Bjorken Sum Rule and (Effective) Strong Coupling

Jian-ping Chen, Jefferson Lab, Virginia, USA

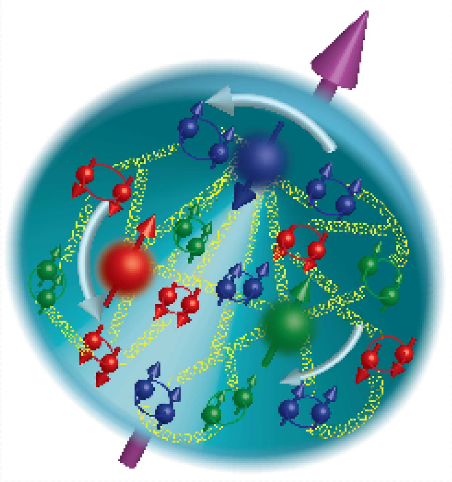
Low-Q-2023, May 15-21, 2023

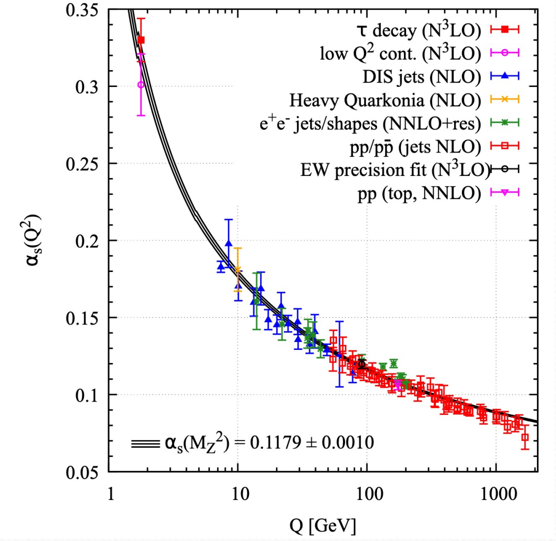
- Introduction
- Bjorken Sum Rule and Strong Coupling
- Experimental Extraction of Bjorken Sum at Low-q
  and (Effective) Strong Coupling
- Summary

Acknowledgment: Thanks to Alexandre Deur and collaborators
for the work in this talk and for providing slides

Introduction

Nucleon Spin Structure and Strong Interaction,





Nucleon Structure and Strong Interaction/QCD

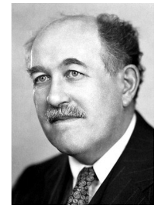
- **Nucleon Structure: discoveries**
  -- anomalous magnetic moment (1943 Nobel)
  -- elastic: form factors (1961 Nobel)
  -- DIS: parton distributions (1990 Nobel)

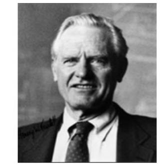
- **Strong interaction, running coupling ~1**
  -- asymptotic freedom (2004 Nobel)
  perturbation calculation works at high energy
  -- interaction significant at intermediate energy, quark-gluon correlations
  -- interaction strong at low energy
  confinement

- **A major challenge in fundamental physics:**
  -- Understand QCD in all regions, including strong (confinement) region

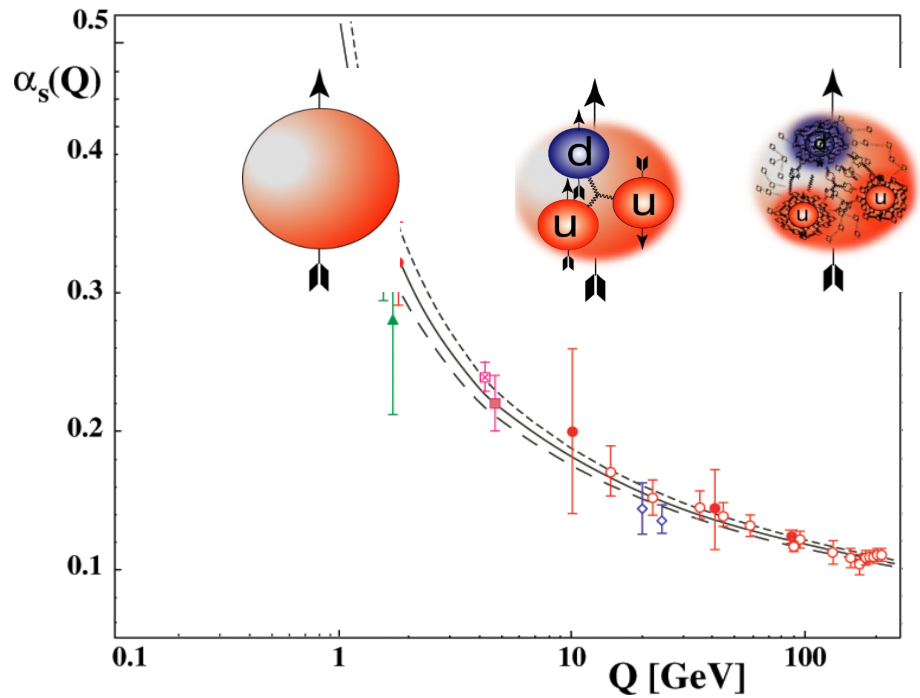
- **Nucleon: most convenient lab to study QCD**

- **Theoretical Tools:**
  pQCD, Lattice QCD, ChEFT, Sum Rules, …

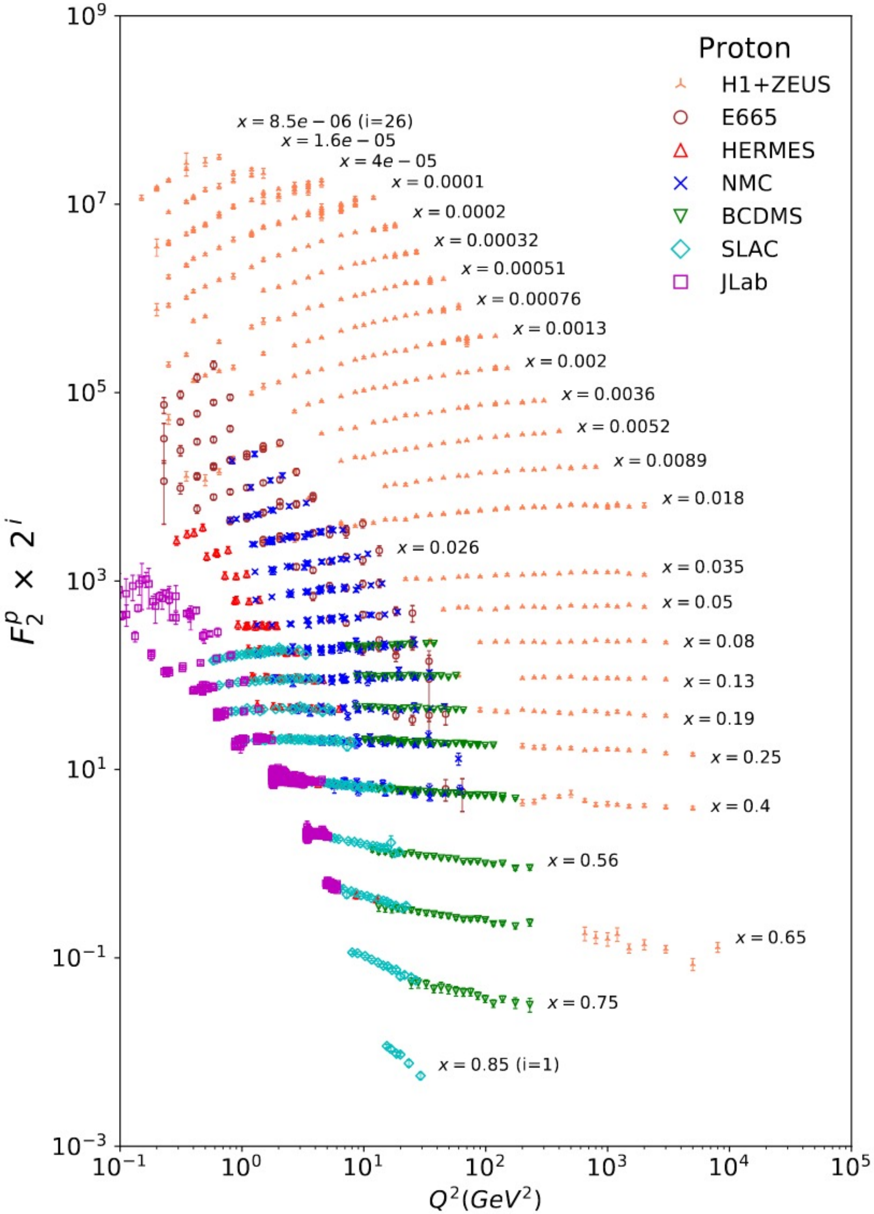


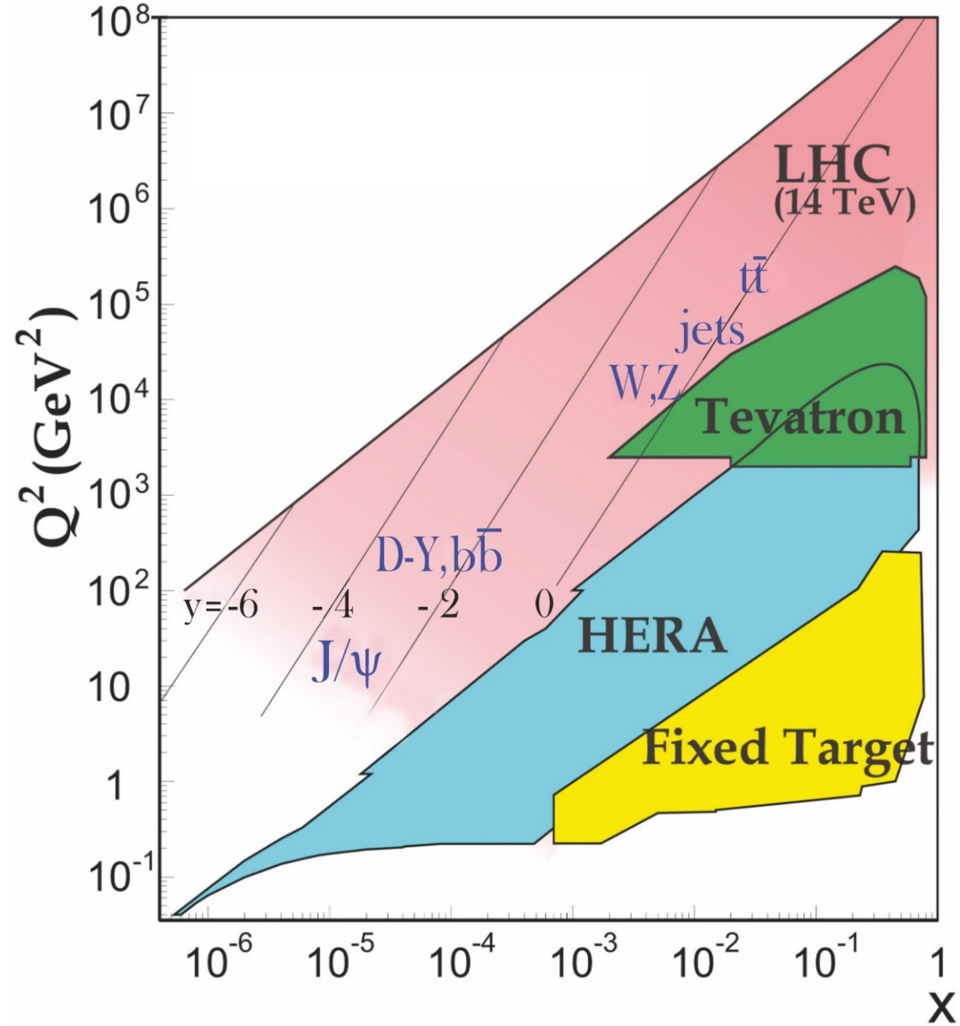




running coupling "constant"



