Hadronic contributions to precision quantities using Sum Rules vs. EFT vs. LQCD

Vladimir Pascalutsa

Institute for Nuclear Physics
University of Mainz, Germany

Volodymir Biloshytskyi
Franziska Hagelstein *et al.*
Vadim Lensky
Marc Vanderhaeghen *et al.*
LQCD: Jeremy Green, Harvey Meyer *et al.*

Constantia Alexandrou, Carl Carlson,
Judith McGovern, Kostas Orginos, ...

**@ 2nd Low-Q Workshop,
Crete, 15—20 May**

Theoretical approaches to low-energy QCD

Lattice QCD

EFTs

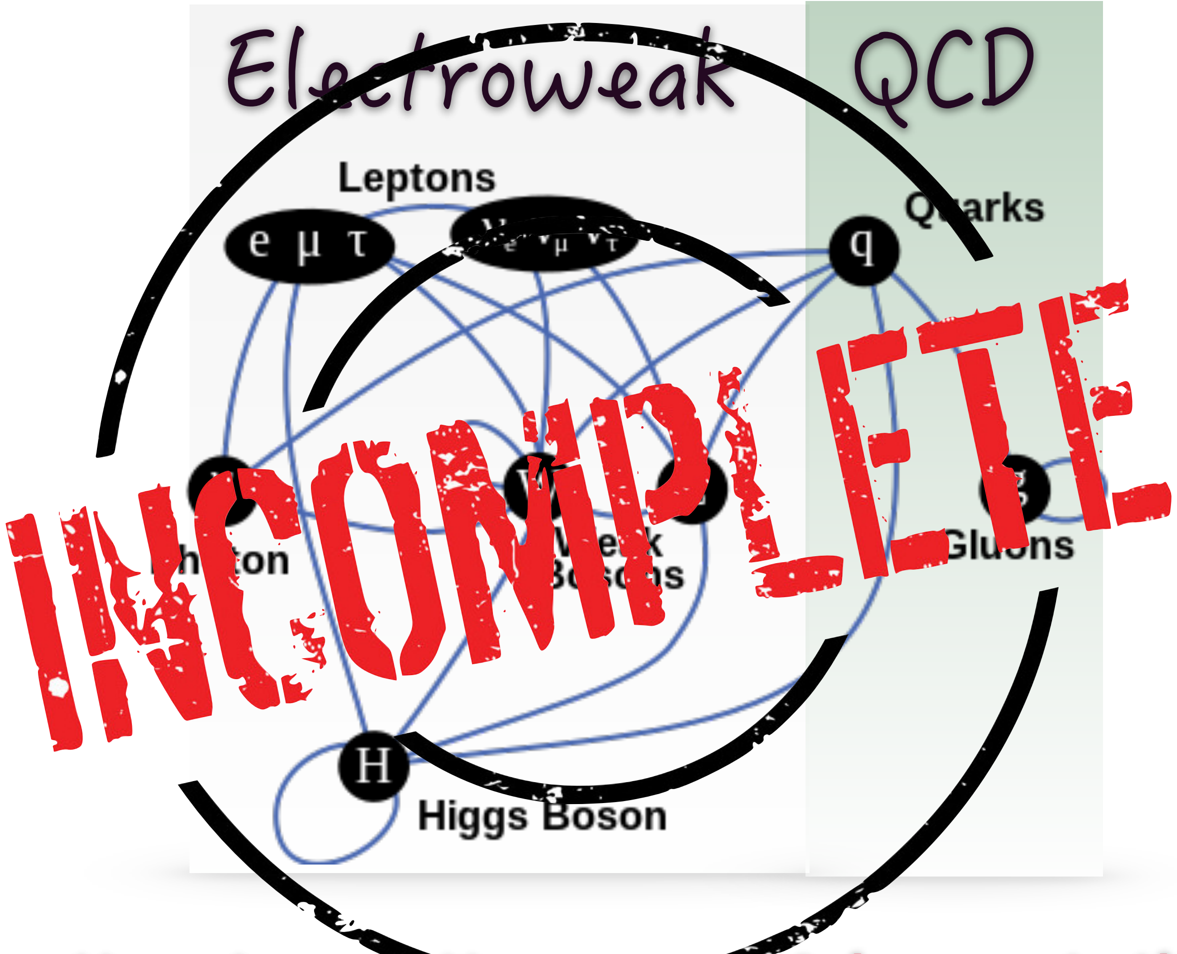
ChPT

SCET

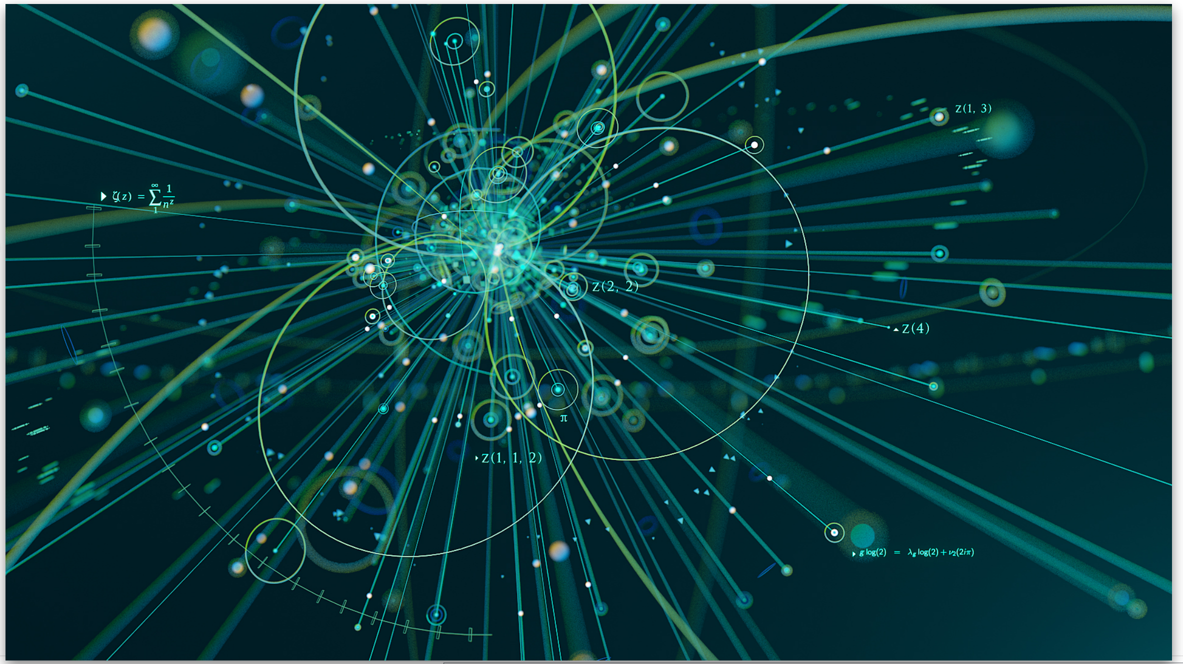
…

Dispersive data-driven

Standard Model



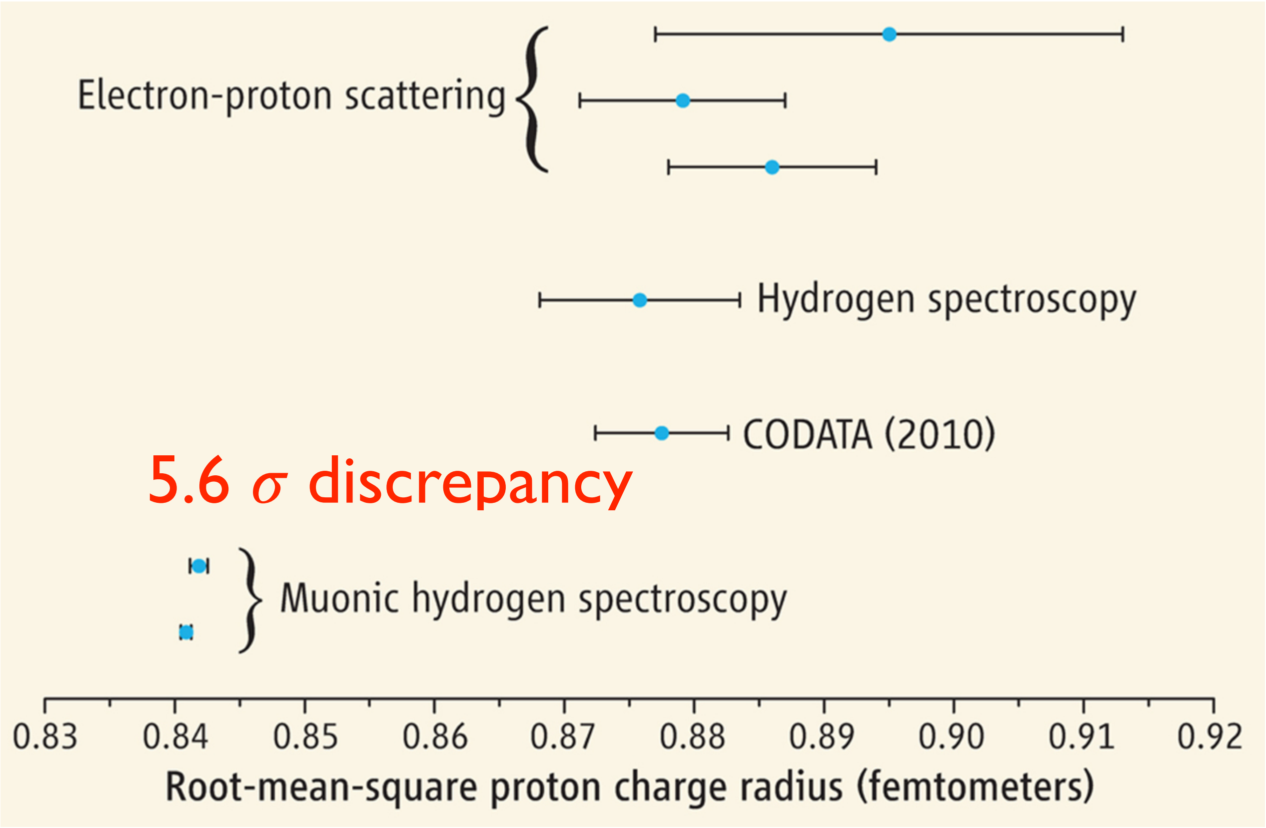
presently the best theory of (nearly!) everything



