

Nucleon spin structure contributions to the Hyperfine Structure determination and related topics

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**Workshop on Nucleon Structure at Low Q
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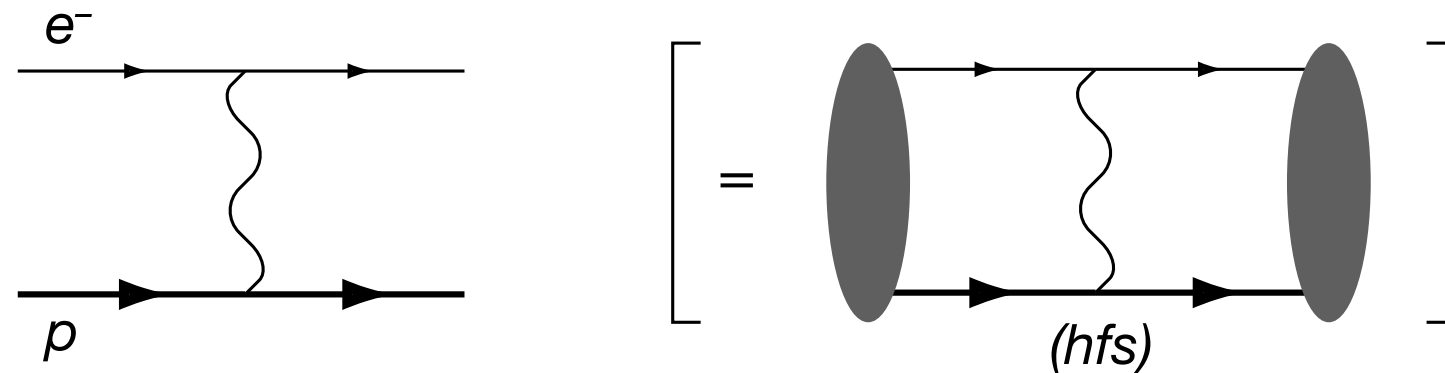
**Talk based on old papers, Nazaryan, Griffioen, Carlson, PRL
2006, CJP 2007, LNP 2008, PRA 2008, 2011, plus recent thinking**

In this talk

- Discussion of accurate calculation of hyperfine splitting (HFS) in hydrogen atom, both eH & μH
- Newly motivated by coming experiments
- Lowest order calculation gives the “Fermi energy” and we will discuss corrections to this

Lowest order (easy)

- UG textbook calculation!



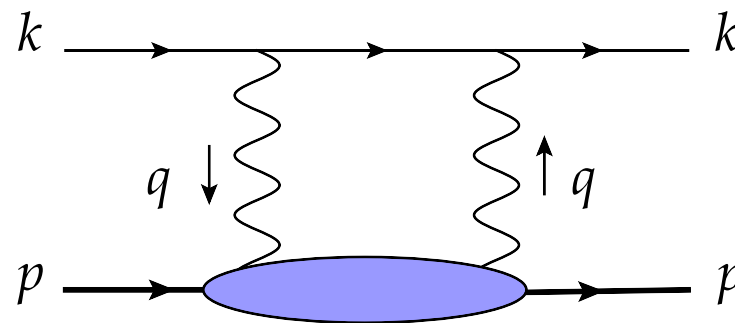
- Get
$$E_F^p = \frac{8\pi}{3} (m_r \alpha)^3 \mu_B \mu_p$$
- $\mu_B = e/(2m_e) =$ Bohr magneton
 $\mu_p =$ exact magnetic moment for proton
- “Fermi energy”
- Can evaluate to about 10-figure accuracy

Next need corrections

- Write as

$$E_{HFS}^p = E_F^p \left(1 + \Delta_{QED} + \Delta_S + \text{some smaller corrections} \right)$$

- Δ_{QED} well calculated, won't discuss here
- “some smaller corrections” won't be mentioned again
- Δ_S = structure dependent corrections,
here meaning corrections from 2- γ exchange,



- Conventionally separate as

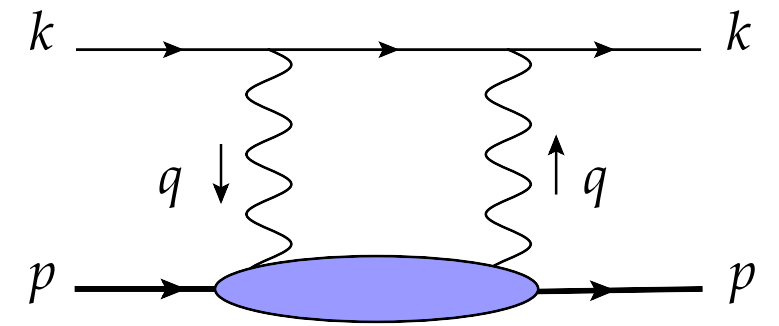
$$\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$$