UNPOLARIZED STRUCTURE FUNCTIONS



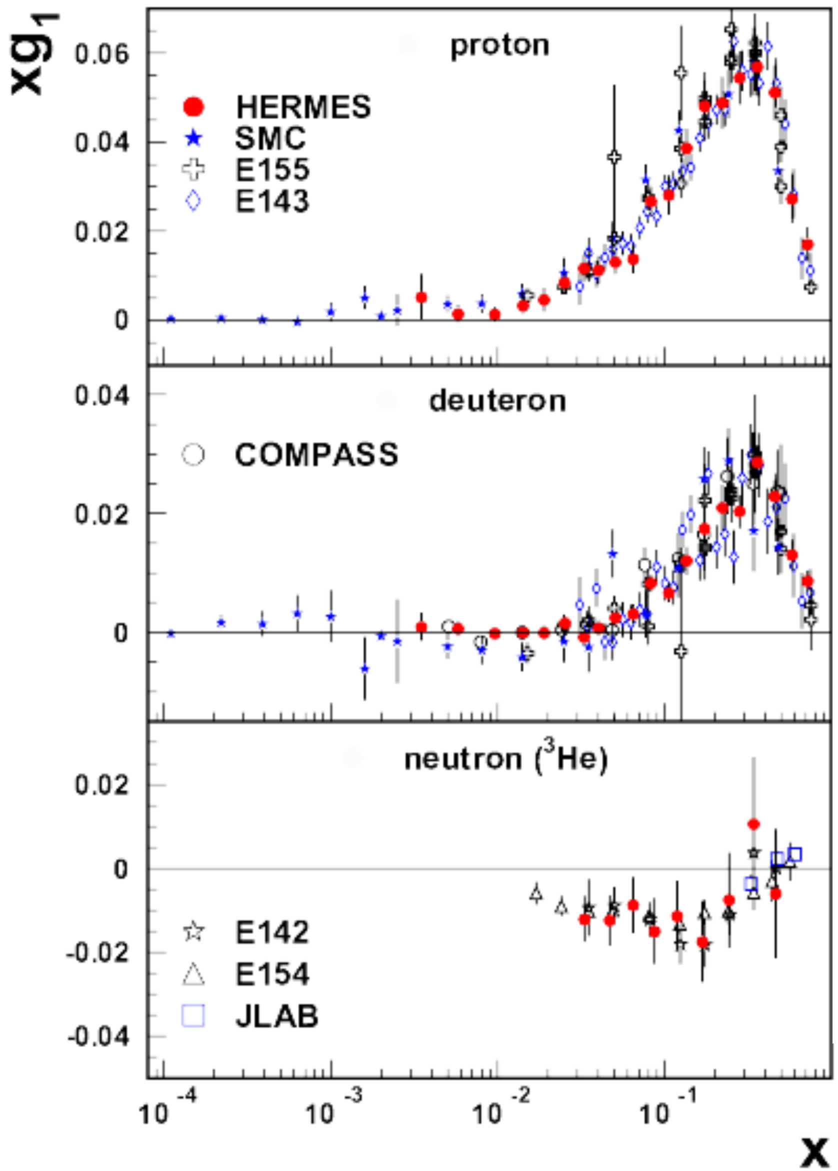


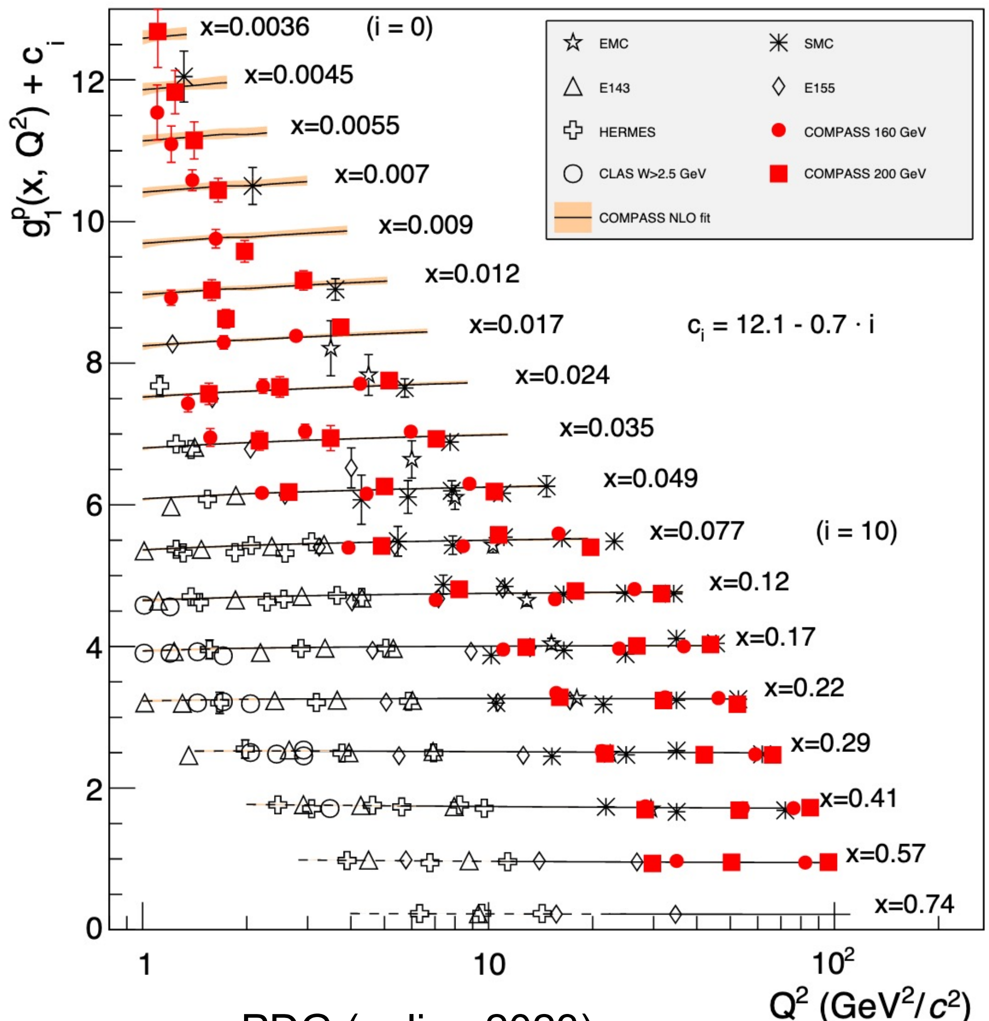
Q^2 evolution: the best test of QCD

Experiment – Theory Dialogue

- Theorist to experimentalist: (some time ago)
  give us spin structure functions in full phase space
  full range of x [0-1], full range of Q^2: $[0, \infty]$,
  we will take care of the rest (comparisons, understanding physics, …)
- Experimentalist: hmm…, we can only measure at limited region with some precision,
  and BTW, we also like to work with you to understand physics
- T: how about moments? we have some predictions at high Q^2 (PQCD) and low Q^2 (ChEFT)
- E: yes, we can measure moments in certain region
- T: we can make predictions on moments with LQCD and
  we are developing a method and might be able to predict x dependence (recently)
- E: great, we are continuing to produce data, let's find out how well data comparison with (PQCD, ChEFT, LQCD, …) predictions
  and how they can help us to understand QCD

POLARIZED STRUCTURE FUNCTIONS





PDG (online 2023)

Experiment Summary ($Q^2 > 0$)

Observable	H target	D target	^{3}He target
g_1, g_2, Γ_1 & Γ_2 at high Q^2	SLAC JLAB SANE	SLAC	SLAC JLAB E97-117 JLAB E01-012 JLAB E06-014
g_1 & Γ_1 at high Q^2 COMPASS RHIC-Spin	SMC HERMES JLAB EG1	SMC HERMES JLAB EG1	HERMES
Γ_1 & Γ_2 at low Q^2	JLab RSS	JLab RSS	JLab E94-010 JLab E97-103
Γ_1 at low Q^2	SLAC HERMES JLAB EG1	SLAC HERMES JLAB EG1	HERMES
Γ_1, $Q^2 << 1$ GeV^2	JLab EG4	JLab EG4	JLab E97-110
Γ_2, $Q^2 << 1$ GeV^2	JLab E08-027		JLab E97-110

JLab12

Q^2=0 **Mainz, Bonn, LEGS, HIGS**

Bjorken Sum Rule and Q^2 dependence

Bjørken Sum Rule

$$\Gamma_1^p(Q^2) - \Gamma_1^n(Q^2) = \int \{g_1^p(x,Q^2) - g_1^n(x,Q^2)\}dx = \frac{1}{6} g_A C_{NS}$$

g_A: axial charge (from neutron β-decay)

C_{NS}: Q^2-dependent QCD corrections (for flavor non-singlet)

- A fundamental relation relating an integration of spin structure functions to axial-vector coupling constant (axial charge)
- Based on Operator Product Expansion within QCD or Current Algebra
- Valid at large Q^2 (higher-twist effects negligible)
- Data are consistent with the Bjørken Sum Rule at 5-10 % level

(Generalized) Bjørken Sum Rule

$$\Gamma_1^{p-n} = \frac{g_A}{6}\left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 + \cdots\right] + \sum_{i=2,3\ldots}^{\infty} \frac{\mu_{2i}^{p-n}(Q^2)}{Q^{2i-2}},$$

- A fundamental relation relating an integration of spin structure functions to axial-vector coupling constant (axial charge)
- Based on Operator Product Expansion within QCD or Current Algebra
- Valid at large Q^2 (higher-twist effects negligible)
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Gerasimov-Drell-Hearn Sum Rule

Circularly polarized photon on longitudinally polarized nucleon

$$\int_{\nu_{in}}^{\infty}\left(\sigma_{1/2}(\nu)-\sigma_{3/2}(\nu)\right)\frac{d\nu}{\nu}=-\frac{2\pi^2\alpha_{EM}}{M^2}\kappa^2$$

- A fundamental relation between the nucleon spin structure and its anomalous magnetic moment

- Based on general physics principles
  - Lorentz invariance, gauge invariance → low energy theorem
  - unitarity → optical theorem
  - casuality → unsubtracted dispersion relation applied to forward Compton amplitude

- Measurements on *proton* up to 800 MeV (Mainz) and up to 3 GeV (Bonn) agree with GDH with assumptions for contributions from un-measured regions

  New measurements on p, d and ^{3}He from LEGS, MAMI(2), …

Generalized GDH Sum Rule

- Many approaches: Anselmino, Ioffe, Burkert, Drechsel, …
- Ji and Osborne (J. Phys. G27, 127, 2001):

  Forward Virtual-Virtual Compton Scattering Amplitudes: $S_1(Q^2,\nu)$, $S_2(Q^2, \nu)$

  Same assumptions: no-subtraction dispersion relation
  optical theorem
  (low energy theorem)

- Generalized GDH Sum Rule

$$S_1(Q^2) = 4\int_{el}^{\infty} \frac{G_1(Q^2,\nu)d\nu}{\nu}$$

Connecting GDH with Bjorken Sum Rules

- Q^2-evolution of GDH Sum Rule provides a bridge linking strong QCD to pQCD
  - Bjorken and GDH sum rules are two limiting cases