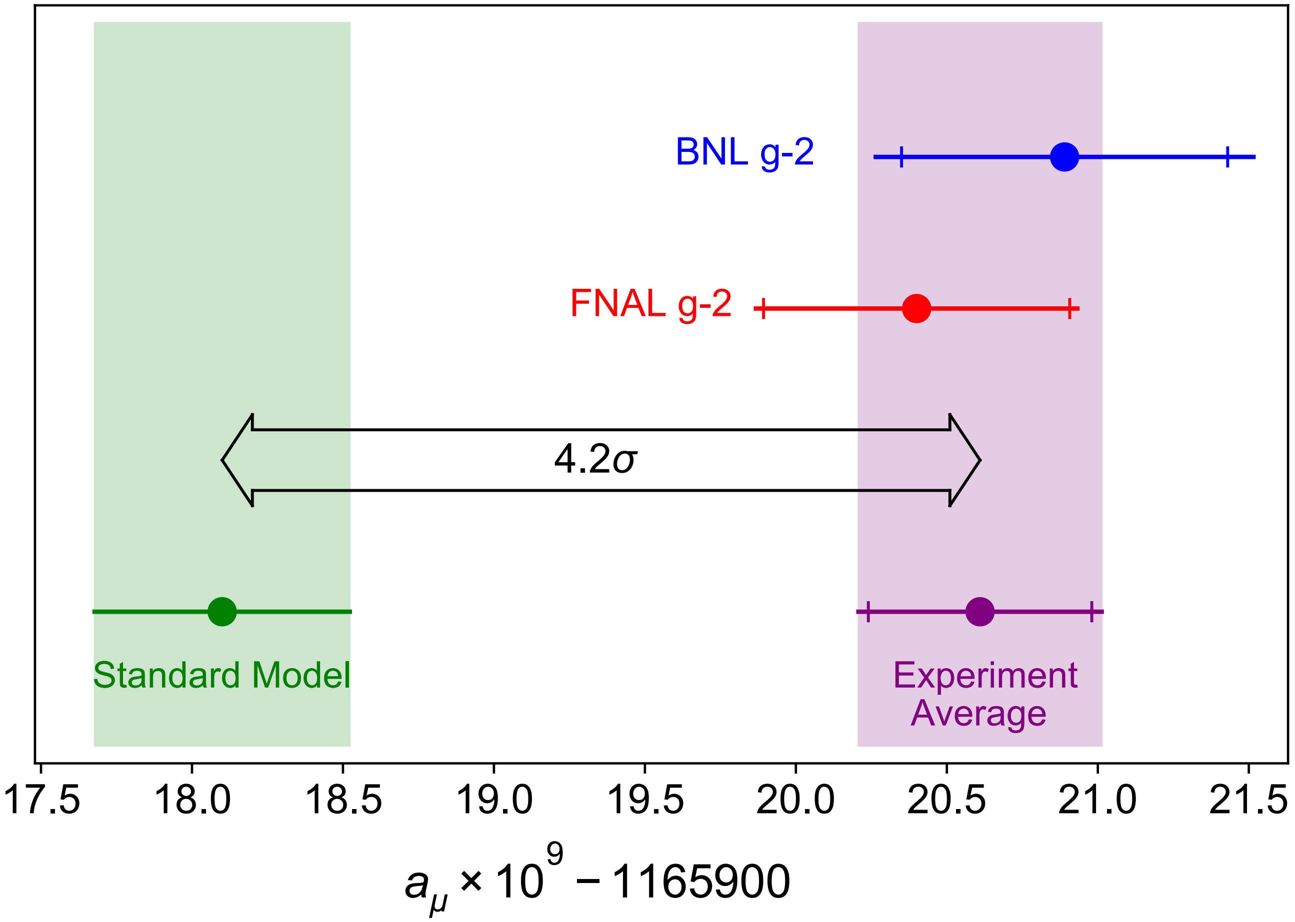
PRECISION SEARCHES FOR NEW PHYSICS

Muon anomalies

Proton radius



Muon g-2



NOW: from Puzzle to Precision

Chapter 1

Sum rules

(2003 — ...)

Kramers-Kronig type of relations

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Physics Reports 378 (2003) 99–205

PHYSICS REPORTS

www.elsevier.com/locate/physrep

ELSEVIER

Dispersion relations in real and virtual Compton scattering

D. Drechsel[a], B. Pasquini[b,c,d], M. Vanderhaeghen[a,*]

[a] *Institut für Kernphysik, Johannes Gutenberg-Universität, D-55099 Mainz, Germany*
[b] *ECT*-European Centre for Theoretical Studies in Nuclear Physics and Related Areas, I-38050 Villazzano (Trento), Italy*
[c] *INFN, Trento, Italy*
[d] *Dipartimento di Fisica, Università degli Studi di Trento, I-38050 Povo, Trento, Italy*

Accepted 10 December 2002
editor: W. Weise

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Physics Letters B 600 (2004) 239–247

PHYSICS LETTERS B

www.elsevier.com/locate/physletb

ELSEVIER

A derivative of the Gerasimov–Drell–Hearn sum rule

Vladimir Pascalutsa [a,b], Barry R. Holstein [a,c], Marc Vanderhaeghen [a,b]

[a] *Theory Group, JLab, 12000 Jefferson Ave, Newport News, VA 23606, USA*
[b] *Department of Physics, College of William & Mary, Williamsburg, VA 23188, USA*
[c] *Department of Physics-LGRT, University of Massachusetts, Amherst, MA 01003, USA*

Received 29 July 2004; received in revised form 3 September 2004; accepted 3 September 2004

Available online 11 September 2004

Editor: H. Georgi

From: Carl E Carlson <carlson@jlab.org>

Subject: Super Gran Père Andreas

Date: 9. June 2014 at 17:08:49 CEST

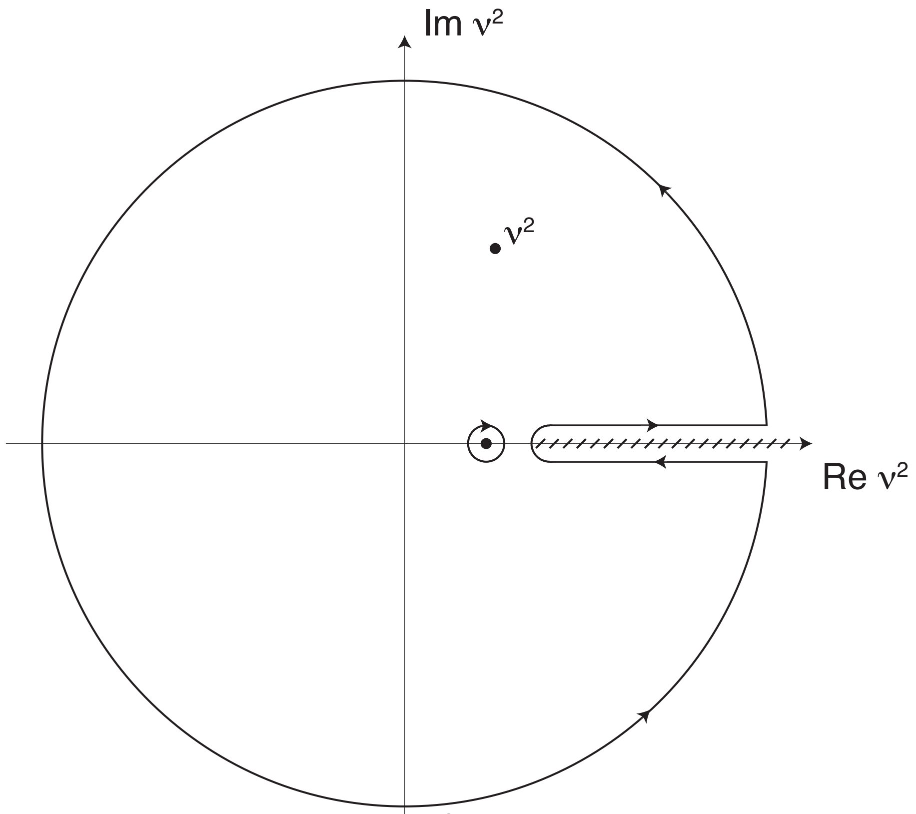
To: Vladimir Pascalutsa <vladipas@kph.uni-mainz.de>

Selected Ph. D. descendents of Andreas von Ettingshausen

0. Andreas von Ettingshausen (Wien,1817)	0. Andreas von Ettingshausen (Wien,1817)
1. Jozef Stefan (Wien, 1858)	1. Francesco Rossetti (Wien, 1857)
2. Ludwig Boltzmann (Wien, 1866)	2. Andrea Naccari (Padova, 1862)
3. Paul Ehrenfest (Wien, 1904)	3. Angelo Batelli (Torino, 1884)
4. Hendrik Kramers (Leiden, 1919)	4. Luigi Puccianti (Pisa, 1898)
5. Nicolaas van Kampen (Leiden, 1952)	5. Enrico Fermi (Pisa, 1922)
6. John Tjon (Utrecht, 1964)	6. Tsung-Dao Lee (Chicago, 1946)
7. Vladimir Pascalutsa (Utrecht, 1998)	7. Carl-Edwin Carlson (Columbia, 1968)

From Carl’s talk:

Dispersion relation



- Work into

$$H_1(\nu,Q^2) = \frac{\text{Res }H_1(\nu,Q^2)\Big|_{el}}{\nu_{el}^2-\nu^2} + \frac{1}{\pi}\int_{cut}\frac{\text{Im}\,H_1(\nu',Q^2)}{\nu'^2-\nu^2}d\nu'^2 + \frac{1}{2\pi i}\int_{|\nu'|=\infty}\frac{H_1(\nu',Q^2)}{\nu'^2-\nu^2}d\nu'^2$$

- Drop the $|\nu| = \infty$ term. O.k. if H_1 falls at high ν.
- Can view as standard or as dramatic assumption.

From Volodymir’s talk:

In these cases the dispersion relation for T_L must be modified as follows:
[Sugawara and Kanazawa, PhysRev (1961)]

$$T_L(\nu,Q^2) - T_L(\infty,Q^2) = \frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu'\,\nu'^2\,\frac{\sigma_L(\nu',Q^2)}{\nu'^2-\nu^2}$$

Bernabeu-Tarrach and a sum rule for the subtraction function

[arXiv:2305.08814]

$$\alpha_{E1} - \frac{\alpha_{\mathrm{em}}\varkappa^2}{4M^3} = \frac{1}{2\pi^2}\int_0^\infty \mathrm{d}\nu \left[\frac{\sigma_L(\nu,Q^2)}{Q^2}\right]_{Q^2\to 0}$$

[Bernabéu and Tarrach, PLB (1975)]

$\varkappa$ is the anomalous magnetic moment of the nucleon.

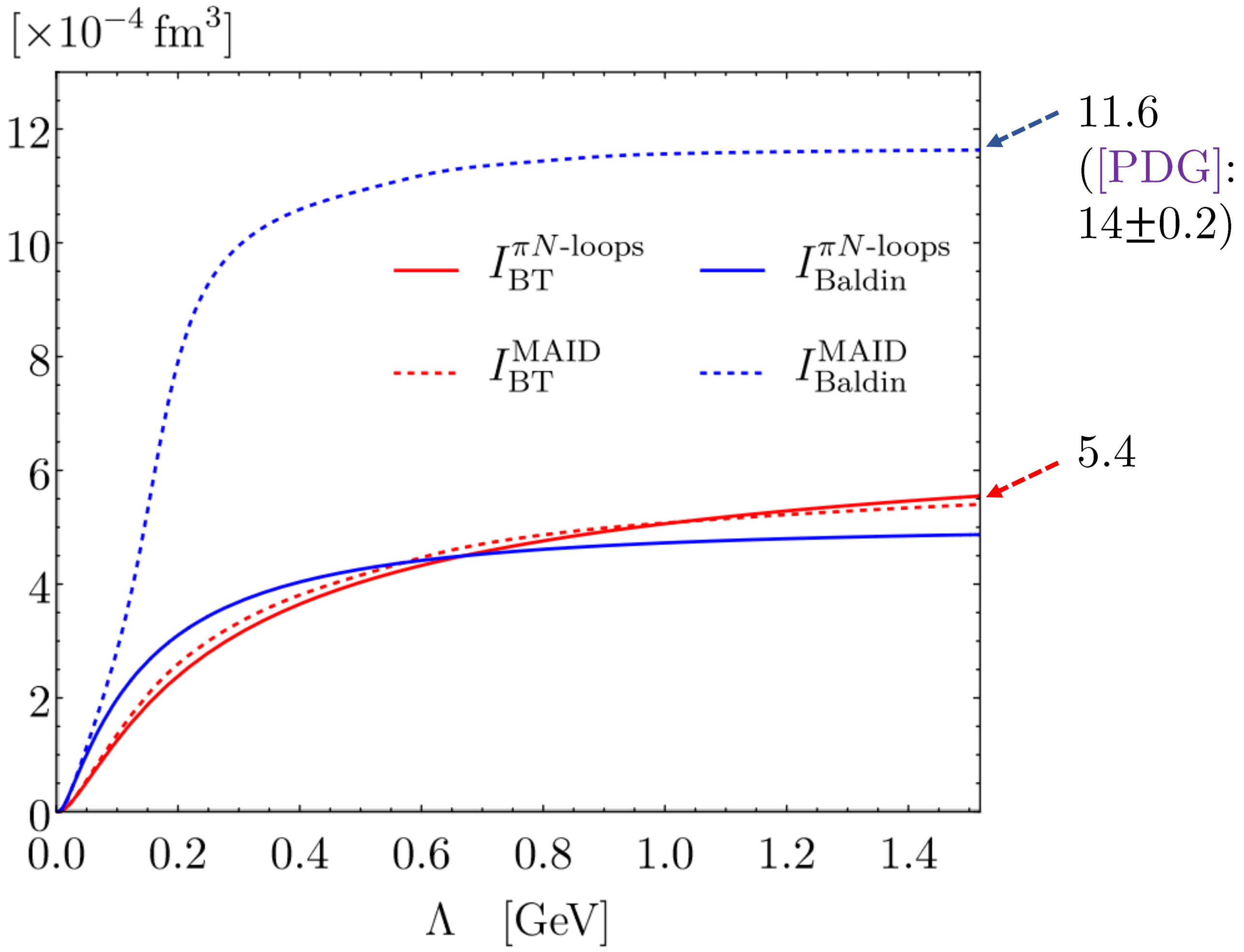
$$T_1(0,Q^2) = \frac{2}{\pi}Q^2\int_{\nu_0}^\infty \frac{\mathrm{d}\nu}{\nu^2+Q^2}\left[\sigma_T - \frac{\nu^2}{Q^2}\sigma_L\right](\nu,Q^2)$$

- The sum rules are invalid only if the integral diverges!

