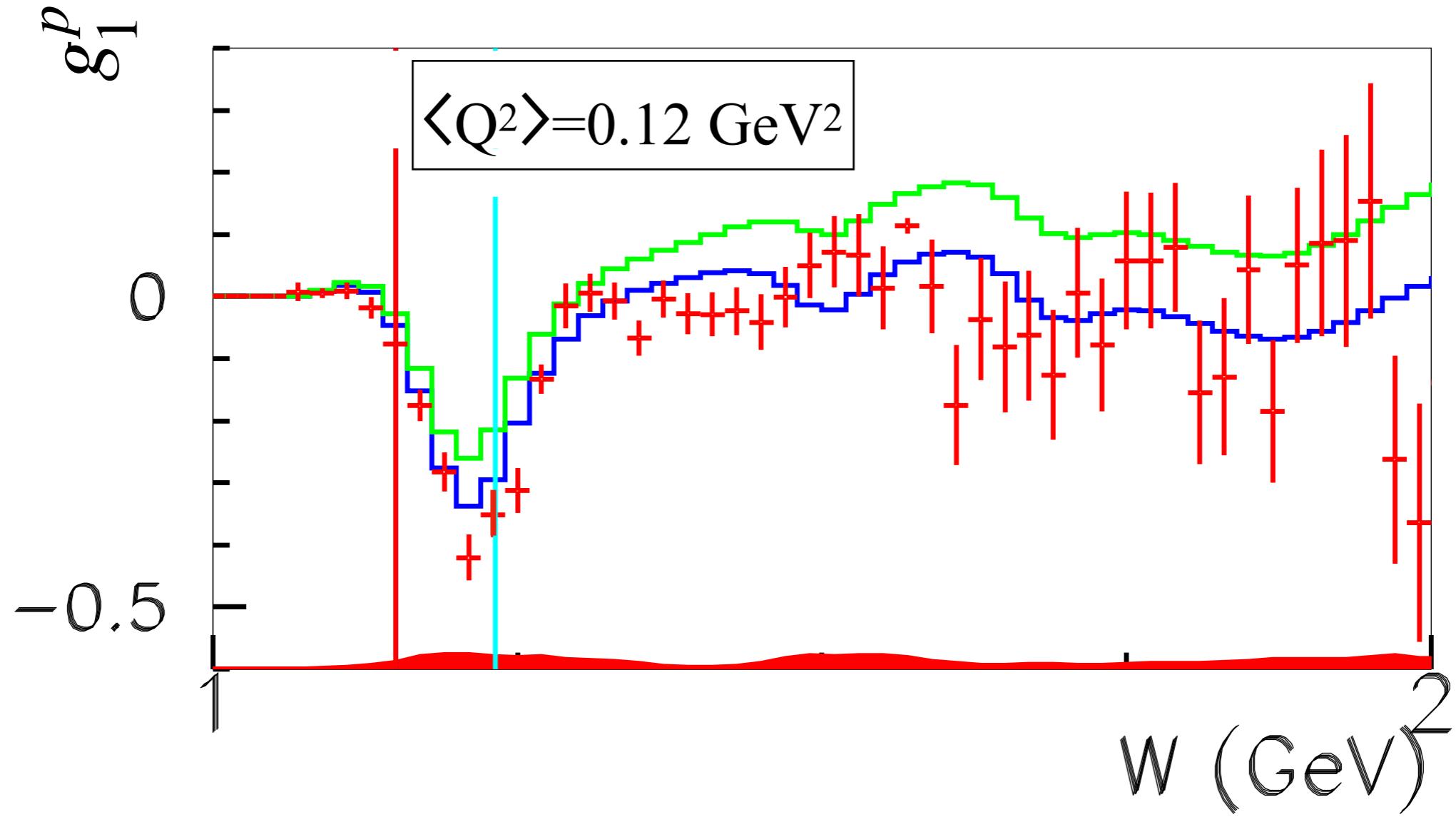
# $g_1^p(x, Q^2)$ from EG4 polarized cross-section difference



X. Zheng *et al.* (CLAS Collaboration),  
Nature Physics, vo. 17 736-741 (2021)

- +
- EG4 data
- “Model” (Fit to EG1b (+ other published data)+extrap. Used as intermediary step to extract  $g_1^p$ )
- Example of “Model” variation: assess uncertainties on extraction method, radiative corrections, ....