NR elastic Rel. elastic Polarizability
“Zemach” Corrections corrections

To be discussed

- How do we get the 2γ corrections from ep scattering data? (General answer: dispersion relations)
- Can we use unsubtracted dispersion relation?
- Comparison with another method: $B\chi$ PT results
- Effect of new data—saw some already in Karl Slifer's talk, and defer further discussion to next talk (David Ruth).

2 γ corrections



- Not calculable *ab initio*.
But lower part is forward Compton scattering of off-shell photons, algebraically gotten from

$$T_{\mu\nu}(q, p, S) = \frac{i}{2\pi m_p} \int d^4\xi \, e^{iq \cdot \xi} \langle pS | j_\mu(\xi) j_\nu(0) | pS \rangle$$

- Spin dependence is in the antisymmetric part

$$T_{\mu\nu}^A = \frac{i}{m_p \nu} \epsilon_{\mu\nu\alpha\beta} q^\alpha \left[(H_1(\nu, Q^2) + H_2(\nu, Q^2)) S^\beta - H_2(\nu, Q^2) \frac{S \cdot q \, p^\beta}{p \cdot q} \right]$$

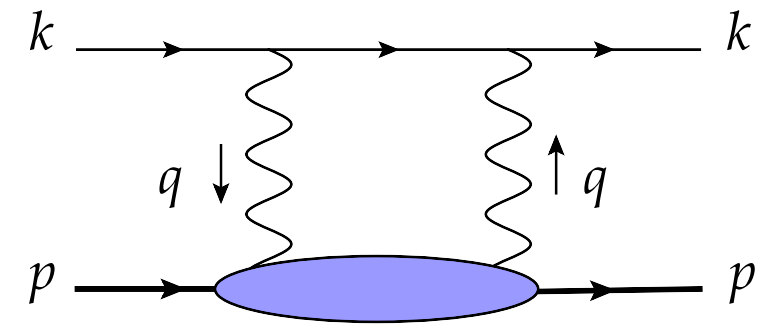
Some use
 $S_{1,2} = 4\pi^2 \alpha H_{1,2}$

- Imaginary part of above is related to polarized inelastic *ep* scattering, with

$$\text{Im } H_1(\nu, Q^2) = \frac{1}{\nu} g_1(\nu, Q^2) \quad \text{and} \quad \text{Im } H_2(\nu, Q^2) = \frac{m_p}{\nu^2} g_2(\nu, Q^2)$$

- Emphasize: g_1 and g_2 are measured at SLAC, HERMES, JLab, ...

2 γ corrections



- Combine electron part of diagram with Compton bottom, and energy from 2 γ exchange

$$\Delta_{\text{pol}} = \frac{E_{2\gamma}}{E_F} \Big|_{\text{inel}} = \frac{2\alpha m_e}{(1 + \kappa_p)\pi^3 m_p} \times \int \frac{d^4 Q}{(Q^4 + 4m_e^2 Q_0^2)Q^2} \{ (2Q^2 + Q_0^2)H_1^{\text{inel}}(iQ_0, Q^2) - 3Q^2 Q_0^2 H_2^{\text{inel}}(iQ_0, Q^2) \}$$

- (Wick rotated). Great, but don't know $H_{1,2}$ from data.
- But do know Im parts, and if no subtraction, simple Cauchy (dispersion relation) gives

$$H_1^{\text{inel}}(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{th}^2}^{\infty} d\nu'^2 \frac{\text{Im } H_1(\nu', Q^2)}{\nu'^2 - \nu^2}$$

and similarly for H_2 .