“Fixed-poles” is a conspiracy theory

Pion-production channel contribution to BT and Baldin sum rules

Saturation of the sum rule integral



$$I_{\mathrm{BT}}(\Lambda) = \frac{1}{2\pi^2}\int_{\nu_0}^{\Lambda} \mathrm{d}\nu \left[\frac{\sigma_L(\nu,Q^2)}{Q^2}\right]_{Q^2\to 0}$$

$$I_{\mathrm{Baldin}}(\Lambda) = \frac{1}{2\pi^2}\int_{\nu_0}^{\Lambda} \mathrm{d}\nu \frac{\sigma_T(\nu)}{\nu^2}.$$

source	$\boldsymbol{\alpha_{E1}}$ **[× 10⁻⁴ fm³]**
I_{BT}(MAID) extrapolated	5.4 ≃7
Kappa term	0.5
resonances*	0.5-1*
total (w/o Regge region)	8-8.5*
[PDG]	11.2±0.4

*Currently, we have no parametrization of the existing data, which has a stable behavior within the limit $Q^2 \to 0$

We need to develop parametrizations of inclusive longitudinal cross-section (akin to Bosted-Christy) with good low-Q limit!

Chapter 2

Effective Field Theory

(2001 — ...)

Chiral EFT (χEFT)

Steven Weinberg, ***Phenomenological Lagrangians,*** **Physica A (1979)**

Origin of EFTs

'Chiral' and 'Perturbative' go together:

pions are Goldstone bosons of spontaneous Chiral Sym. Breaking,
interaction goes with powers of energy, vanishes at E=0
perturbative expansion in energy and pion mass (but not a series expansion!)

$$\frac{p^\mu}{4\pi f_\pi}, \quad \text{or} \quad \frac{|\vec{p}|}{4\pi f_\pi}, \frac{m_\pi}{4\pi f_\pi}$$

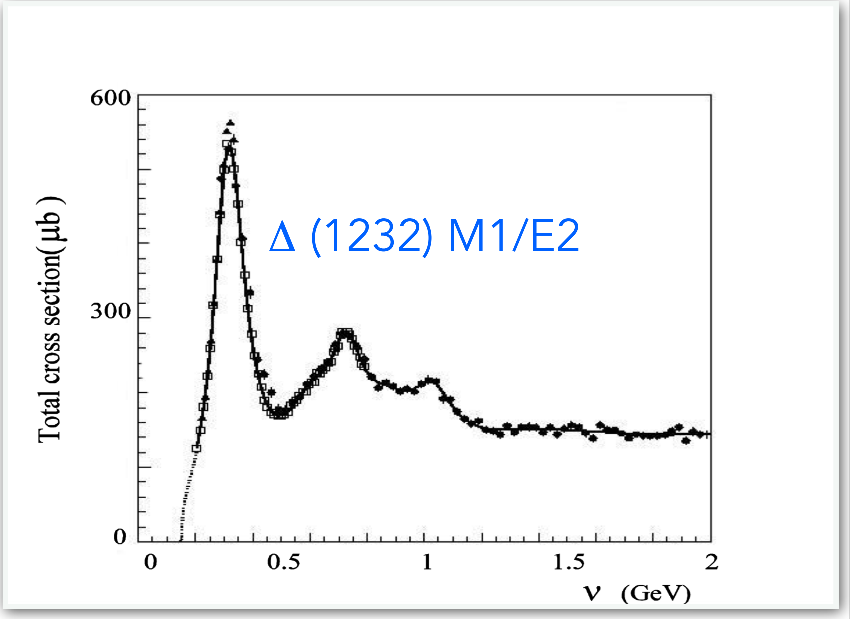
Most general Lagrangian (allowed by symmetries), hence infinitely many constants (LECs) parametrising the short-range physics.

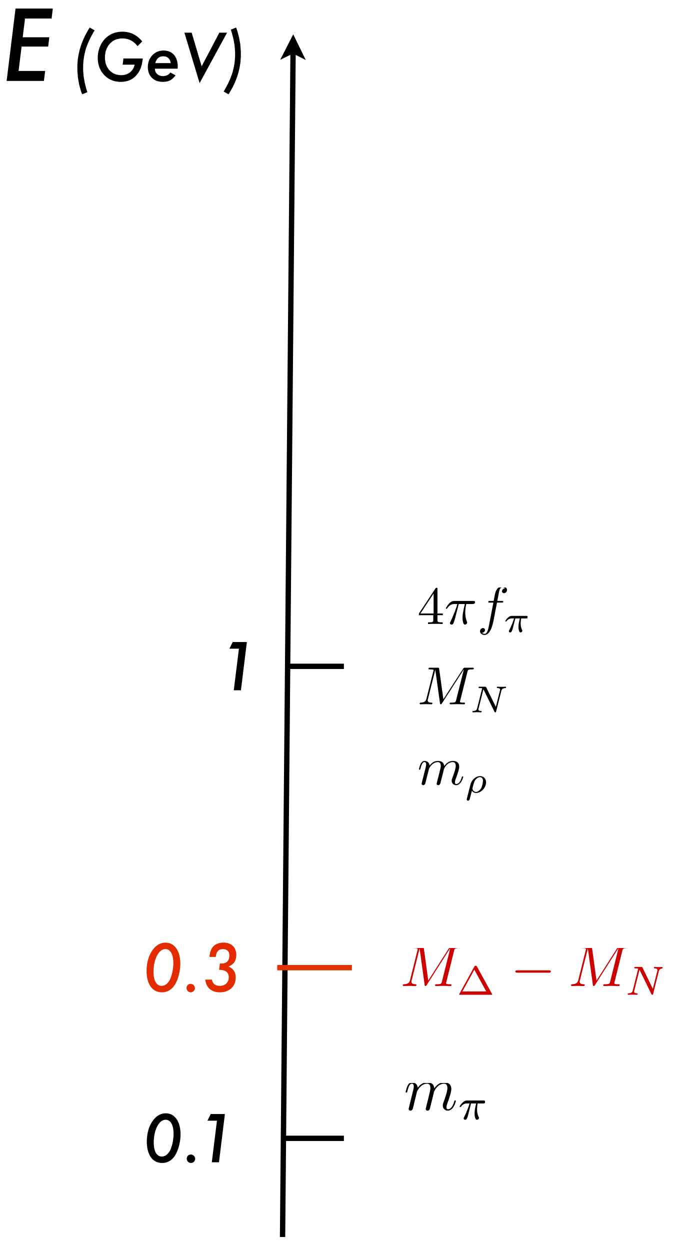
Predictive, provided: *Hierarchy of scales and Naturalness*

Baryon χPT

χPT + Nucleons + Delta(1232)

Jenkins & Manohar PLB (1991)
Hemmert, Holstein & Kambor JPhys G (1998)
VP & Phillips PRC (2003)



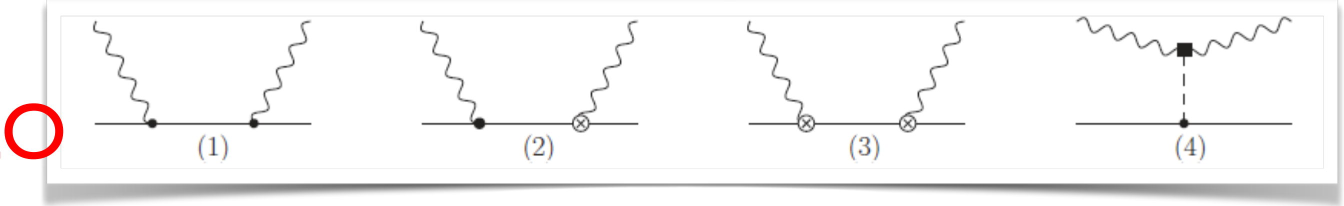


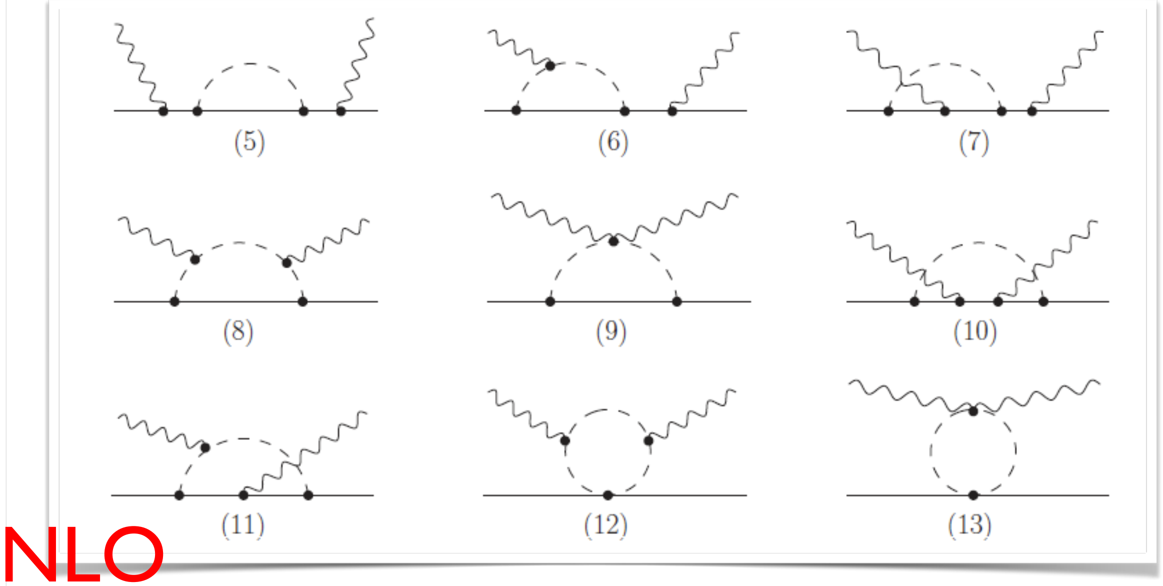
- The 1st nucleon excitation — Delta(1232) is within reach of chiral perturbation theory (293 MeV excitation energy is a light scale)
- Include into the chiral effective Lagrangian as explicit dof
- Power-counting for Delta contributions (SSE or ``delta-counting”) depends on what chiral order is assigned to the excitation scale.

