

The proton radius puzzle

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Center for Frontiers
in Nuclear Science



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Stony Brook
University

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What is “stuff”?

The matter around us is described by non-perturbative quantum chromodynamics. npQCD is hard.

Simplest QCD system to study: Protons



100 years of protons!

Proton is a composite system. It must have a size!

How big is it?

What shape is it?

Knowledge of proton radius limits QED tests

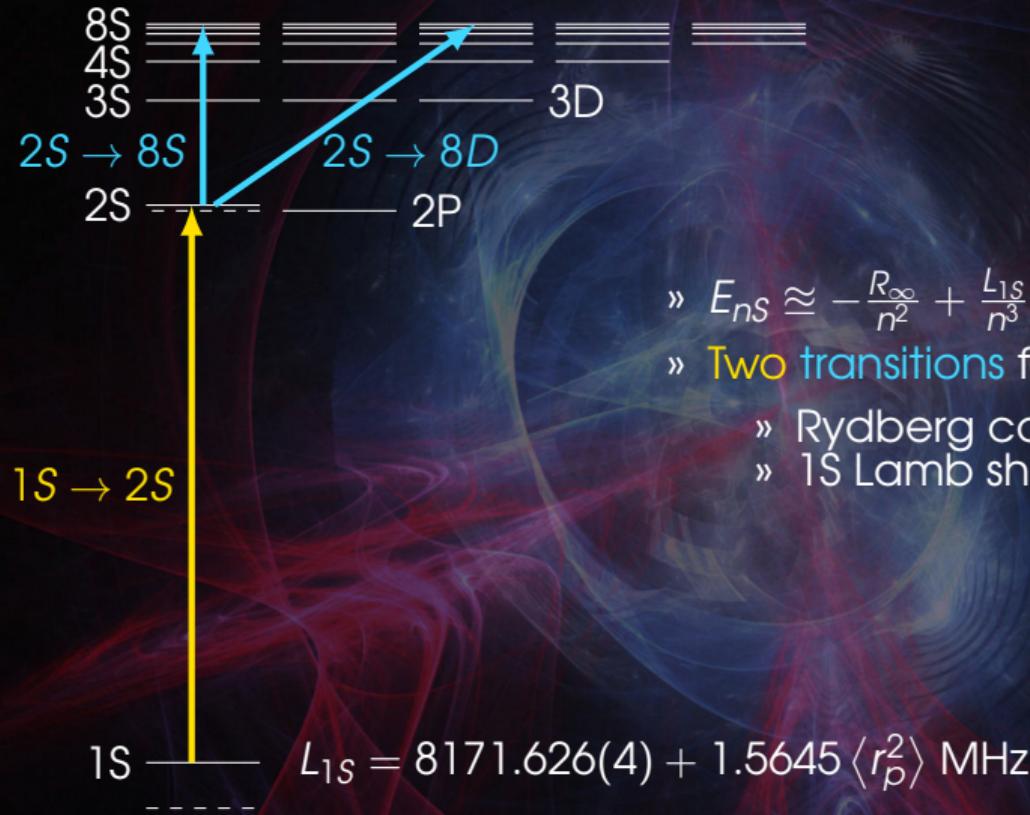


2S ----- 2P

$$\gg E_{nS} \approx -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

$$1S ----- L_{1S} = 8171.626(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz}$$

Knowledge of proton radius limits QED tests



» $E_{nS} \approx -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$

» Two transitions for two unknowns:

- » Rydberg constant R_∞
- » 1S Lamb shift \Rightarrow radius

Knowledge of proton radius limits QED tests



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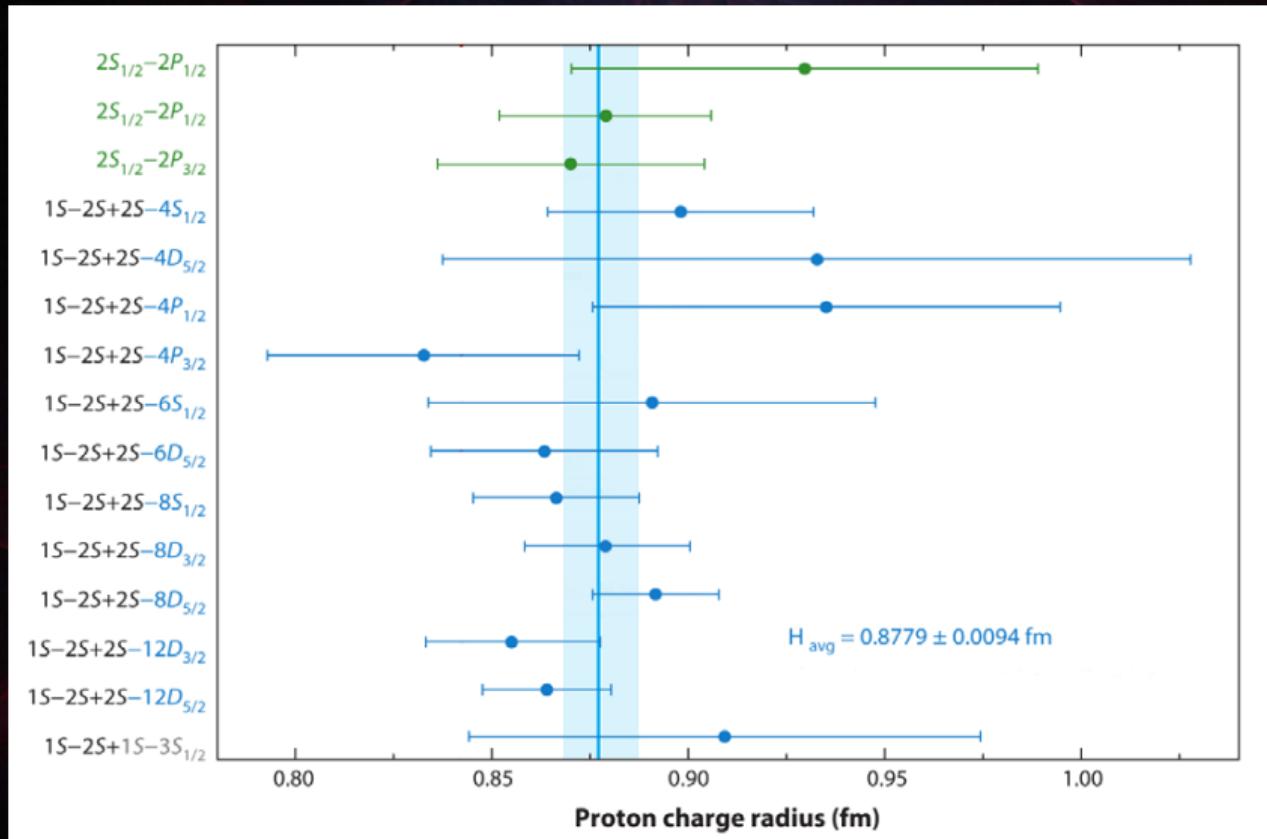
» Two transitions for two unknowns:

- » Rydberg constant R_∞
- » 1S Lamb shift \Rightarrow radius

» Direct Lamb shift $2S \rightarrow 2P$

$$1S \quad L_{1S} = 8171.626(4) + 1.5645 \langle r_p^2 \rangle \text{ MHz}$$

"Normal" Hydrogen Spectroscopy Results (before 2010)



Elastic lepton-proton scattering

Method of choice: Lepton-proton scattering

- » Point-like probe
- » No strong force
- » Lepton interaction “straight-forward”

Cross section for elastic scattering

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} = \frac{1}{\varepsilon(1+\tau)} \left[\varepsilon G_E^2(Q^2) + \tau G_M^2(Q^2) \right]$$

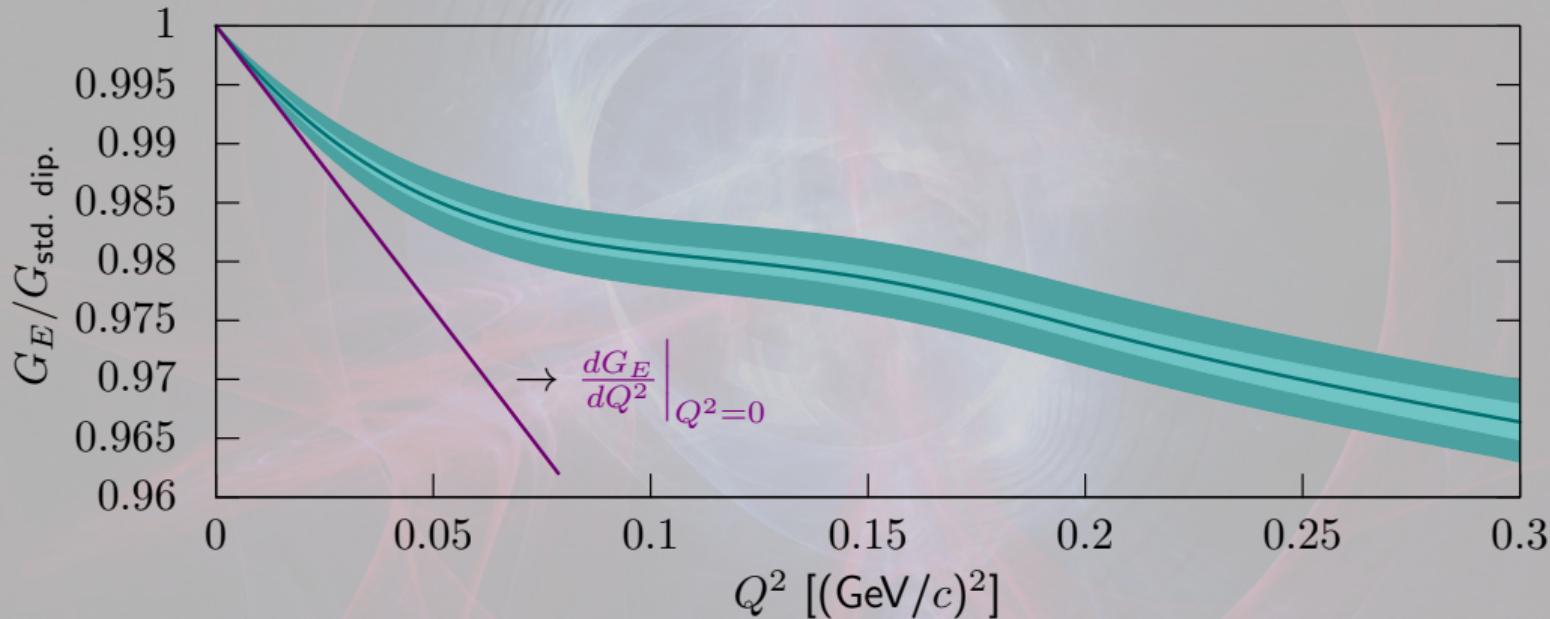
with:

$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2(1+\tau) \tan^2 \frac{\theta_e}{2} \right)^{-1}$$

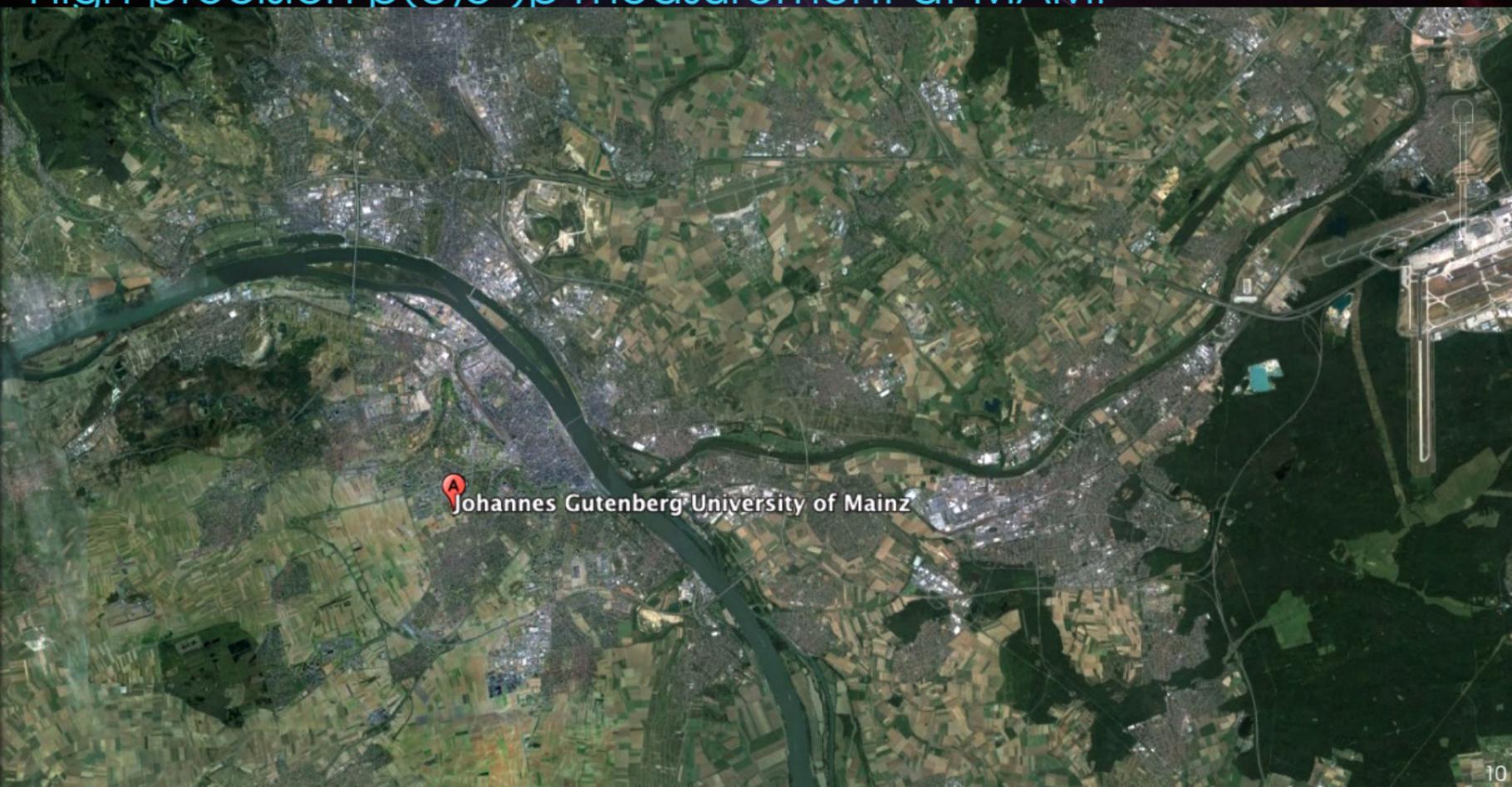
- » Rosenbluth formula
- » Electric and magnetic form factor encode the shape of the proton
- » Fourier transform (almost) gives the spatial distribution, in the Breit frame