    High Q^2, Operator Product Expansion : $S_1(p\text{-}n) \sim g_A$ → Bjorken

    $Q^2 \rightarrow 0$, Low Energy Theorem: $S_1 \sim \kappa^2$ → GDH

  - High Q^2 (> ~1 GeV^2): Operator Product Expansion
  - Intermediate Q^2 region: Lattice QCD calculations?
  - Low Q^2 region (< ~0.1 GeV^2): Chiral Perturbation Theory

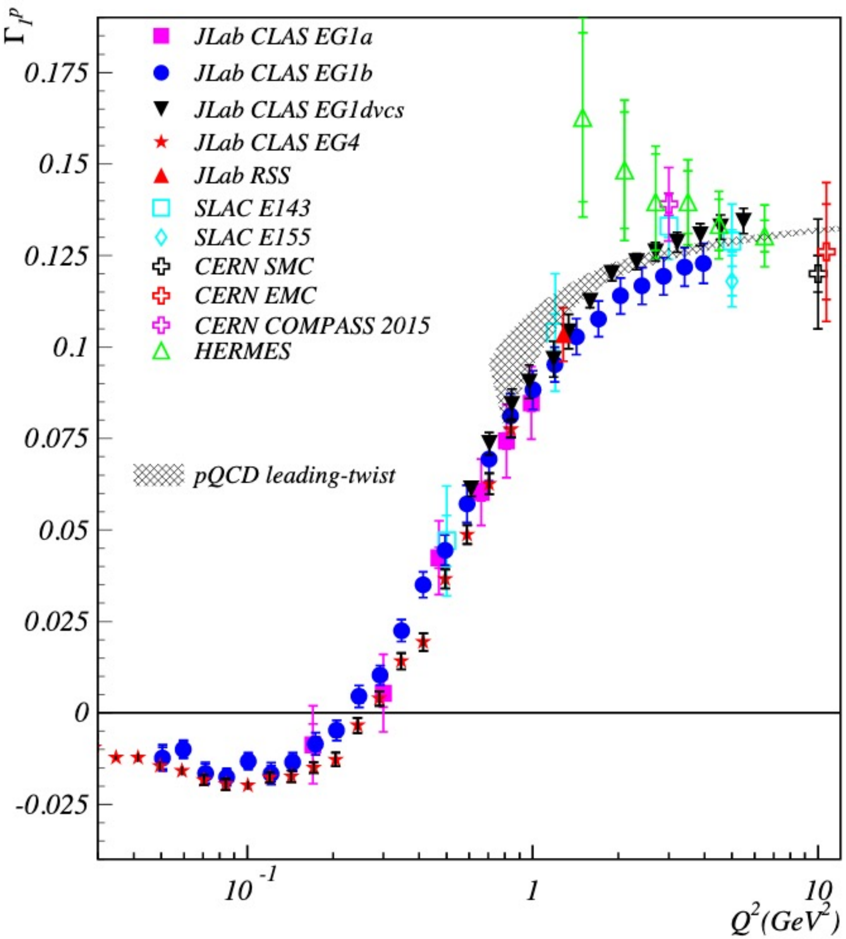
Calculations: BχPT: Ji, Kao,…,Vanderhaeghen,…

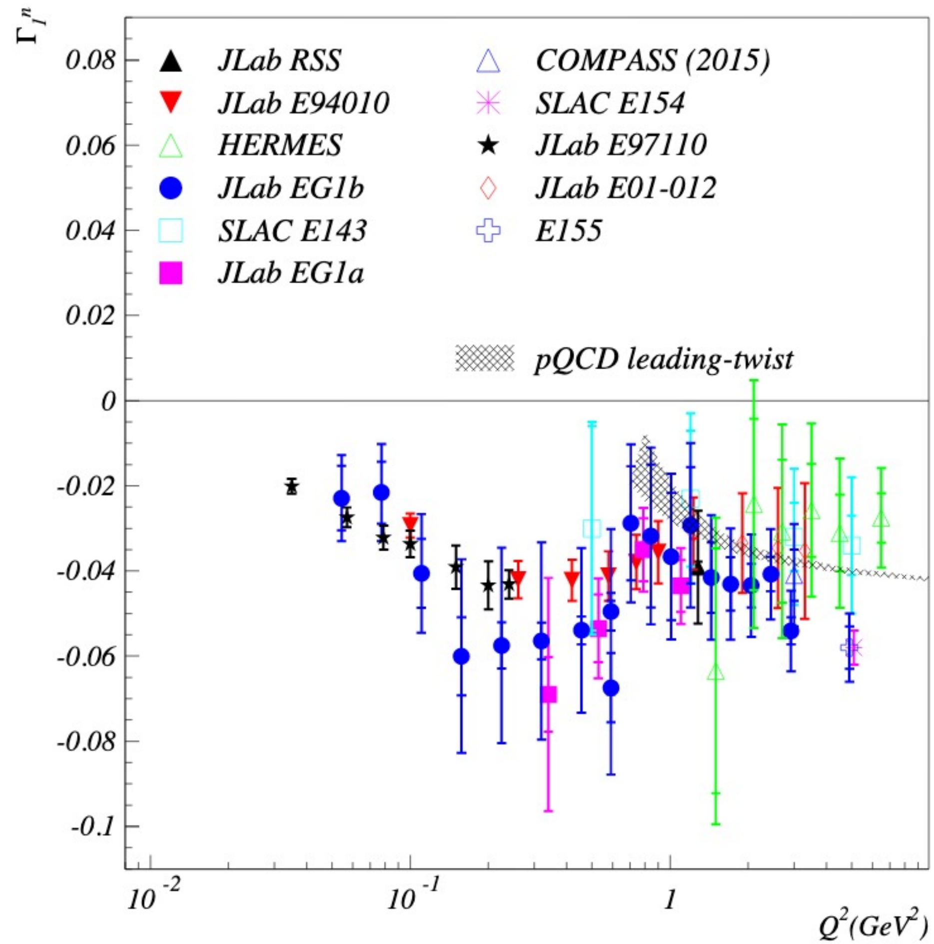
Lensky, Alarcon & Pascalutsa

Bernard, Hemmert, Meissner

World data on Γ_1 for proton and neutron

Previous Publications and
New Low-Q data: talks on EG4 (A. Deur for M. Ripani onTuesday
and E97-110 (A. Deur, next)



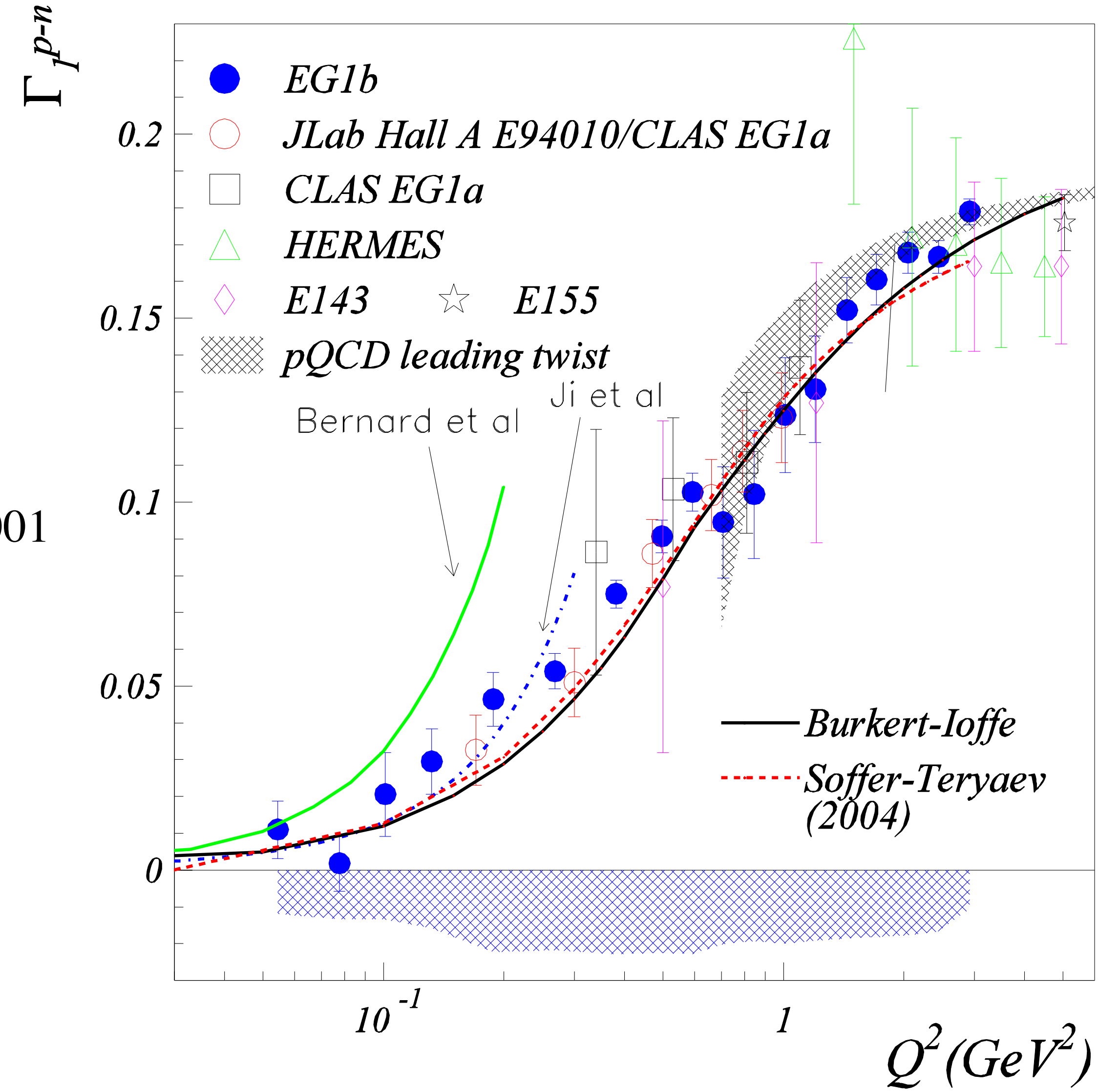


Bjorken Sum: Γ_1 of *p-n* (before new low-Q data)

A. Deur, *et al*.

EG1b, PRD 78, 032001 (2008)