→ McGovern

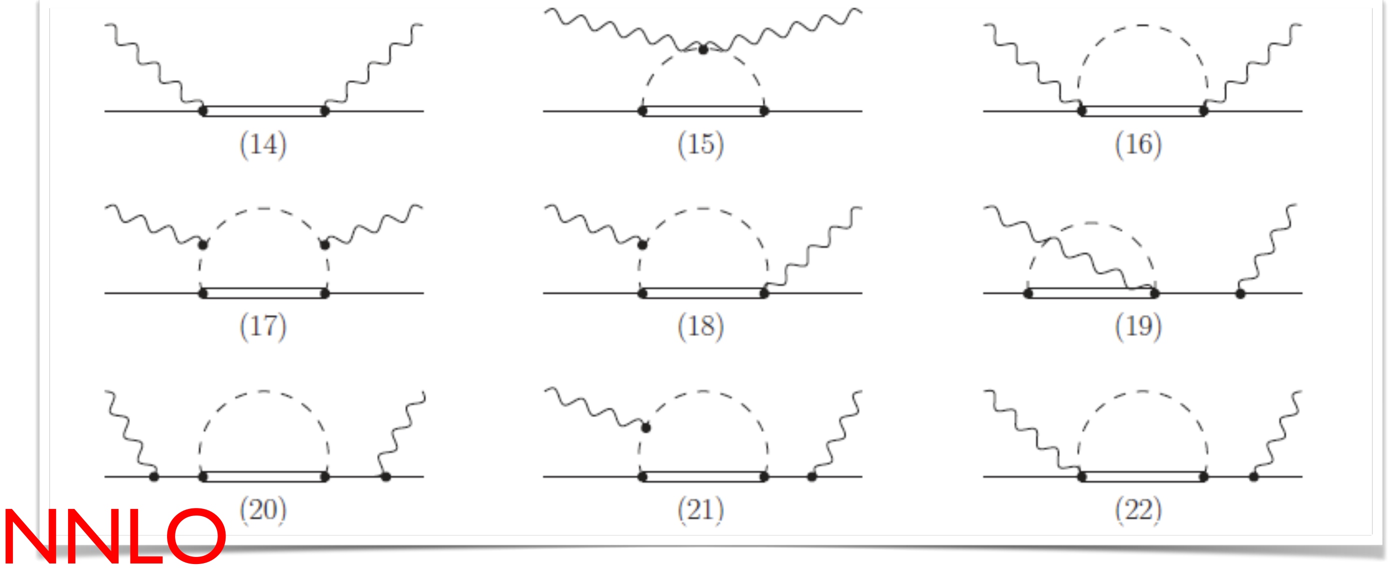
BχPT of (Real) Compton scattering on the nucleon

LO





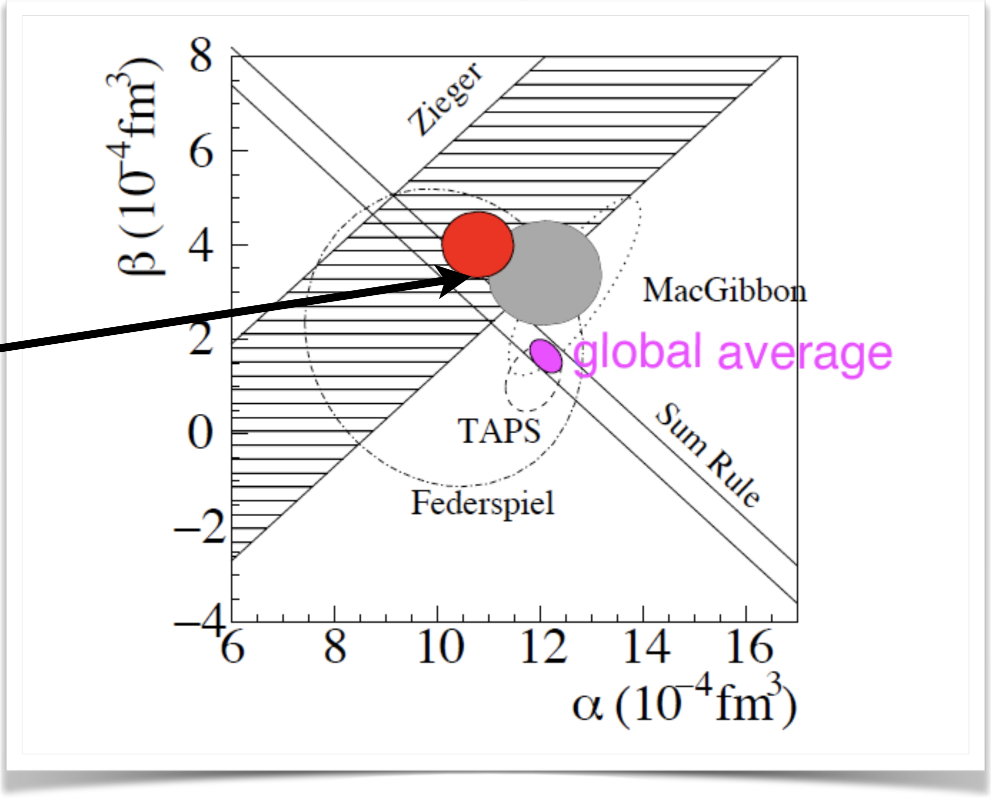
NLO



NNLO

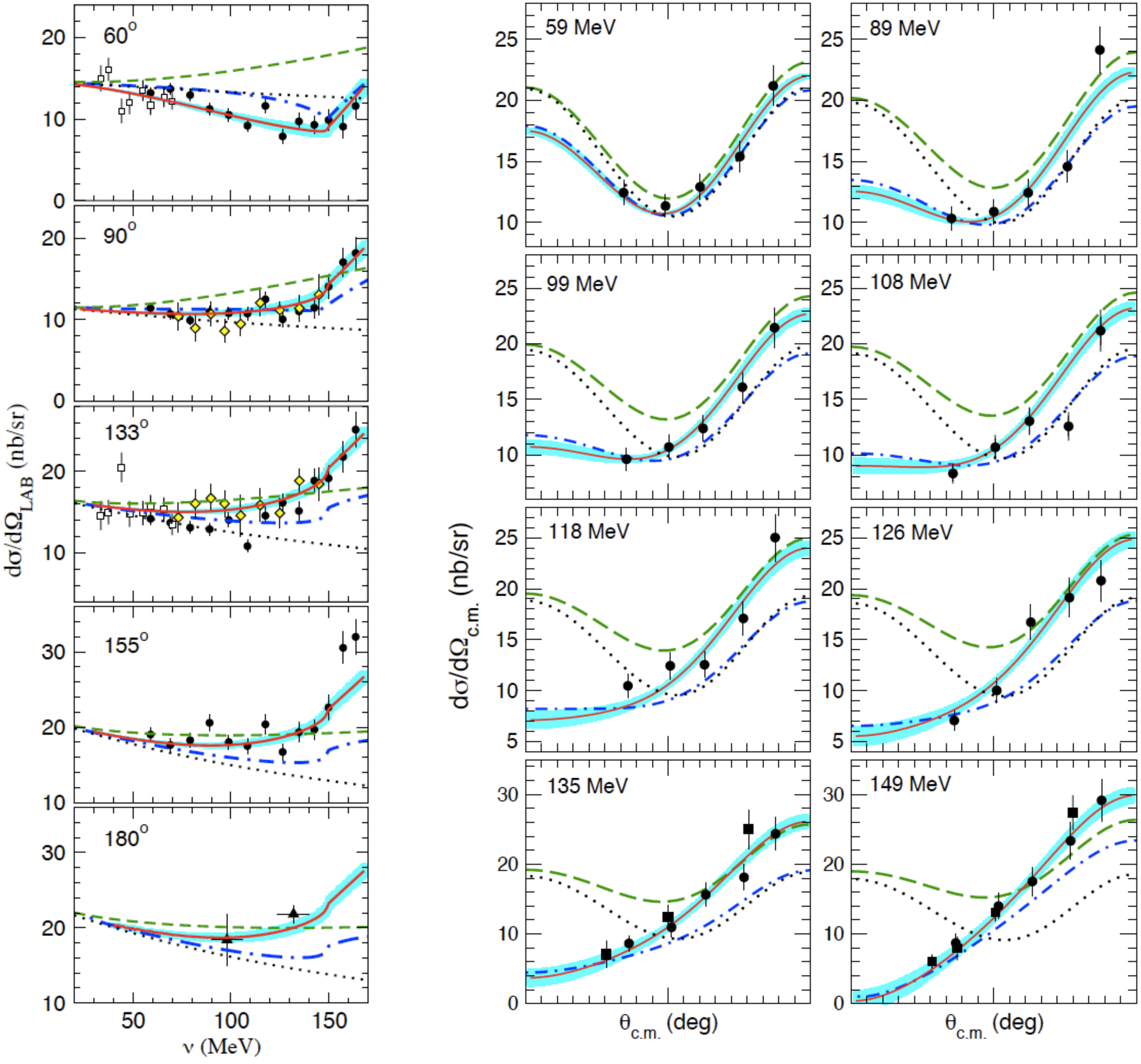
$\mathcal{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV$\cdot$fm
$\mathcal{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathcal{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathcal{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$

size of the red blob



Lensky & VP, EPJC (2010)
Lensky, McGovern & VP, EPJC (2015)

Unpolarized cross sections



Data points:
MAMI/TAPS (2001)
SAL (1993)
Illinois (1991)

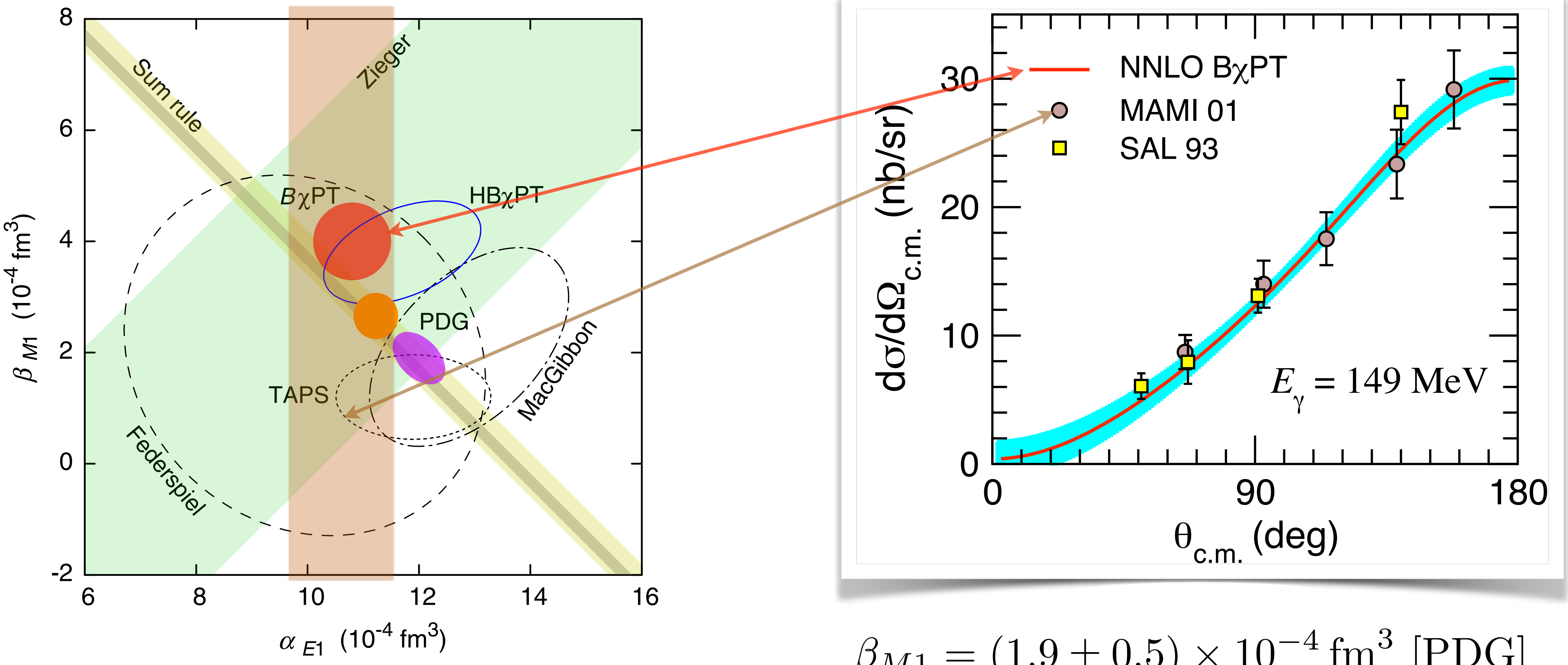
Curves:

- …………… Klein-Nishina
- – – – – Born + WZW
- – · – · – + p-qube
- ━━ Total NNLO

Lensky & V.P., EPJC (2010)

Proton polarizabilities

Dream of BT sum rule



$$\beta_{M1} = (1.9 \pm 0.5) \times 10^{-4}\,\mathrm{fm}^3 \;\; [\mathrm{PDG}]$$

$$\beta_{M1} = (4.0 \pm 0.7) \times 10^{-4}\,\mathrm{fm}^3 \;\; [\mathrm{BChPT@NNLO}]$$

BChPT

`Lensky,`
`Pascalutsa(2010)`

HBChPT

`Griesshammer,`
`McGovern,`
`Phillips(2013)`

→ McGovern, Mornacci, Howell, Pedroni

(Straightforward) Extensions to

Virtual Compton scattering (VCS)

Lensky, VP & Vanderhaeghen, EPJC (2017)

