NUCLEON STRUCTURE IN LIGHT MUONIC ATOMS

Franziska Hagelstein (JGU Mainz & PSI Villigen)

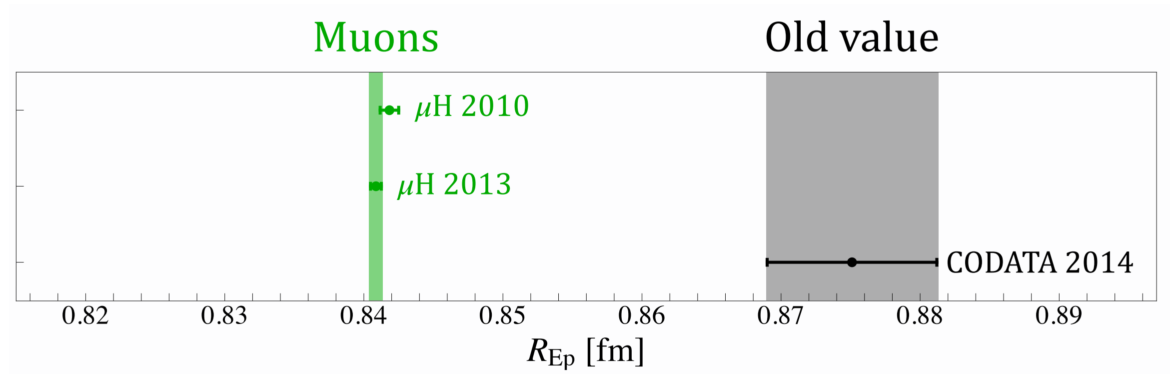
in collaboration with

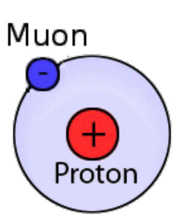
Volodymyr Biloshytskyi, Vadim Lensky and Vladimir Pascalutsa (JGU)

2305.09633

PROTON RADIUS PUZZLE

5.6 σ discrepancy





μH spectroscopy

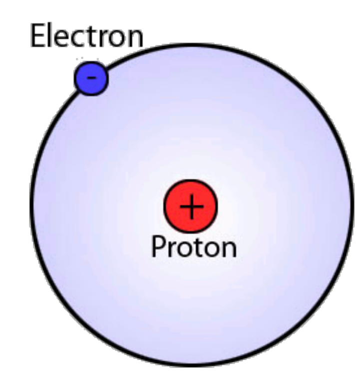
$$[R_{Ep}^{\mu\mathrm{H}} = 0.84087(39)\ \mathrm{fm}]$$

R. Pohl, A. Antognini et al., Nature **466**, 213 (2010)
A. Antognini et al., Science **339**, 417 (2013)

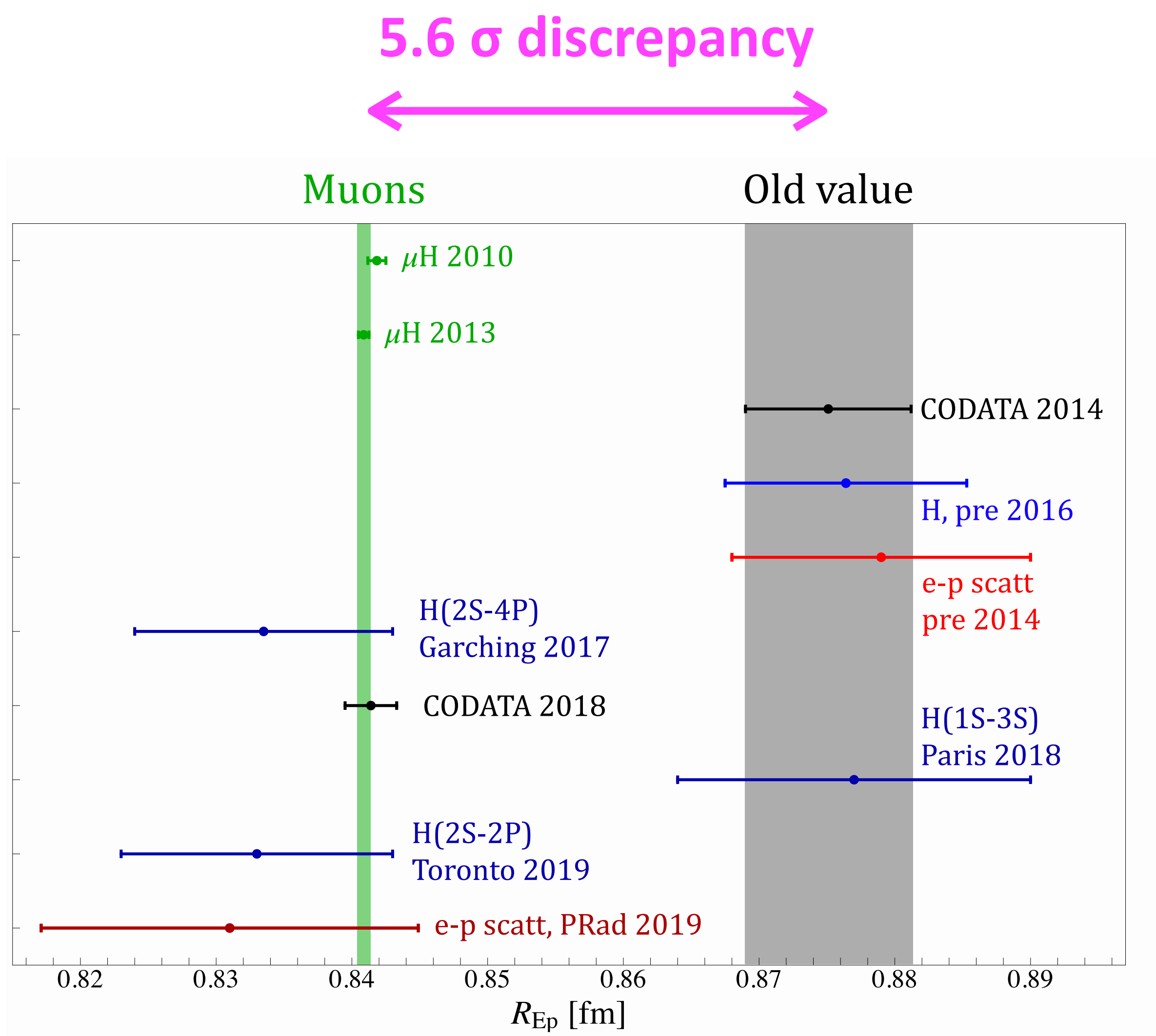
ep scattering, eH spectroscopy

$$[R_{Ep}^{\mathrm{CODATA\ 2014}} = 0.8751(61)\ \mathrm{fm}]$$

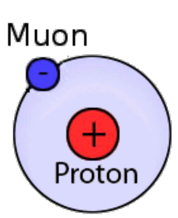
P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012)