Proton radius from scattering

$$\left\langle r_E^2 \right\rangle = -6\hbar^2 \frac{dG_E}{dQ^2} \Big|_{Q^2=0} \quad \left\langle r_M^2 \right\rangle = -6\hbar^2 \frac{d(G_M/\mu_p)}{dQ^2} \Big|_{Q^2=0}$$



High-precision $p(e,e')p$ measurement at MAMI



Johannes Gutenberg University of Mainz

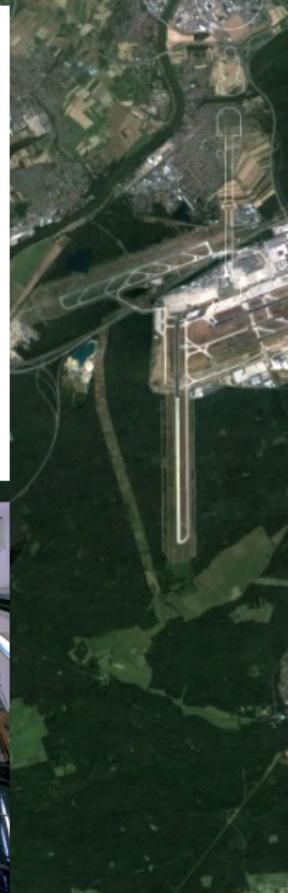
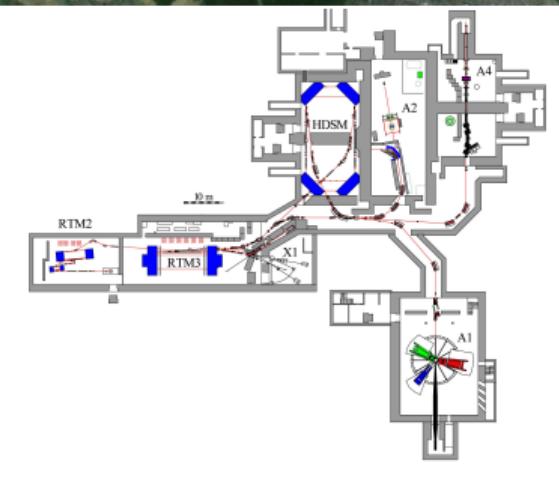
High-precision p(e,e')p measurement at MAMI

Mainz Microtron

- » cw electron beam
- » $10 \mu\text{A}$ polarized,
 $100 \mu\text{A}$ unpolarized
- » MAMI A+B: 180-855 MeV
- » MAMI C: 1.6 GeV



Johannes Gutenberg University



High-precision p(e,e')p measurement at MAMI

Mainz Microtron

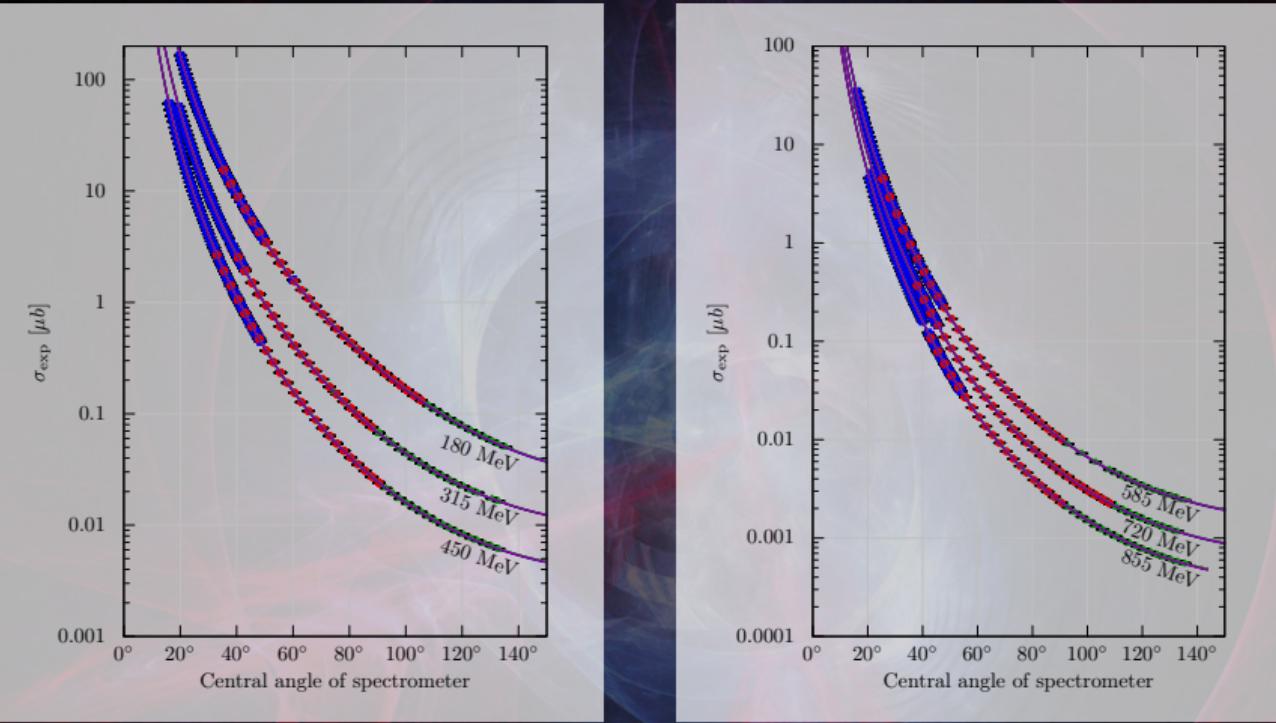
- » cw electron beam
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 $100 \mu\text{A}$ unpolarized
- » MAMI A+B: 180-855 MeV
- » MAMI C: 1.6 GeV

A1 3-spectrometer facility

- » 28 msr acceptance
- » angle resolution: 3 mrad
- » momentum res.: 10^{-4}



Cross sections



Polynomial
Poly. + dip
Poly. × dip

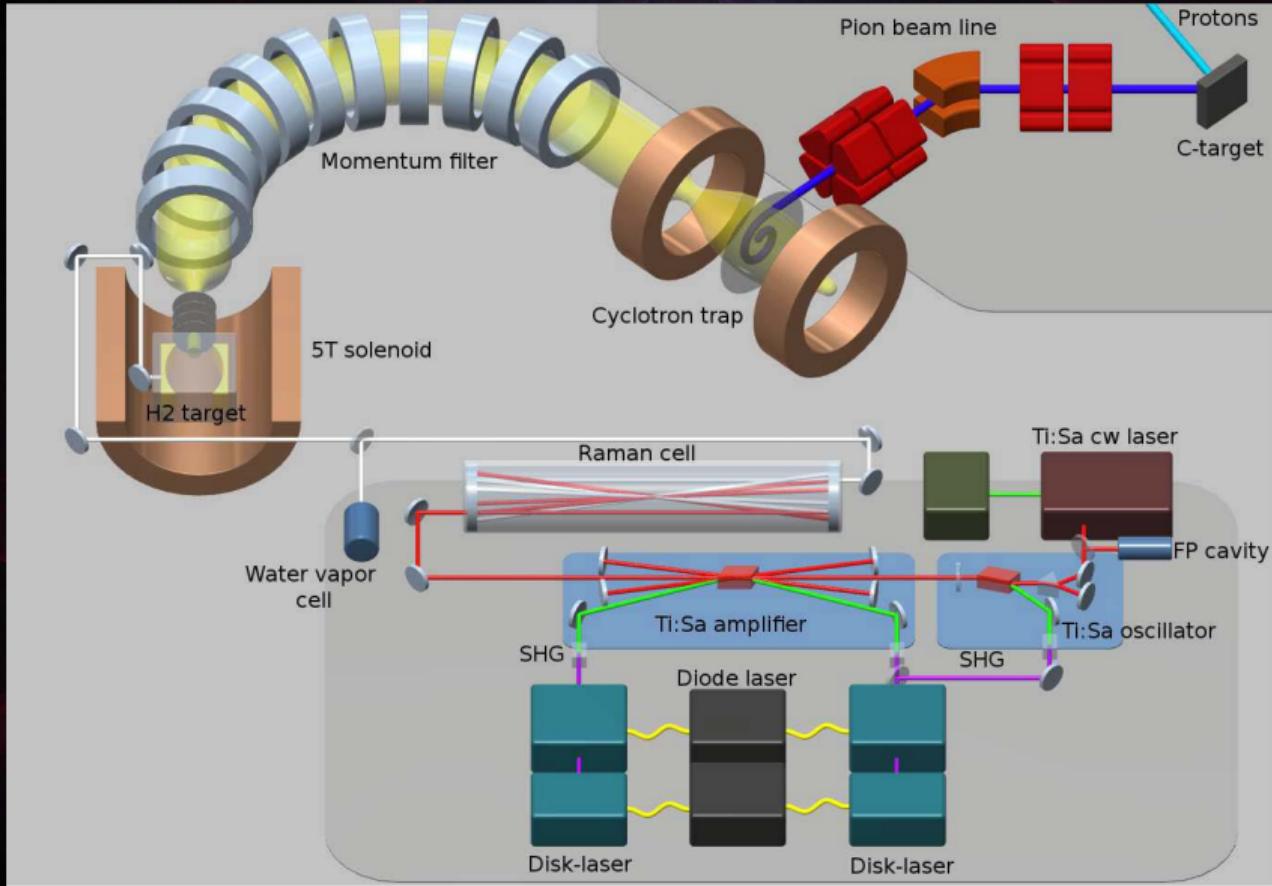
Inv. poly.
Spline
Spline × dip

Friedrich-Walcher
Double dipole
Extended G.K.

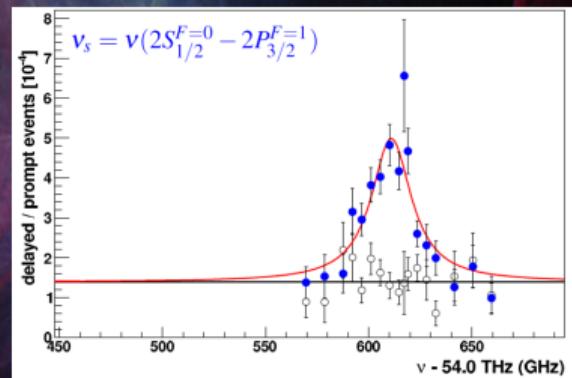
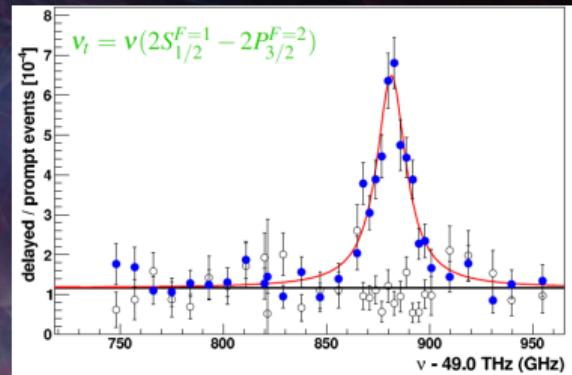
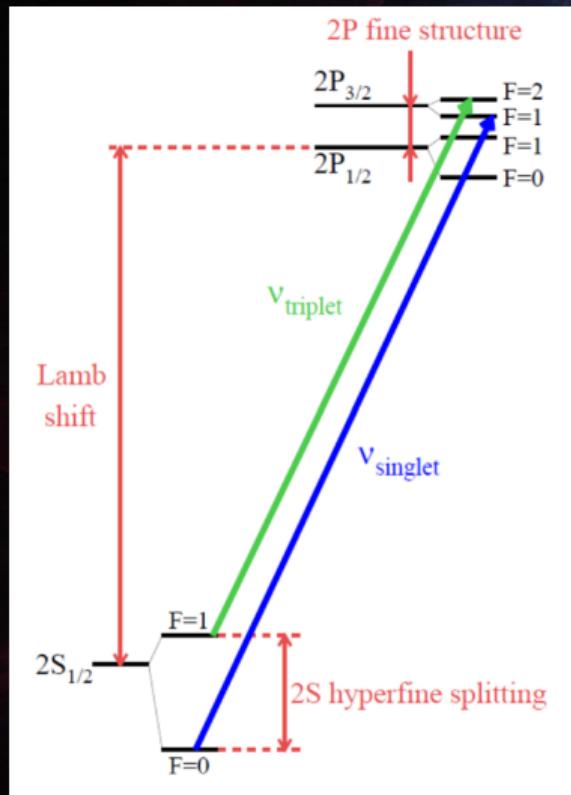
Muonic Hydrogen Spectroscopy

- » Replace **electron** with **muon**
- » 200 times heavier \Rightarrow 200 times smaller orbit
- » Probability to be “inside” 200^3 higher!