E94-010 + EG1a: PRL 93 (2004) 212001

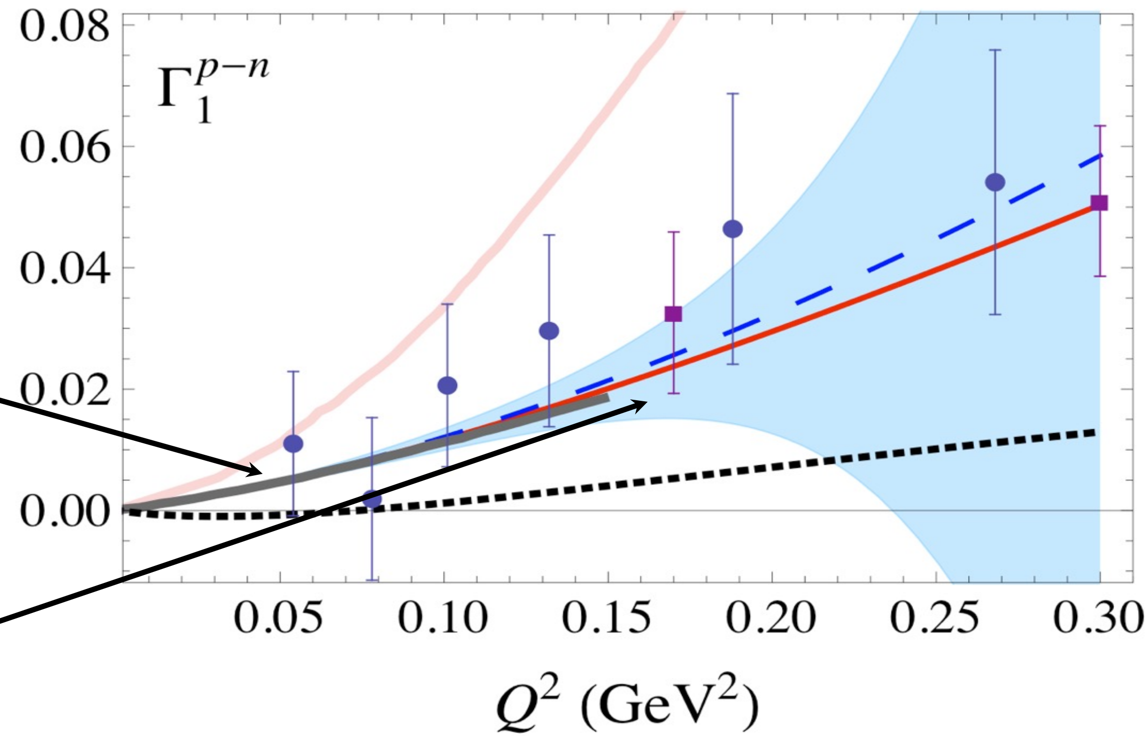


Bjorken Sum (p-n) (before new low-Q)

- Low Q^2: test of χpt calculations

Bernard et al., PRD 87 (2013)

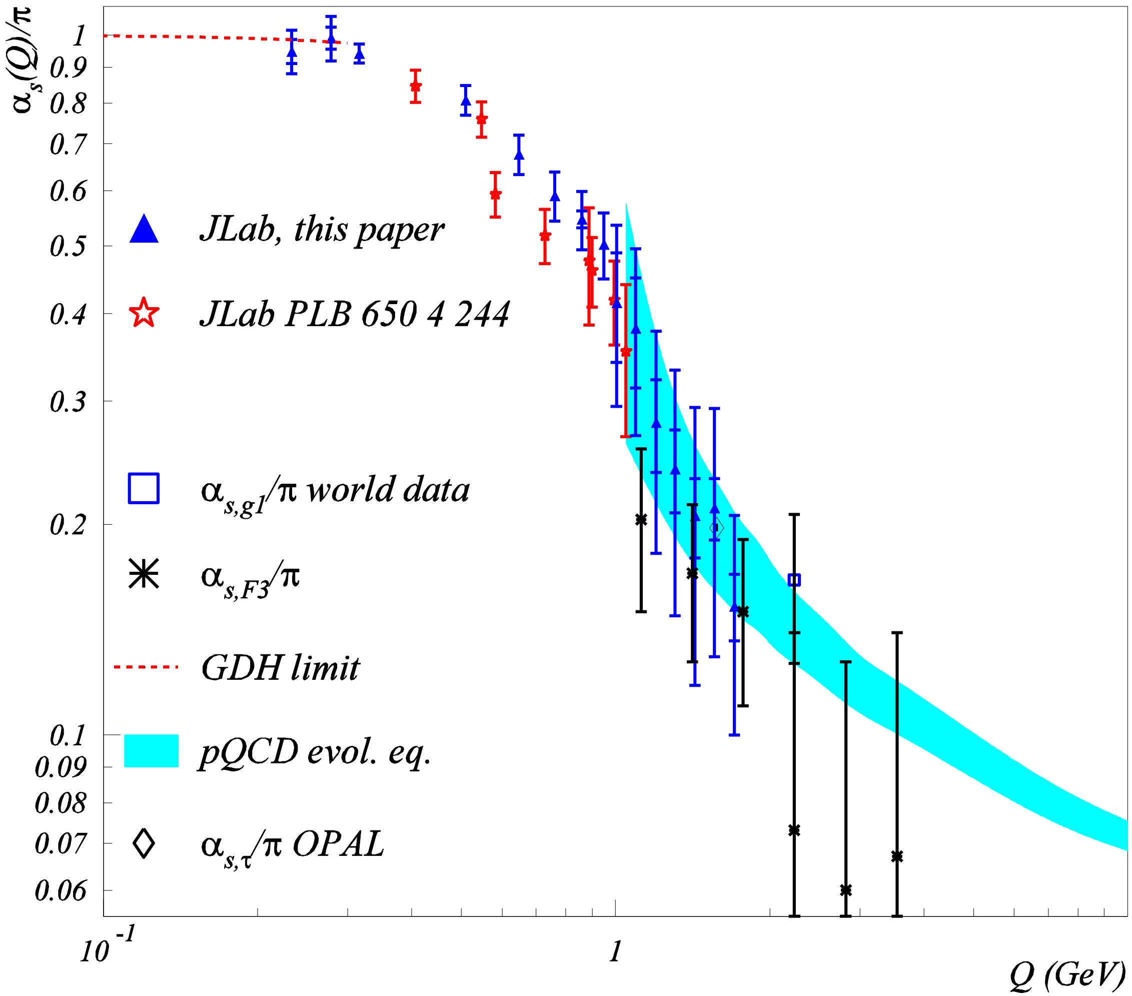
Lensky, Alarcon & Pascalutsa
PRC 90 055202 (2014)



Effective α_s Extracted from Bjorken Sum (before new low-Q)

A. Deur, V. Burkert, J. P. Chen and W. Korsch
PLB 650, 244 (2007) and PLB 665, 349 (2008)

α_s/π



The strong coupling α_s at short distances (large Q^2)

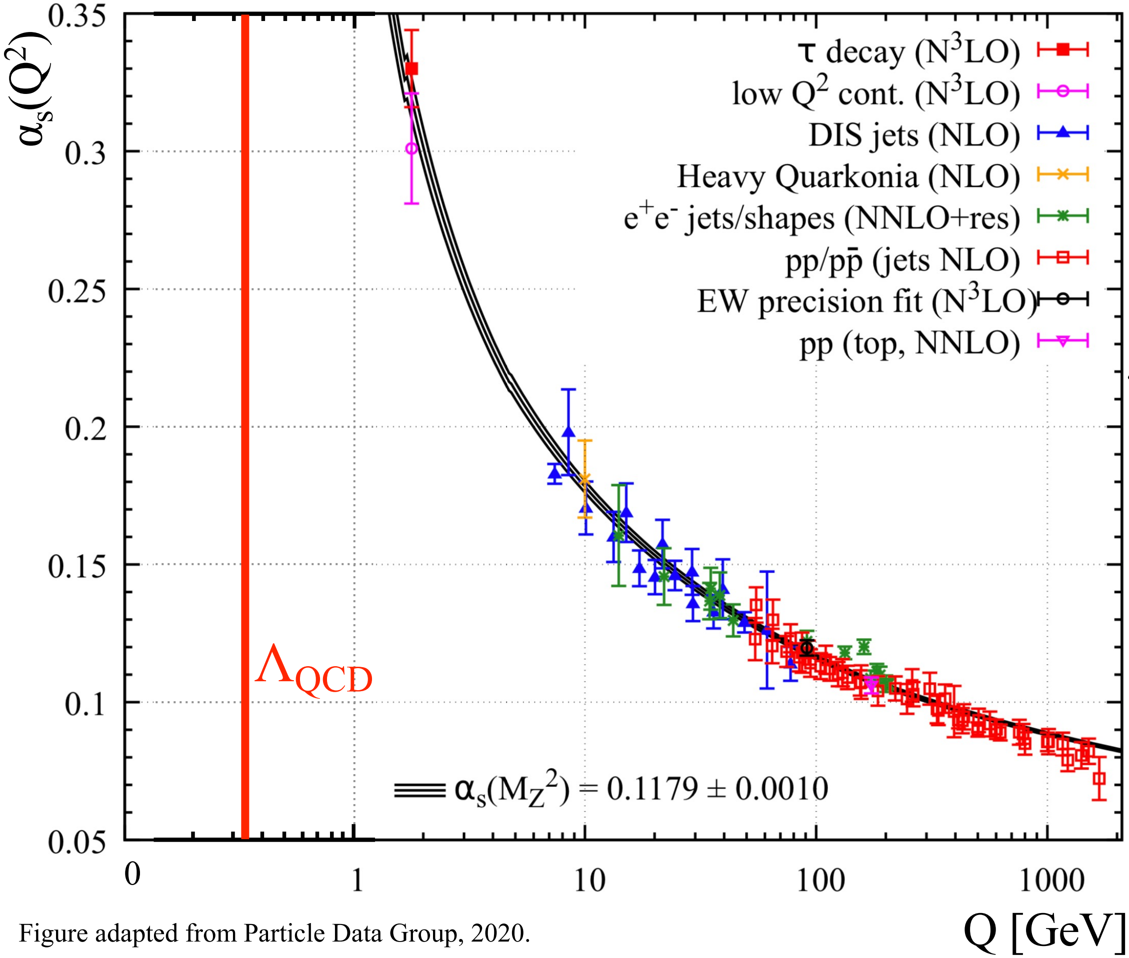
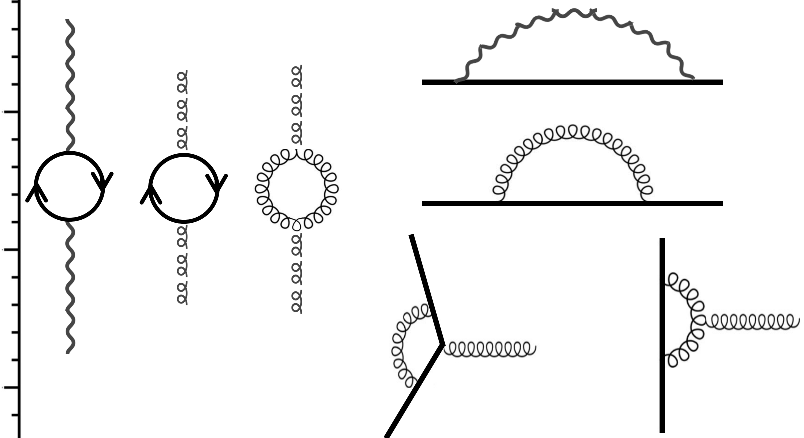


Figure adapted from Particle Data Group, 2020.

α_s is not constant due to loops in gluon propagator, fermion self-energy, and vertex corrections:



α_s becomes small at short distances (large Q^2)

$\Rightarrow$ Asymptotic freedom: perturbative treatment of QCD (pQCD). $\alpha_s(Q^2)$ is well defined within pQCD.