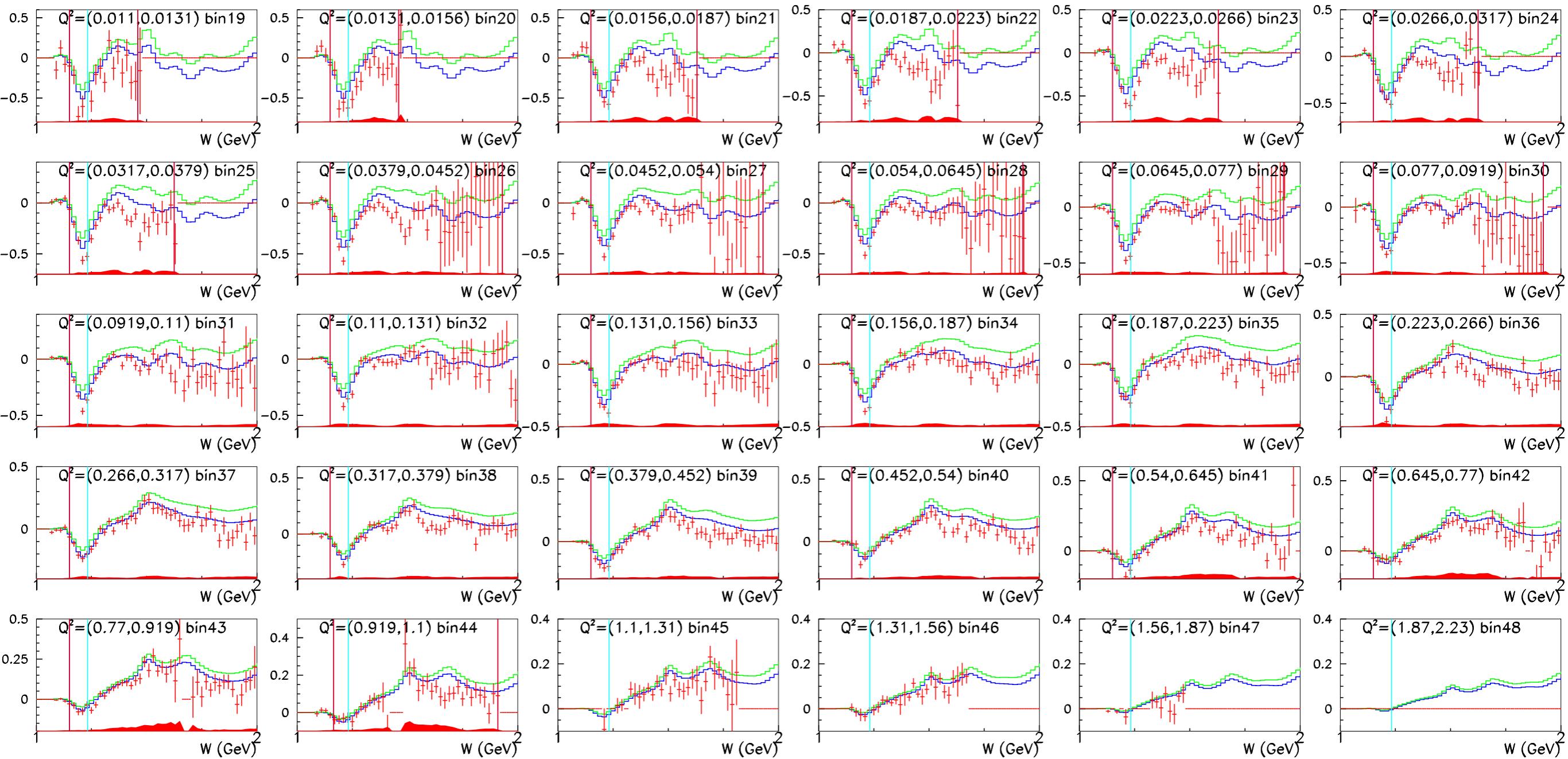
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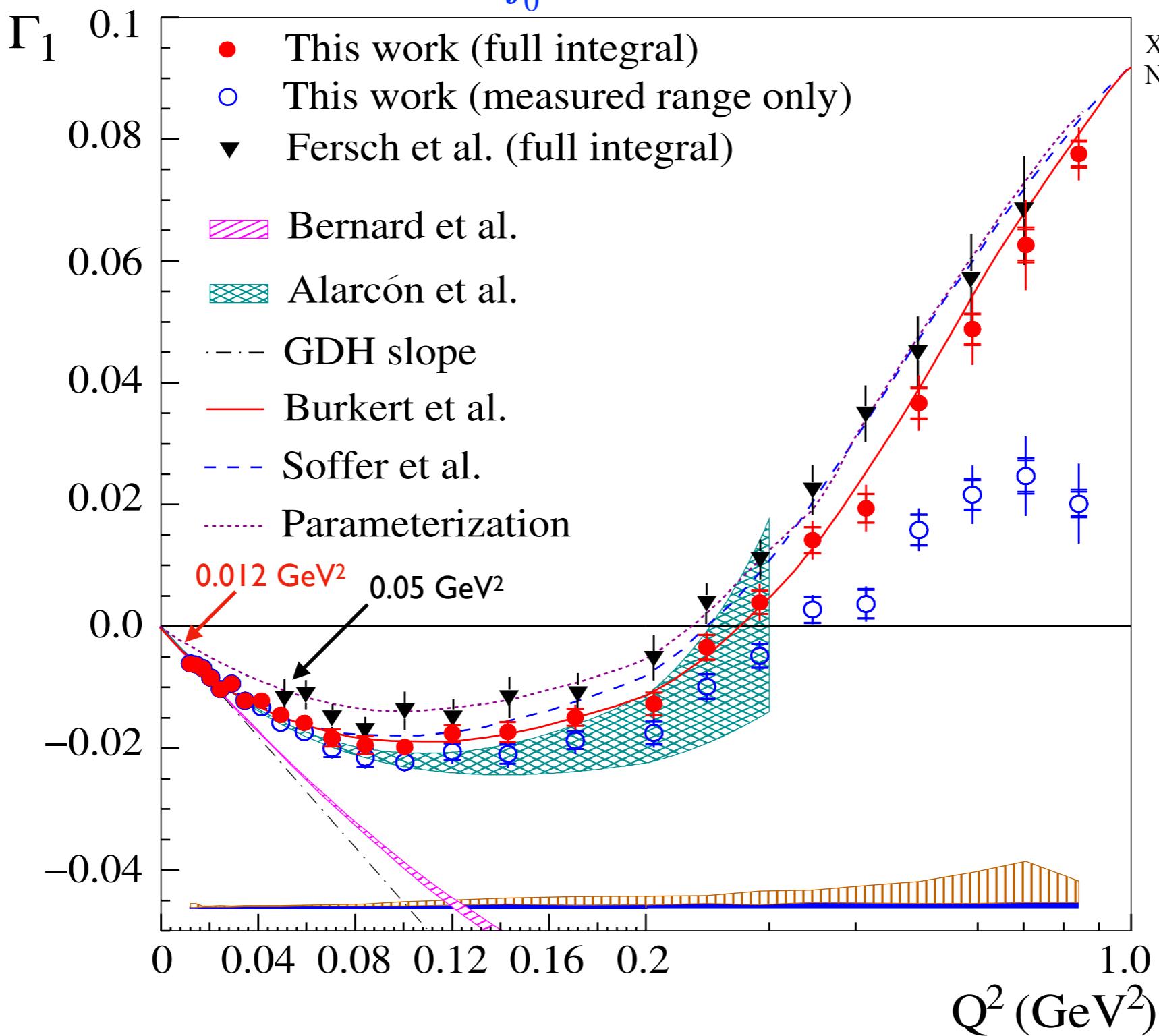
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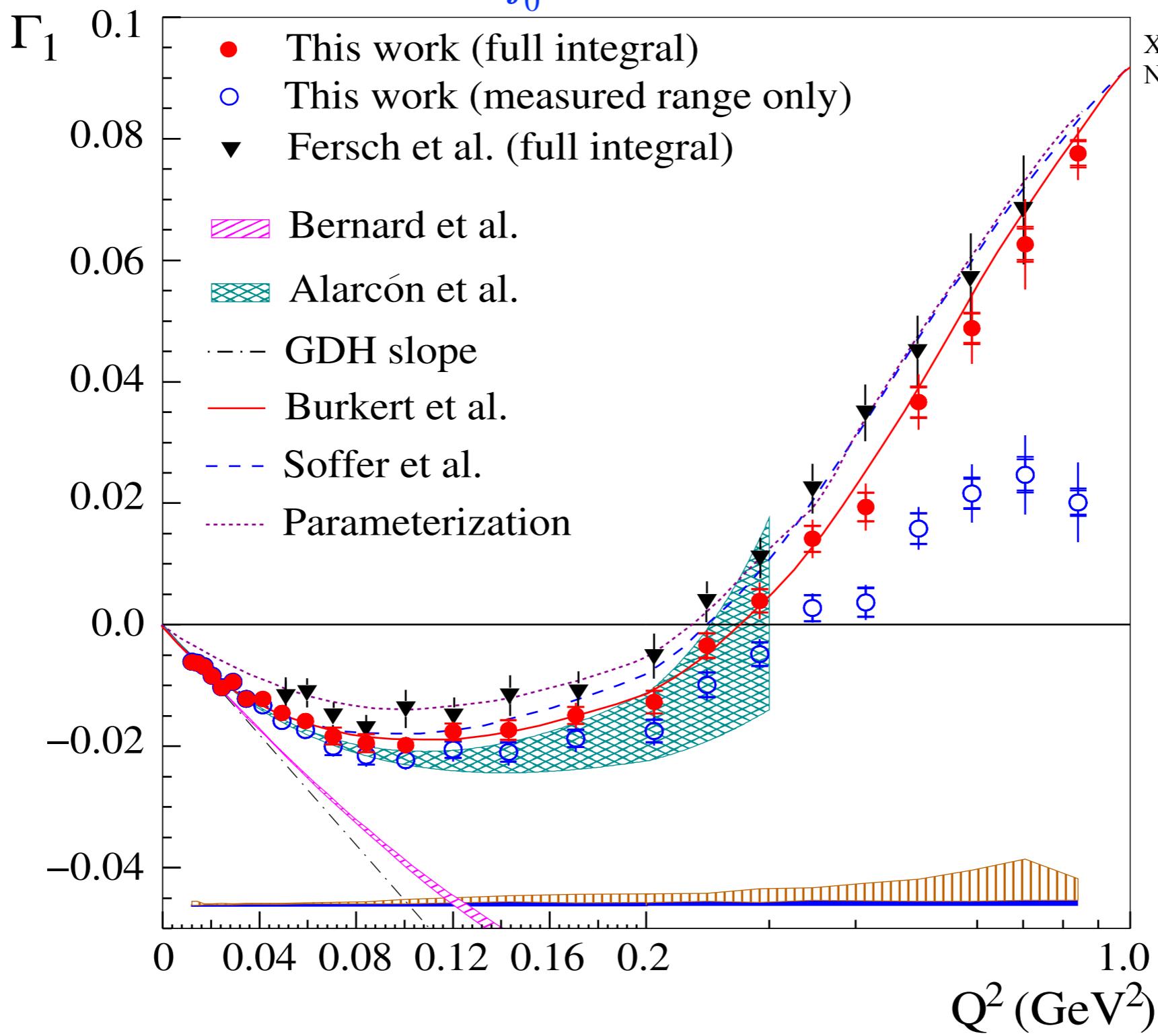
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X. Zheng *et al.* (CLAS Collaboration),  
 Nature Physics, vo. 17 736-741 (2021)