PROTON RADIUS PUZZLE



Is it still a puzzle?



μH spectroscopy

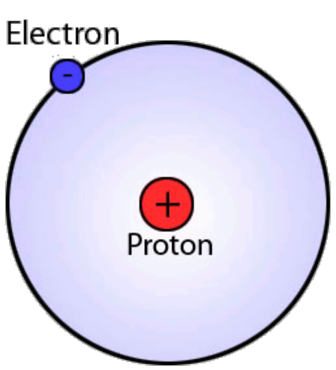
$$[R_{Ep}^{\mu\mathrm{H}} = 0.84087(39)\ \mathrm{fm}]$$

R. Pohl, A. Antognini et al., Nature **466**, 213 (2010)
A. Antognini et al., Science **339**, 417 (2013)

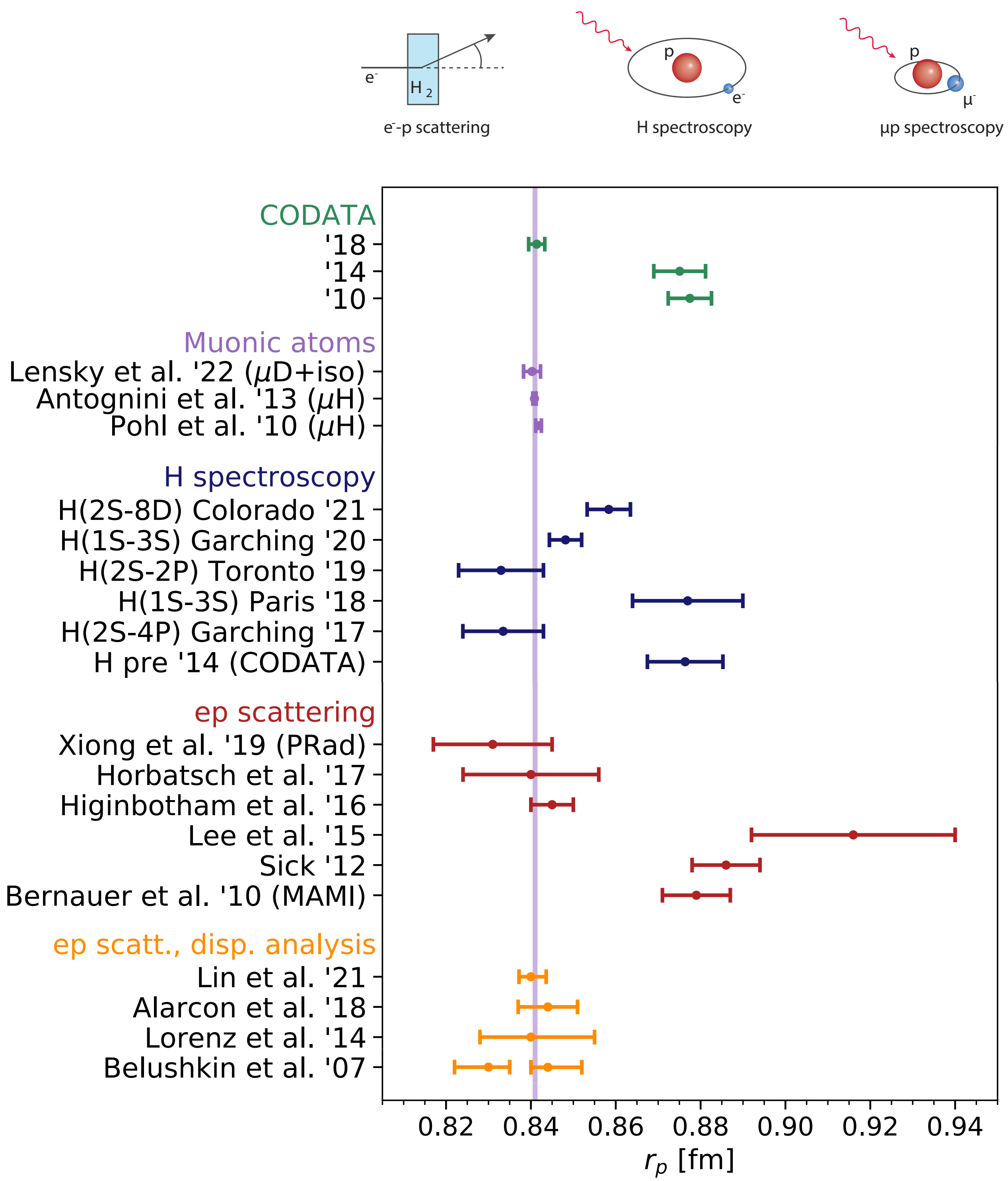
ep scattering, eH spectroscopy

$$[R_{Ep}^{\mathrm{CODATA\ 2014}} = 0.8751(61)\ \mathrm{fm}]$$

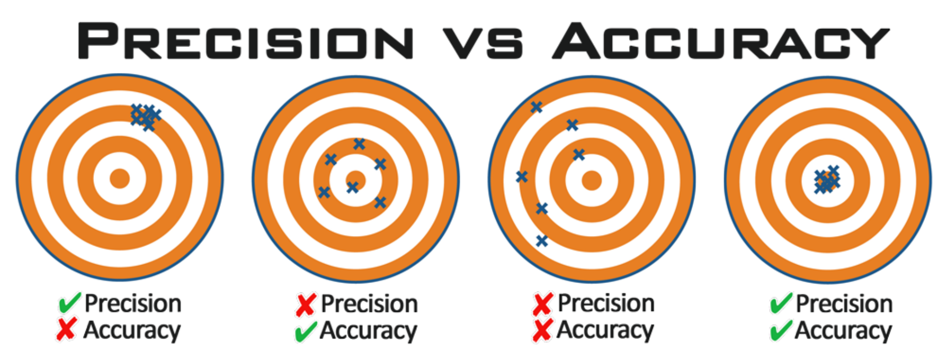
P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012)