The strong coupling α_s at short distances (large Q^2)

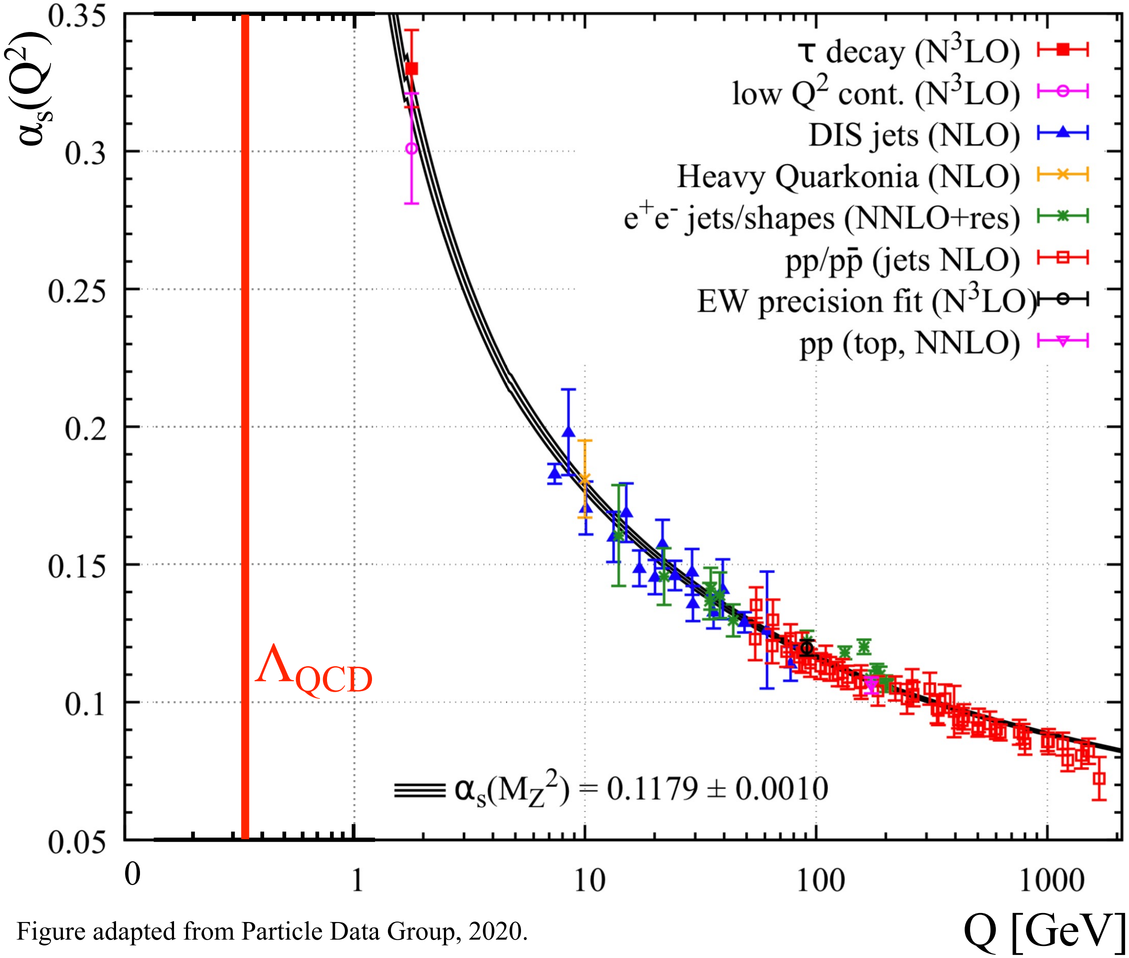


Figure adapted from Particle Data Group, 2020.

$\alpha_s(Q^2)$ $\Rightarrow$ needs data or non-perturbative methods to get $\alpha_s(Q^2)$.

Lattice calculations: currently most accurate determination of $\alpha_s(M_Z^2)$.

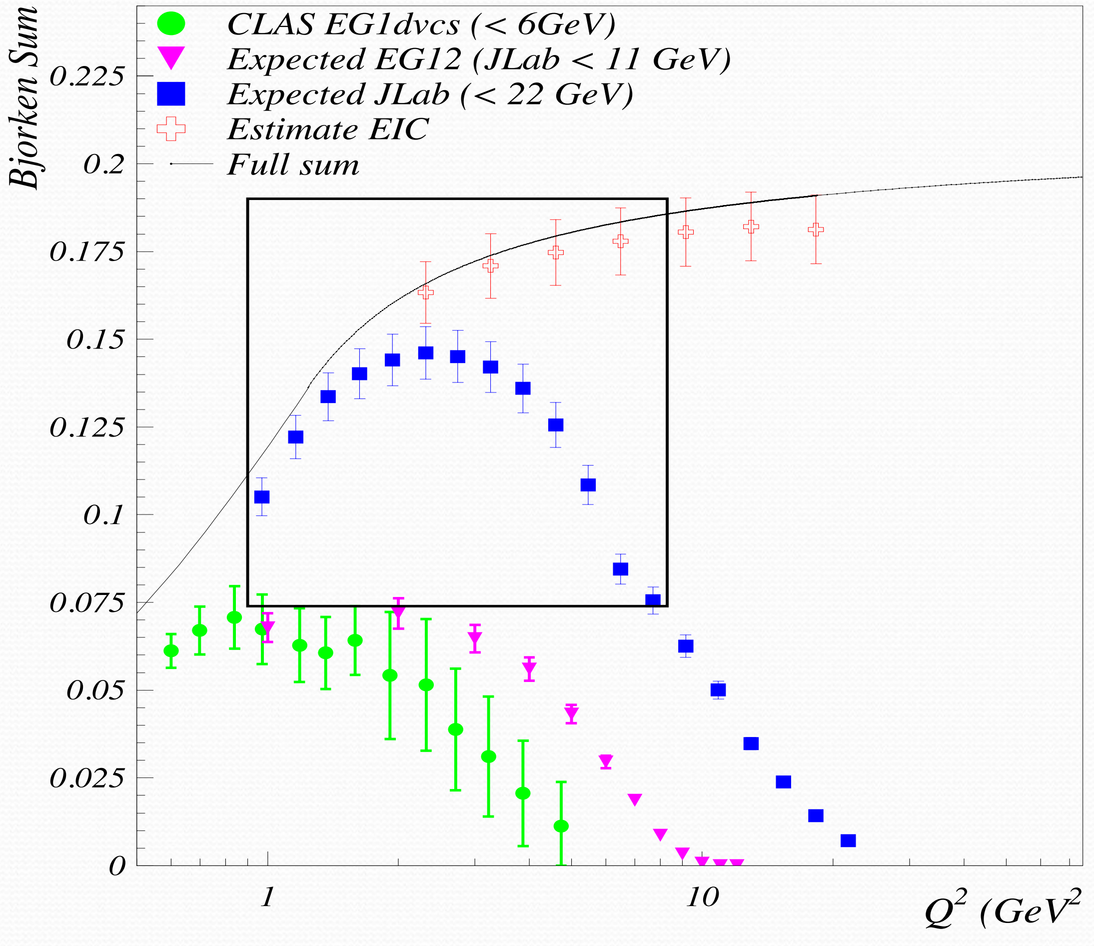
Otherwise, $\alpha_s(Q^2)$ is extracted from data, e.g. Bjorken sum rule:

$$\int (g^p_1 - g^n_1)dx = \frac{1}{6} g_A\left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right)$$

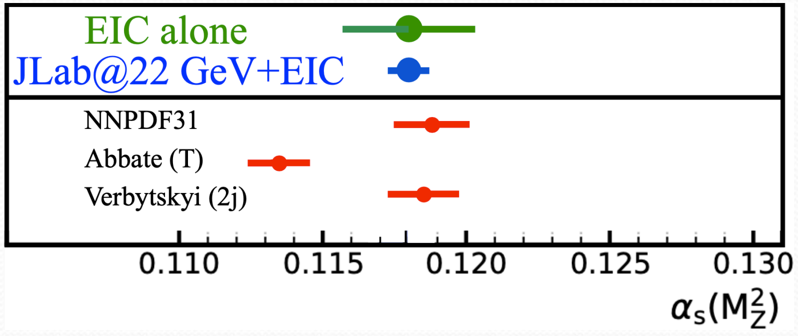
Projection of JLab22 (+ EIC) on Extraction of α_s

JLab22 + EIC can make a significant improvement in the extraction of α_s

A.Deur, contribution to the JLab22 Whitepaper (to be published)



$$\Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 - 175.7 \left(\frac{\alpha_s}{\pi}\right)^4 - \sim 893 \left(\frac{\alpha_s}{\pi}\right)^5 \right] + \frac{a}{Q^2}.$$



α_s from high to low Q^2

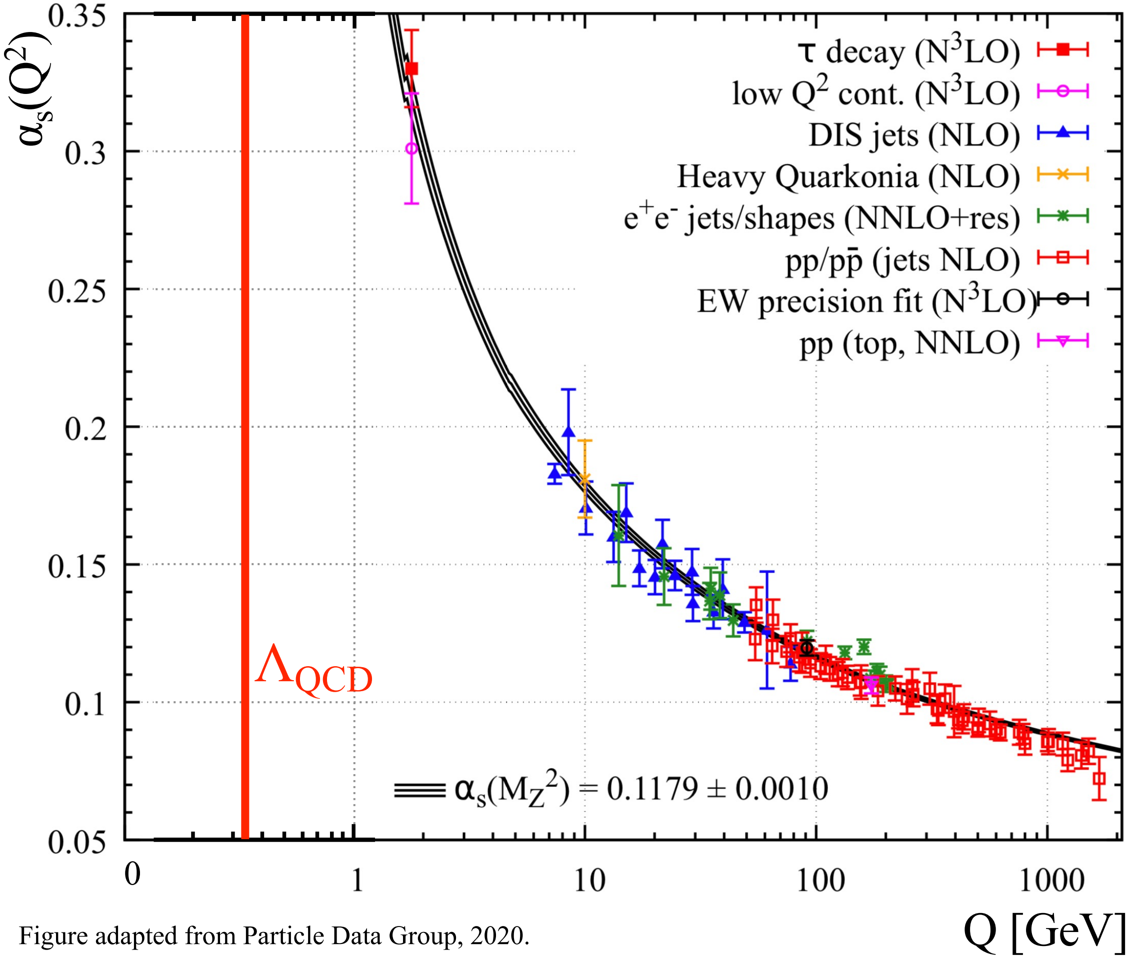


Figure adapted from Particle Data Group, 2020.

At $Q^2 \lesssim 1\text{GeV}^2$, pQCD cannot be used to define α_s: if pQCD is trusted, $\alpha_s \rightarrow \infty$ when $Q \rightarrow \Lambda_{QCD}$.