→ Sparveris

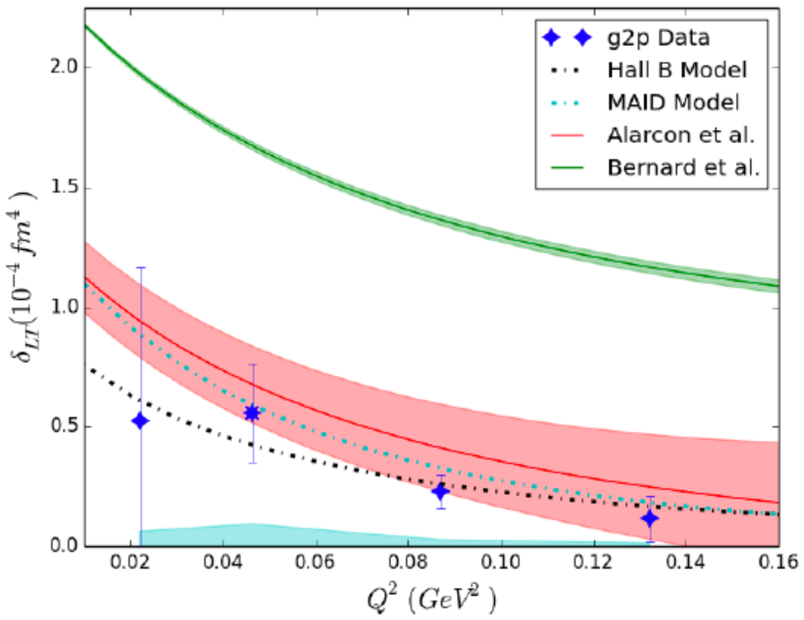
Forward doubly-virtual-CS (VVCS)

Lensky, Alarcon & VP, PRC (2014)

Alarcon, Hagelstein, Lensky & VP, PRD (2020)

→ Vanderhaeghen, Deur, Slifer, J-P Chen

$\delta_{LT}{}^{p}$ **JLab/Hall A** data



Ruth et al, Nat. Phys. (2022)

Two-photon exchange in the Lamb shift and hfs

Alarcon, Lensky & VP, EPJC (2014)

Hagelstein & VP, PoS CD15 (2016)

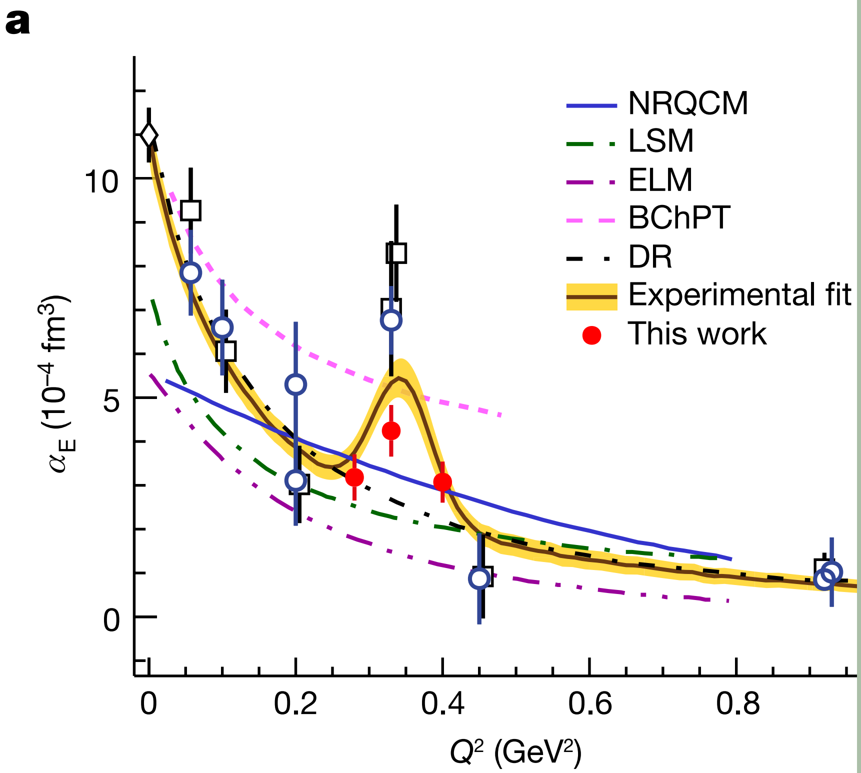
Hagelstein, PhD thesis (2017)

Hagelstein *et al., arXiv* (May 17, 2023)

→ Hagelstein

PROTON "STRECHINESS" ?

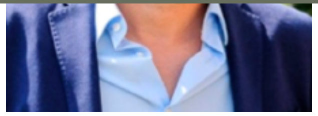
- Electric dipole polarizability extracted from virtual Compton scattering differs from theoretical expectation



tion,
is

NewScie

Judith McGover
most people took [th
really seriously, I thin
that it would go awa
quite honest, I think most
people will still assume that it
will go away."



skeptical as a theorist that this thing
is going to stay."

Chapter 3

Lattice QCD

(2005 — ...)

⟶ Alexandrou, Orginos, Constantinou, Frank Lee, Xu Feng

The end

Thank you !

(Now)

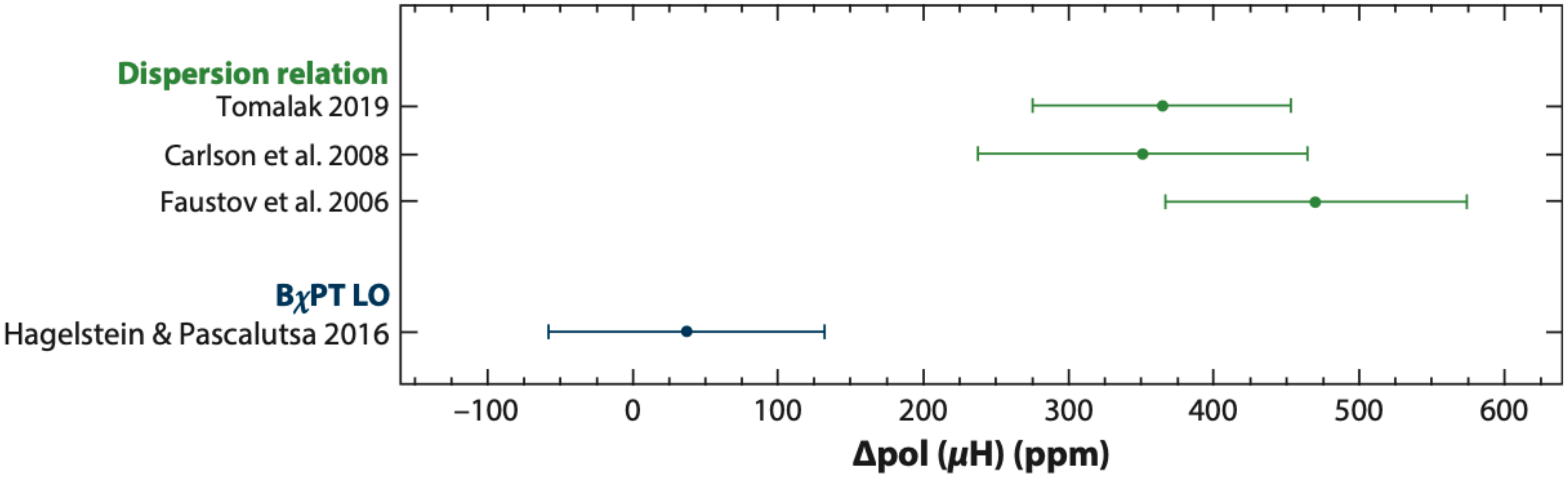
One more remark on Carl’s talk

(Now or Saturday?)

From Carl’s talk:

Polarizability discrepancy

- Plot from Antognini, Hagelstein, Pascalutsa (2022), similar one in Hagelstein, Pascalutsa, Lensky (2022),



- Numbers explicit, $\Delta_{\text{pol}}\,(\text{Tomalak}) = 364(89)$ ppm

$\Delta_{\text{pol}}\,(\text{H \& P}) \quad = \;\; 29(90)$ ppm

Difference $= 322$ ppm

- Bad: polarizability corrections calculated in different ways do not agree.

- (Happens that different authors results for total HFS are in decent agreement, because Zemach terms also different. That “agreement” seems like luck. Want individual pieces to agree.)

Best constraint for hfs in μH

Antognini, Hagelstein & VP, Ann. Rev. Nucl. Part. ***72*** *(2022)[arXiv:2205.10076]*

$$E^{\mathrm{Z+pol}}_{1S\text{-HFS}}(\mathrm{H}) = E_{\mathrm{F}}(\mathrm{H})\left[b_{1S}(\mathrm{H})\,\Delta_{\mathrm{Z}}(\mathrm{H}) + c_{1S}(\mathrm{H})\,\Delta_{\mathrm{pol}}(\mathrm{H})\right] = -54.900(71)\,\mathrm{kHz}$$

The coefficients b and c are well-known in both hydrogens.

The ***non-recoil*** *$O(\alpha^5)$ effects have simple scaling*

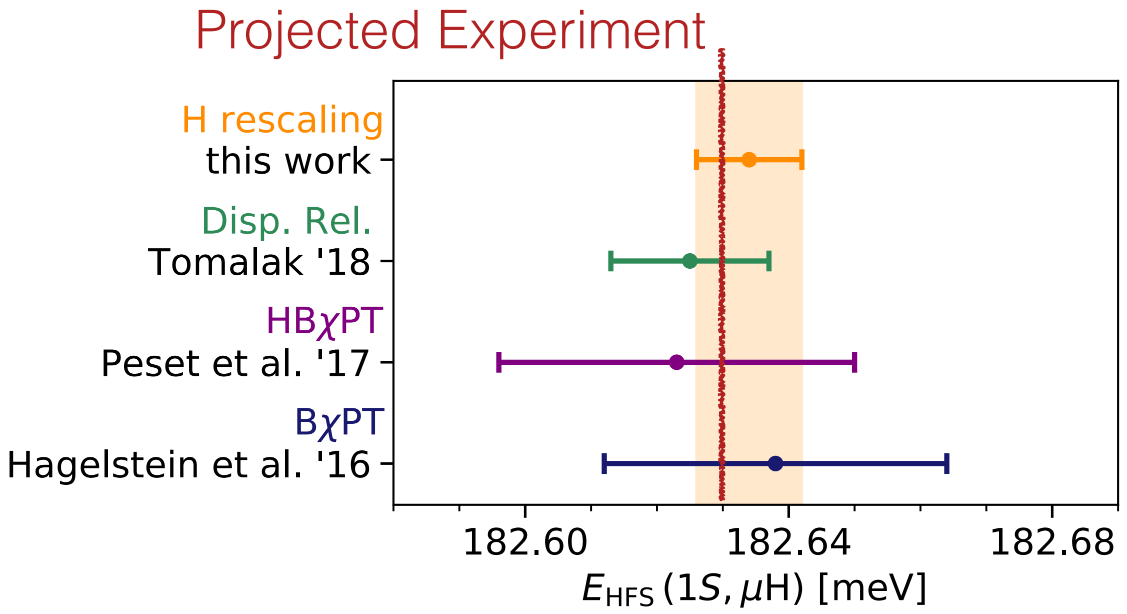
$$\frac{\Delta_i(\mathrm{H})}{m_r(\mathrm{H})} = \frac{\Delta_i(\mu\mathrm{H})}{m_r(\mu\mathrm{H})}, \qquad i = \mathrm{Z}, \mathrm{pol}.$$