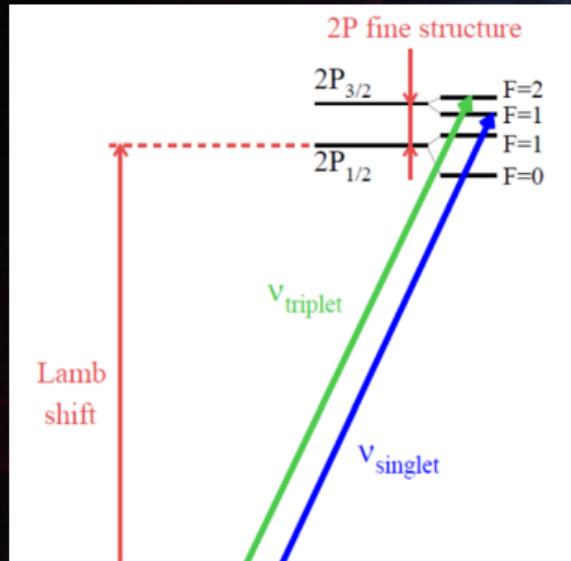
PSI setup (CREMA)



Muonic Hydrogen Spectroscopy Results



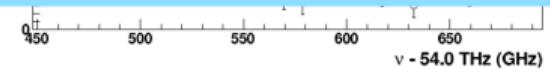
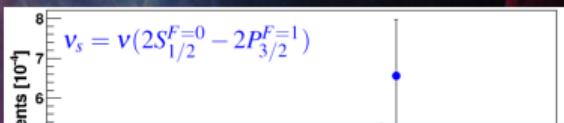
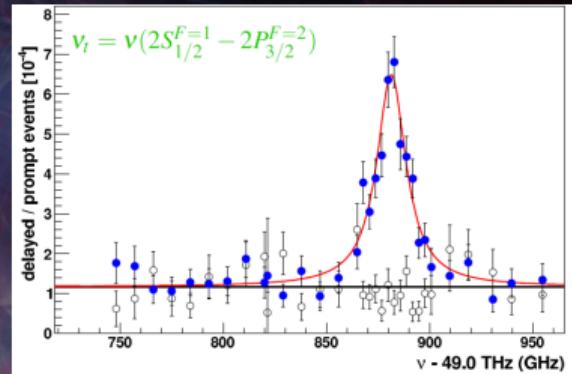
Muonic Hydrogen Spectroscopy Results



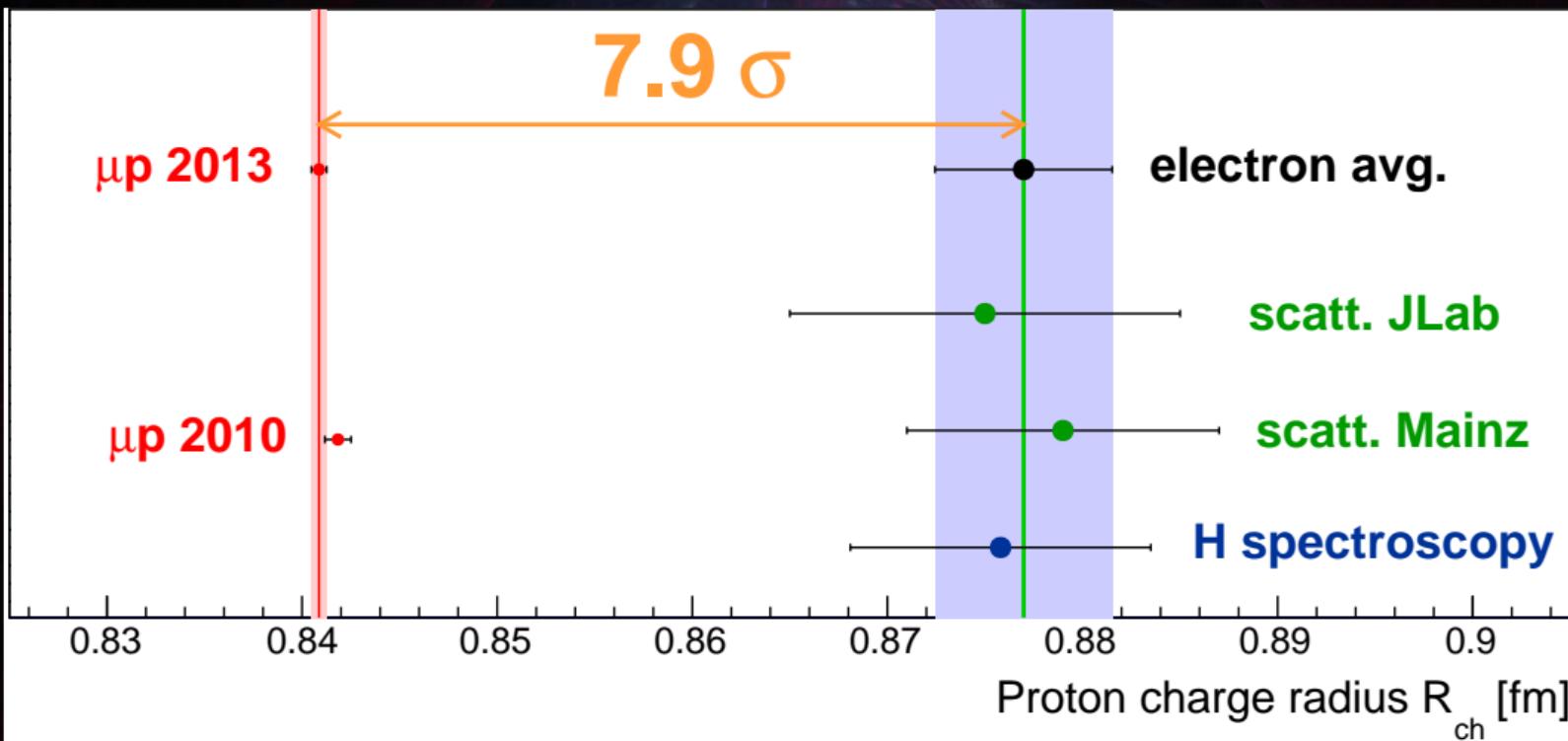
Result

- » Two semi-independent measurements
- » Consistent results

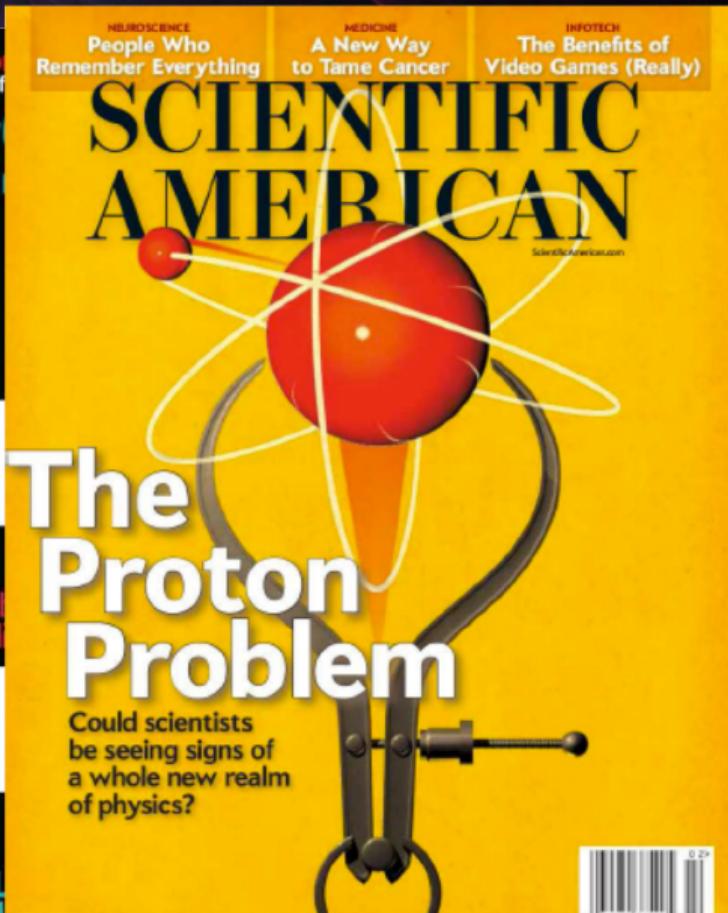
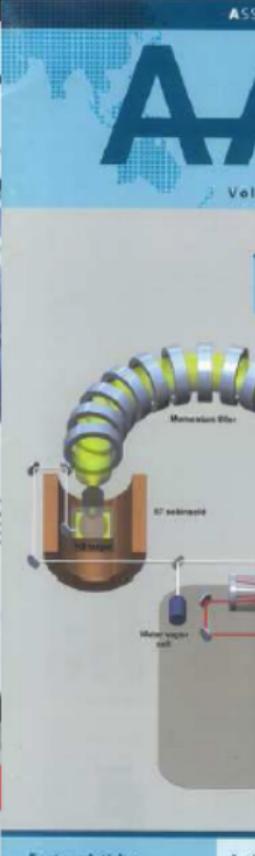
$F=0$



The Proton Radius puzzle



The Proton Radius puzzle



What's going on?

- » Are we measuring the same thing: Yes! G. Miller, Phys. Rev. C 99, 035202
- » Muon spectroscopy wrong?
 - » Data is robust
 - » A lot of theory required! Checked extensively.
 - » But: arxiv:2304.07035
- » Electron data wrong?
 - » Spectroscopy and scattering?
 - » Data or theory? Or Fit?
- » **BSM physics?** Still alive and kicking, E.g.: Liu, Cloet, Miller Nucl. Phys. B 944 114638 (also explains $g_\mu - 2$)

The face puzzle that launched a thousand ships experiments



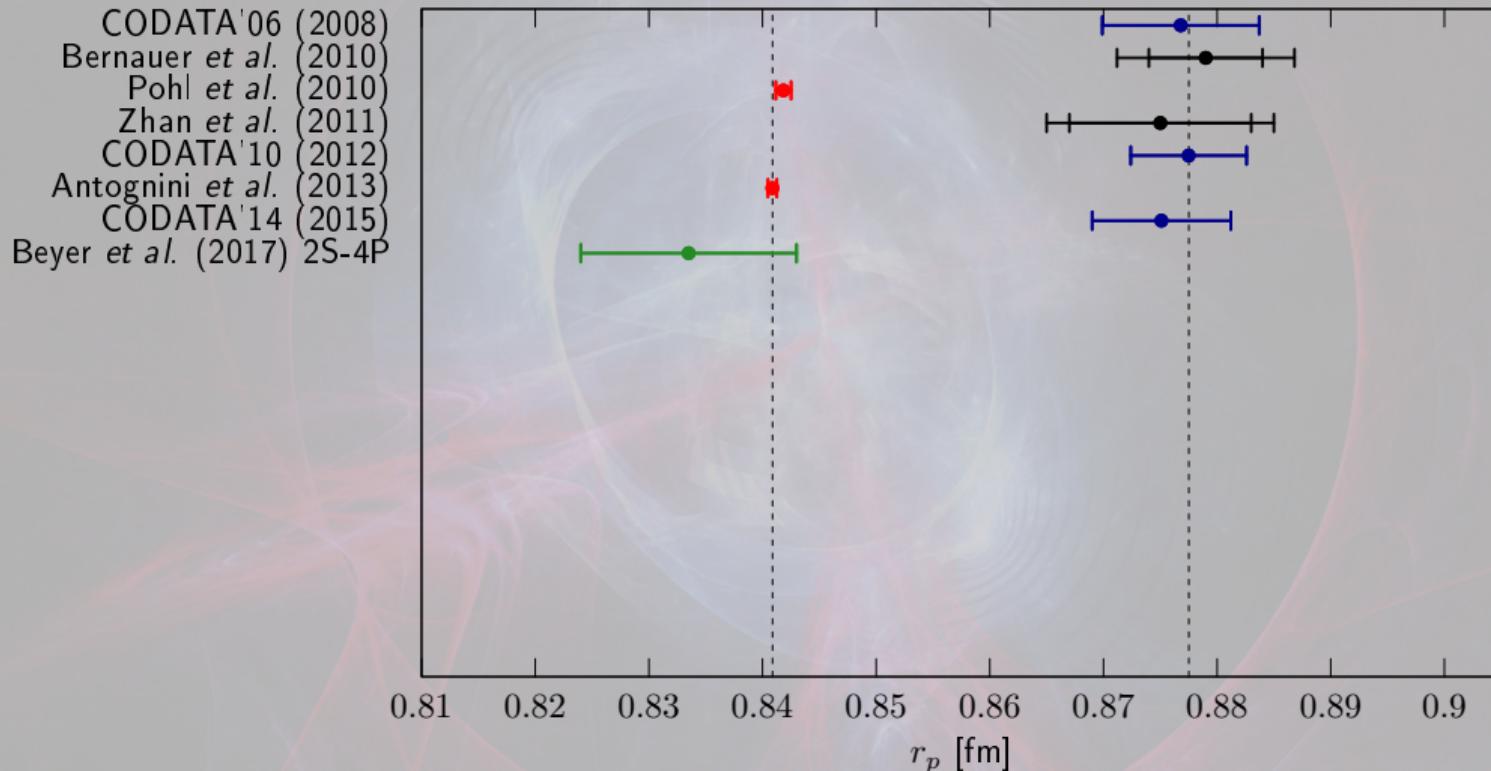
Spectroscopy:

- » MPQ
- » York University
- » Paris
- » + measurements on $d, {}^3He, {}^4He, \dots$