- Contradict the perturbative hypothesis;
- The divergence (Landau pôle) is unphysical..

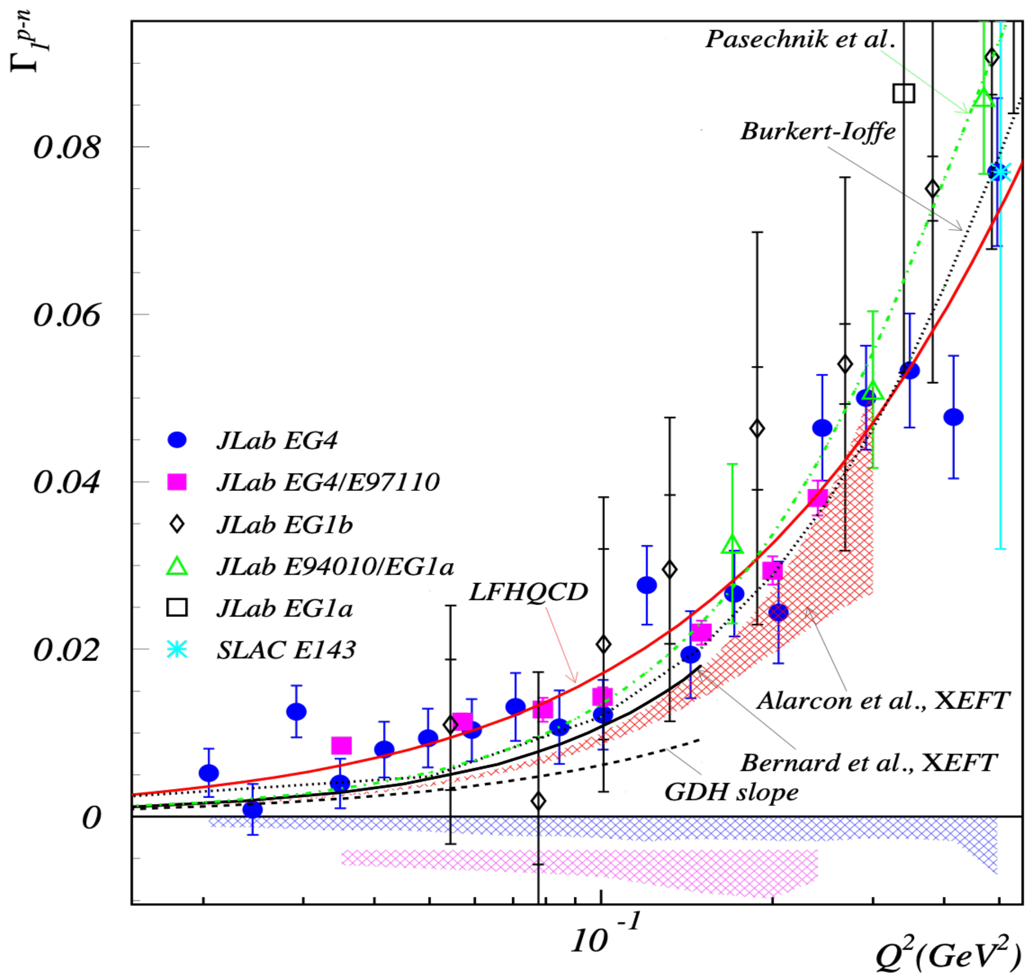
Definition and computation of α_s at long distance?

Bjorken Sum at Low-Q and Effective α_s

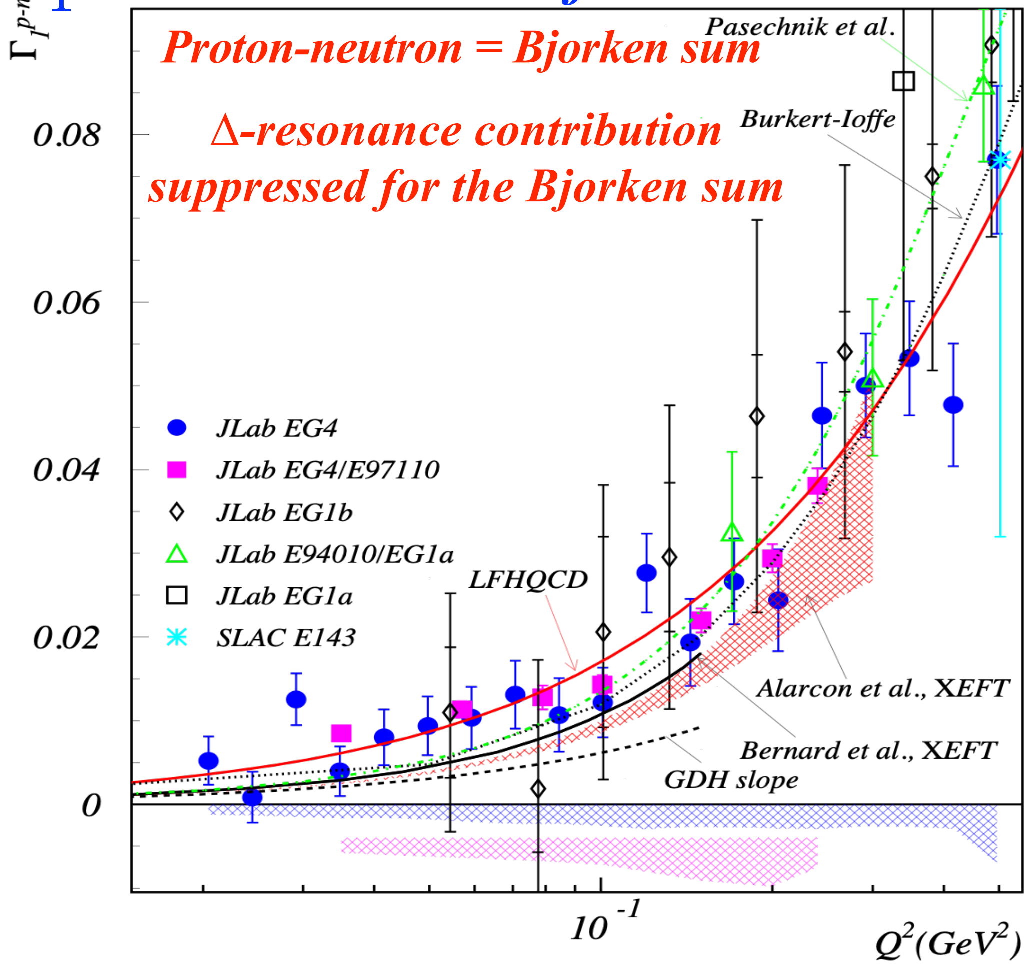
Bjorken Sum: Γ_1 of p-n (EG4 and E97-110)

A. Deur, *et al*.

EG4 and E97-110,
Phys. Lett. B 825 (2022) 136878



Γ_1^{p-n} measurements from E97-110 and EG4



Fit $\Gamma_1 = bQ^2+cQ^4$:

χEFT prediction

Data set	$(b \pm uncor \pm cor)$ [GeV^{-2}]	$c \pm uncor \pm cor$ [GeV^{-4}]
World data	$0.182 \pm 0.016 \pm 0.034$	$\boxed{-0.117 \pm 0.091 \pm 0.095}$
GDH Sum Rule	0.0618	-
χEFT Bernard *et al.*	0.07	0.3
χEFT Alarcón *et al.*	0.066(4)	0.25(12)
Burkert-Ioffe	0.09	0.3
Pasechnik *et al.*	0.09	0.4
LFHQCD	0.177	-0.067

α_s at long distance (low Q)

Prescription: Define effective couplings from an observable's perturbative series truncated to first order in α_s.

G. Grunberg, PLB B95 70 (1980); PRD 29 2315 (1984); PRD 40 680(1989).

Ex: Bjorken sum rule:

$$\int (g^p_1 - g^n_1)dx \triangleq \Gamma_1^{p-n} = \frac{1}{6} g_A \left(1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - \ldots\right) + \frac{M^2}{9Q^2}[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s)] + \ldots$$

Nucleon axial charge.

pQCD corrections (gluon bremsstrahlung)

Higher Twists: $1/Q^{2n}$ corrections.
Non-perturbative quantities. Express correlations between parton distributions and confinement forces.

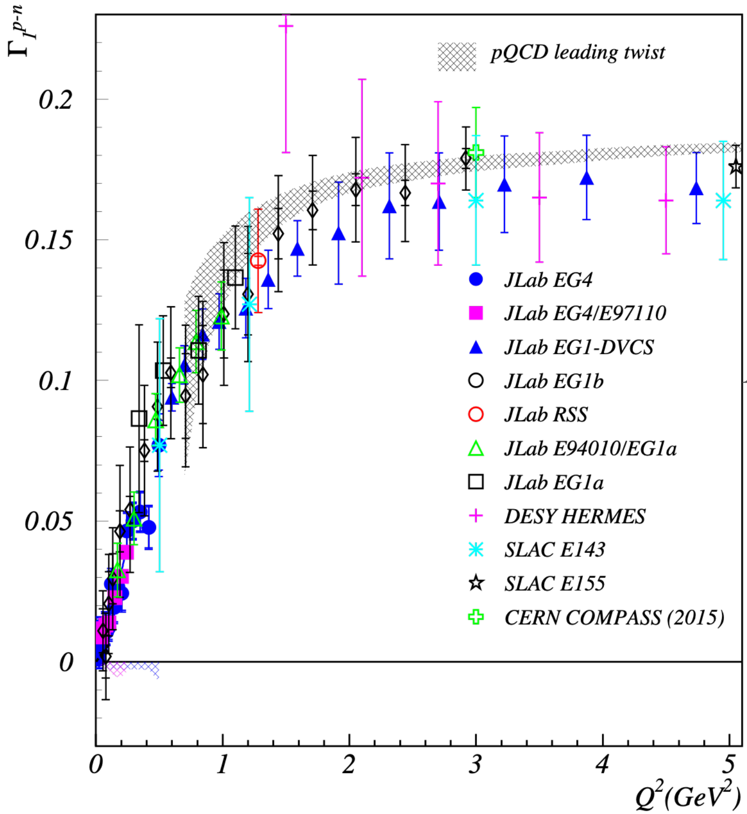
$$\Rightarrow \boxed{\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A \left(1 - \frac{\alpha_{g1}}{\pi}\right)}$$

This means that additional short distance effects, and long distance confinement force and parton distribution correlations are now folded into the definition of α_s.

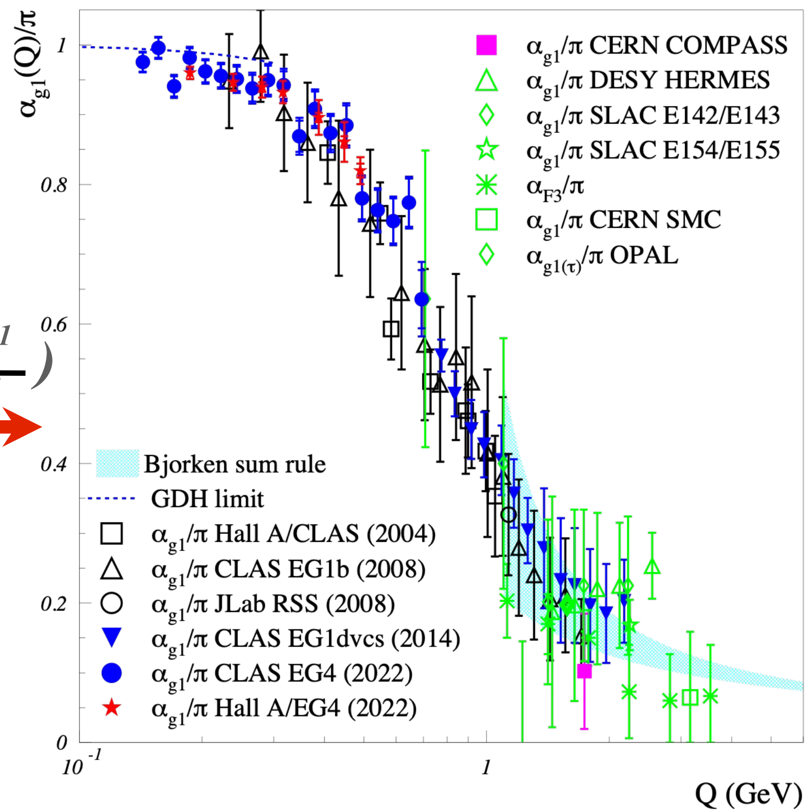
Analogy with the original coupling constant becoming an effective coupling when short distance quantum loops are folded into its definition.