PROTON CHARGE RADIUS



- Muonic atoms allow for PRECISE extractions of nuclear charge and Zemach radii
- CODATA since 2018 included the μH result for r_p
- Still open issues: H(2S-8D) and H(1S-3S)
- Question:



FROM PUZZLE TO PRECISION

- Several experimental activities ongoing and proposed:
  - 1S hyperfine splitting in μH and μHe (CREMA, FAMU, J-PARC)
  - Improved measurement of Lamb shift in μH, μD and μHe$^+$ possible ($\times 5$)
  - Medium- and high-Z muonic atoms

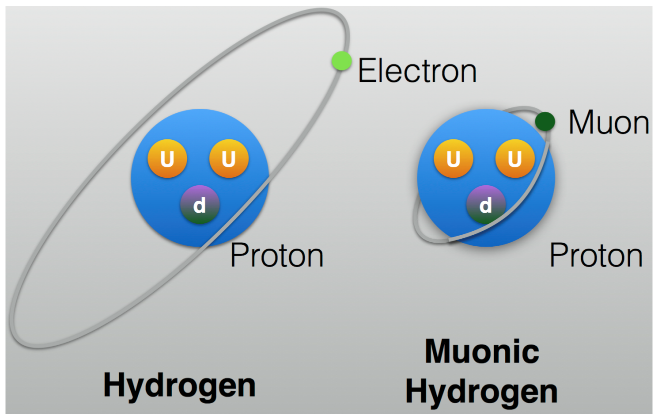
- **Theory support** is needed!

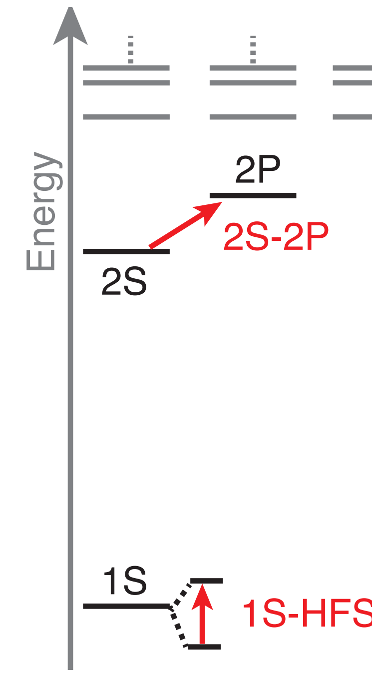
Muonic Atom Spectroscopy Theory Initiative

- First brainstorming meeting October 2022 @ PSI
- Initials objectives:
  - Accurate theory predictions for light muonic atoms to test fundamental interactions by comparing to electronic atoms
  - Community consensus on SM predictions
  - Emphasis on the hyperfine splitting in μH
- **Join us Saturday to discuss hadronic contributions to atomic spectra**
- Kick-off meeting (PREN & μASTI 2023): 26.06.2023 - 30.06.2023 @ JGU, Mainz
- Updates and mailing list on https://asti.uni-mainz.de

NUCLEAR STRUCTURE EFFECTS

Why muonic atoms ?





Lamb shift
μH, μD, μ^3He$^+$, μ^4He$^+$
⇒ Charge radii

Hyperfine splitting (HFS)
μH, μ^3He$^+$
⇒ Zemach radii, magnetic properties

- **Lamb shift:**

wave function at the origin

$$\Delta E_{nl}(\text{LO+NLO}) = \delta_{l0}\,\frac{2\pi Z\alpha}{3}\,\frac{1}{\pi (an)^3}\left[R_E^2 - \frac{Z\alpha m_r}{2} R^3_{E(2)}\right]$$

NLO becomes appreciable in μH

- **HFS:**

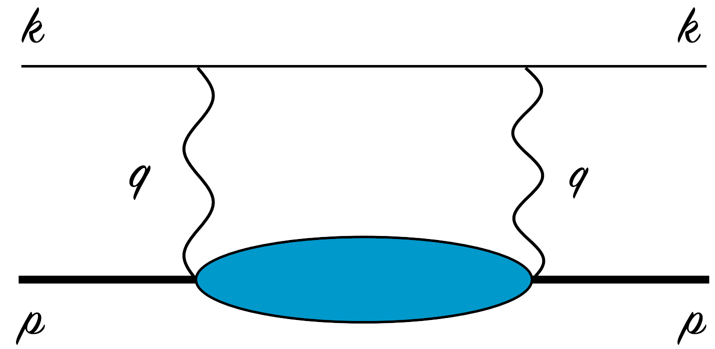
$$\Delta E_{nS}(\text{LO}+\text{NLO}) = E_F(nS)\,[1 - 2\,Z\alpha m_r\,R_Z]$$

Fermi energy:
$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$
with Bohr radius $a = 1/(Z\alpha m_r)$