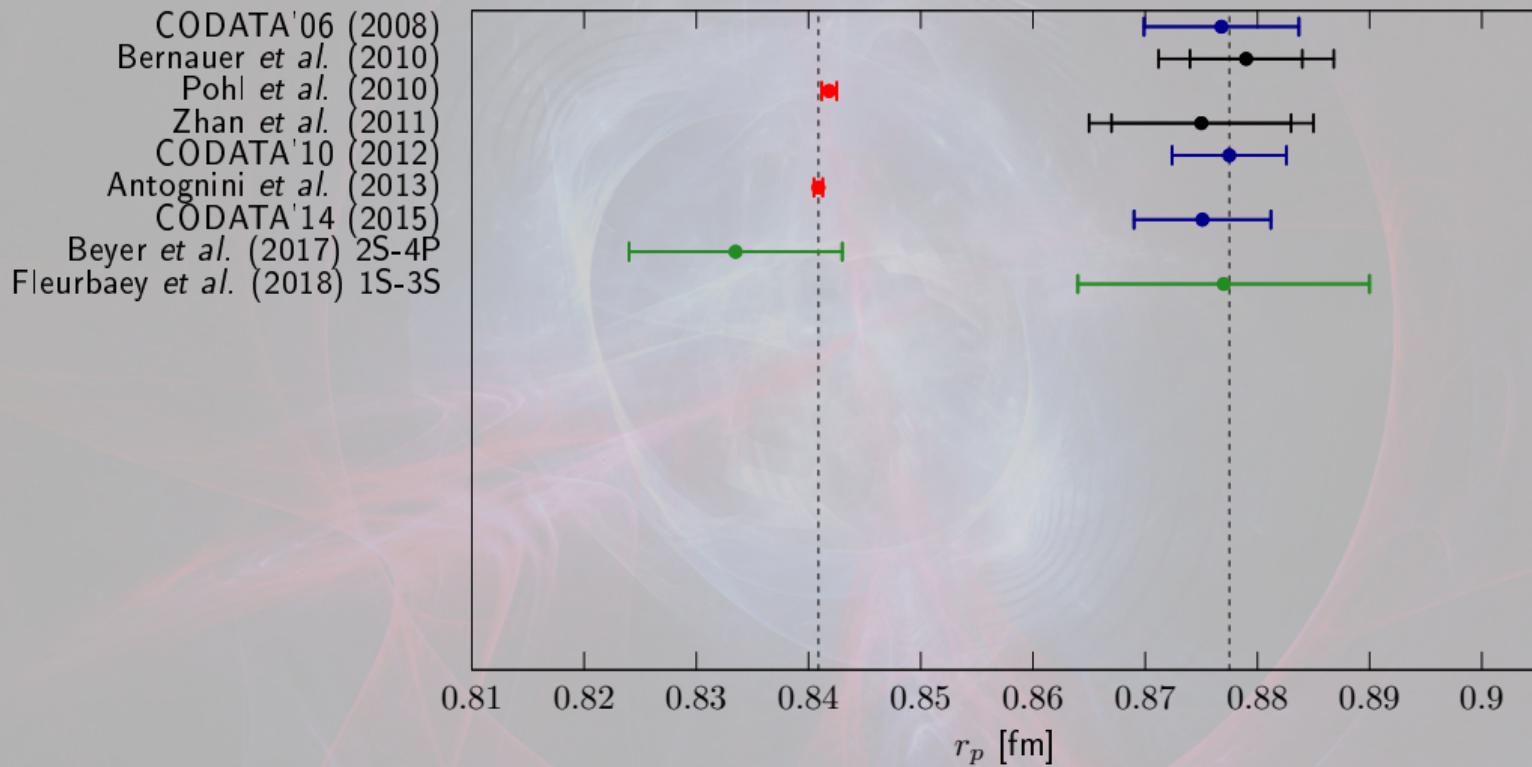
Scattering:

- » PRad (Jefferson Lab)
- » Mainz: ISR, H-TPC, Next-gen FF
- » AMBER@CERN
- » ULQ2 Sendai
- » MUSE

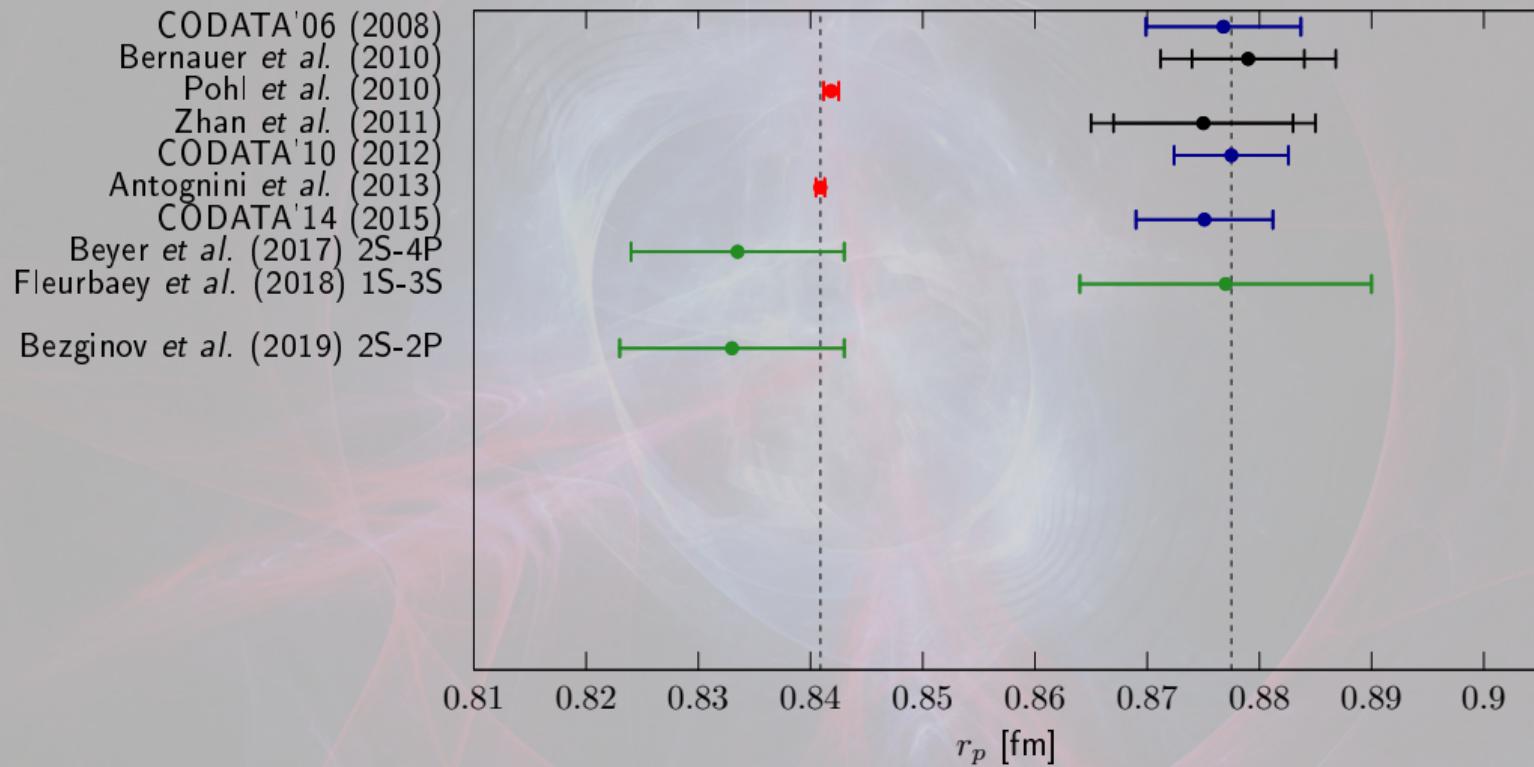
New results: MPQ (A. Beyer et al., Science 358, 79 (2017))



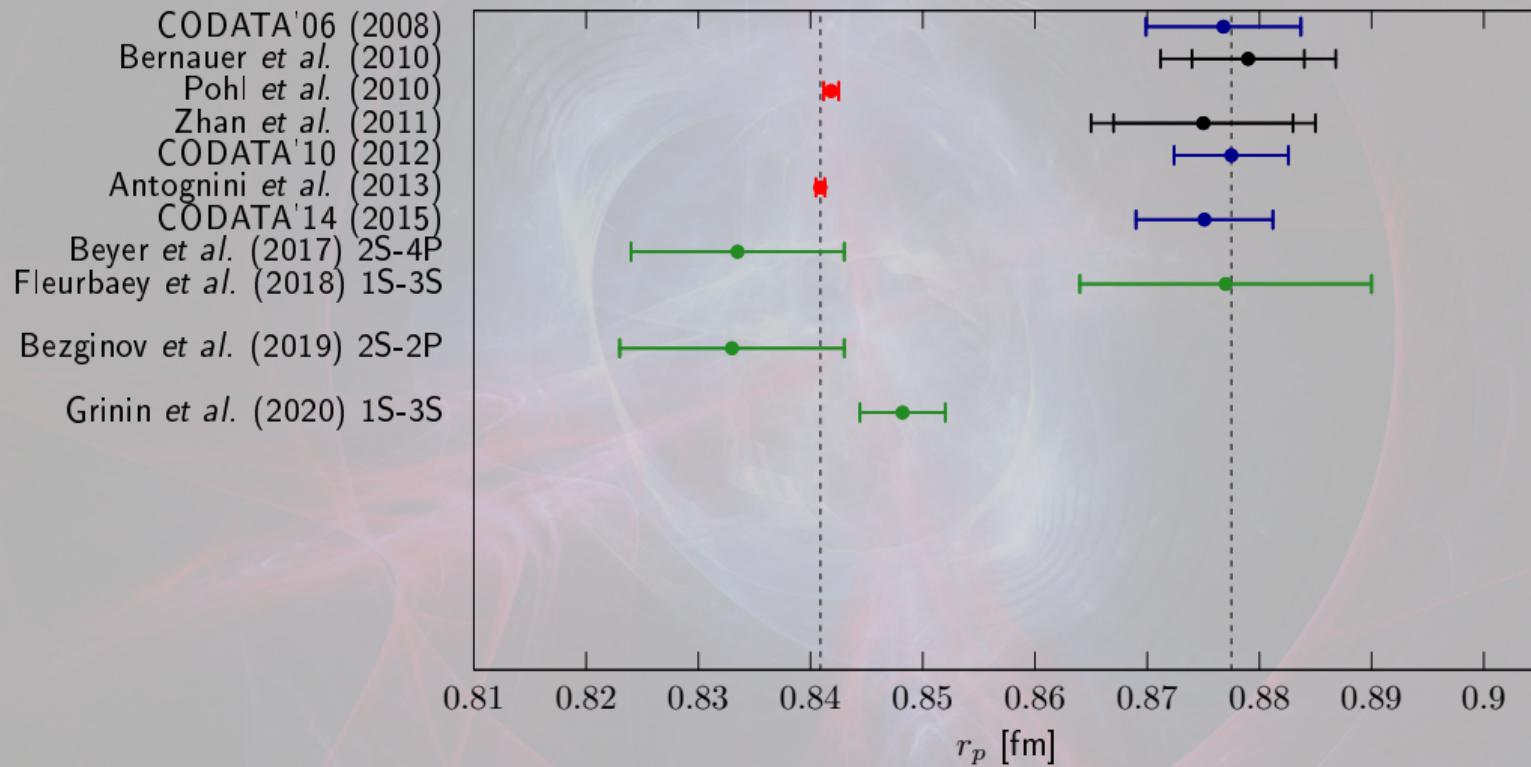
New results: Paris (Fleurbaey et al., Phys. Rev. Lett. 120, 183001 (2018))



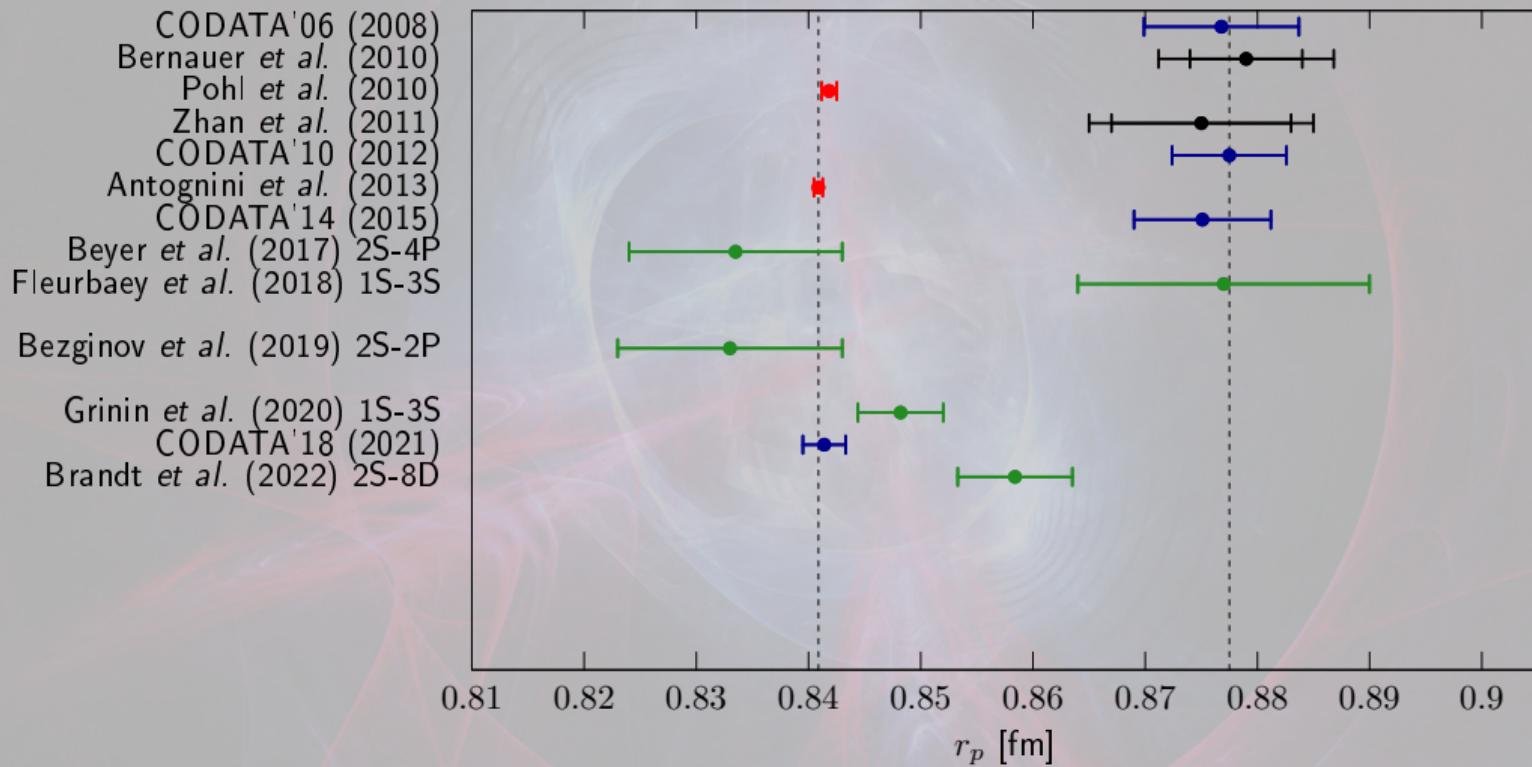
New results: York (Bezginov et al., Science 365, 1007–1012 (2019))