Do some integrals analytically, getting

- $$\Delta_{\text{pol}} = \frac{\alpha m_e}{2(1 + \kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

- $$\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2^2(Q^2) + 4m_p \int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2} \beta(Q^2/\nu^2) g_1(\nu, Q^2) \right\}$$

- $$\Delta_2 = -12m_p \int_0^\infty \frac{dQ^2}{Q^2} \int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2} \beta_2(Q^2/\nu^2) g_2(\nu, Q^2)$$

- $$\beta(\tau) = \frac{4}{9} \left[-3\tau + 2\tau^2 + 2(2 - \tau)\sqrt{\tau(\tau + 1)} \right] \quad (\text{for } m_e = 0)$$

- $$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)}$$

Comments

- Early history: begun by Iddings (1965),
finalized by Drell and Sullivan (1967),
put in present notation by de Rafael (1971).
No spin-dependent data existed,
no nonzero evaluation for > 30 years,
until Faustov and Martynenko (2002),
then modern era starts
- Someone added something: the F_2^2 term. Not inelastic.
(Put in here, taken out somewhere else.)
Thought convenient in 1967, still there.
- Term as written finite in $m_e \rightarrow 0$ limit, because of known sum
rule, $4m_p \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2} g_1(\nu, 0) = -\kappa_p^2$

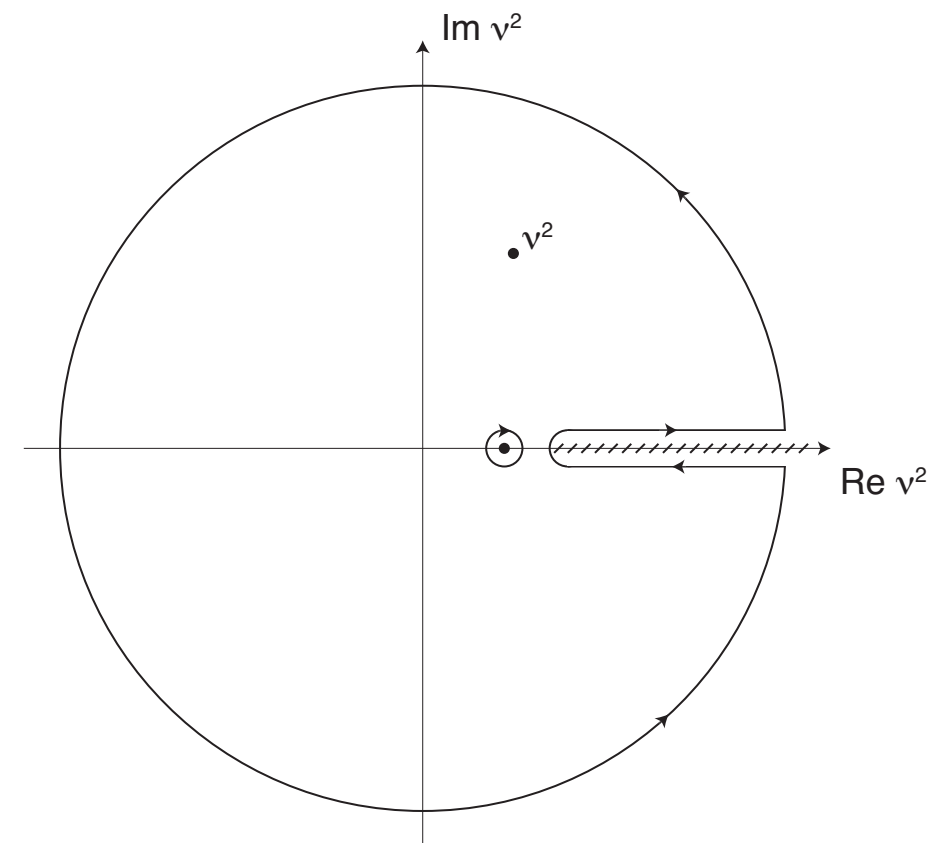
Get results

- Use data, modeling regions where data is scarce
- From CNG 2008, mostly using JLab 2003 data
$$\Delta_{pol}(eH, 2S) = 1.88 (0.07) (0.60) (0.20) \text{ ppm}$$
$$\Delta_{pol}(\mu H, 2S) = 351.0 (12.0) (107.0) (36.0) \text{ ppm}$$
- Improved by Tomolak and by Peset and Pineda (2018). They realized that the experimental $E_{HFS}^p(eH)$ is known to 13 figures and the bulk of the μH calculation just scales with the m_μ/m_e mass ratio, known to 10 figures. Just need to calculate the smaller pieces that done scale this way, leader to a final result with smaller overall uncertainty. Will see again soon.
- Want to proceed to discuss subtracted or unsubtracted dispersion relation for $H_{1,2}$

Unsubtracted dispersion relation (DR)?