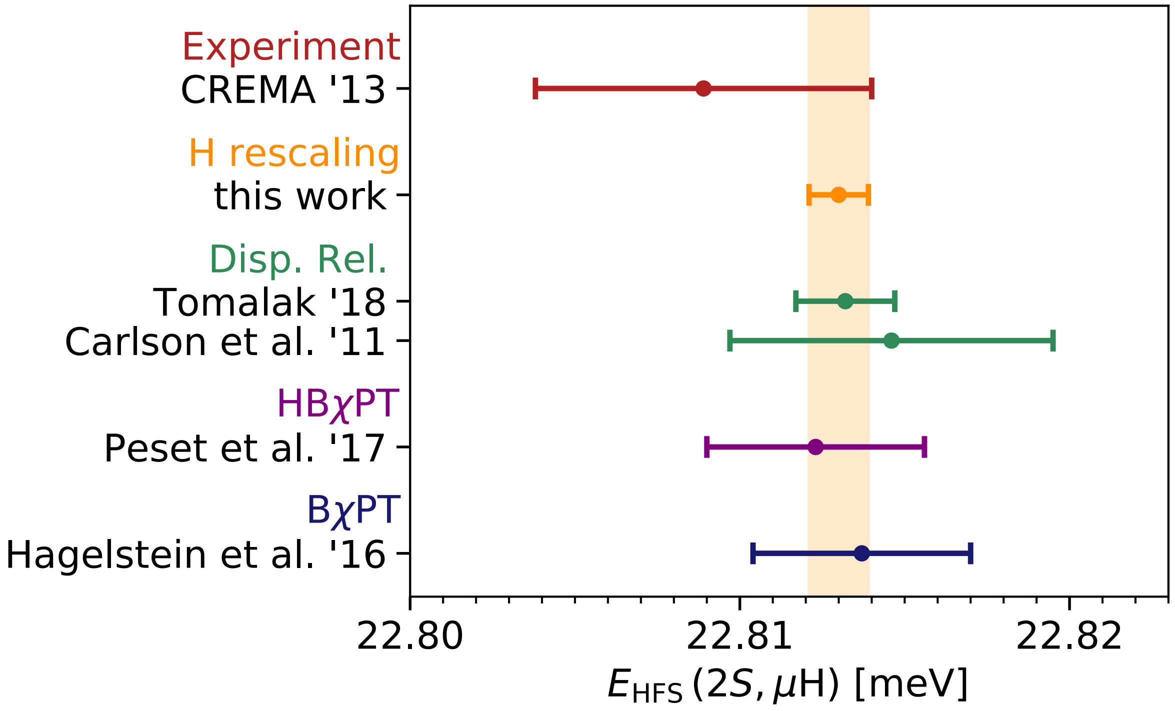
Hence,

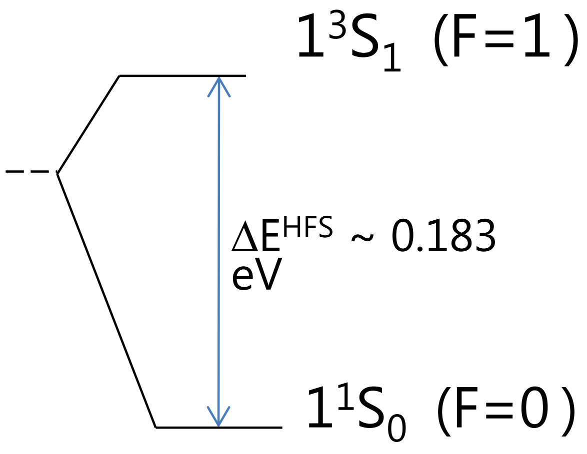
$$E^{\mathrm{Z+pol}}_{nS\text{-HFS}}(\mu\mathrm{H}) = \frac{E_{\mathrm{F}}(\mu\mathrm{H})\,m_r(\mu\mathrm{H})\,b_{nS}(\mu\mathrm{H})}{n^3 E_{\mathrm{F}}(\mathrm{H})\,m_r(\mathrm{H})\,b_{1S}(\mathrm{H})}\,E^{\mathrm{Z+pol}}_{1S\text{-HFS}}(\mathrm{H}) - \frac{E_{\mathrm{F}}(\mu\mathrm{H})}{n^3}\,\Delta_{\mathrm{pol}}(\mu\mathrm{H})\underbrace{\left[c_{1S}(\mathrm{H})\frac{b_{nS}(\mu\mathrm{H})}{b_{1S}(\mathrm{H})} - c_{nS}(\mu\mathrm{H})\right]}_{\substack{=-6\times 10^{-5}\text{ for } n=1\\ =-5\times 10^{-5}\text{ for } n=2}}$$

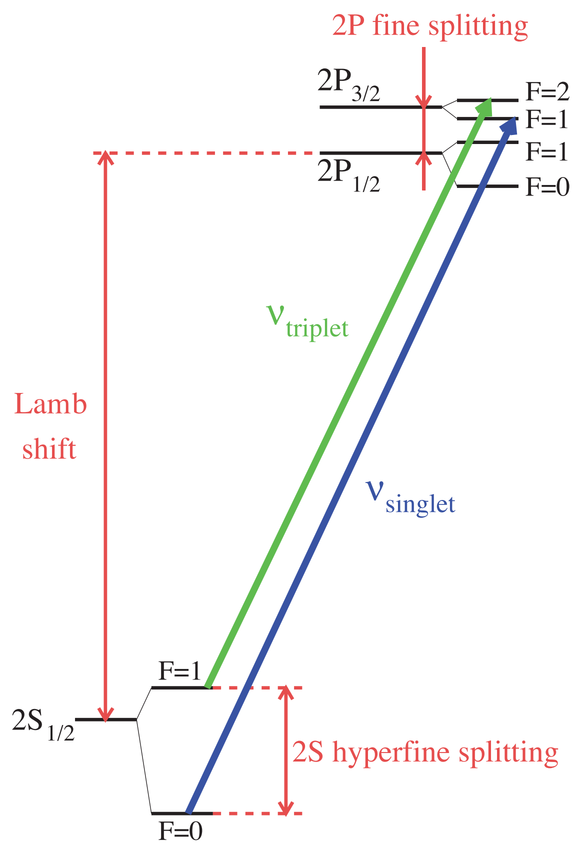
The effect in muonic hydrogen is expressed through the effect in H !

Hence prediction for muonic-hydrogen hfs









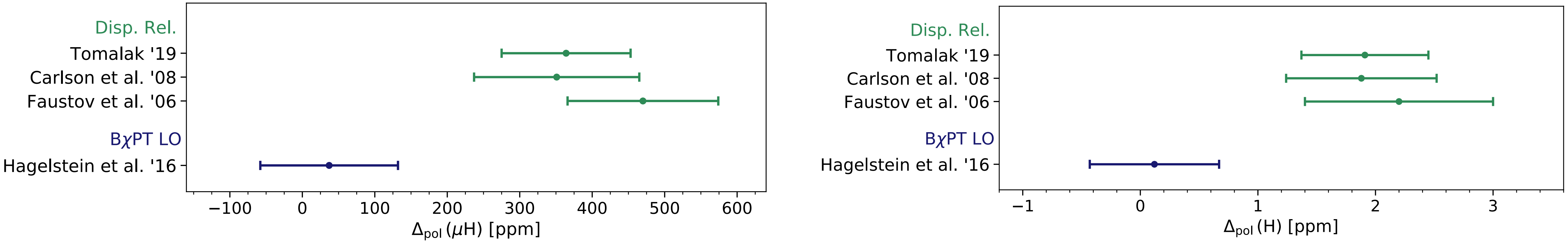
Backup slides

POLARIZABILITY EFFECT IN THE HFS

$$\begin{aligned}
\Delta_{\mathrm{pol}} &= \frac{\alpha m}{2\pi(1+\kappa)M}\left[\Delta_1+\Delta_2\right] \\
\Delta_1 &= 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left(\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)+F_2^2(Q^2)\right]-\frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\,x^2 g_1(x,Q^2)\right. \\
&\quad\left.\times\left\{\frac{1}{(v_l+\sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})(1+v_l)}\left[4+\frac{1}{1+\sqrt{1+x^2\tau^{-1}}}+\frac{1}{v_l+1}\right]\right\}\right) \\
\Delta_2 &= 96M^2\int_0^\infty\frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\,g_2(x,Q^2)\left\{\frac{1}{v_l+\sqrt{1+x^2\tau^{-1}}}-\frac{1}{v_l+1}\right\}
\end{aligned}$$

- Polarizability effect on the HFS is completely constrained by empirical information
- ChPT calculation puts the reliability of dispersive calculations (and ChPT) to the test ?!

Tension between the BChPT prediction and data-driven dispersive results:



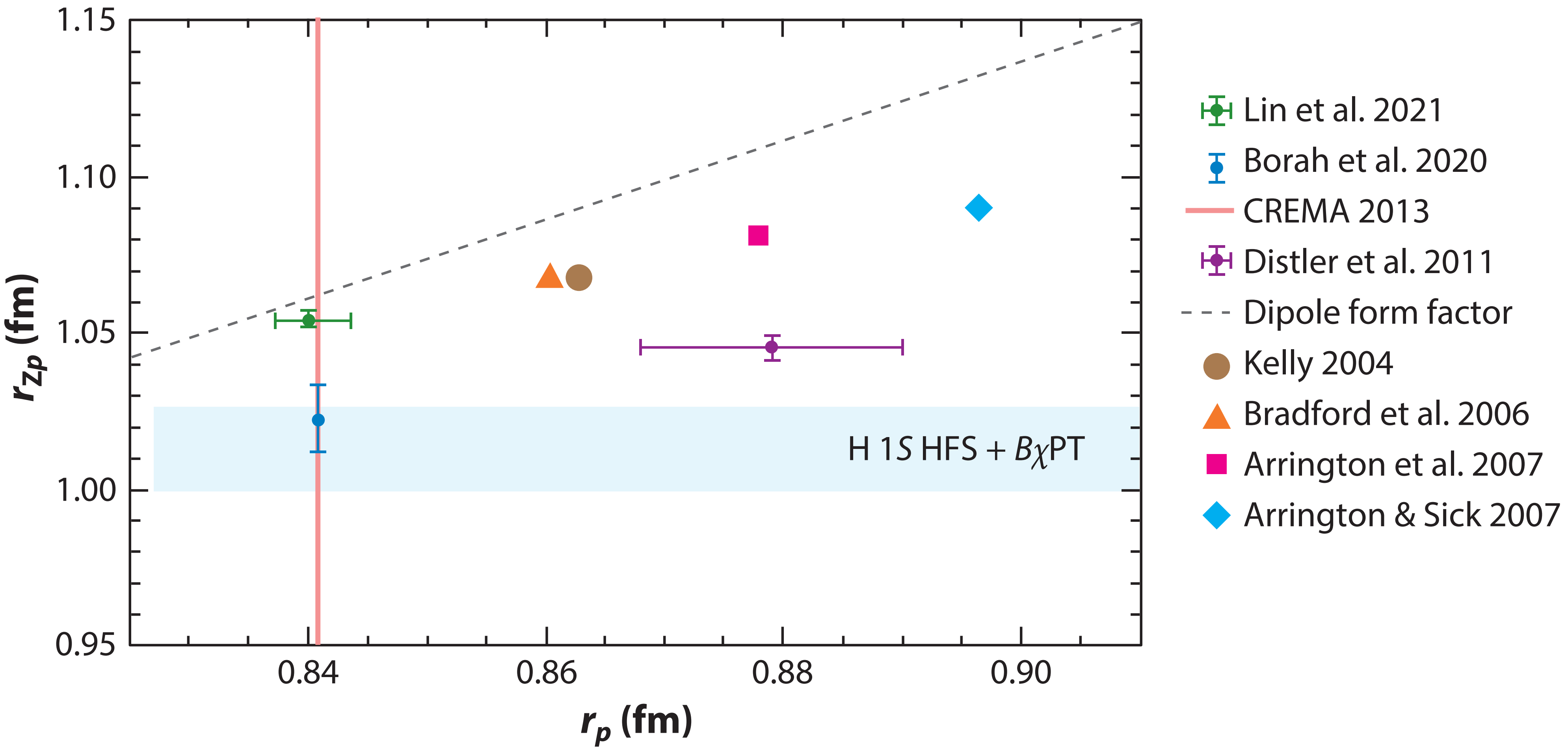
ZEMACH RADIUS

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_Z$$

- Smaller polarizability results in Zemach radius

Table 2 Determinations of the proton Zemach radius r_{Zp}, in units of fm.

ep scattering		μH 2*S* hfs		H 1*S* hfs	
Lin *et al.* (26)	Borah *et al.* (91)	Antognini *et al.* (2)	BχPT (62)	Volotka *et al.* (92)	BχPT (62)
$1.054^{+0.003}_{-0.002}$	1.0227(107)	1.082(37)	1.041(31)	1.045(16)	1.012(14)