New results: MPQ again (Grinin et al., Science 370, 1061-1066 (2020))



New results: CSU (Brandt et al., Phys. Rev. Lett. 128, 023001 (2022))



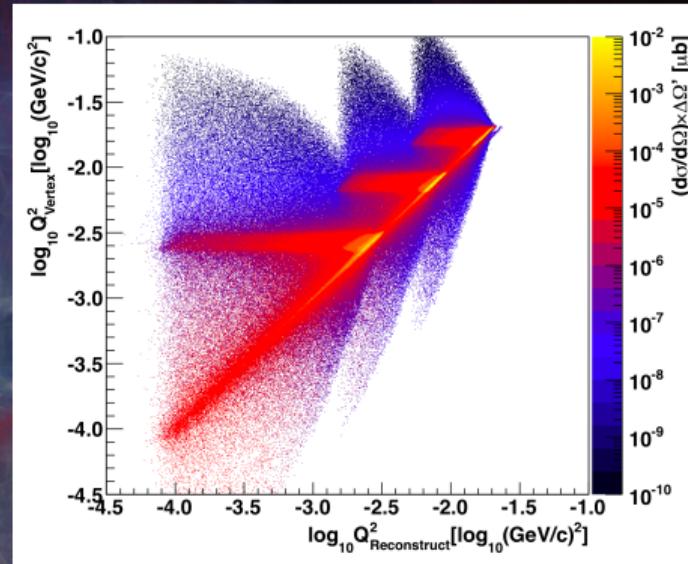
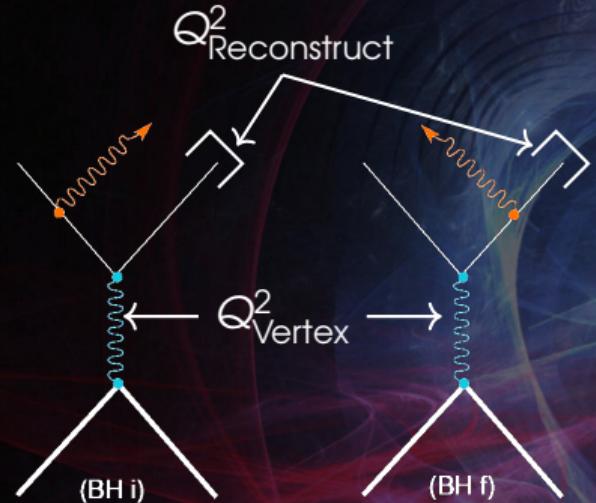
Need to extrapolate Mainz to $Q^2 = 0$

Need a fit to extract slope at $Q^2 = 0$

- » Much written about this
- » A lot of fits get small radius:
 - » Choice of Q^2 cut-off
 - » Choice of fit function
 - » Fits not in global minimum
- » Fits with all data mostly get large radius
- » Except when function is constrained (e.g. from theory). But then, bad χ^2
- » N.B.: A polynomial fit is not a Taylor expansion. Completely different convergence behavior!

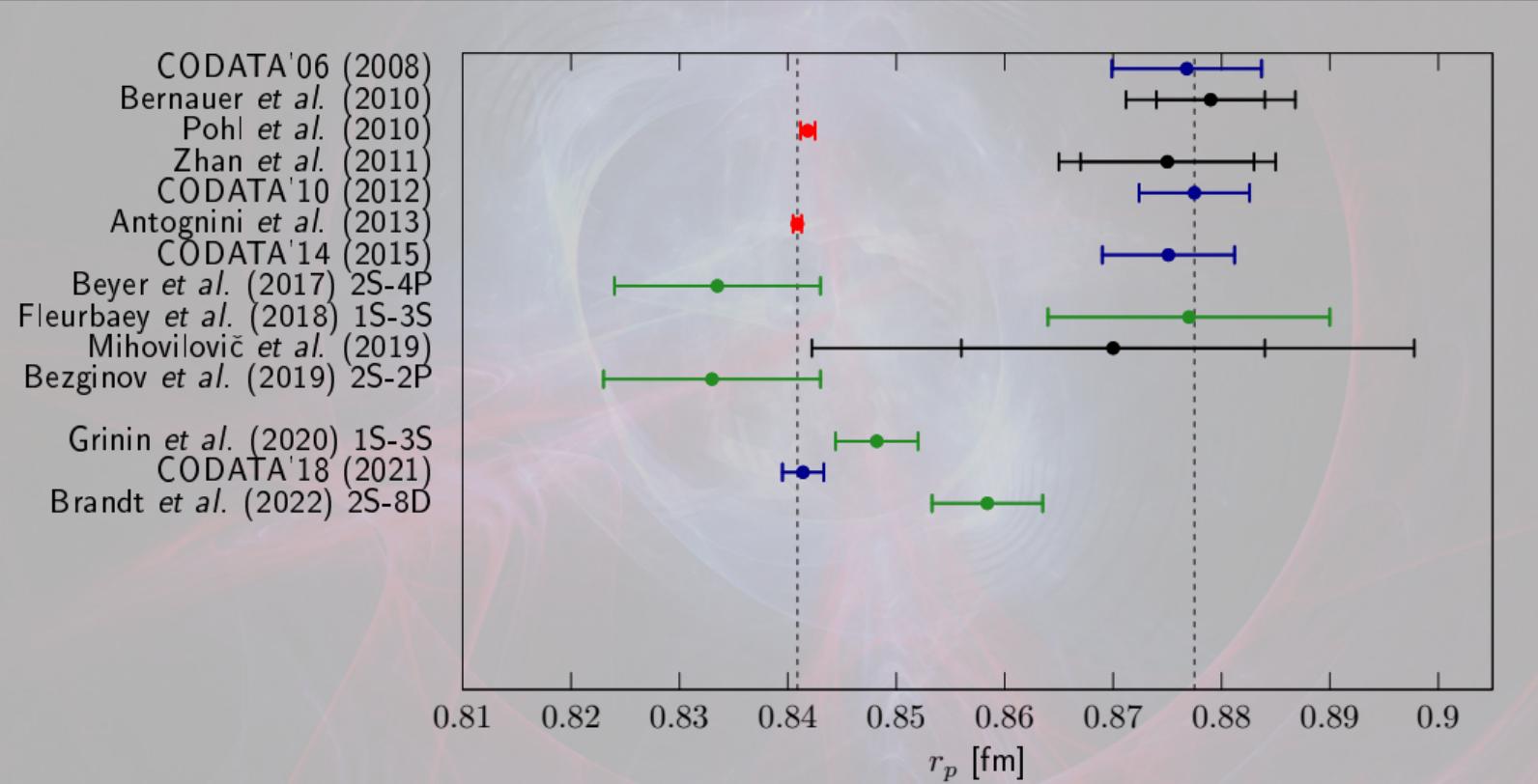
Want data at smaller Q^2 .

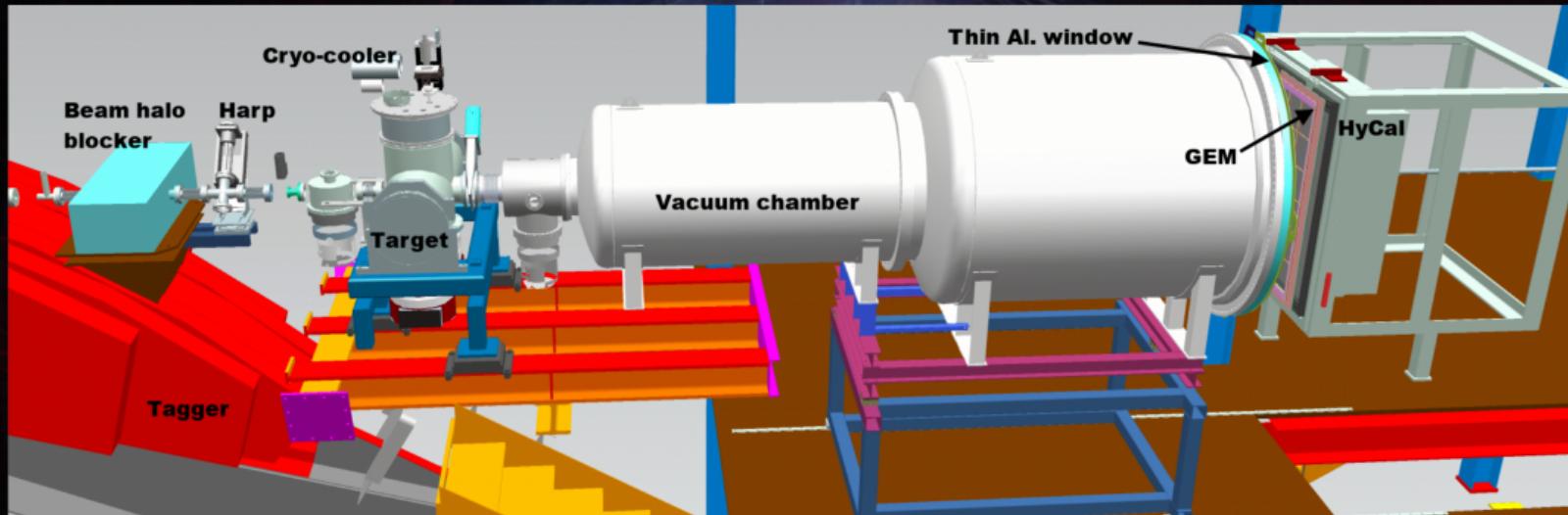
ISR method



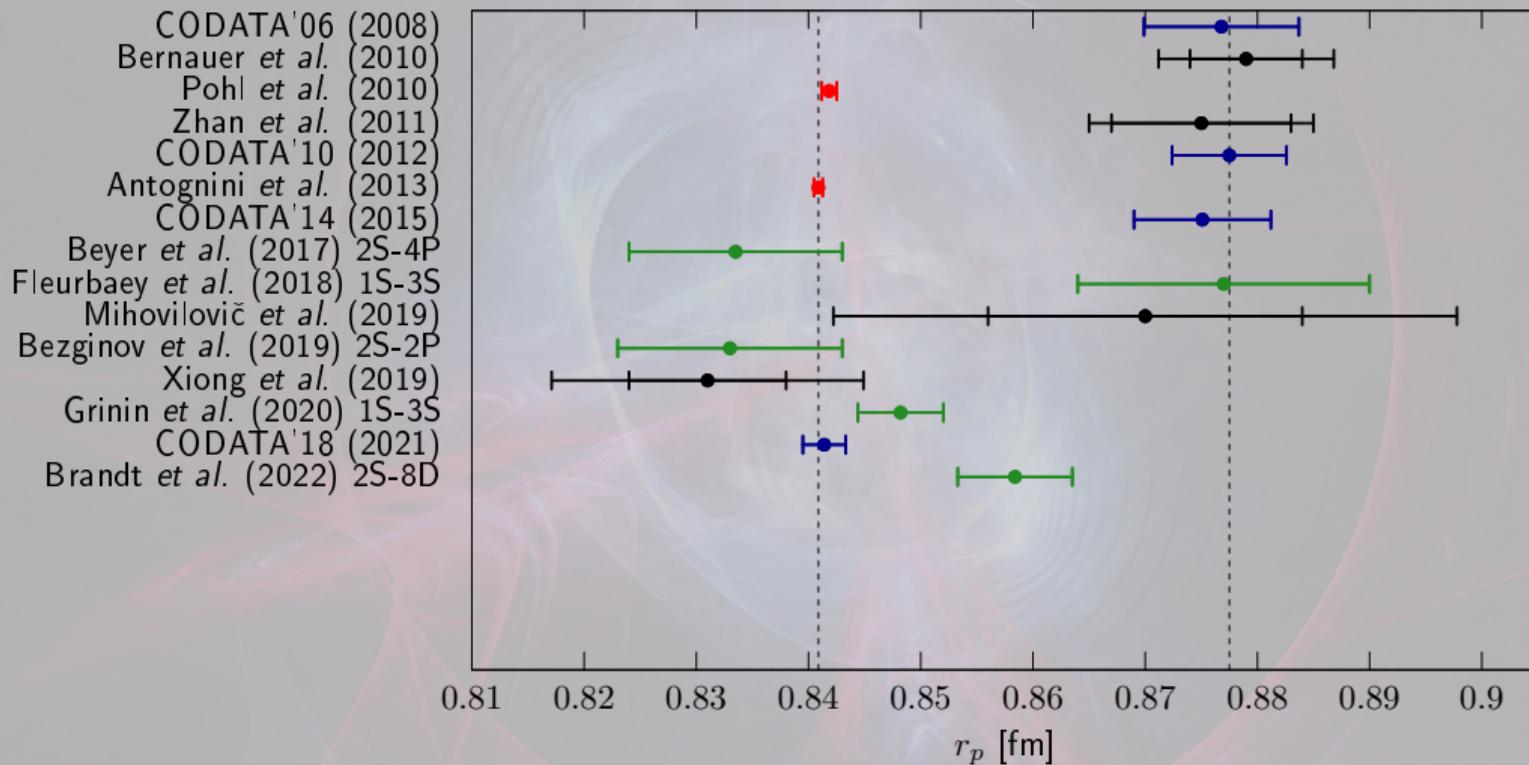
- » Use initial state radiation to reduce effective beam energy
- » Have to subtract FSR