- Small unmeasured low-x contribution
- Lowest Q<sup>2</sup> decreased by factor of ~4  $\Rightarrow$  Clean test of  $\chi$ EFT
- Much improved precision

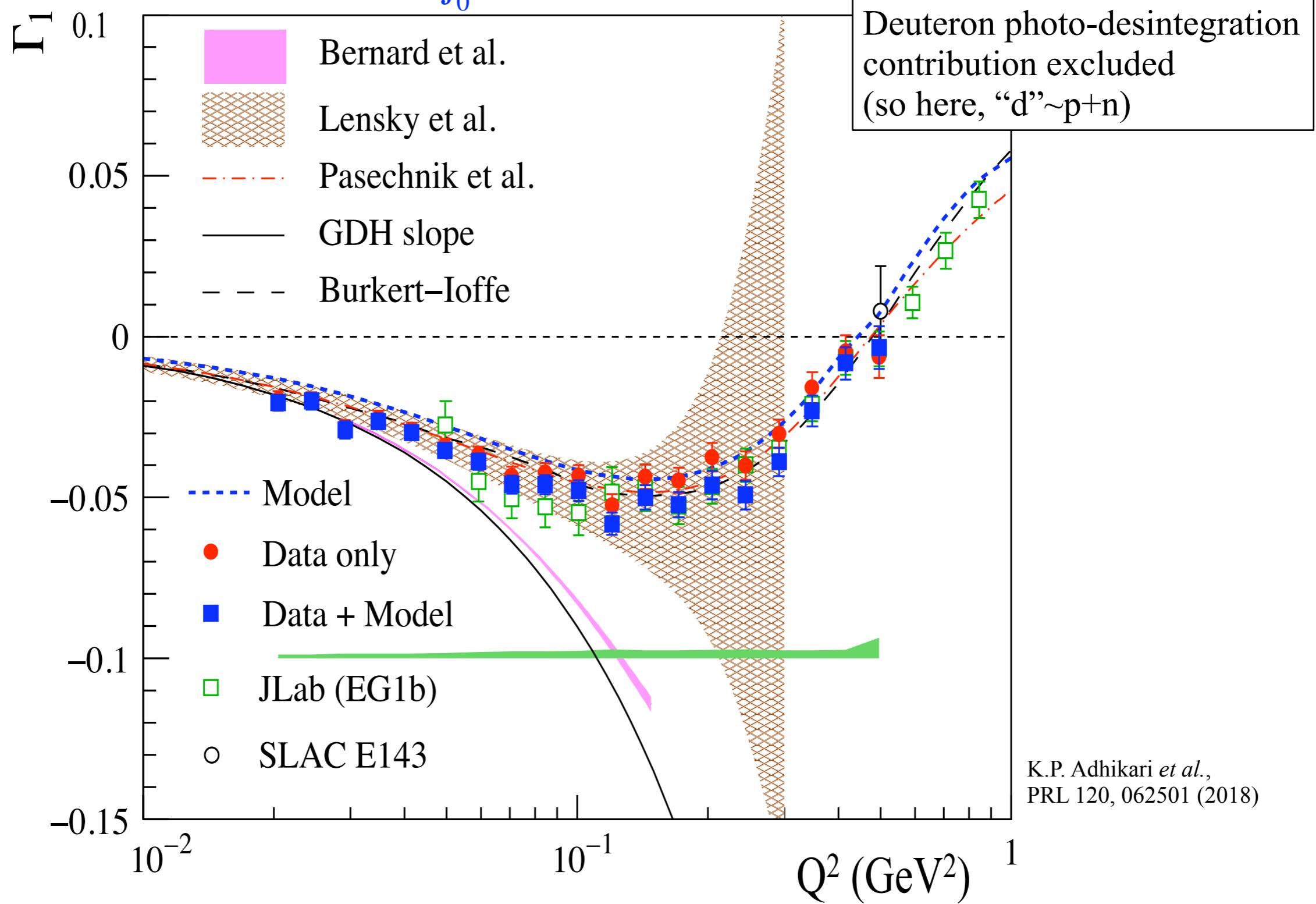
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X. Zheng *et al.* (CLAS Collaboration),  
Nature Physics, vo. 17 736-741 (2021)

- Slight tension between EG4 and EG1 above  $Q^2 \sim 0.1$  GeV $^2$ .
- EG4 and  $\chi$ EFT agree up to  $Q^2 \sim 0.04$  GeV $^2$  (Bernard et al) or  $Q^2 > 0.2$  GeV $^2$  (Alarcón et al.)
- Phenomenological models (Pasechnik et al, Burkert-Ioffe) agree well.