α_{g1} Extracted from the Bjorken Sum data

Bjorken sum Γ_1^{p-n} measurements



$$\Gamma_1^{p-n} \triangleq \frac{1}{6} g_A\left(1-\frac{\alpha_{g1}}{\pi}\right)$$



Low Q limit

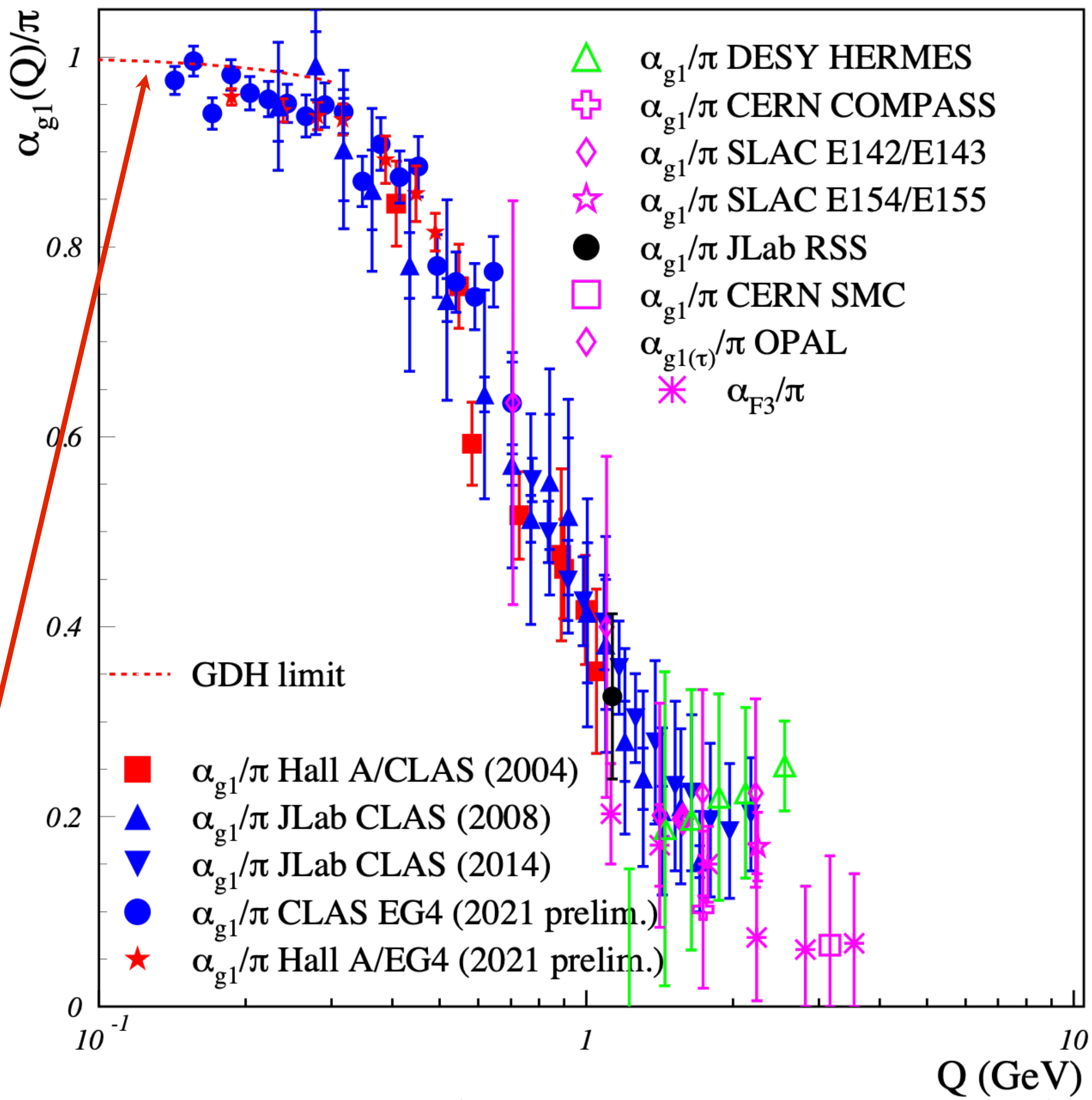
At $Q^2 = 0$, a sum rule related to the Bjorken sum rule exists: the Gerasimov-Drell-Hearn (GDH) sum rule:

At $Q^2 = 0$, GDH sum rule:

$$\Gamma_1 = \frac{-\kappa^2 Q^2}{8M^2}$$

$\Rightarrow Q^2 = 0$ constraints:

$$\Rightarrow \boxed{\begin{cases} \alpha_{g1} = \pi \\ \dfrac{d\alpha_{g1}}{dQ^2} = \dfrac{3\pi}{4g_A}\left(\dfrac{\kappa_n^2}{M_n^2} - \dfrac{\kappa_p^2}{M_p^2}\right) \end{cases}}$$

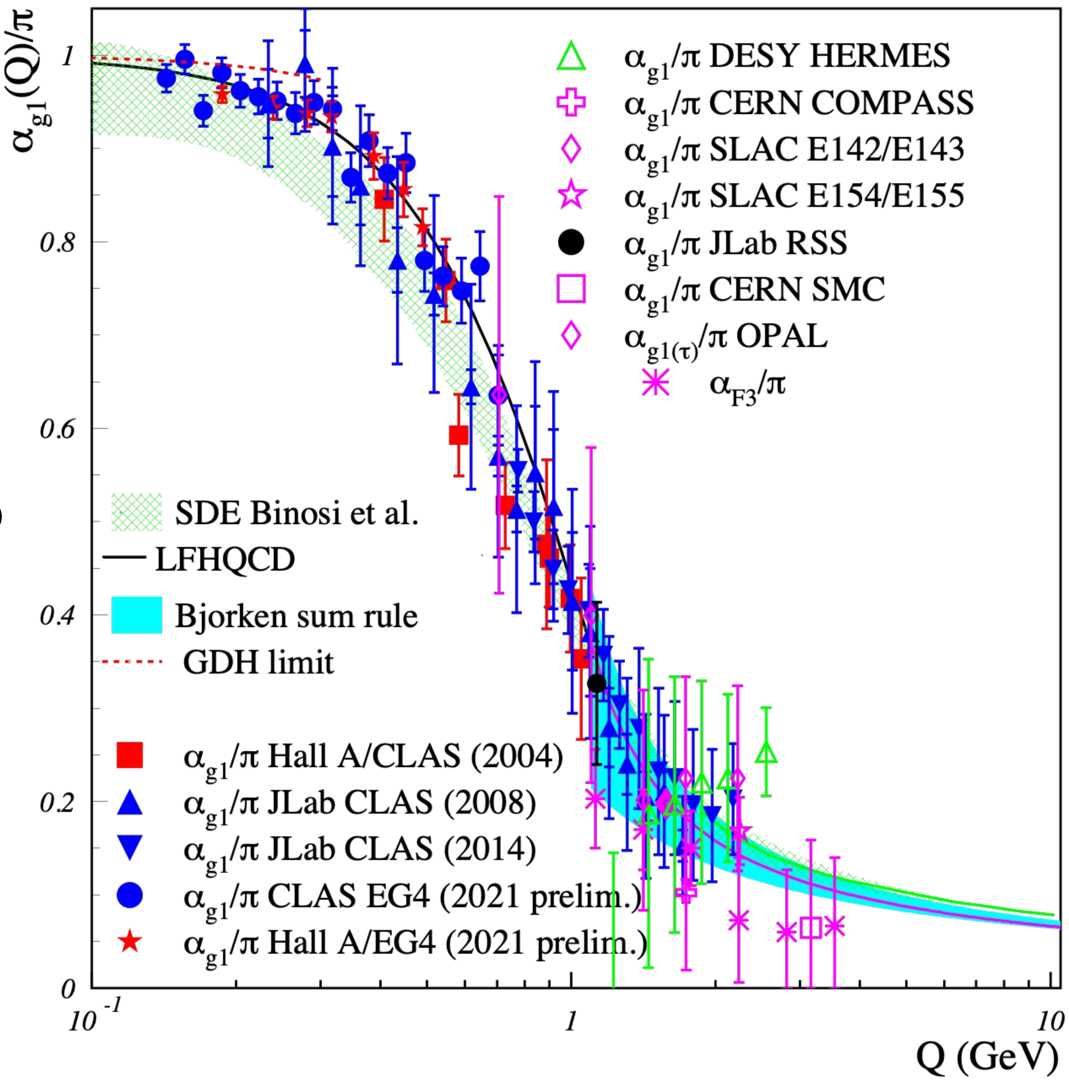


First experimental evidence of nearly *conformal behavior* (i.e. no Q^2-dependence) of QCD at low Q^2.

Comparisons with SDE and LFHQCD Calculations

Binosi et al. PRD 96, 054026 (2017)

Brodsky, de Téramond, Dosch, Lorcé, PLB 759, 171 (2016)



⇒ SDE, LFHQCD and data agree very well.