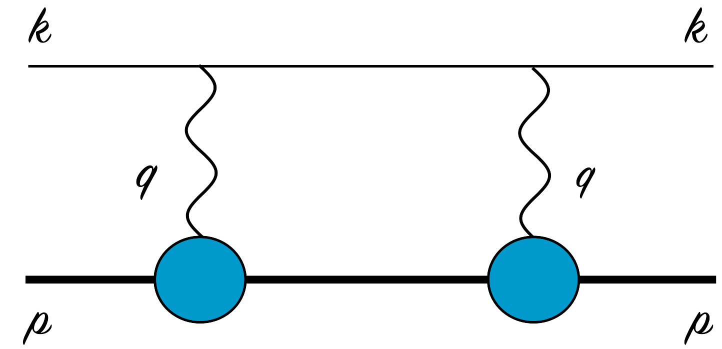
STRUCTURE EFFECTS THROUGH 2γ

- Proton-structure effects at subleading orders arise through multi-photon processes

forward two-photon exchange (2γ)



polarizability contribution
(non-Born VVCS)



elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

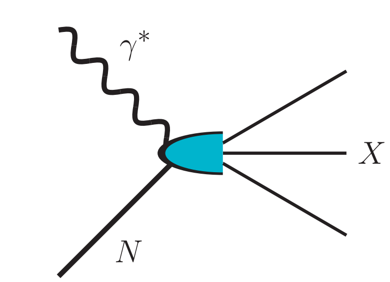
- "Blob" corresponds to doubly-virtual Compton scattering (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

- Proton structure functions: $\boxed{f_1(x,Q^2),\ f_2(x,Q^2)}$, $\boxed{g_1(x,Q^2),\ g_2(x,Q^2)}$

Lamb shift

Hyperfine splitting (HFS)



2γ EFFECT IN THE LAMB SHIFT

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\, \phi_n^2\, \frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

dispersion relation & optical theorem:

$$T_1(\nu,Q^2) = \boxed{T_1(0,Q^2)} + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\, \frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\, \frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

- Caution: in the data-driven dispersive approach the $T_1(0,Q^2)$ subtraction function is modelled!

low-energy expansion:

$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

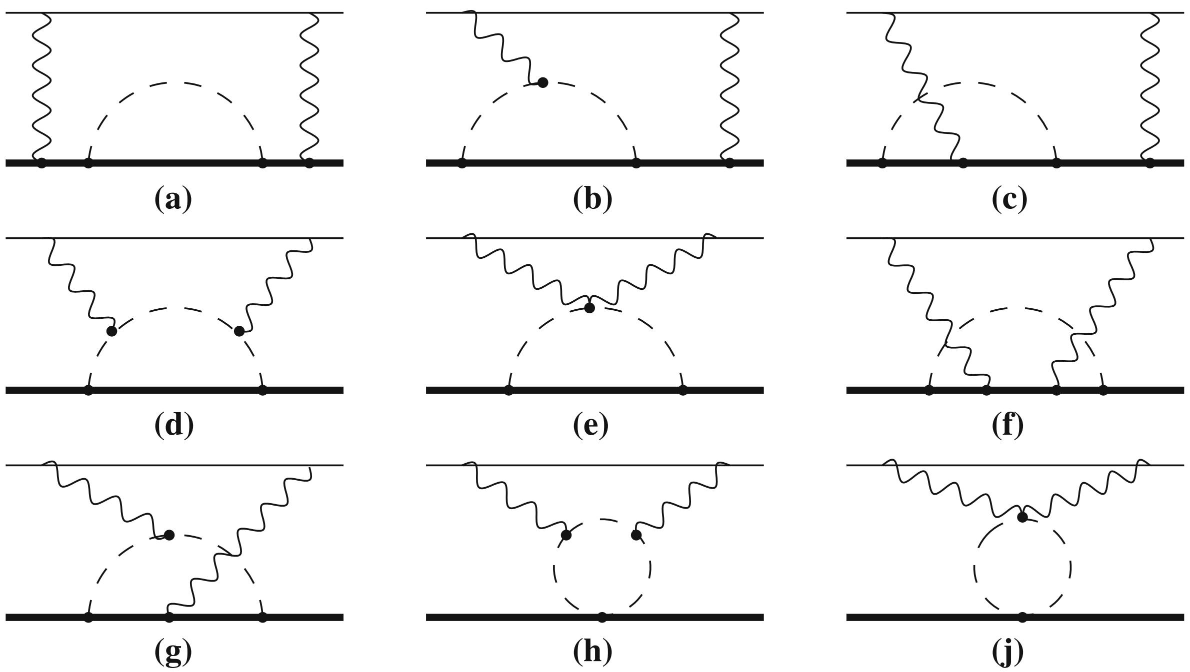
modelled Q^2 behavior:

$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\, Q^2/\left(1+Q^2/\Lambda^2\right)^4$$

see talks by V. Pascalutsa and V. Biloshytskyi

Assuming ChPT is working, it should be best applicable to atomic systems, where the energies are very small !

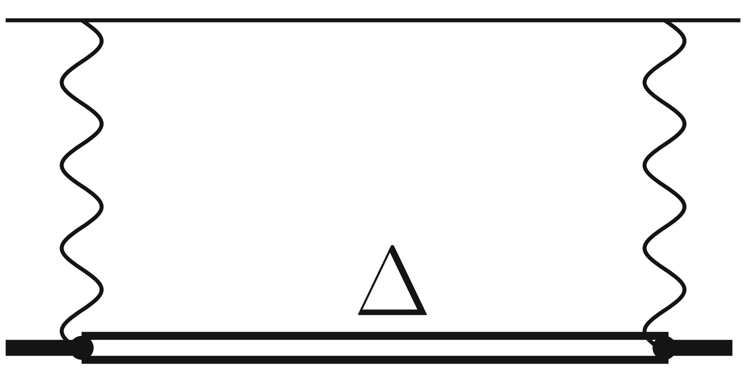
POLARIZABILITY EFFECT FROM BCHPT



J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

- LO BChPT prediction with pion-nucleon loop diagrams:

$$\Delta E^{\langle \mathrm{LO}\rangle \mathrm{pol}}(2S,\mu\mathrm{H}) = -\,9.6^{+1.4}_{-2.9}\,\mu\mathrm{eV}$$



V. Lensky, FH, V. Pascalutsa, M. Vanderhaeghen,
Phys. Rev. D **97** (2018) 074012

- Δ prediction from Δ(1232) exchange:
  - Uses large-N_c relations for the Jones-Scadron N-to-Δ transition form factors
  - Small due to the suppression of β_{M1} in the Lamb shift but important for the T_1 subtraction function

$$\Delta E^{\langle \Delta-\mathrm{excit}\rangle \mathrm{pol}}(2S,\mu\mathrm{H}) = 0.95 \pm 0.95\,\mu\mathrm{eV}$$

POLARIZABILITY EFFECT IN LAMB SHIFT

BChPT result is in good agreement with dispersive calculations !!!