$$\Gamma_1^d = \int_0^{1^-} g_1^d(x, Q^2) dx$$

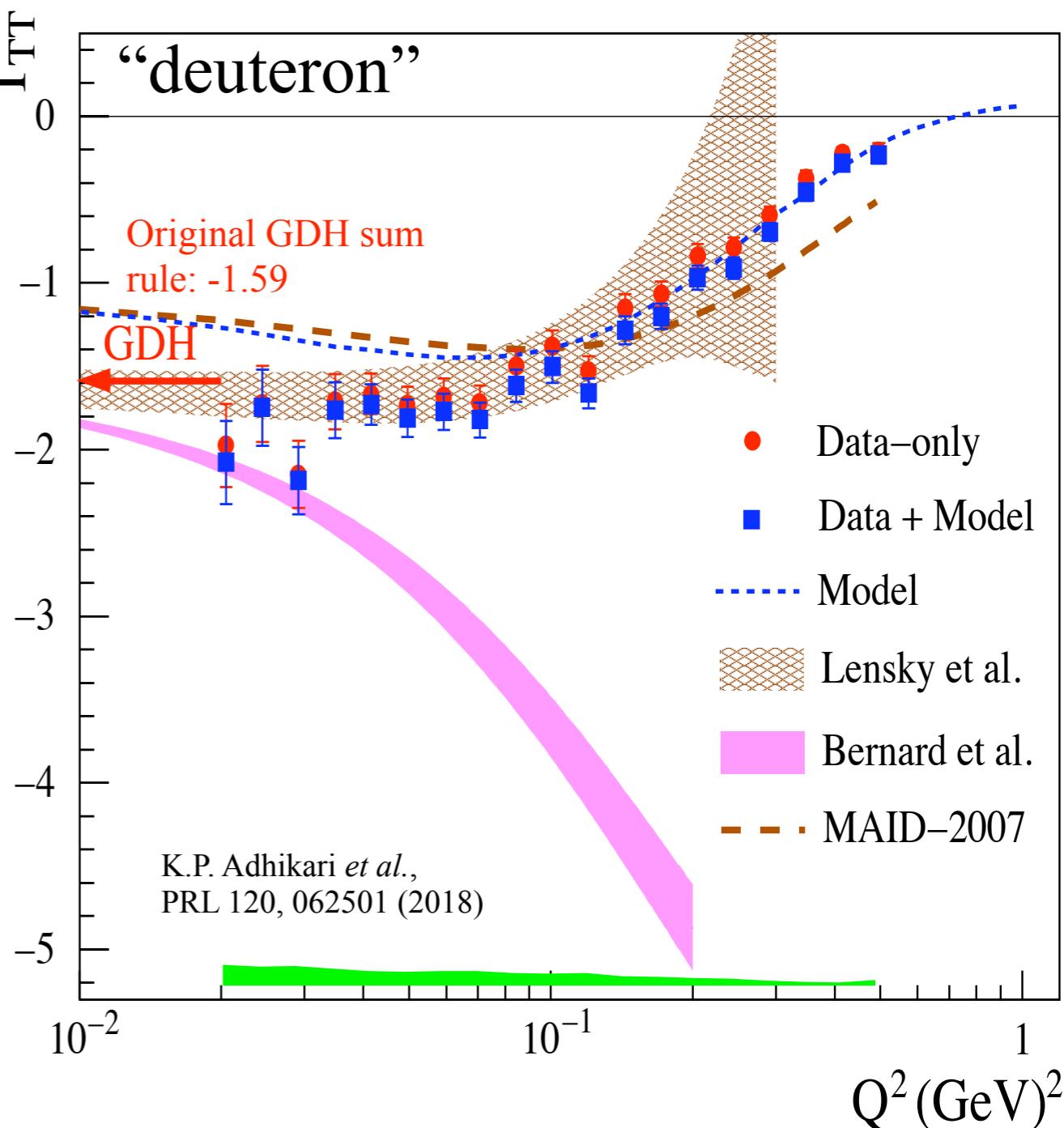
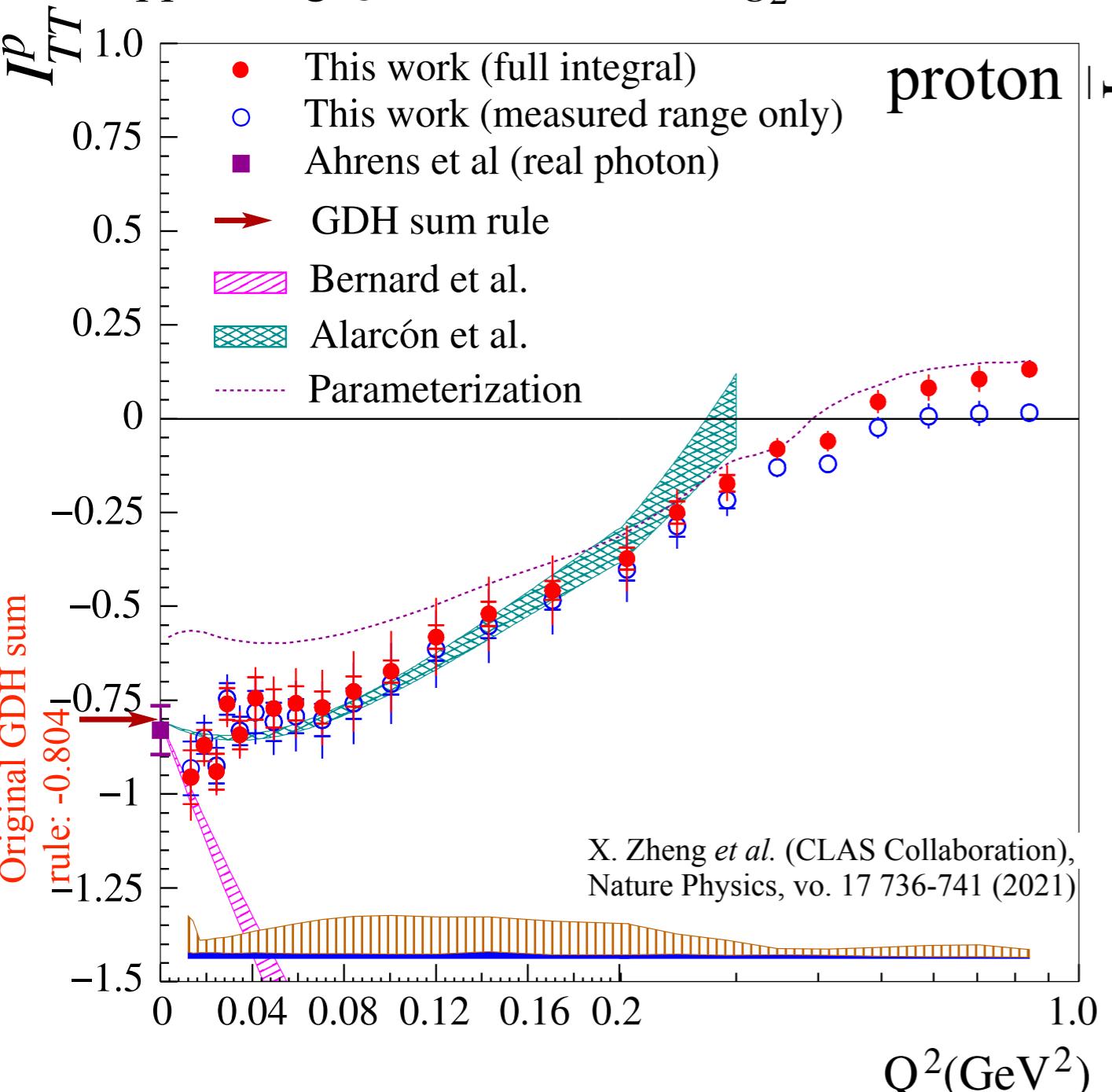


- EG4 and EG1 agree well.
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$$\text{Another generalization of GDH sum: } I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha} \int_{\nu_{thr}}^{\infty} \frac{K \sigma_A - \sigma_P}{\nu} d\nu$$