New results: Mainz ISR (Mihovilović et al., Eur.Phys.J.A 57 (2021) 3, 107)

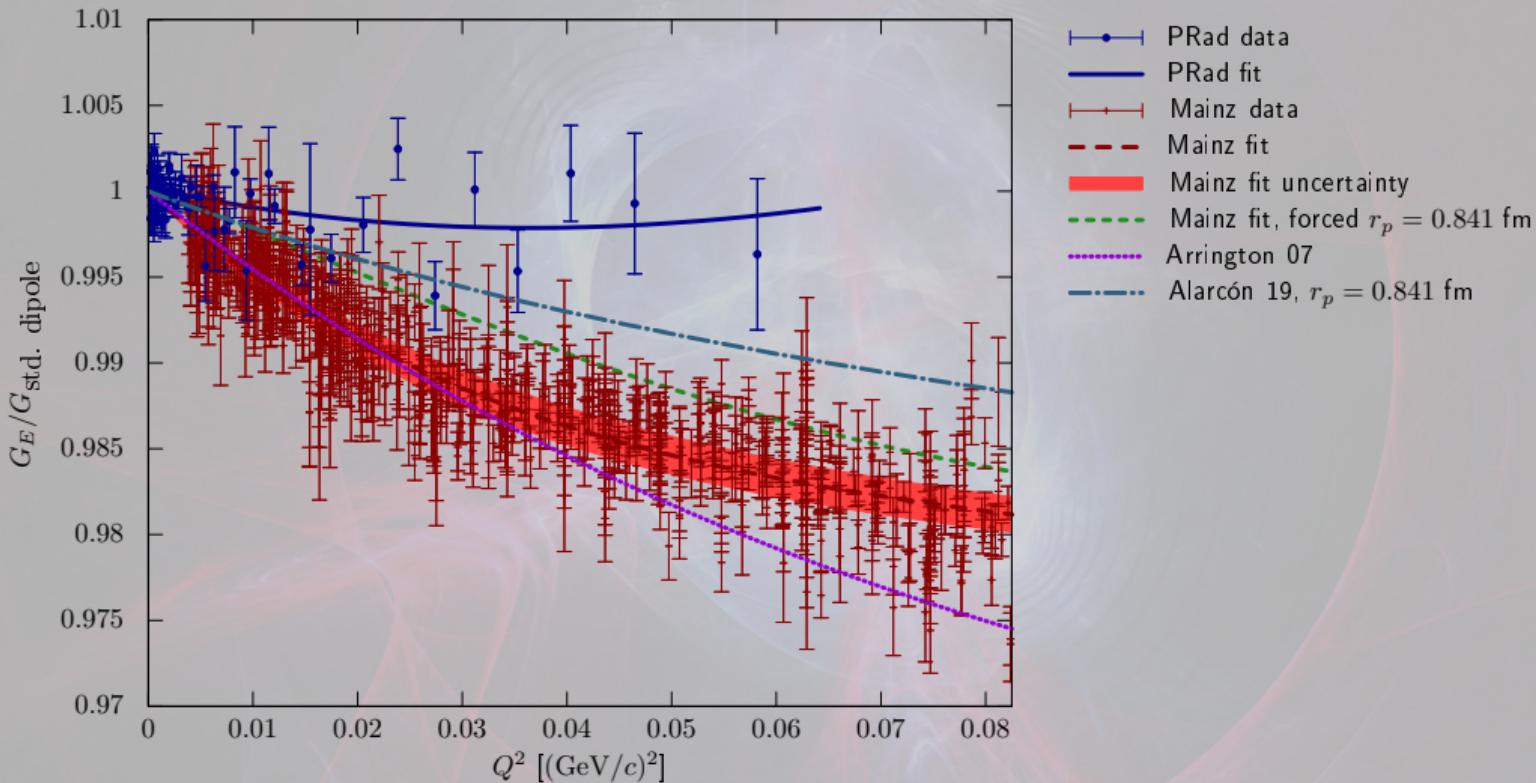




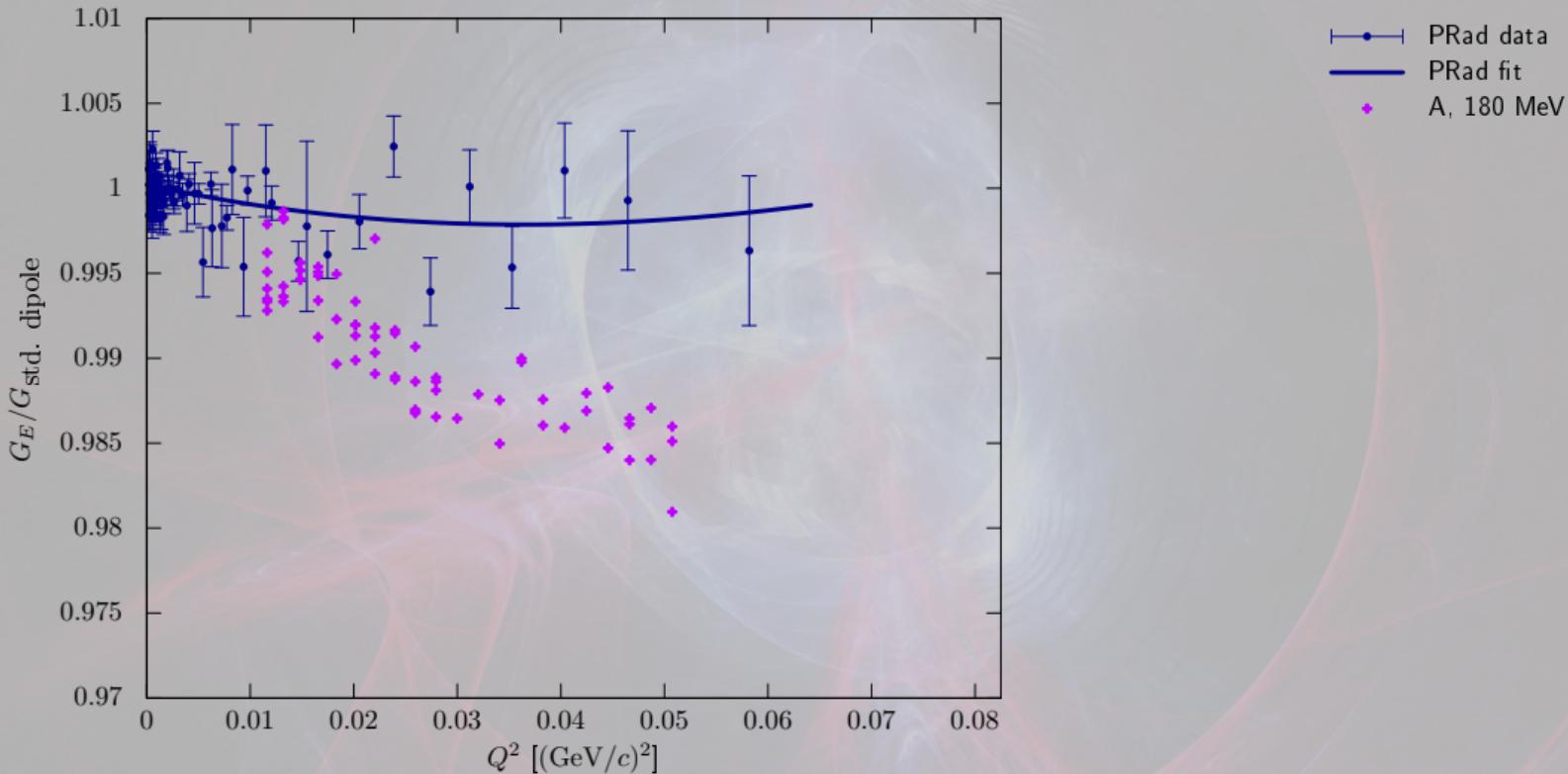
New results: PRad (Xiong et al., Nature 575 7781, 147-150 (2019))