MAMI vs HIGS - proton polarizabilities

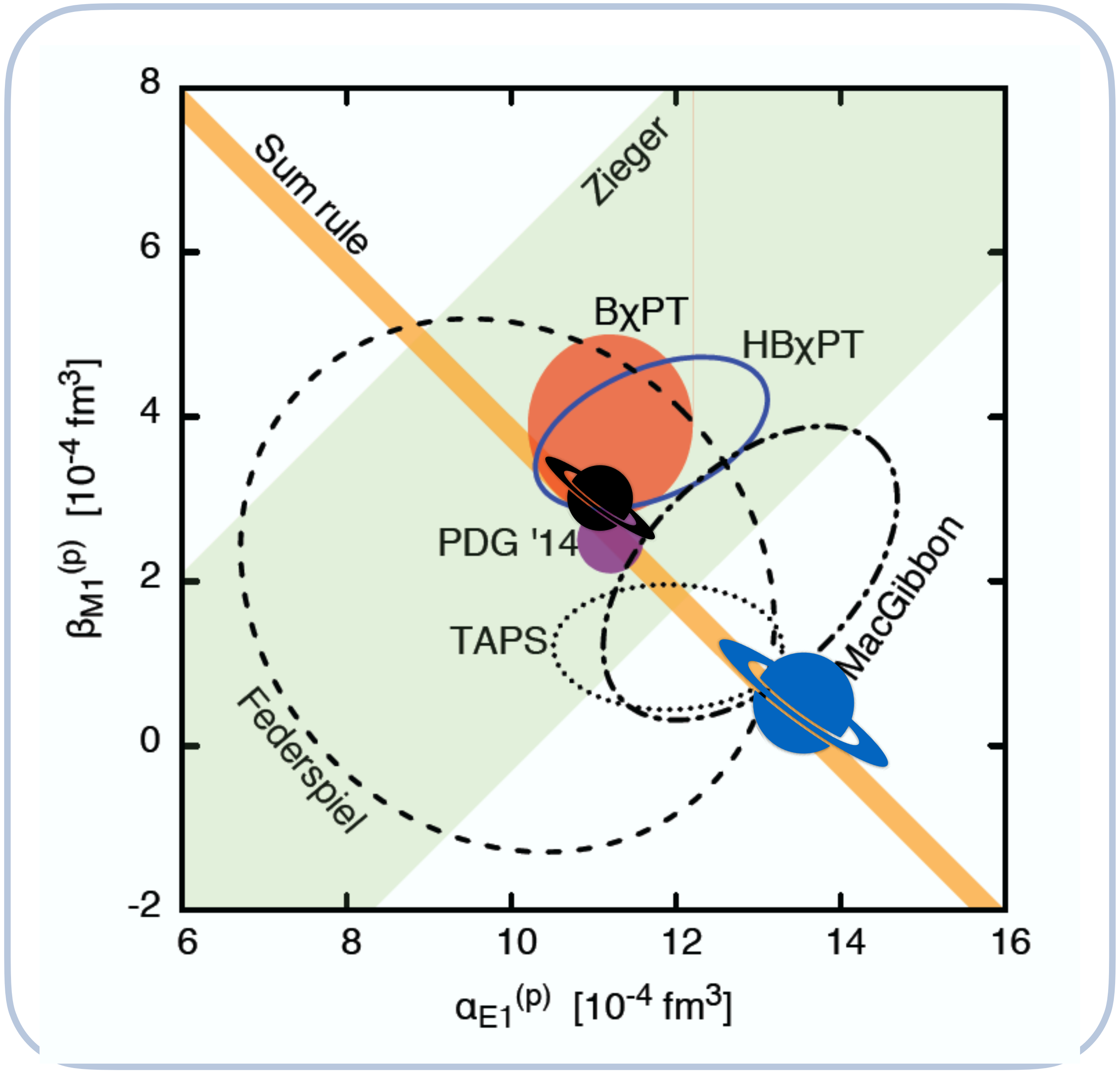
Phys.Rev.Lett. 128 (2022)

$\alpha_E = 11.0 \pm 1.2 \pm 0.1 \pm 0.3$

$\beta_M = 3.2 \mp 1.2 \pm 0.1 \pm 0.3$

$\alpha_E = 13.8 \pm 1.2 \pm 0.1 \pm 0.3$

$\beta_M = 0.2 \mp 1.2 \pm 0.1 \pm 0.3$



Theory analyses:

BChPT

Lensky, Pascalutsa (2010)

HBChPT

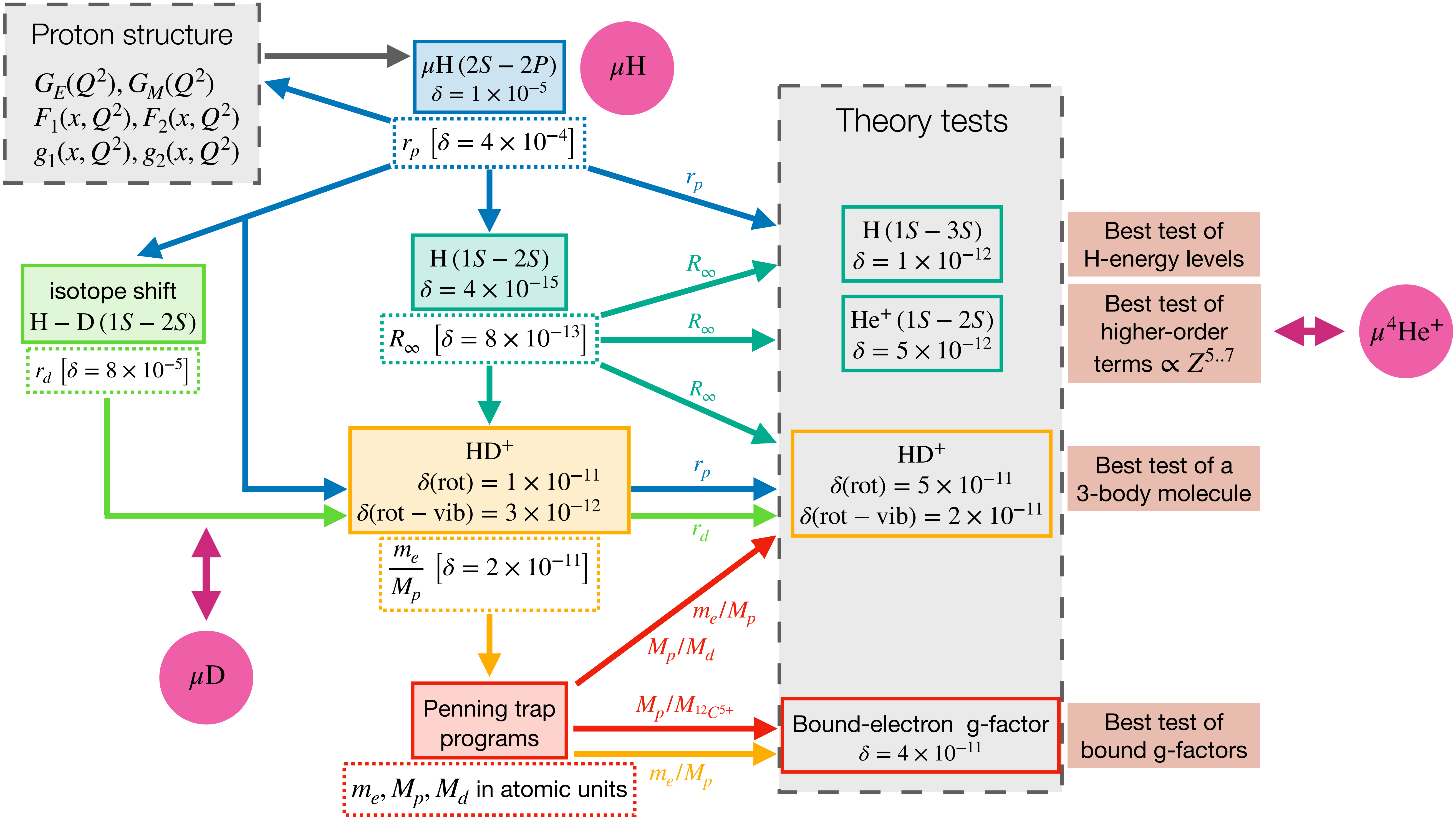
Griesshammer, McGovern, Phillips (2013)

PDG '14 values:

$\alpha_E = (11.2 \pm 0.2) \times 10^{-4}\text{ fm}^3$

$\beta_M = (2.5 \pm 0.4) \times 10^{-4}\text{ fm}^3$

INTERPLAY AND IMPACTS



Proton structure

$G_E(Q^2), G_M(Q^2)$

$F_1(x,Q^2), F_2(x,Q^2)$

$g_1(x,Q^2), g_2(x,Q^2)$

$\mu H\,(2S-2P)$ $\delta = 1\times 10^{-5}$

μH

r_p $[\delta = 4\times 10^{-4}]$

r_p

isotope shift $H-D\,(1S-2S)$

r_d $[\delta = 8\times 10^{-5}]$

$H\,(1S-2S)$ $\delta = 4\times 10^{-15}$

R_∞ $[\delta = 8\times 10^{-13}]$

R_∞

R_∞

R_∞

HD^+ $\delta(\mathrm{rot}) = 1\times 10^{-11}$ $\delta(\mathrm{rot-vib}) = 3\times 10^{-12}$

r_p

r_d

$\dfrac{m_e}{M_p}$ $[\delta = 2\times 10^{-11}]$

μD

Penning trap programs

m_e/M_p

M_p/M_d

$M_p/M_{^{12}C^{5+}}$

m_e/M_p

m_e, M_p, M_d in atomic units

Theory tests

$H\,(1S-3S)$ $\delta = 1\times 10^{-12}$

$He^+\,(1S-2S)$ $\delta = 5\times 10^{-12}$

HD^+ $\delta(\mathrm{rot}) = 5\times 10^{-11}$ $\delta(\mathrm{rot-vib}) = 2\times 10^{-11}$

Bound-electron g-factor $\delta = 4\times 10^{-11}$

Best test of H-energy levels

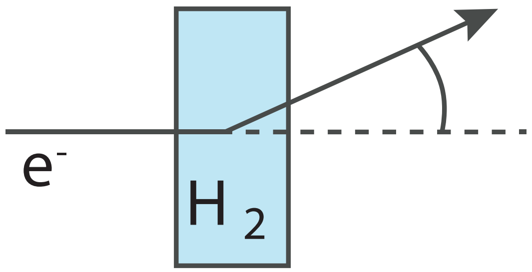
Best test of higher-order terms $\propto Z^{5..7}$

$\mu\,^4He^+$

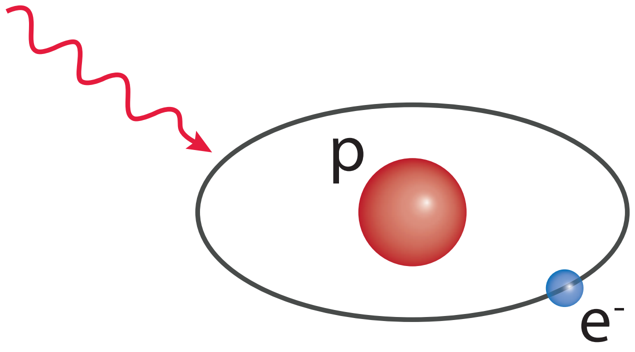
Best test of a 3-body molecule

Best test of bound g-factors

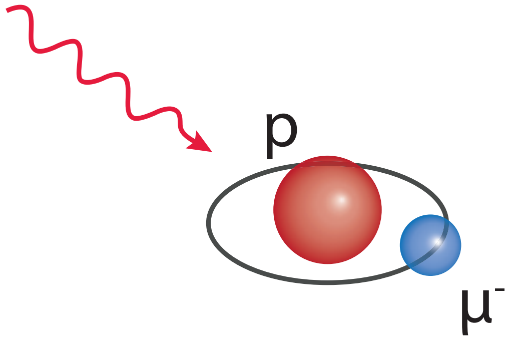
From: *Antognini, Hagelstein & VP, Ann. Rev. Nucl. Part.* ***72*** *(2022)[arXiv:2205.10076]*