Agreement also for the contribution of the T_1 subtraction function !!!

Table 1 Forward 2γ-exchange contributions to the 2S-shift in μH, in units of μeV.

Reference	$E_{2S}^{(\mathrm{subt})}$	$E_{2S}^{(\mathrm{inel})}$	$E_{2S}^{(\mathrm{pol})}$	$E_{2S}^{(\mathrm{el})}$	$E_{2S}^{\langle 2\gamma \rangle}$
DATA-DRIVEN					
(73) Pachucki '99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(74) Martynenko '06	2.3	−16.1	−13.8(2.9)		
(75) Carlson *et al.* '11	5.3(1.9)	−12.7(5)	−7.4(2.0)		
(76) Birse and McGovern '12	4.2(1.0)	−12.7(5)	−8.5(1.1)	−24.7(1.6)	−33(2)
(77) Gorchtein et al.'13 [a]	−2.3(4.6)	−13.0(6)	−15.3(4.6)	−24.5(1.2)	−39.8(4.8)
(78) Hill and Paz '16					−30(13)
(79) Tomalak'18	2.3(1.3)		−10.3(1.4)	−18.6(1.6)	−29.0(2.1)
LEADING-ORDER BχPT					
(80) Alarcòn *et al.* '14			$-9.6^{+1.4}_{-2.9}$		
(81) Lensky *et al.* '17 [b]	$3.5^{+0.5}_{-1.9}$	−12.1(1.8)	$-8.6^{+1.3}_{-5.2}$		
LATTICE QCD					
(82) Fu *et al.* '22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.
[b] Partially includes the $\Delta(1232)$-isobar contribution.

see talk by Xu Feng

LAMB SHIFT IN MUONIC ATOMS

THEORY **EXPERIMENT**

	$\Delta E_{TPE} \pm \delta_{theo}\,(\Delta E_{TPE})$	Ref.	$\delta_{exp}(\Delta_{LS})$	Ref.
μH	33 μeV $\pm$ 2 μeV	Antognini et al. (2013)	2.3 μeV	Antognini et al. (2013)
μD	1710 μeV $\pm$ 15 μeV	Krauth et al. (2015)	3.4 μeV	Pohl et al. (2016)
μ^3He$^+$	15.30 meV $\pm$ 0.52 meV	Franke et al. (2017)	0.05 meV	
μ^4He$^+$	9.34 meV $\pm$ 0.25 meV $-$0.15 meV $\pm$ 0.15 meV (3PE)	Diepold et al. (2018) Pachucki et al. (2018)	0.05 meV	Krauth et al. (2020)

μH: present accuracy comparable with experimental precision

μD, μ³He⁺, μ⁴He⁺: present accuracy factor 5-10 worse than experimental precision

$$r_p = 0.84087(12)_{\rm sys}(23)_{\rm stat}(29)_{\rm theory}\ \ {\rm fm}$$

(25) 2PE (mainly subtraction term)
(15) QED

$$r_d = 2.12562(5)_{\rm sys}(12)_{\rm stat}(77)_{\rm theory}\ \ {\rm fm}$$

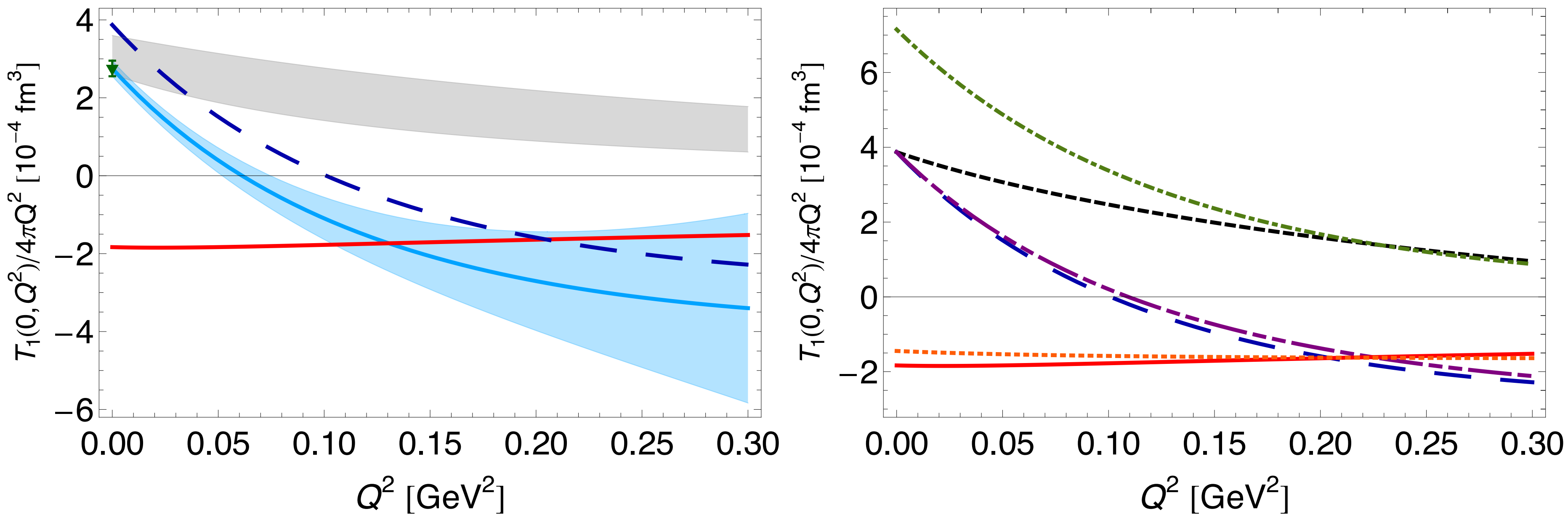
basically only nuclear 2PE

$$r_\alpha = 1.67824(2)_{\rm sys}(13)_{\rm stat}(82)_{\rm theory}\ \ {\rm fm}$$

(70) 2PE (elastic 25, nuclear inelastic 36, nucleon inelastic 56)
(42) 3PE (inelastic contribution missing)
(4) QED

from talk by M.Vanderhaeghen

SUBTRACTION FUNCTION



NLO BChPT δ-exp.