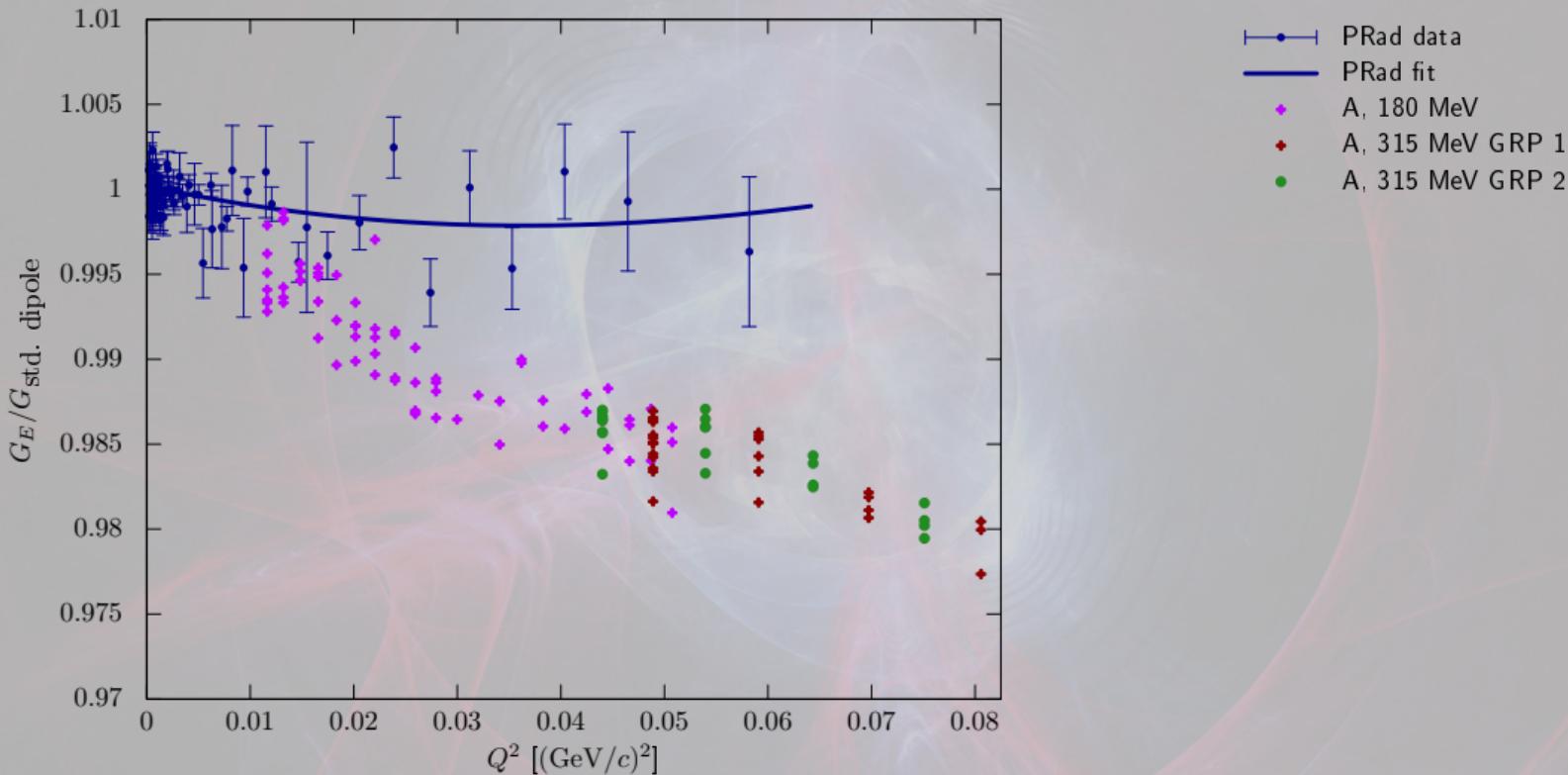
A new puzzle



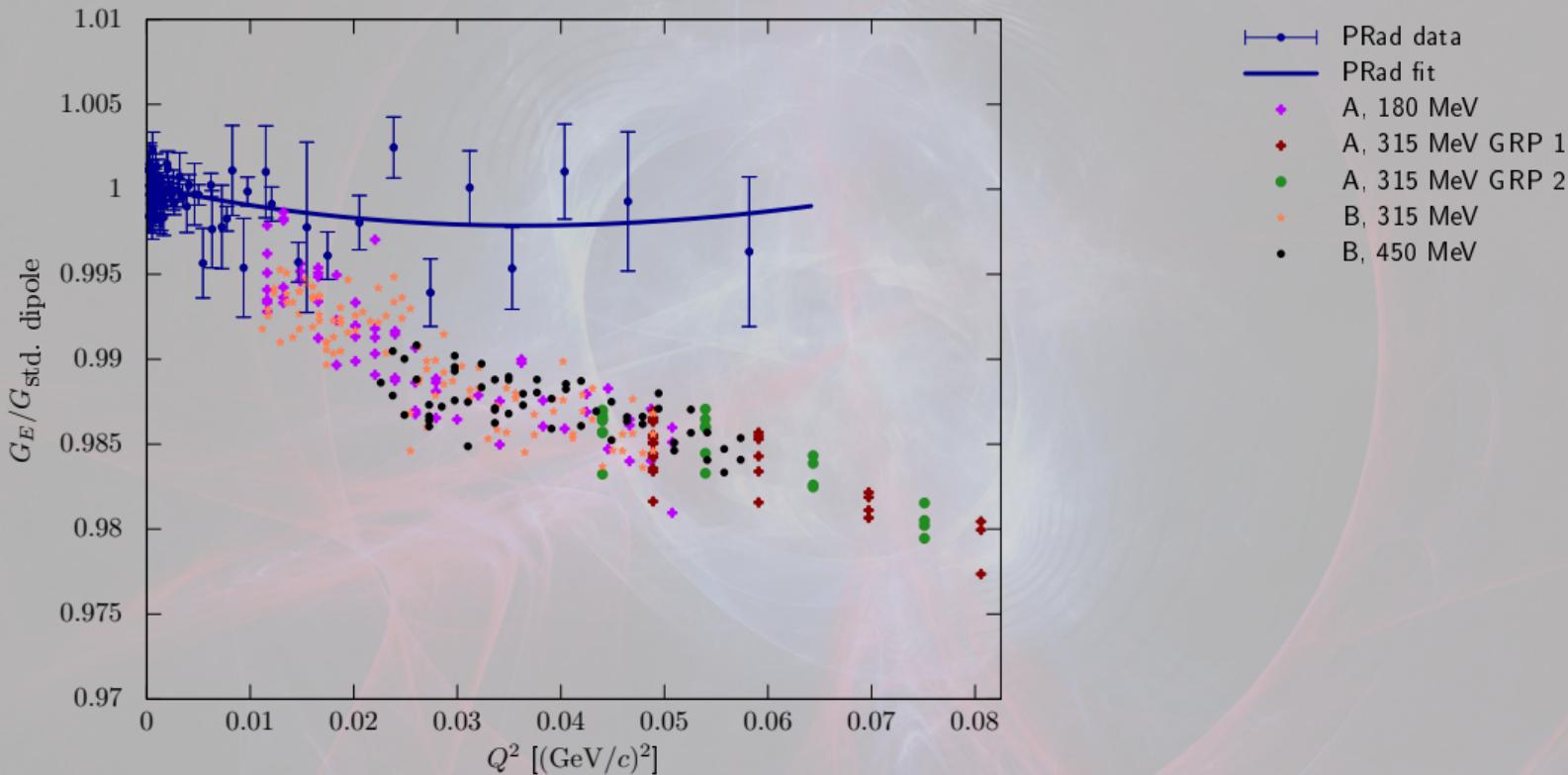
A new puzzle



A new puzzle

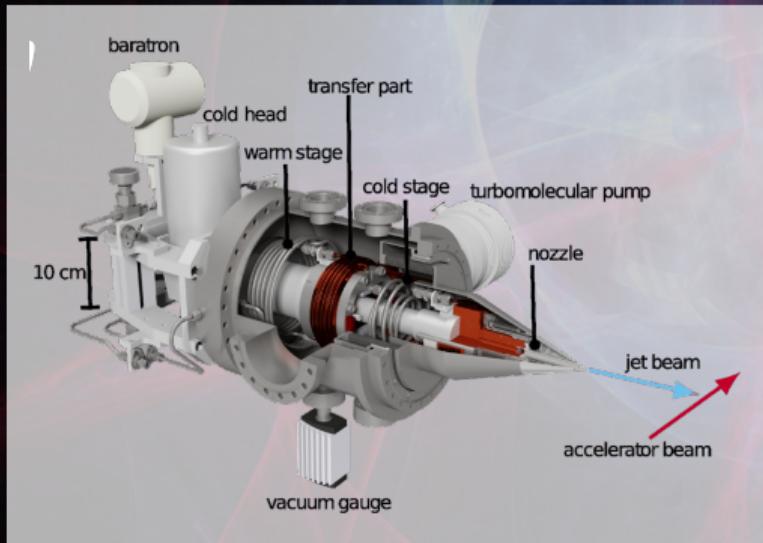


A new puzzle



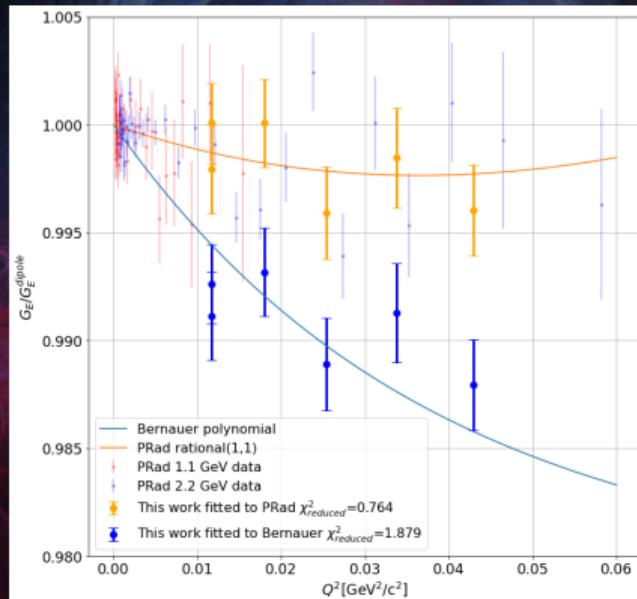
Upgraded target at Mainz

- » Gas-Jet target: pure hydrogen target, point-like. Eliminate major background.
- » Designed for MAGIX, but run at A1 as a prototype exp.



Upgraded target at Mainz

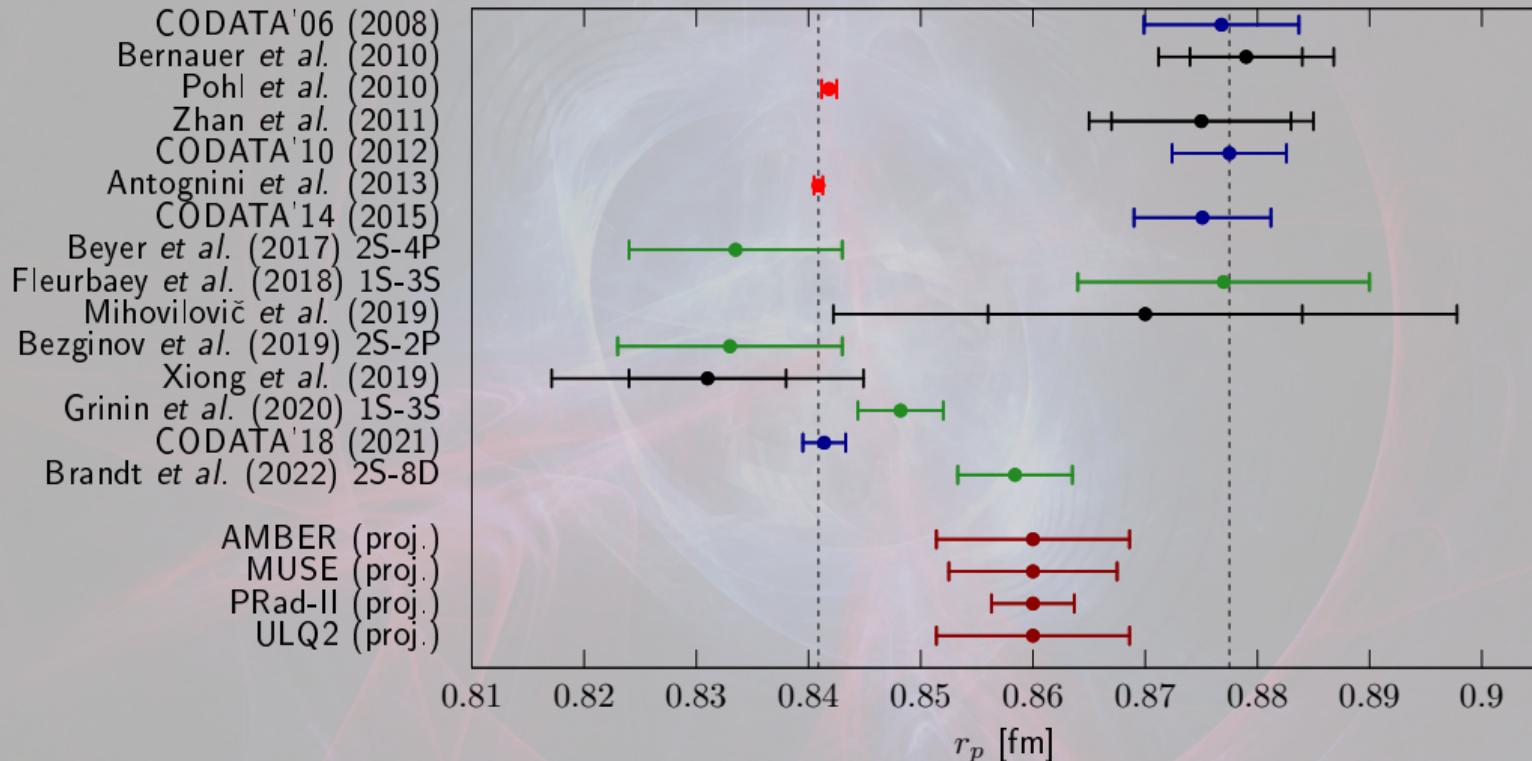
- » Gas-Jet target: pure hydrogen target, point-like. Eliminate major background.
- » Designed for MAGIX, but run at A1 as a prototype exp.
- » COVID limited reach: Prefers PRad, but not decisive.



Ongoing/future experiments

- » ULQ2@Sendai: $e p$ with CH₂ target. (See talk by Toshimi on Friday)
- » AMBER@CERN: μp , detecting proton, at many-GeV beam energies
- » MUSE@PSI: $e/\mu p$, direct lepton universality check (see talks by Win and Tanvi later today)
- » PRad-II: larger momentum range (see talk by Douglas (next!))
- » A1@MAMI + MAGIX@MESA: gas-jet, also measure G_M and magnetic radius

Projections



Scattering will cover (almost) all approaches

	Large E, small θ	small E, large θ
e^-	PRad, PRad-II	A1+MAGIX, MUSE, ULQ2
e^+	<your exp here?>	MUSE
μ^+, μ^-	AMBER	MUSE

Kinematics tests systematics

	Large E, small θ	small E, large θ
e^-	PRad, PRad-II	A1+MAGIX, MUSE, ULQ2
e^+	<your exp here?>	MUSE
μ^+, μ^-	AMBER	MUSE

- » Different kinematics tests radiative corrections, G_M dependence

Charge switch tests TPE

	Large E, small θ	small E, large θ
e^-	PRad, PRad-II	A1+MAGIX, MUSE, ULQ2
e^+	<your exp here?>	MUSE
μ^+, μ^-	AMBER	MUSE

- » Test TPE with ratio/asymmetry
- » Eliminate TPE+charge-odd corrections with charge-average

Beam species tests lepton universality, rad. corr.

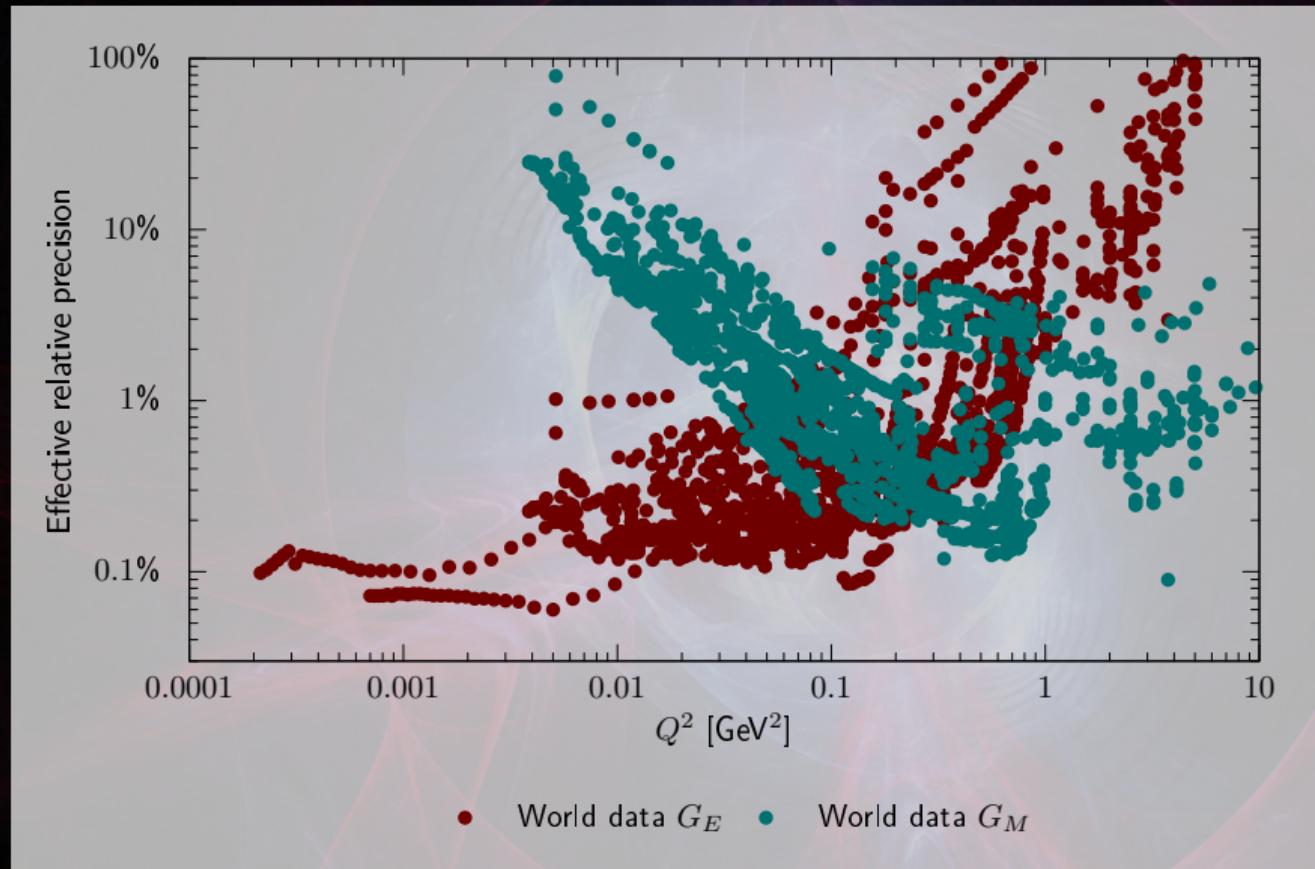
	Large E, small θ	small E, large θ
e^-	PRad, PRad-II	A1+MAGIX, MUSE, ULQ2
e^+	<your exp here?>	MUSE
μ^+, μ^-	AMBER	MUSE

- » Direct test of lepton universality
- » Also: rad. corr. different, smaller for muons