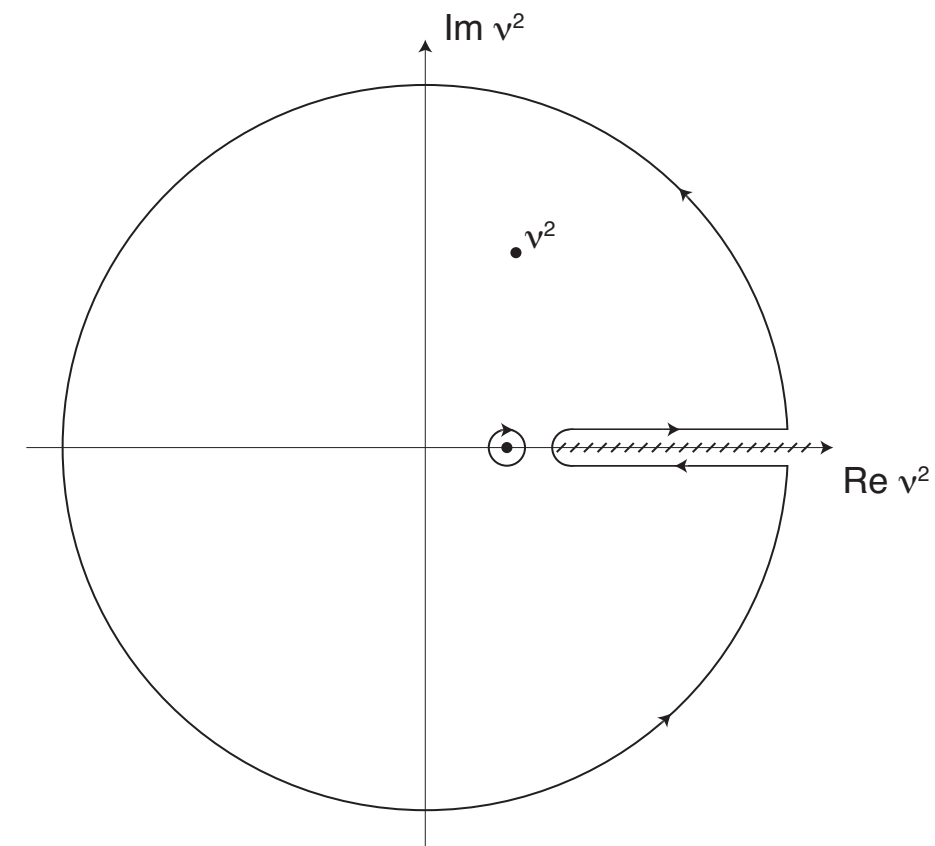
- Was once openly discussed (< 2006, say), now seems generally thought o.k.
- DR comes from Cauchy integral formula applied with some contour (closed integration path)

$$H_1(\nu, Q^2) = \frac{1}{\pi} \oint \frac{\text{Im } H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2$$



- (DR in ν (or ν^2) with Q^2 fixed)

Dispersion relation



- Work into

$$H_1(\nu, Q^2) = \frac{\text{Res } H_1(\nu, Q^2) \big|_{el}}{\nu_{el}^2 - \nu^2} + \frac{1}{\pi} \int_{cut} \frac{\text{Im } H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2 + \frac{1}{2\pi i} \int_{|\nu'|=\infty} \frac{H_1(\nu', Q^2)}{\nu'^2 - \nu^2} d\nu'^2$$

- Drop the $|\nu| = \infty$ term. O.k. if H_1 falls at high ν .
- Can view as standard or as dramatic assumption.

H_1

- The elastic term can be worked out, sticking on-shell form factors at the γp vertices,

$$H_1^{el} = \frac{2m_p}{\pi} \left(\frac{Q^2 F_1(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2 \nu^2} - \frac{F_2^2(Q^2)}{4m_p^2} \right)$$

- The second term does not fall with ν at fixed Q^2 .
- Unsubtracted DR fails for H_1^{el} alone. Overall success requires exact cancelation between elastic and inelastic contributions.