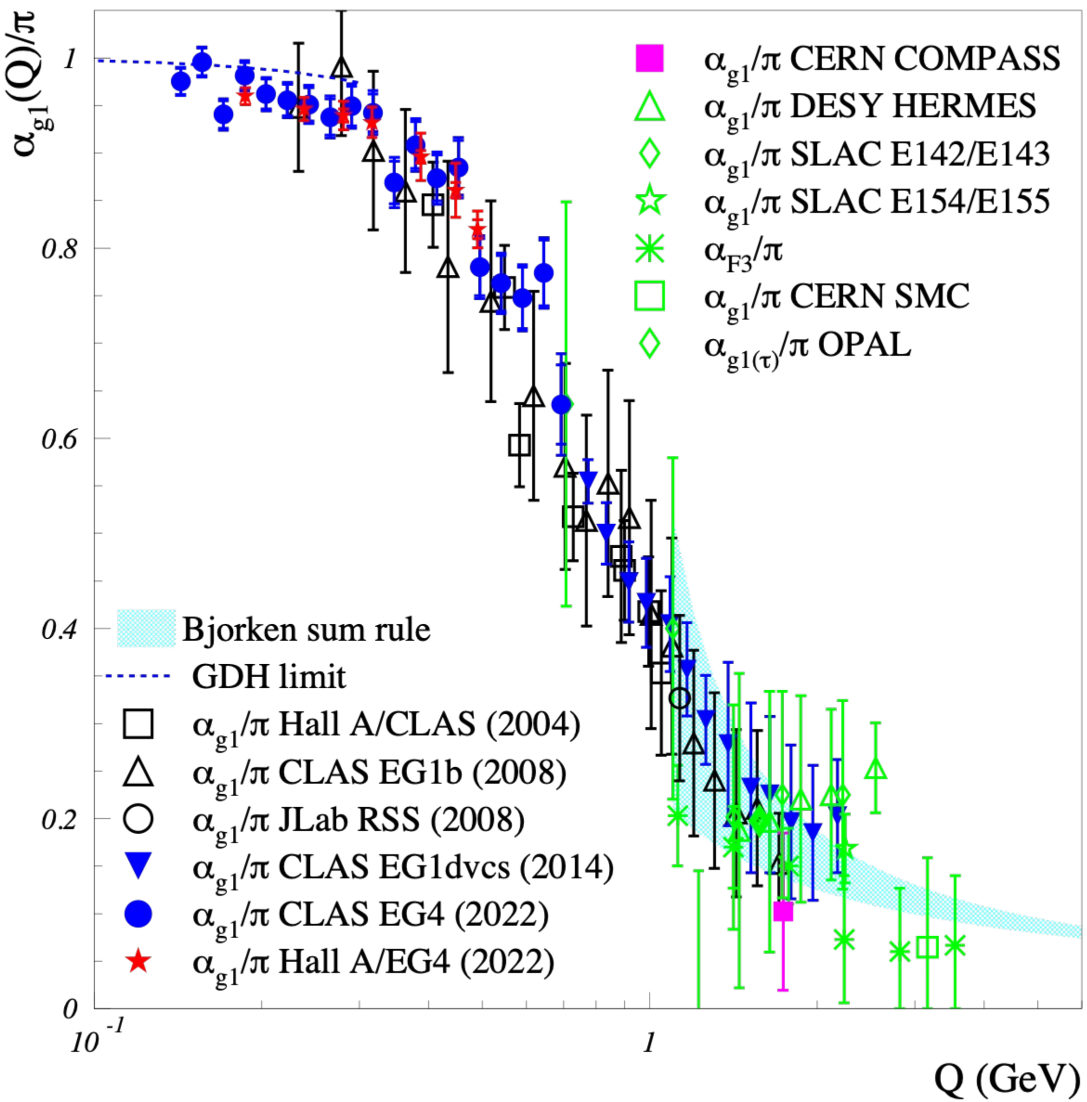
Effective Coupling and Impact

Featured as Cover
Featured in JLab News
https://phys.org/news/2022-08-strength-strong.html
Featured in YouTube
https://www.youtube.com/watch?v=8BTZOz850GI&t=497s
hailed as
"accidental discovery"
"pretty major breakthrough"

Base for understanding of emergence of hadron properties, can have impact on:

hadron spectroscopy
PDFs and GPDs
quark mass functions
pion decay constant
scale of QCD, Λs
QCD Phase/Hot QCD
…

A. Deur, V. Burkert, J. P. Chen and W. Korsch
Particles, 5-171 (2022)



Summary

- Bjorken Sum Rule : Link flavor non-singleton (isovector) part of the nucleon spin structure moment with the axial charge
- Generalized Bjorken/GDH Sum Rules provide a tool to study QCD in full Q^2 range
  - Extractions of (effective) strong coupling α_s (α_{g1})
- Experimental Data on Bjorken Sum Over a Wide Q^2 range
- High Q2: PQCD, extraction of strong coupling α_s, potential of JLab22 + EIC
- Intermediate Q2: Transition from PQCD to Strong QCD region
- Low Q2: Strong QCD region, 1st extraction of effective strong coupling α_{g1}

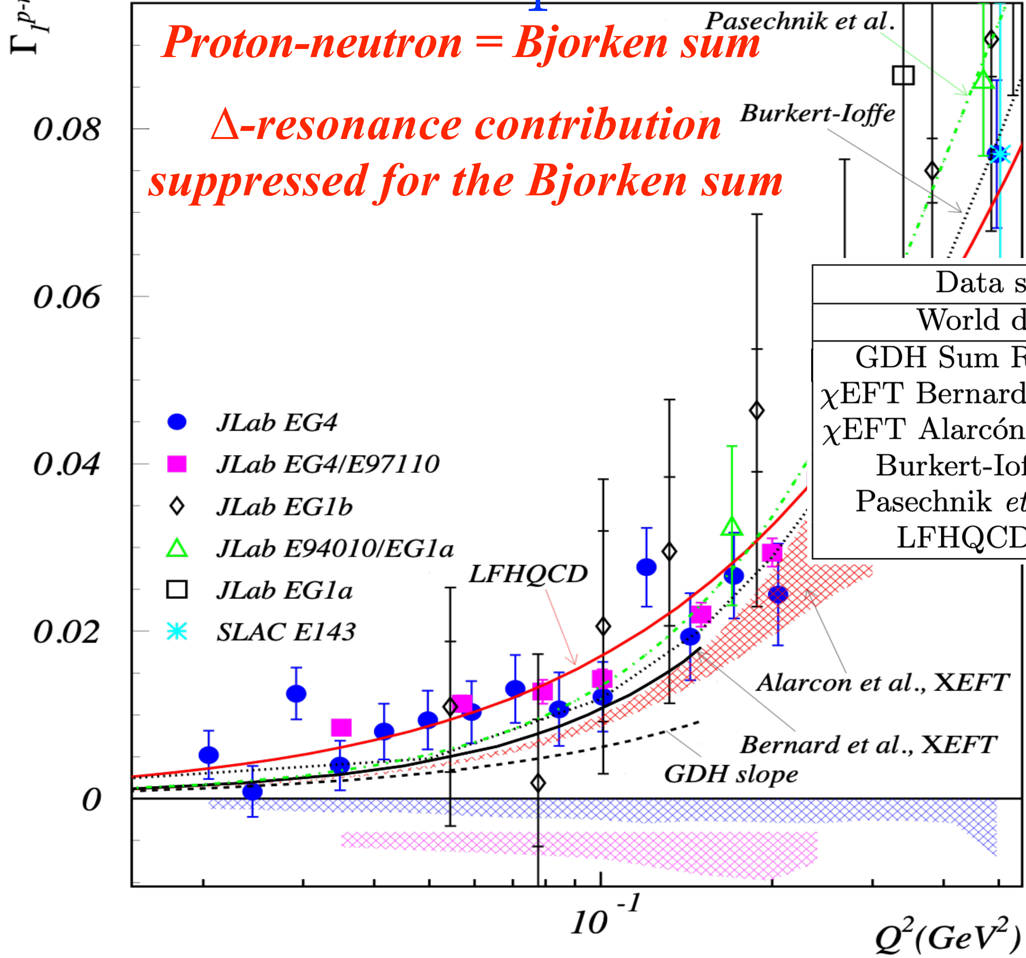
Extracted effective strong coupling from the new JLab low-Q data

→ conformal behavior,

providing a potential base for understanding strong QCD

significant impact

Γ_1^{p-n} measurements from E97-110 and EG4



χEFT prediction

Fit $\Gamma_1 = bQ^2+cQ^4$:

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χEFT prediction

Fits $\Gamma_1 = a + bQ^2+cQ^4+dQ^6$:

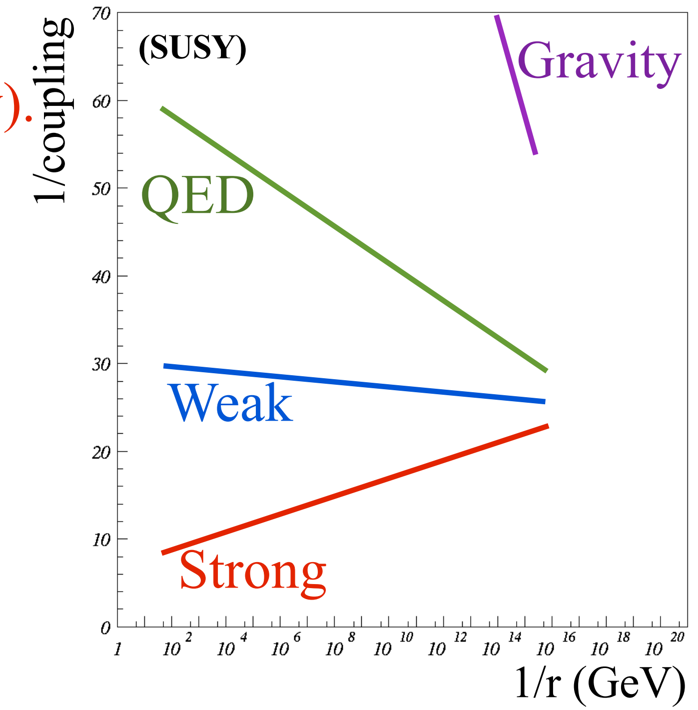
Data set	$(a \pm uncor \pm cor)$	$(b \pm uncor \pm cor)$ [GeV^{-2}]	$c \pm uncor \pm cor$ [GeV^{-4}]	$d \pm uncor \pm cor$ [GeV^{-6}]	$\chi^2/n.d.f.$
EG4, no low-x	NA	$0.093 \pm 0.032 \pm 0.000$	$-0.137 \pm 0.191 \pm 0.000$	NA	1.24
EG4/E97110, no low-x	NA	$0.112 \pm 0.022 \pm 0.028$	$-0.123 \pm 0.118 \pm 0.078$	NA	1.00
EG4	NA	$0.170 \pm 0.032 \pm 0.000$	$-0.046 \pm 0.191 \pm 0.000$	NA	1.04
EG4/E97110	NA	$0.185 \pm 0.023 \pm 0.027$	$-0.144 \pm 0.123 \pm 0.075$	NA	1.00
World data	NA	$\mathbf{0.182 \pm 0.016 \pm 0.034}$	$\mathbf{-0.117 \pm 0.091 \pm 0.095}$	NA	**1.00**
World data	NA	$b^{GDH} \equiv 0.0618$	$1.41 \pm 0.17 \pm 0.39$	$-4.30 \pm 0.80 \pm 1.48$	1.97
World data	$(4.3 \pm 1.8 \pm 0.1) \times 10^{-3}$	$0.092 \pm 0.042 \pm 0.031$	$0.213 \pm 0.167 \pm 0.086$	NA	0.82

Coupling constants

When charges are quantized: (coupling constant)$^{1/2}$ normalizes the unit charge to 1 (e.g. α
⇒ set the magnitude of the force (classical domain) or the probability amplitude to emit a

Force=coupling constant × charge$_1$ × charge$_2$ × f(r)
(static case)

α (QED), α_s (QCD), G_F (Weak Force), G_N (gravity).
Quantum effects induce an energy dependence.
(effective couplings: the couplings are “running”)



α_s at long distance (low Q^2)

The effective coupling is then:

- Extractable at any Q^2;
- Free of divergence;
- Renormalization scheme independent.

But it is:

- Process dependent.

⇒There is *a priori* a different α_s for each different process.

However these α_s can be related (Commensurate Scale Relations).

S. J. Brodsky & H. J. Lu, PRD 51 3652 (1995)
S. J. Brodsky, G. T. Gabadadze, A. L. Kataev, H. J. Lu, PLB 372 133 (1996)

⇒ pQCD retains it predictive power.

Such definition of α_s using a particular process is equivalent to a particular choice of renormalization scheme.

(process dependence) ⇔ (scheme dependence)

$\alpha_{g1} = \alpha_s$ in the "g_1 scheme".
Relations between g_1 scheme and other schemes are known in pQCD domain, e.g.
$\Lambda_{g_1} = 2.70\Lambda_{\overline{MS}} = 1.48\Lambda_{MOM} = 1.92\Lambda_V = 0.84\Lambda_\tau$.