total without g_M dipole

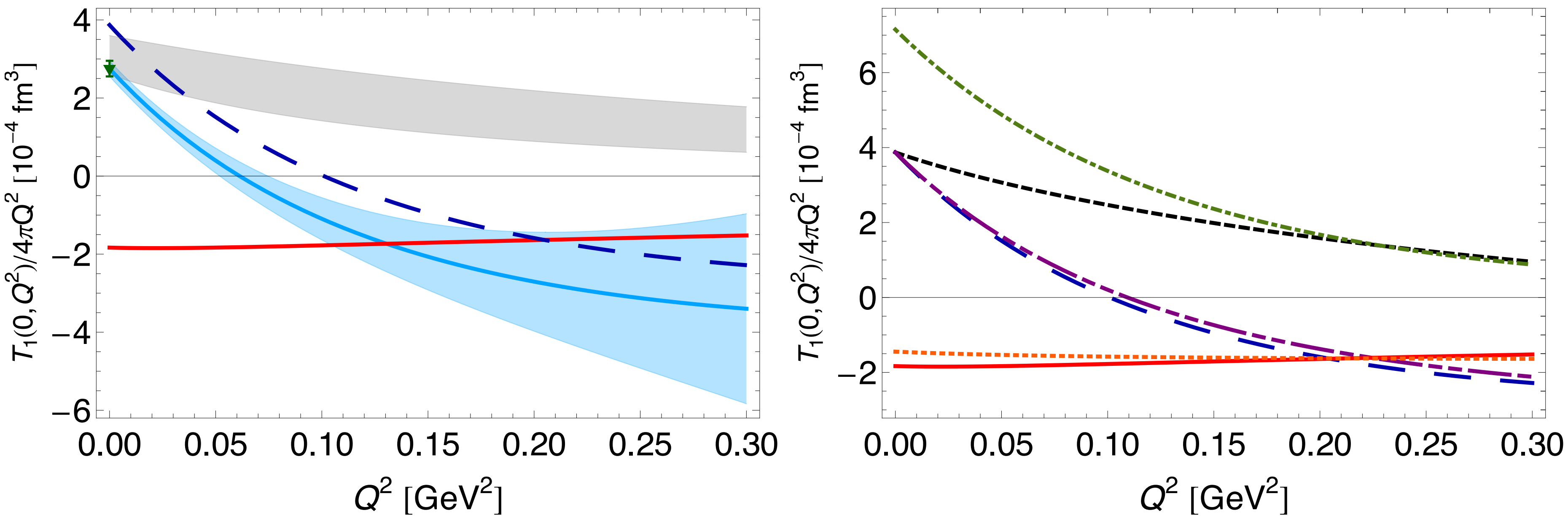
πN loops

πΔ loops

Δ-exchange

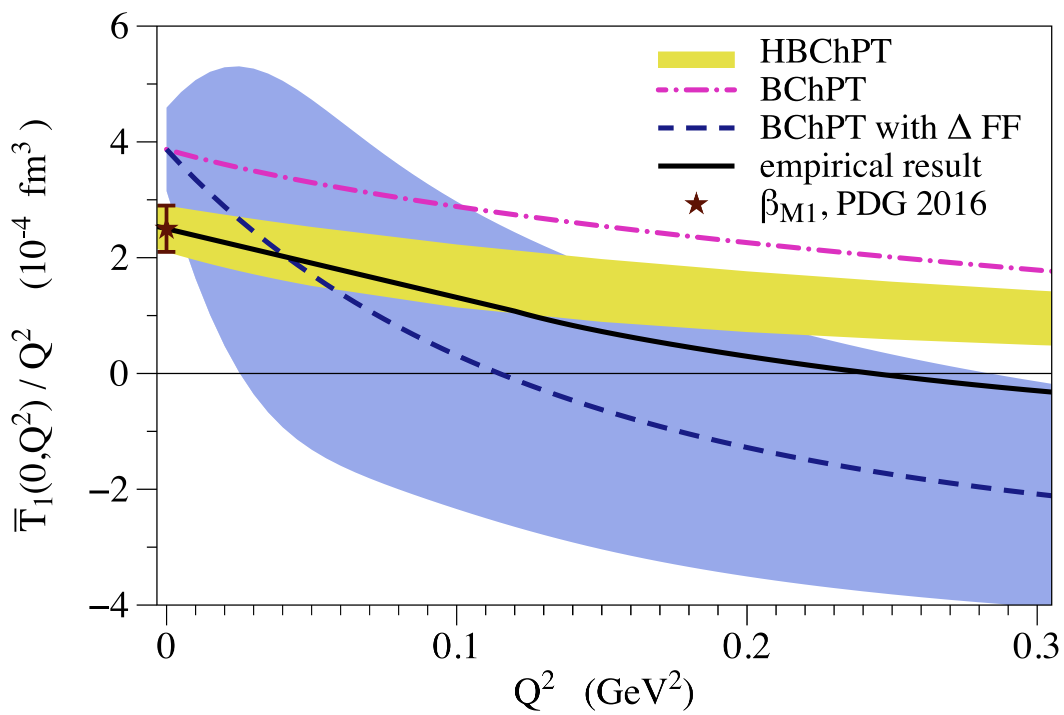
J. Alarcon, FH, V. Lensky and V. Pascalutsa, Phys. Rev. D **102** (2020) 114026; ibid. **102** (2020) 114006