- (In case of interest: $H_2^{el} = -\frac{2m_p}{\pi} \frac{m_p \nu F_2(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2 \nu^2} .$)

But then,

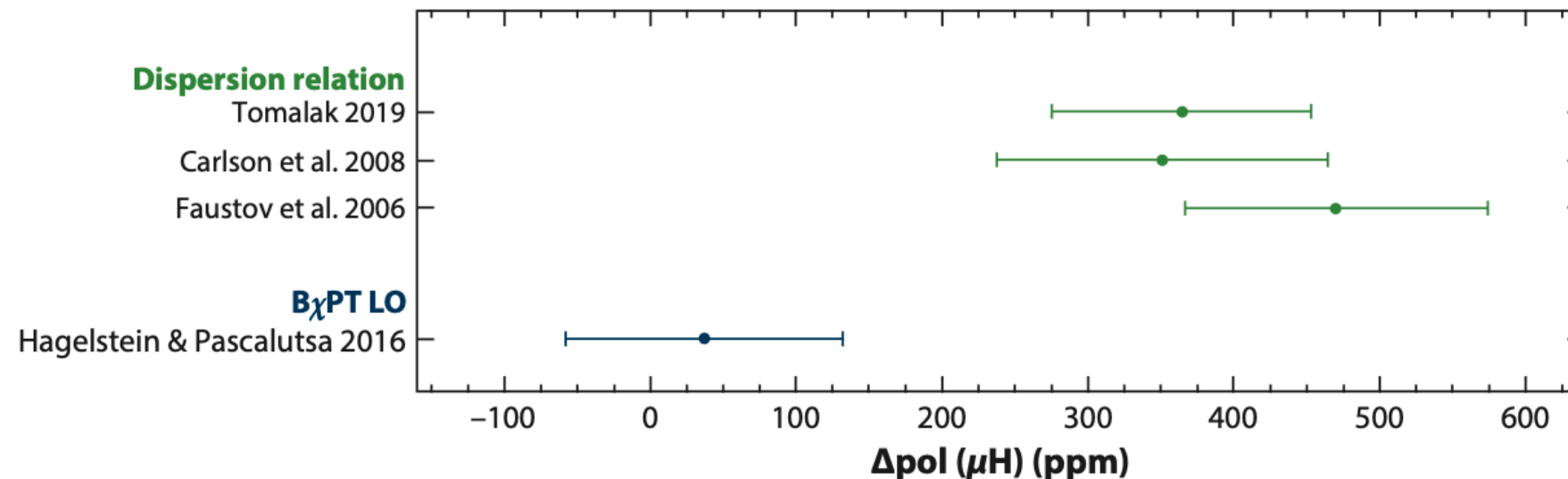
- Free quarks if there is at least one large momentum scale. So at high ν , Compton amplitude for proton should be sum of Compton amplitudes for free quarks, which have zero F_2 .
- Regge theory suggests H_1 must fall with ν . See Abarbanel and Nussinov (1967), who show $H_1 \sim \nu^{\alpha-1}$ with $\alpha < 1$.*
- Very similar DR derivation gives GDH sum rule, which is checked experimentally and works, within current experimental uncertainty.
- GDH sum rule also checked in LO and NLO order perturbation theory in QED. Appears to work.

Resolution?

- In modern times, authors who use experimental scattering data and DR to calculate the 2γ corrections assume an unsubtracted DR works for all of H_1 .
- Reevaluation always possible.
- Proceed to next topic, comparison of data driven evaluations of HFS to evaluations using B χ PT to obtain $H_{1,2}$.
- See if subtraction comments come into play.

Polarizability discrepancy

- Plot from Antognini, Hagelstein, Pascalutsa (2022), similar one in Hagelstein, Pascalutsa, Lensky (2022),



- Numbers explicit, $\Delta_{\text{pol}}(\text{Tomalak}) = 364(89) \text{ ppm}$
 $\Delta_{\text{pol}}(\text{H \& P}) = 29(90) \text{ ppm}$
Difference = 322 ppm
- Bad: polarizability corrections calculated in different ways do not agree.
- (Happens that different authors results for total HFS are in decent agreement, because Zemach terms also different. That “agreement” seems like luck. Want individual pieces to agree.)