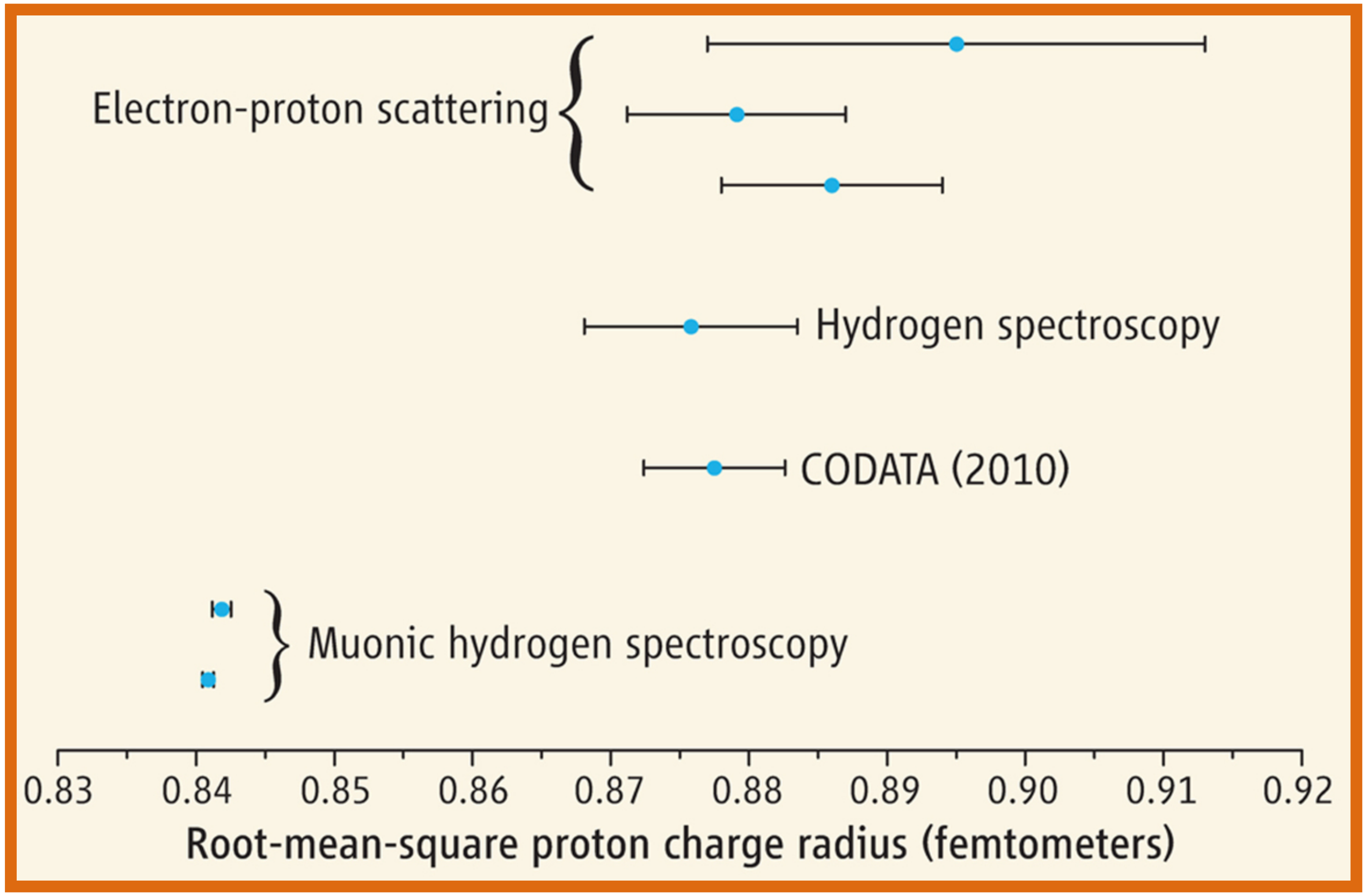
e^--p scattering



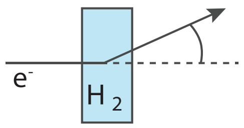
H spectroscopy



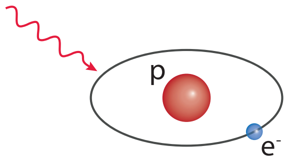
μp spectroscopy



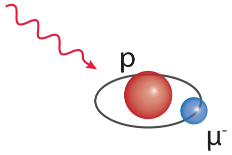
Summary of proton and deuteron radii



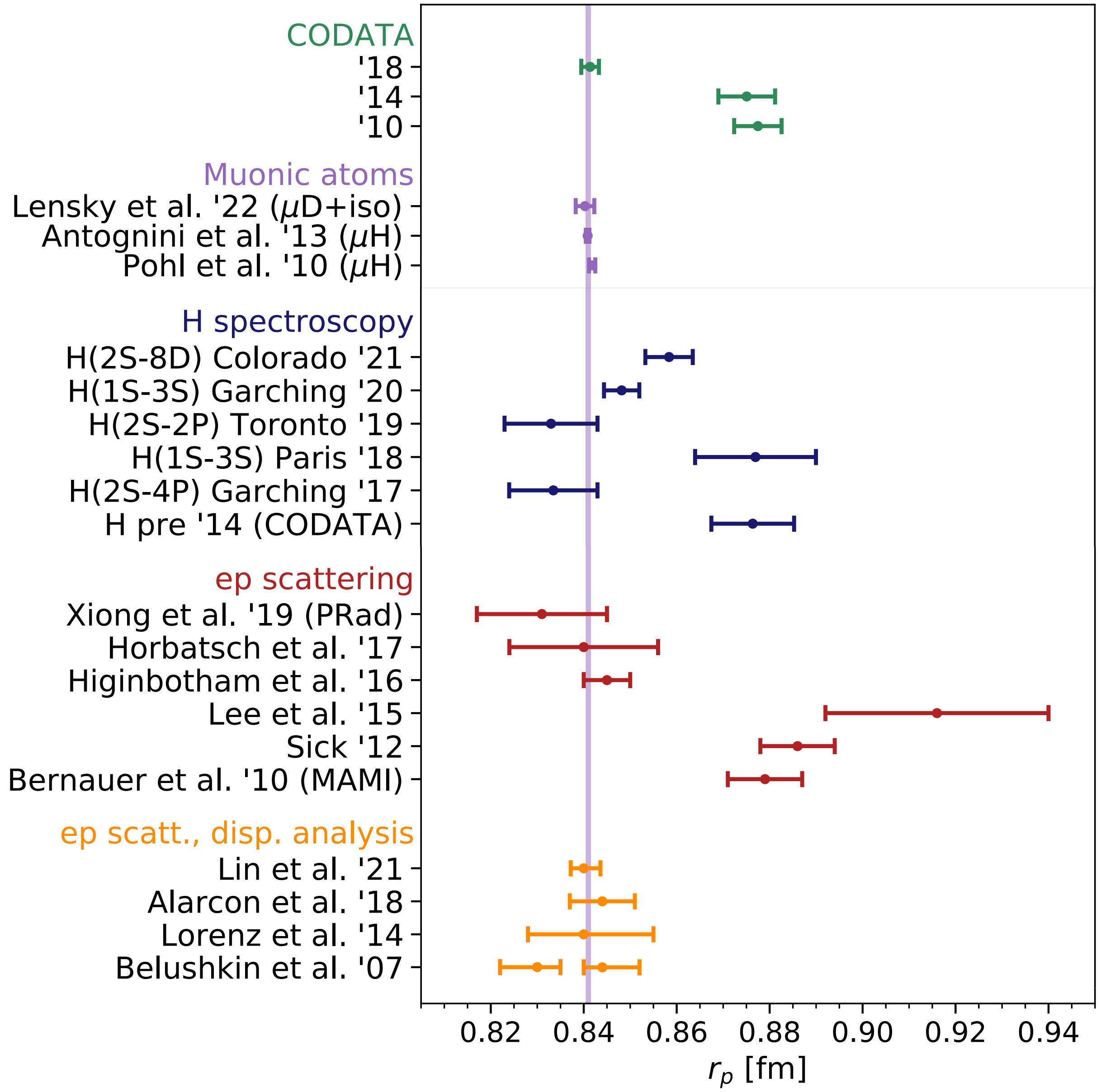
e⁻-p scattering

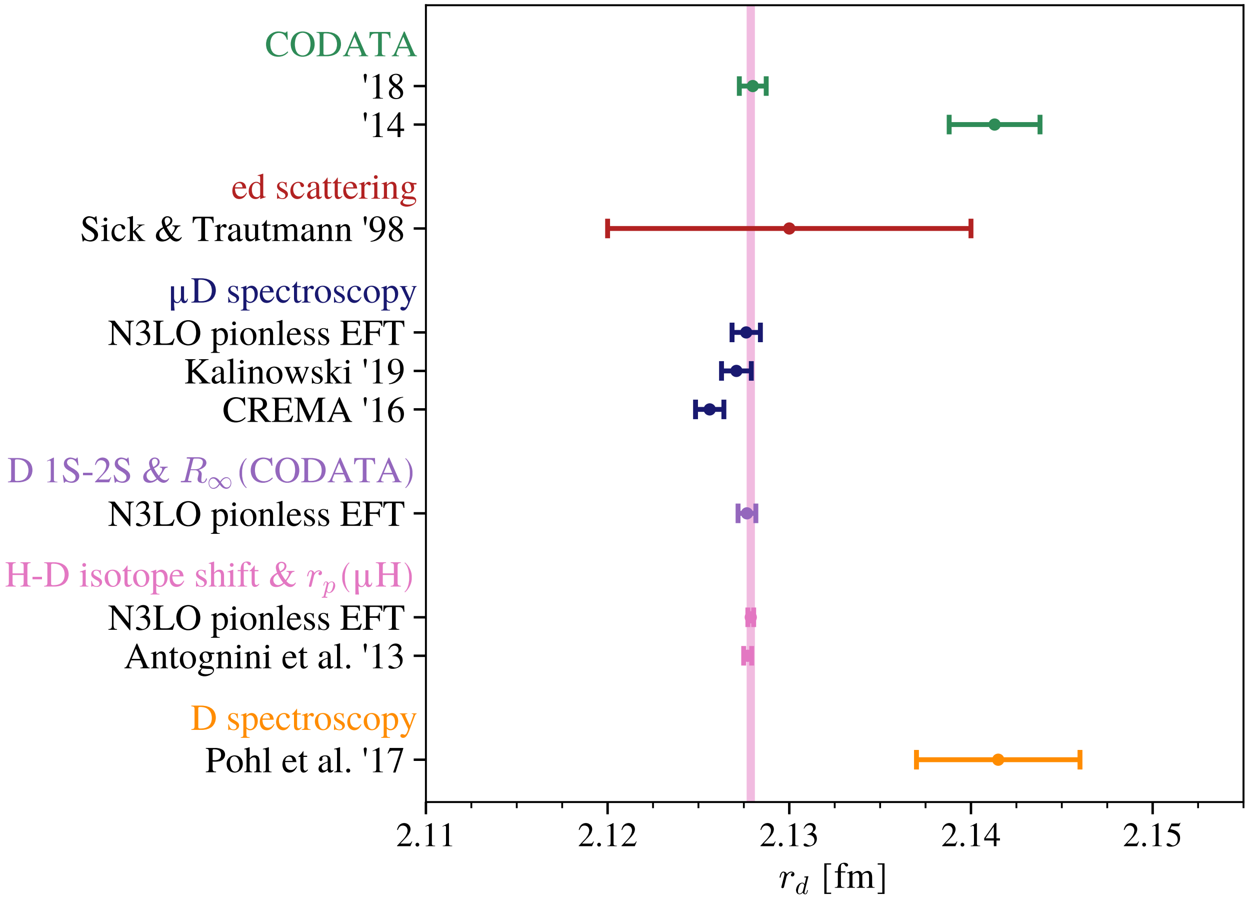


H spectroscopy

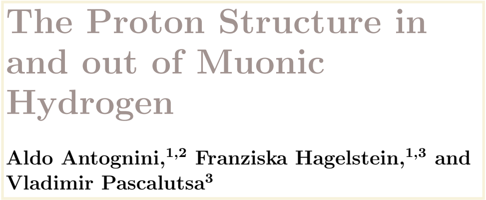


μp spectroscopy





Figures from


The Proton Structure in and out of Muonic Hydrogen

Aldo Antognini,[1,2] Franziska Hagelstein,[1,3] and Vladimir Pascalutsa[3]

Annu. Rev. Nucl. Part. Sci. 2022. 72:1–31

Proton radius puzzle: what could it mean ?

$$\Delta E_{LS} = 206.0336\,(15)\ -\ 5.2275\,(10)\ R_E^2\ +\ \Delta E_{TPE}\qquad \text{meV}$$

different radii

Lamb shift

difference of

≈ 330 μeV

μH expt. wrong ?

eH theory wrong ?

μH theory wrong ?

- QED bound state corrections
- hadronic corrections
- check with different targets

eH expt. wrong ? -> R_∞ wrong

+ ep scattering wrong ?

- radiative corrections
- 2γ corrections
- low Q^2 extrapolation

new physics ?

see PPNP review Carlson (2015)