K: virtual photon flux

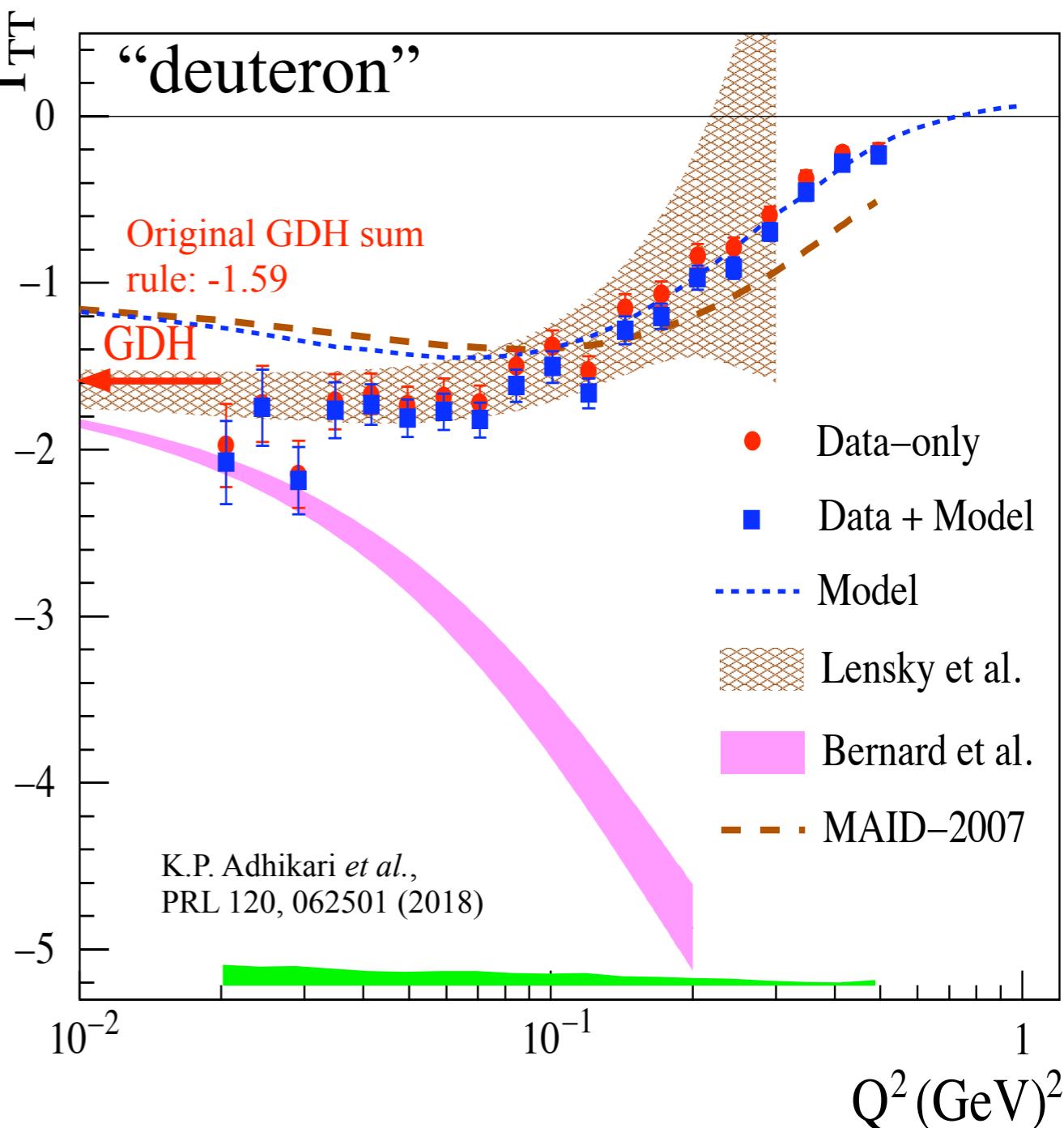
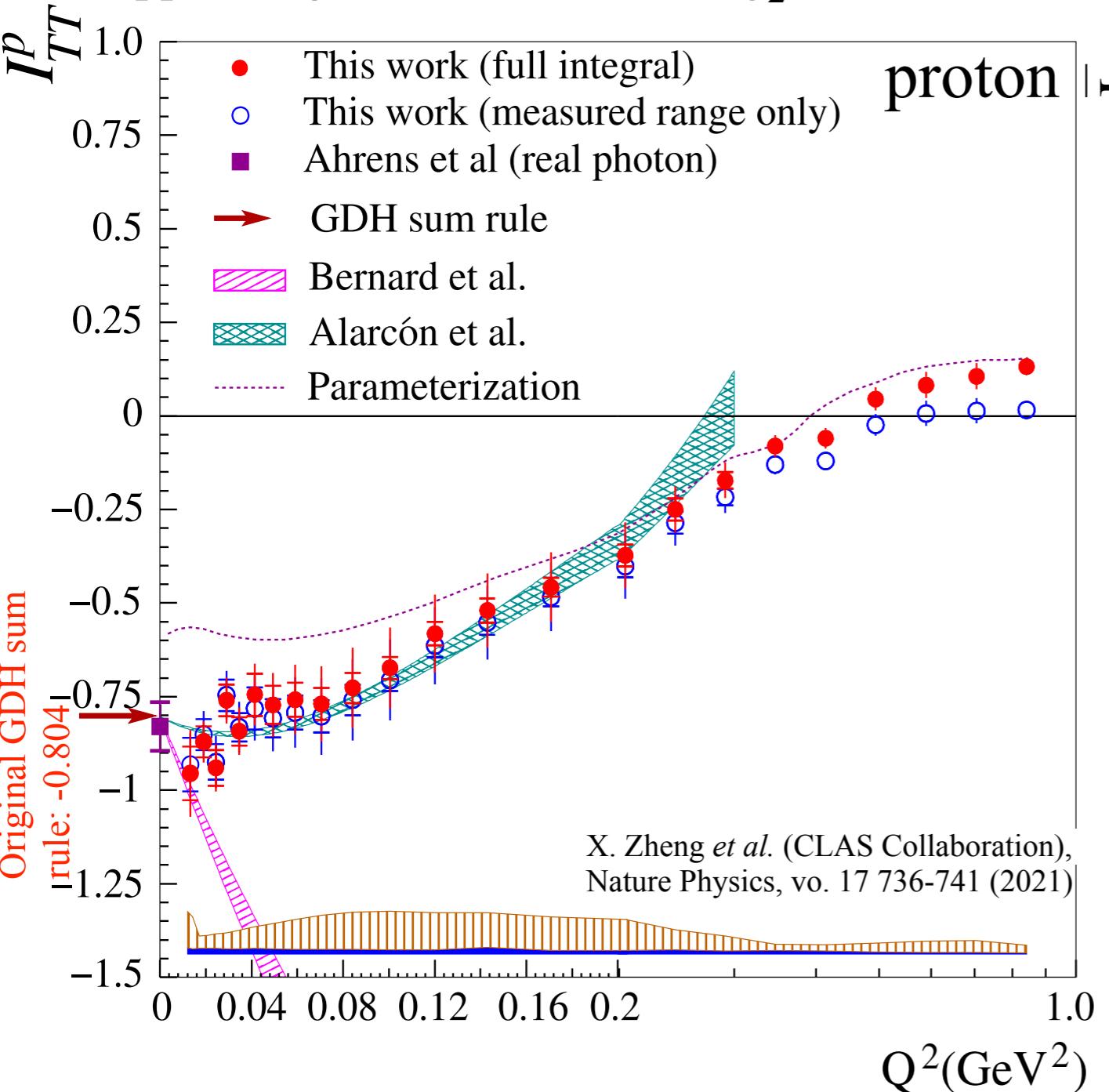
No suppressing  $Q^2$  factor. Contains  $g_2$  (not measured by EG4)



Extrapolating the (very low  $Q^2$ ) data to  $Q^2=0$  provides an independent check of the GDH SR validity, with a different method (inclusive data) than photoproduction experiments (exclusive data).