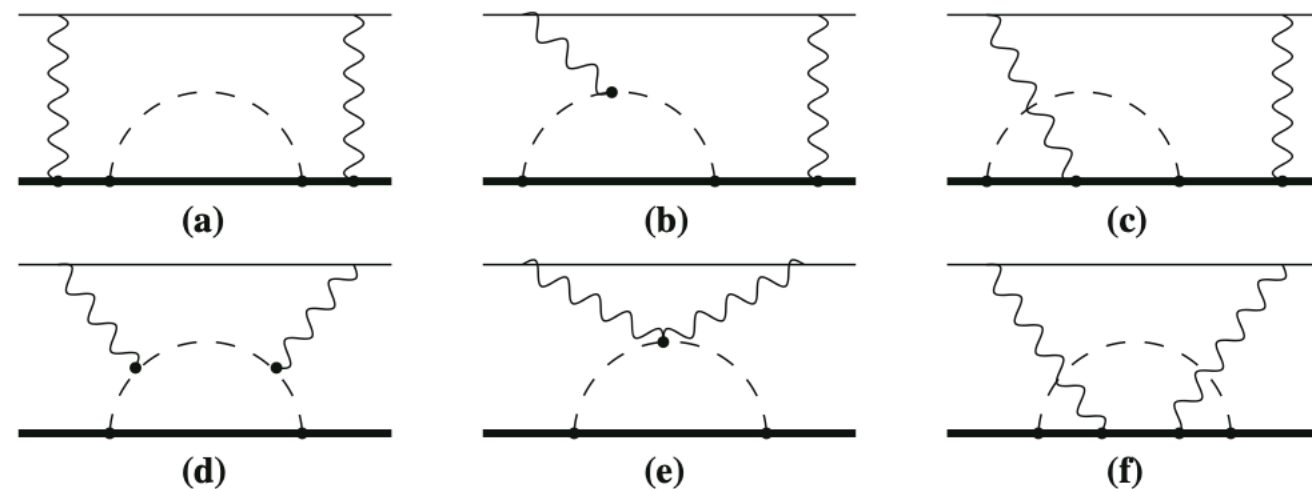
Side note: how good need we be?

- New measurements of HFS in μH in 1S state are planned.
- May measure to 0.1 ppm (as fraction of Fermi energy).
But need theory prediction to help determine starting point of laser frequency scan.
- From 2018 conference at MITP (Mainz), want theory prediction to 25 ppm or better. Better is what we should look for.
- Believe state of art for HFS in 1S μH is from Antognini, Hagelstein, Pascalutsa (2022),
$$E_{\text{HFS}}^{1\text{S}} = 182.634(8) \text{ meV}$$

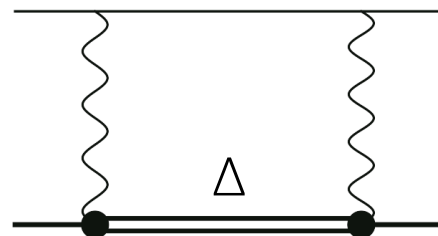
or 44 ppm.

Application of $B\chi PT$

- Using chiral perturbation theory, one can calculate beyond the elastic case diagrams like