SUBTRACTION FUNCTION



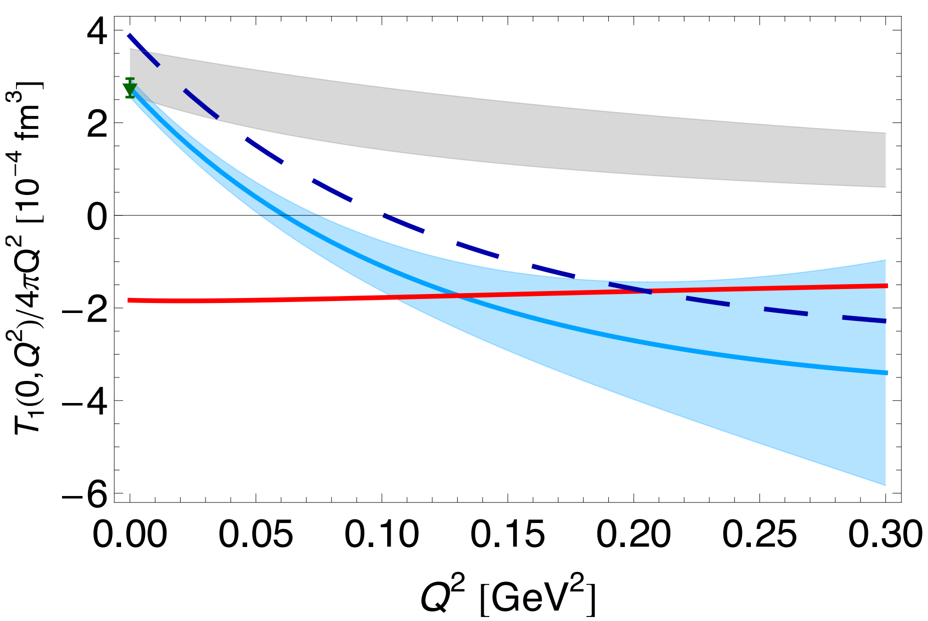
NLO BChPT δ-exp.
total without g_M dipole
πN loops
πΔ loops
Δ-exchange

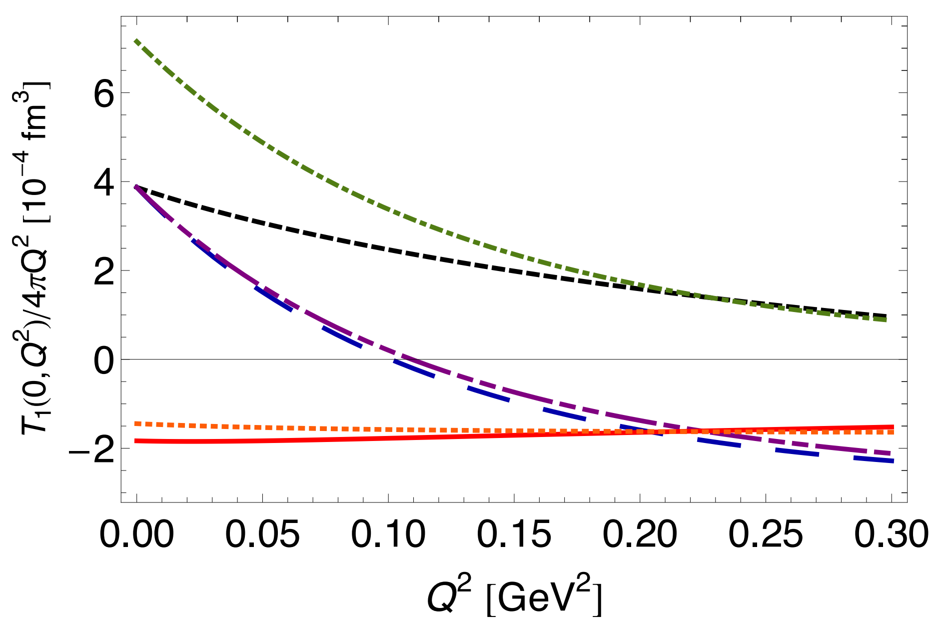
J. Alarcon, FH, V. Lensky and V. Pascalutsa, Phys. Rev. D **102** (2020) 114026; ibid. **102** (2020) 114006