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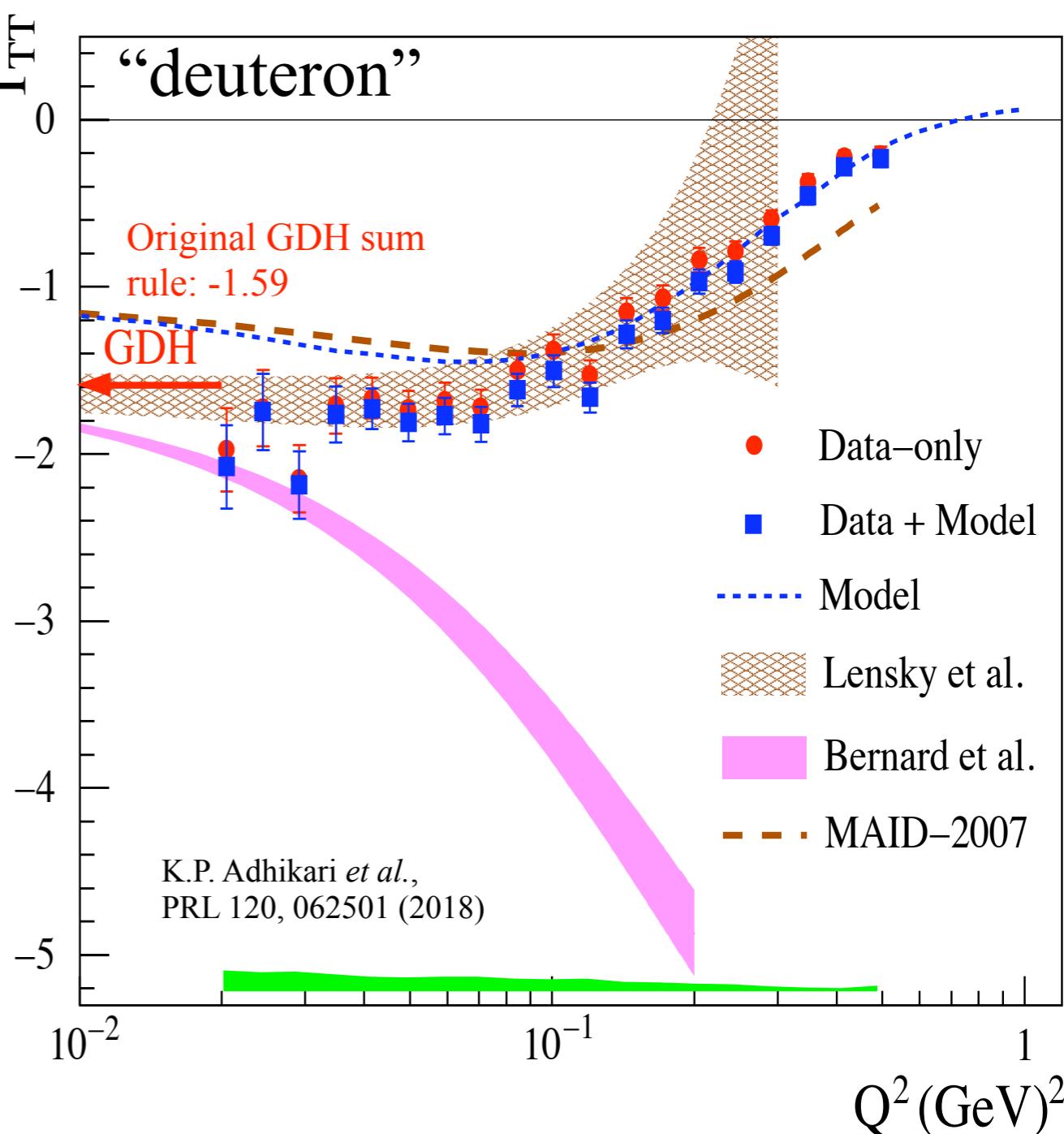
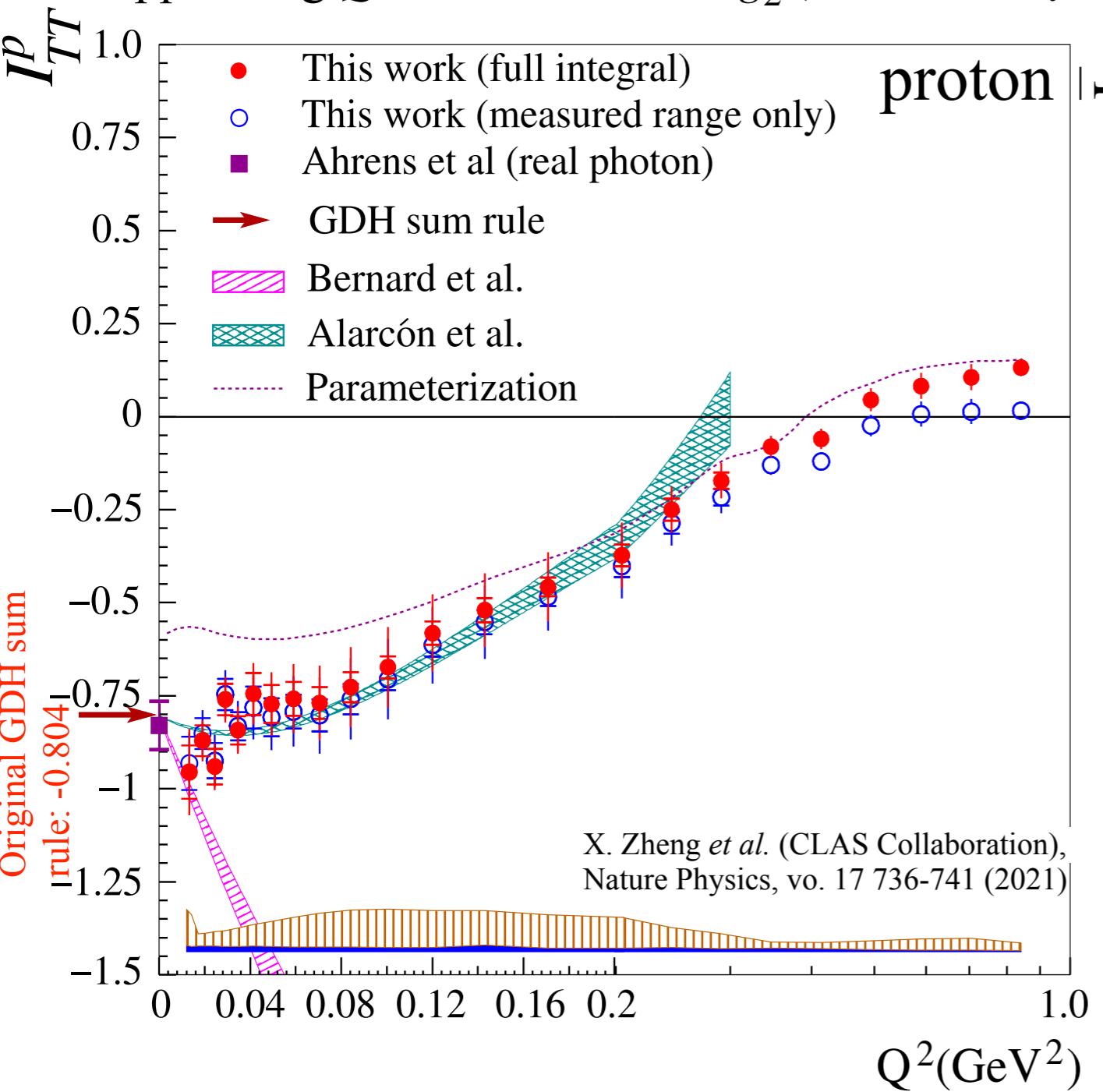
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$K$ : virtual photon flux

No suppressing  $Q^2$  factor. Contains  $g_2$  (not measured by EG4)