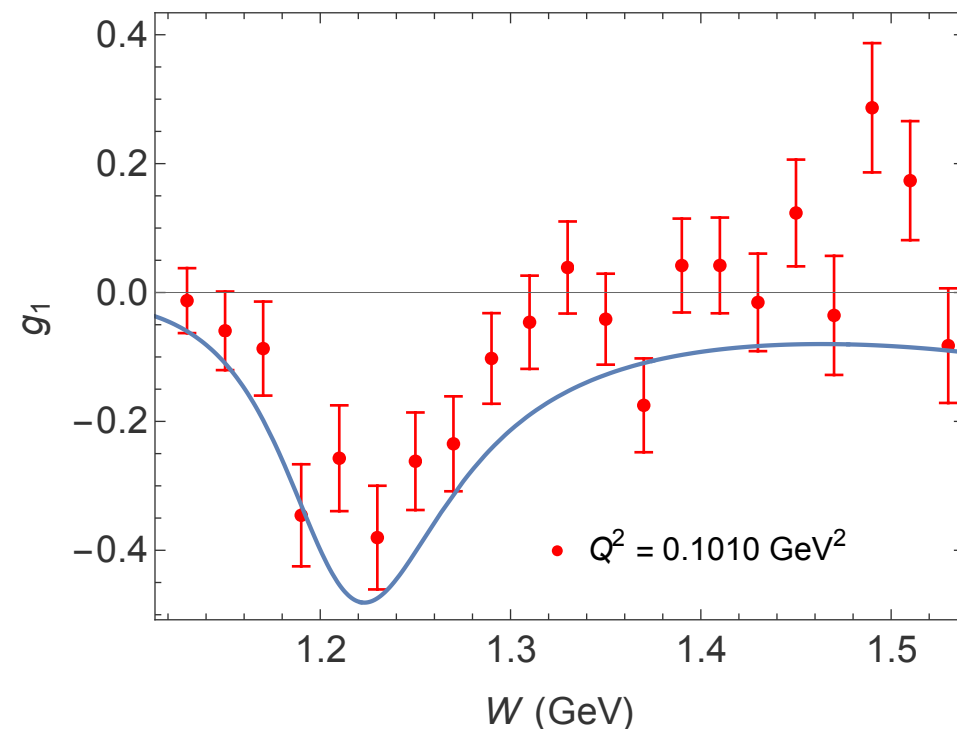
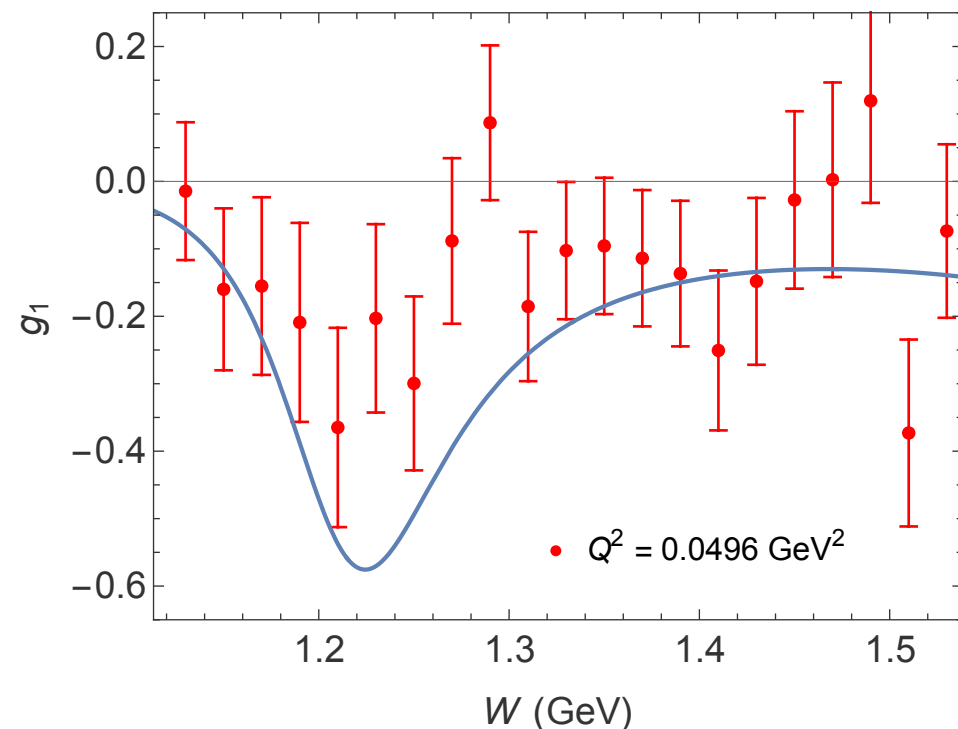
- Or diagrams where there is a Δ -baryon on the hadronic leg,



- These can be used to calculate $H_{1,2}$, at low Q^2 and CM energy W not too far from threshold. Also can get $\gamma^* N \rightarrow \pi N$ or $\gamma^* N \rightarrow \Delta$ from them obtain $g_{1,2}$ at similarly low kinematics.

g_1 comparison

- Compare g_1 from B χ PT (blue lines) to actual JLab data



- Plots are “unofficial”: Made by me* and involve spreading Δ pole out using Lorentzian of same total area.

*With greatest thanks to Pascalutsa and Hagelstein for providing code for their $\gamma N \rightarrow \pi N$

- O.k. This won't explain difference in Δ_{pol} results.

Non-pole terms

- Non-pole means ν independent terms in $H_{1,2}$.
- Recall elastic $H_1^{el} = \frac{2m_p}{\pi} \left(\frac{Q^2 F_1(Q^2) G_M(Q^2)}{(Q^2 - i\epsilon)^2 - 4m_p^2 \nu^2} - \frac{F_2^2(Q^2)}{4m_p^2} \right)$.
- The B χ PT results for H_1 with π - N and Δ intermediate states also have non-pole terms.
- To calculate energies for the non-pole terms, cannot use the DR (at least not un-subtracted ones), but can use the expressions on slide 7, which were before any Cauchy trickery was used

Pole and non-pole

- One part: The Δ contribution to μH HFS for 2S state*

$$\begin{aligned} E_{pol}^{HFS} &= -40.69 \mu\text{eV} && \text{pole} \\ &= 39.54 \mu\text{eV} && \text{non-pole} \\ &= -1.15 \mu\text{eV} && \text{total} \end{aligned}$$

- Lot of cancellation.
- But from asymptotic freedom, or from Regge analysis, or from success of DHG sum rule, expect zero non-pole term. Totality, from elastic and resonances and inelastic terms, needs to add to zero for the ν independent terms.
- Something to talk about.