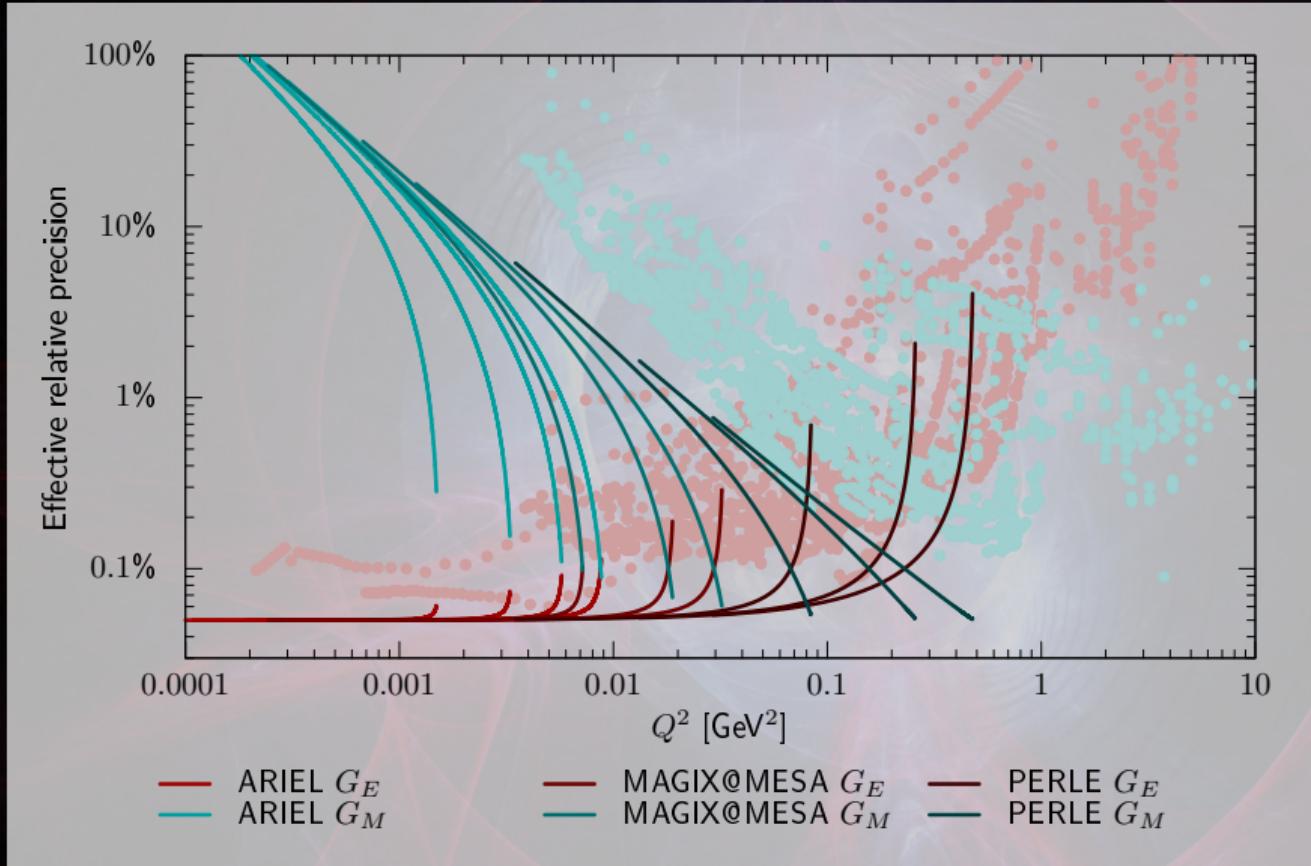
More orthogonality

- » Small acceptance vs. large acceptance:
 - » SA: efficiency is normalization effect
 - » SA: but need to control acceptance, luminosity when moved
 - » LA: luminosity is a normalization effect
 - » LA: but have to control acceptance, efficiency across detector
- » Lepton detection vs. proton/coincidence detection
 - » Fundamentally difference systematics

But what about G_M ?



But what about G_M ?



Conclusion

- » New data available and in the pipeline, but puzzle unresolved
- » Has changed shape:
 - » ~~Muon spectorscopy \leftrightarrow everybody else~~
 - » Spectroscopy many new low results, but not consistent
 - » Scattering not consistent
- » Scattering results: PRad finds small radius
 - » But not extrapolation as the problem
 - » Strong discrepancy in overlap region
- » Magnetic low-Q FF essentially unknown!
 - » Accessible only with low E at back angles.

Backup slide

Hic sunt dracones

Test your models!

- » Generate many pseudo data set from input parametrizations “known truth”
 - » permute with uncertainty model (statistics+systematics!)
- » Fit with your model
- » Repeat with different parametrizations
- » Look at bias, uncertainty.
- » **Do it before you have your data in hand!**

Also useful to get meaningful errors: Use fit on data as source. Gets proper error estimates including all correlations.

Taylor expansions and polynomial fits

It's a common theme that a polynomial fit is related to a Taylor expansion around 0, sharing important traits, mainly radius of convergence.

- » "We will fit ... a simple Taylor series expansion." R.J. Hill and G. Paz, Phys. Rev. D 82, 113005 (2010)
- » "correct inclusion of the lowest singularity" I. Lorenz and U.G-Meißner, Phys. Lett. B 737, 57 (2014)
- » "Maclaurin fits", D. W. Higinbotham et al., Phys.Rev. C93, 055207 (2016)
- » "We do not advocate using polynomial fits.... since convergence ... is not assured..." K. Griffioen et al., arxiv:1509.06676

This is wrong.

Traits of Taylor, Weierstrass, Fits

Taylor expansion

- » Is correct in all orders (upto truncated order) at x_0 .
- » Converges on a radius up to the next pole.
- » Error is $R_k = \frac{f^{(k+1)}(\xi_c)}{k!}(x - \xi_c)^k(x - x_0)$

Weierstrass theorem

- » Any function continuos over $[a, b]$ can be approximated with a polynomial in that range.
- » The convergence is uniform:
 $\forall \epsilon > 0, \exists \text{ poly.}, \text{ so that } ||f(x) - p(x)||_\infty < \epsilon, x \in [a, b]$

Polynomial fit

- » Minimizes L2-norm over the points: $||f(x) - p(x)||_2$
- » Will converge to the function, NOT to the Taylor expansion of the function

We have no choice

Taylor expansion

- » Is correct in all order (to truncated order) at x_0 .
- » Converges on a radius unto the next pole.
- » Error is $R_k = \frac{f^{(k+1)}(\xi_c)}{k!}(x - \xi_c)^k(x - a)$

Weierstrass theorem

- » Any function continuous over $[a, b]$ can be approximated with a polynomial in that range.
- » The convergence is uniform:
 $\forall \epsilon > 0, \exists \text{ poly. } . \text{so that } \|f(x) - p(x)\|_\infty < \epsilon, x \in [a, b]$

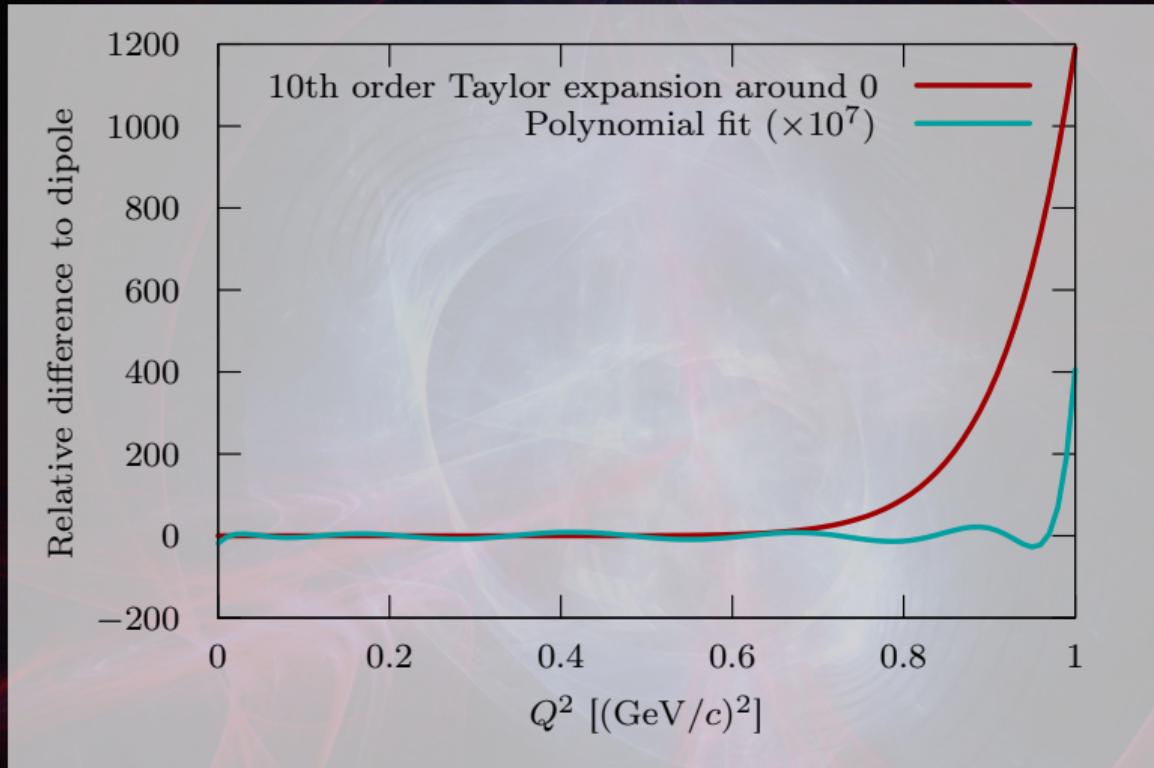
Polynomial fit

- » Minimizes L2-norm over the points: $\|f(x) - p(x)\|_2$
- » Will converge to the function, NOT to the Taylor expansion of the function

What does that mean in reality?

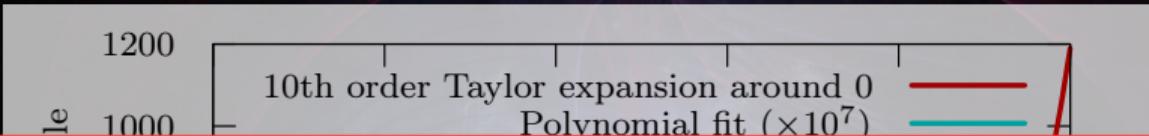
- » Let's fit perfect pseudo data
- » Compare with Taylor expansion
- » Input function: dipole, i.e. pole at $Q^2 = -0.71 \text{ (GeV/c)}^2$

Fit results



Fit within 40 ppm over data range, better than expansion for $Q^2 > 0.15 (\text{GeV}/c)^2$

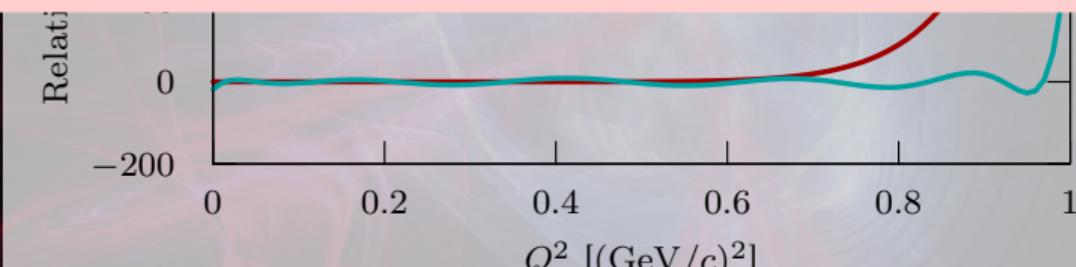
Fit results



Conclusion:

Taylor convergence radius

- » has no consequence for polynomial fit
- » is not a reason to use conformal mapping
- » is not a reason to limit Q^2 range



Fit within 40 ppm over data range, better than expansion for $Q^2 > 0.15 (\text{GeV}/c)^2$

Extrapolation to $Q^2 = 0$

Have to extrapolate form factor to $Q^2 = 0$.

Mainz lowest $Q^2 = 0.0033 \text{ (GeV/c)}^2$.

We use a 10th order polynomial to fit data up to 1 (GeV/c)^2 .

Can we fit just a linear term to a reduced range?

Can a linear fit work?

$$\frac{d\sigma}{d\Omega} \propto 1 - \underbrace{A}_{\mathcal{O}(6)} \cdot Q^2 + \underbrace{B}_{\mathcal{O}(30)} \cdot Q^4 + \dots$$

(Q in units of GeV/c)

We want to measure the radius ($\sim \sqrt{A}$) to within 0.5%, without knowing B . So:

$$B/A \cdot Q^2 \ll 0.01 \rightarrow Q^2 \ll 0.002 (\text{GeV}/c)^2$$

Can a linear fit work?

$$\frac{d\sigma}{d\Omega} \propto 1 - \underbrace{A}_{\mathcal{O}(6)} \cdot Q^2 + \underbrace{B}_{\mathcal{O}(30)} \cdot Q^4 + \dots$$

(Q in units of GeV/c)

We want to measure the radius ($\sim \sqrt{A}$) to within 0.5%, without knowing B . So:

$$B/A \cdot Q^2 \ll 0.01 \rightarrow Q^2 \ll 0.002 (\text{GeV}/c)^2$$

But: Need to measure A to 1%, so measure $\frac{d\sigma}{d\Omega}$ to $6 \cdot 0.002 \cdot 0.01 = 0.012\%$.

How many orders?

- » "As many as needed" → Knee in χ^2 , F-test, AIC, ...
 - » Only tells you that the data can't prove you need more.
 - » Doesn't tell you about bias
- » Orthogonal basis?
 - » Depends on data points
 - » Cannot construct orthogonal basis where slope is only in one function!

Observation: Fit as many data with as many orders as possible. Highest order will have strongest bias, lower orders insulated.