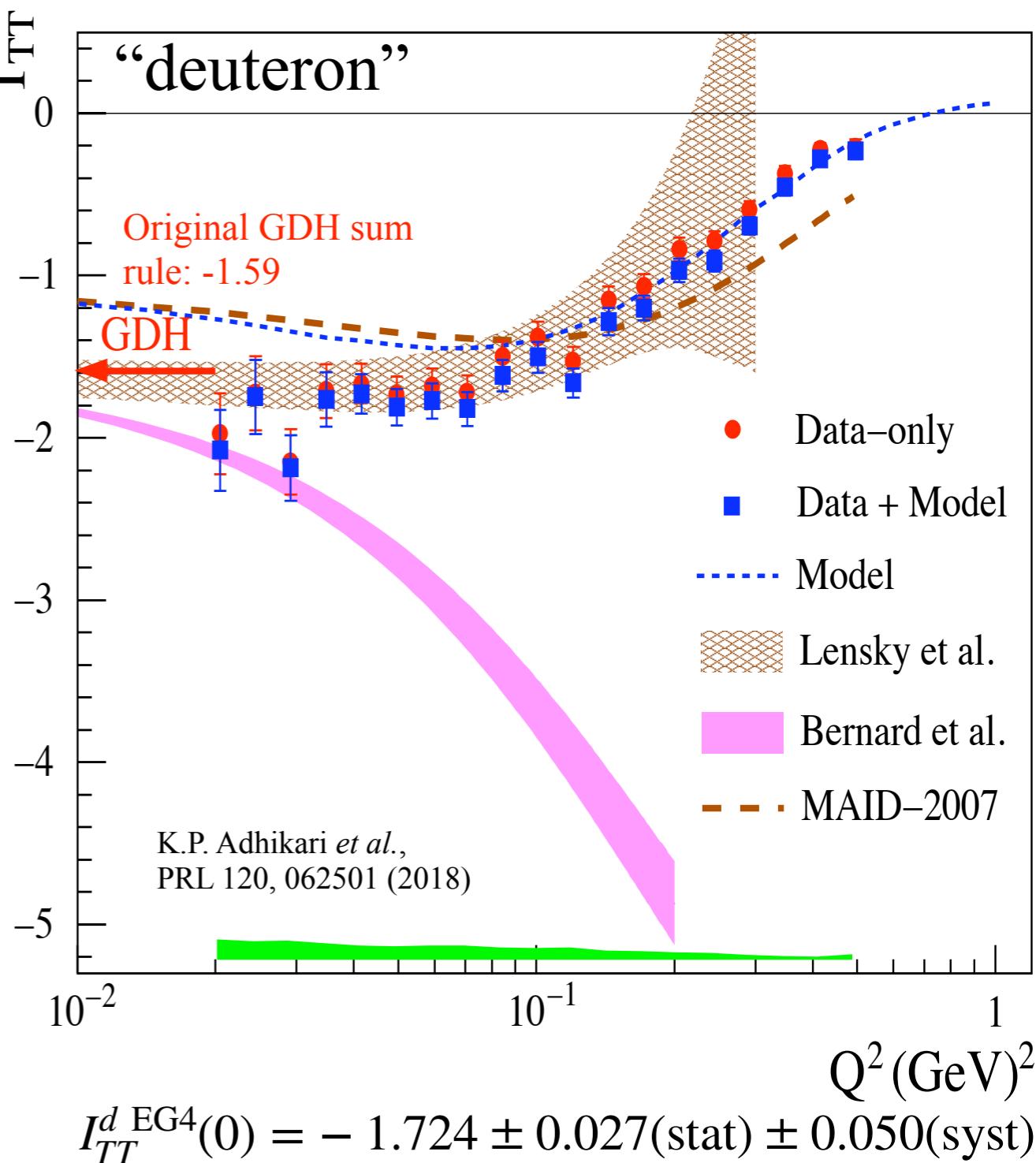
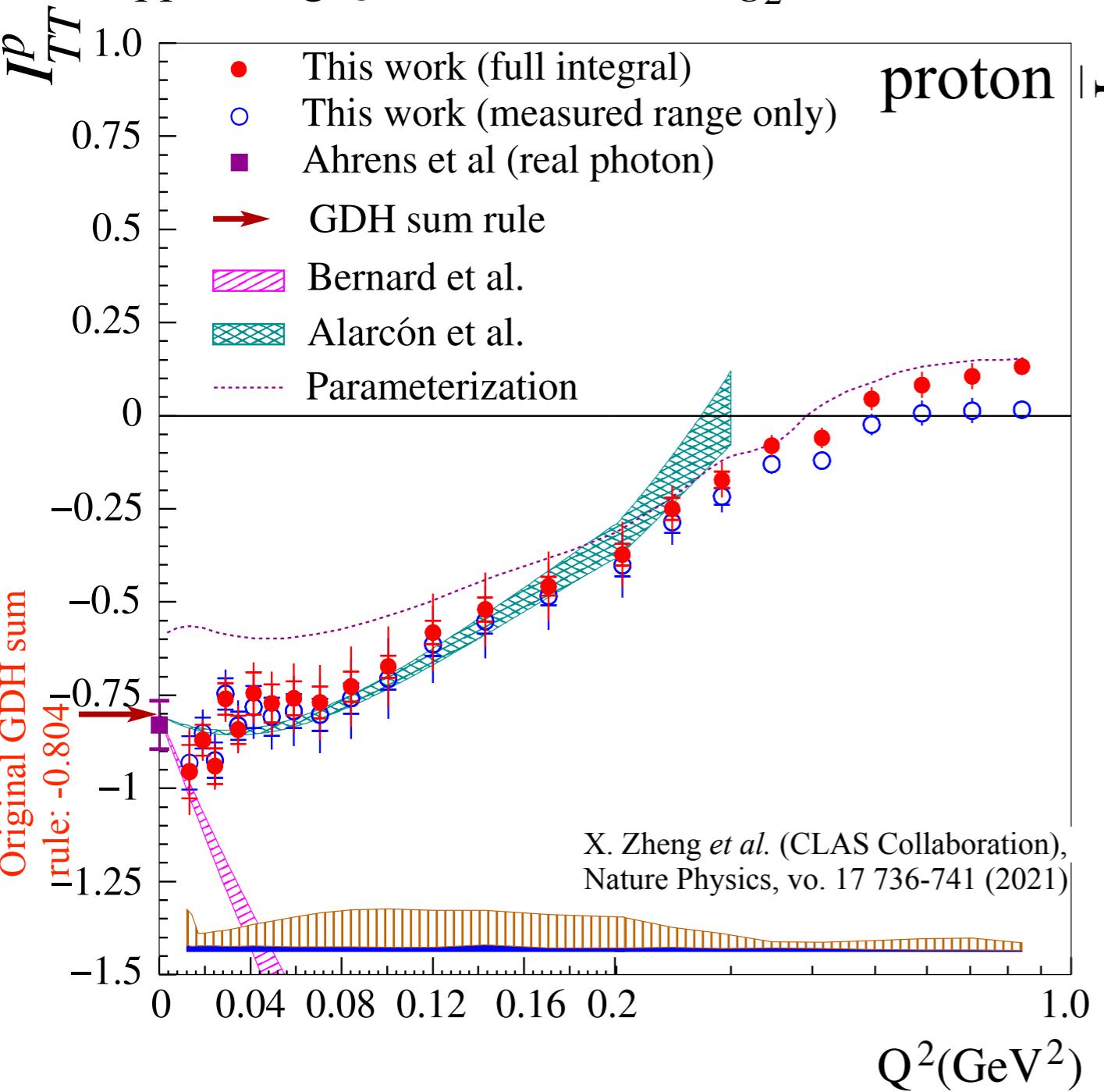


Extrapolated data agree with the GDH SR, with precision similar to photoproduction method:  
 $I_T^p \text{ MAMI}(0) = -0.832 \pm 0.023(\text{stat}) \pm 0.063(\text{syst})$

$$\text{Another generalization of GDH sum: } I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha} \int_{\nu_{thr}}^{\infty} \frac{K}{\nu} \frac{\sigma_A - \sigma_P}{\nu} d\nu$$