One point

- How should one deal with non-zero non-pole terms that result from partial information, when one knows that the non-pole terms are zero when one has complete information?

Δ_{pol} with newest $g_{1,2}$

- Defer to David Ruth (next talk).
- Except for comment on handling regions outside the data range.
- Mostly, because of the kinematic factors, the need is for data at low Q^2 and low ν (or W near threshold), and this is where the data is.
- Again, mostly, where there is no data and we use models or interpolations, the contributions to $\Delta_{1,2}$ are not great and the accruing uncertainty is not great.

Δ_{pol} with newest $g_{1,2}$

- An exception may be the very low Q^2 region, where there is no data. For the 2003 data, this was $Q^2 < 0.0492 \text{ GeV}^2$.

- What we did: reminder

$$\Delta_1 = \frac{9}{4} \int_0^\infty \frac{dQ^2}{Q^2} \left\{ F_2^2(Q^2) + \frac{8m_p^2}{Q^2} B_1(Q^2) \right\}$$

with
$$B_1(Q^2) = \frac{4}{9} \int_0^{x_{th}} dx \beta_1(\tau) g_1(x, Q^2) \quad .$$

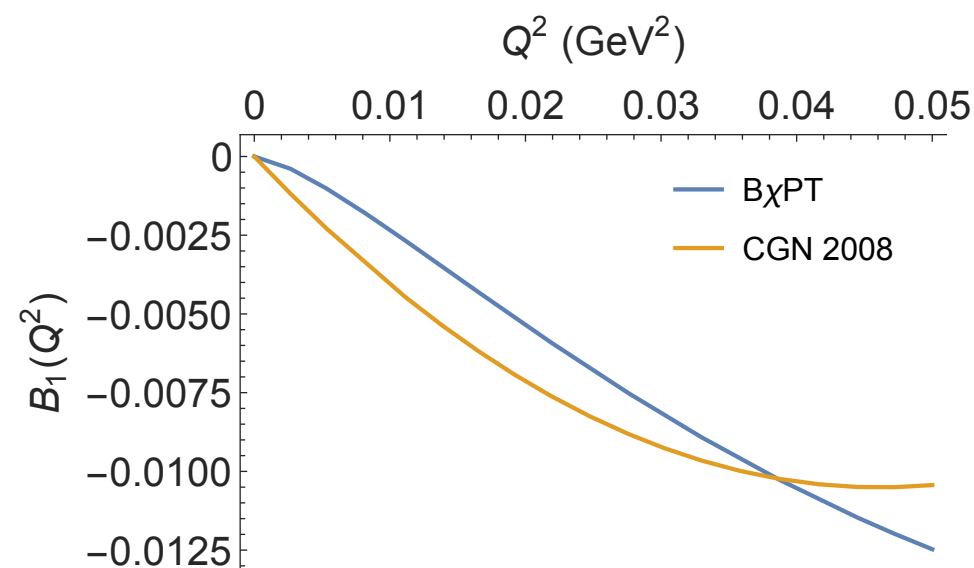
- For very low Q^2 we used

$$B_1(Q^2) = -\frac{\kappa_p^2}{8m_p^2} Q^2 + c_{1B} Q^4 = -\frac{\kappa_p^2}{8m_p^2} Q^2 + 4.94 Q^4 / \text{GeV}^4$$

got by fitting to data $Q^2 < 0.3 \text{ GeV}^2$

Δ_{pol} with newest $g_{1,2}$

- The region $Q^2 < 0.0492 \text{ GeV}^2$ contributed about 15% of Δ_1 and (by our estimate) 30% of the error.
- Now can check using $B\chi PT$ to do low Q^2 integrals for B_1 . Get



- Believe the below-available-data low Q^2 contributions can be included accurately.

Summary

- Dispersive calculation, assuming no subtractions are needed, is complete, well defined, and unambiguous.
- Gets value of HFS using spin-dependent *ep* scattering data as input.
- Really pleased about new data.
- EFT calculations should also be totally fine, but there is a “tension” that requires resolution.