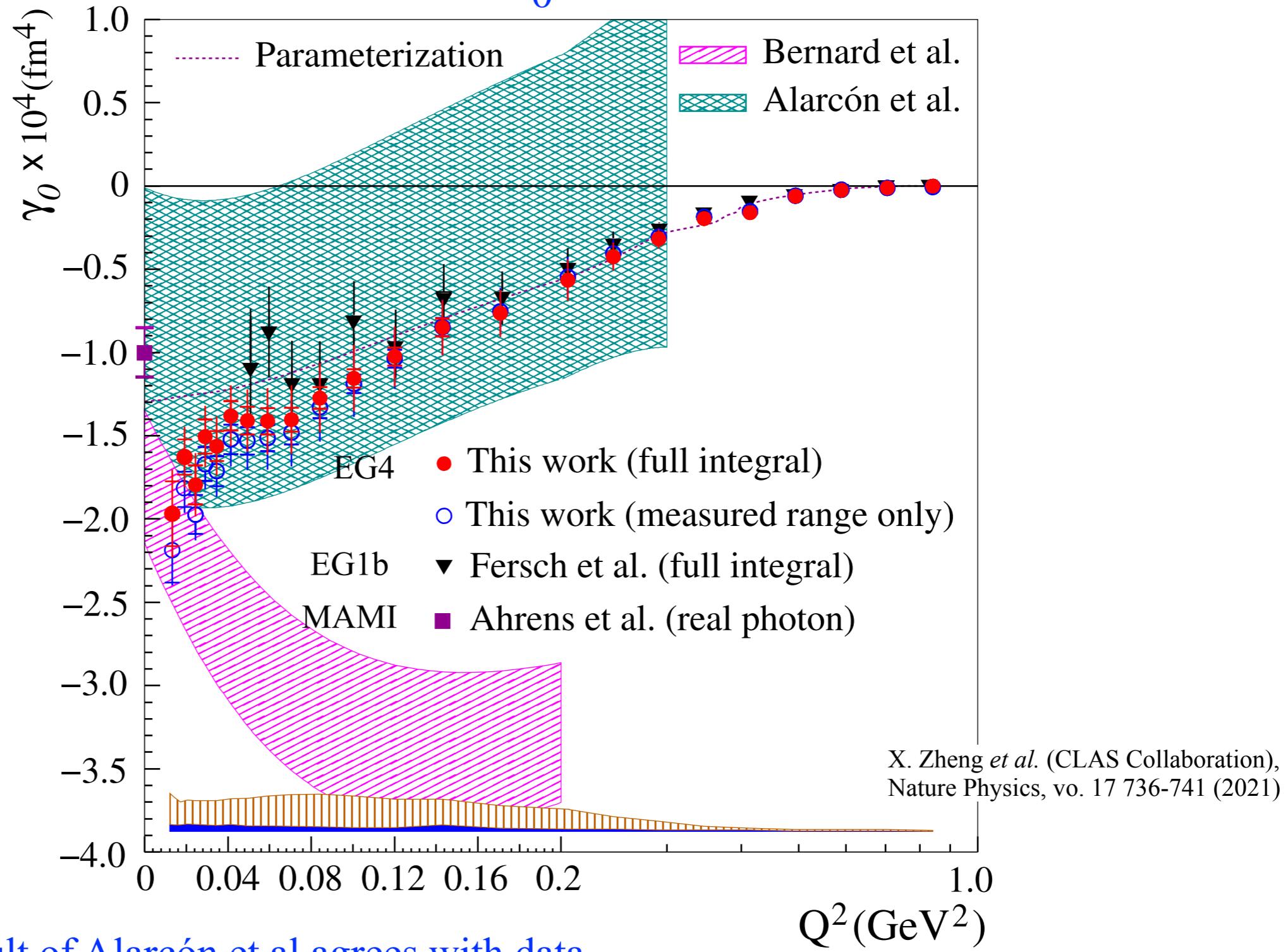
K: virtual photon flux

No suppressing  $Q^2$  factor. Contains  $g_2$  (not measured by EG4)



# EG4 results on $\gamma_0^p(Q^2)$

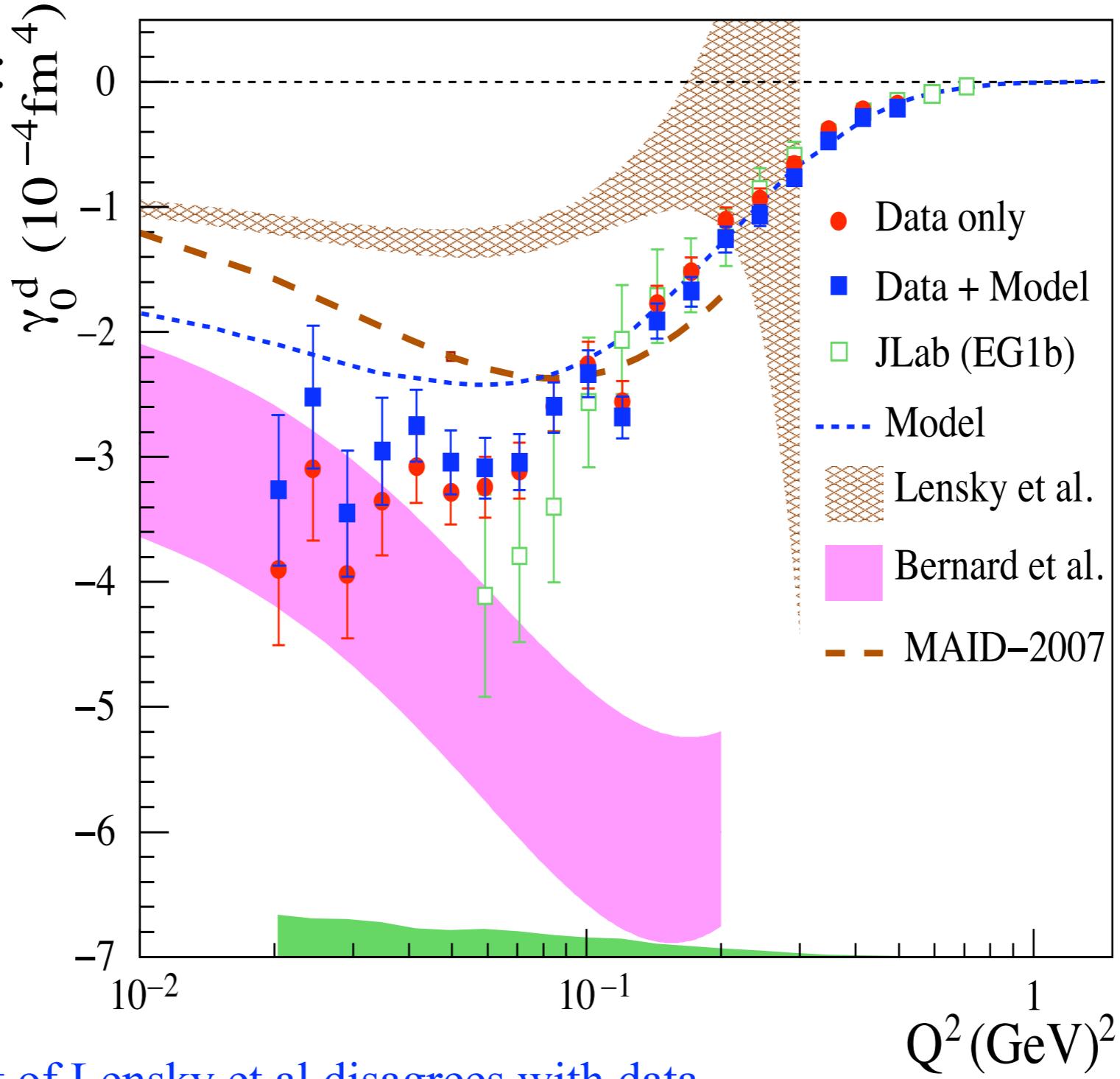
EG4 proton,  $\gamma_0^p$ :



- $\chi$ EFT result of Alarcón et al agrees with data.
- Bernard et al.  $\chi$ PT calculation agrees for lowest  $Q^2$  points. Large slope at low  $Q^2$  supported by the MAMI+EG4 data

# EG4 results on $\gamma_0^d(Q^2)$

EG4 “deuteron”,  $\gamma_0^d$ :



- $\chi$ EFT result of Lensky et al disagrees with data.
- Bernard et al.  $\chi$ PT calculation agrees for lowest  $Q^2$  points.

K.P. Adhikari *et al.*,  
PRL 120, 062501 (2018)

## Summary and conclusion

- EG4: Low  $Q^2$  measurement using polarized e- on polarized p and d, over a large x-range in order to study spin sum rules.
- New detector necessary to reach these kinematics.
- Main goal: unambiguous test of  $\chi$ EFT.
- Doubly polarized inclusive cross-section analysis.
- Exclusive data on  $\pi^+$  and  $\pi^-$  spin-dep. electroprod. on p published in 2016 (asym. analysis).  
X. Zheng *et al.* (CLAS Collaboration), PRC 94, 045206 (2016)
- Inclusive analysis on d published in 2018. K.P. Adhikari *et al.* (CLAS Collaboration). PRL 120, 062501 (2018)
- Inclusive analysis on p published in 2021. X. Zheng *et al.* (CLAS Collaboration), Nature Physics, vo. 17 736 (2021)
- Archival long paper is being written. (Will contain the neutron information).
- Data on  $\Gamma_1(Q^2)$ ,  $I_{TT}(Q^2)$ , and  $\gamma_0(Q^2)$  and  $\chi$ EFT compare with mixed success, depending on the  $\chi$ EFT method and observable.
- Original GDH sum rule seems fine for p and d (and n).