V. Lensky, FH, V. Pascalutsa and M. Vanderhaeghen Phys. Rev. D **97** (2018) 074012

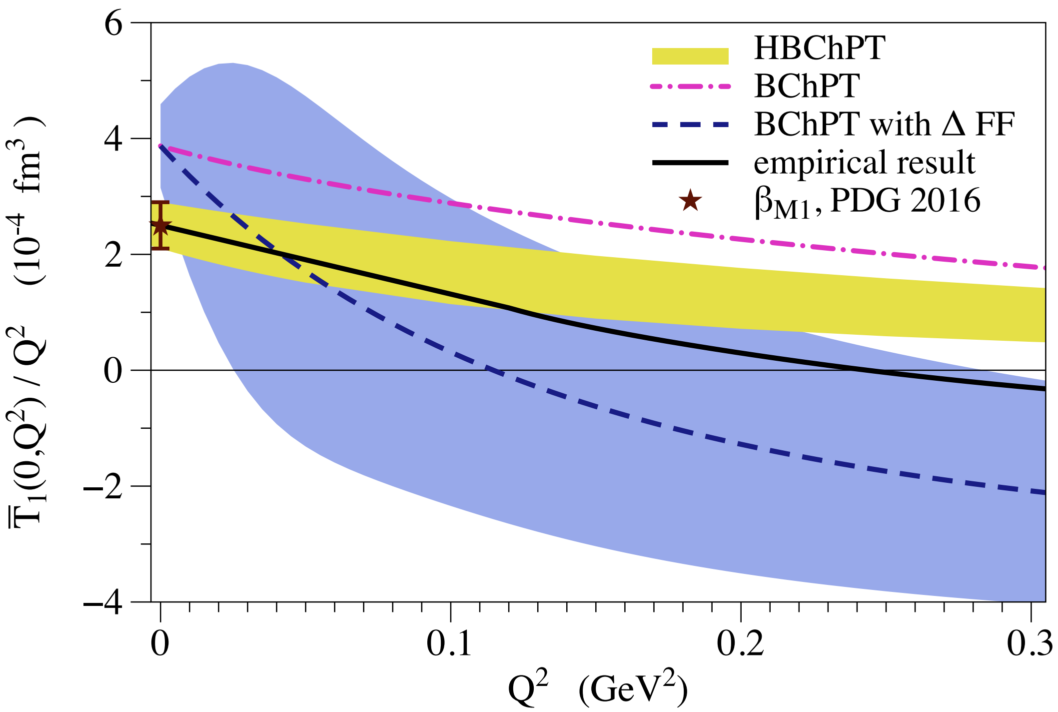
SUBTRACTION FUNCTION





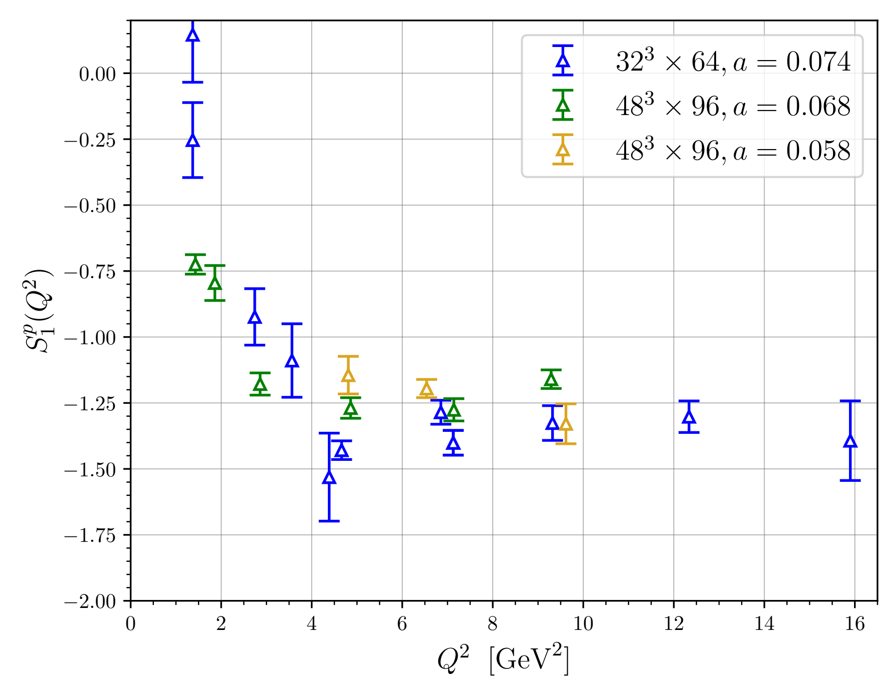
NLO BChPT δ-exp.
total without g_M dipole
πN loops
πΔ loops
Δ-exchange

J. Alarcon, FH, V. Lensky and V. Pascalutsa,
Phys. Rev. D **102** (2020) 114026;
ibid. **102** (2020) 114006



V. Lensky, FH, V. Pascalutsa and M. Vanderhaeghen
Phys. Rev. D **97** (2018) 074012

First lattice results!



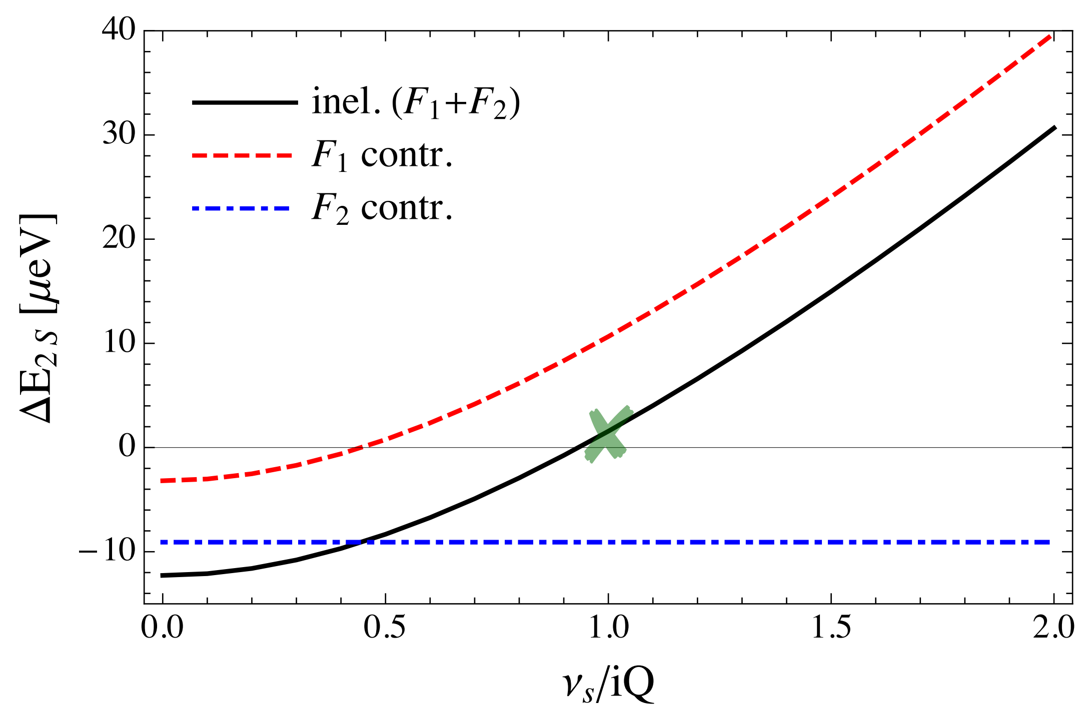
CSSM-QCDSF-UKQCD Collaboration, 2207.03040.

EUCLIDEAN SUBTRACTION FUNCTION

- Once-subtracted dispersion relation for $\overline{T}_1(\nu, Q^2)$ with subtraction at $\nu_s = iQ$
- Dominant part of polarizability contribution:

$$\Delta E_{nS}^{\prime\,(\mathrm{subt})} = \frac{2\alpha m}{\pi}\phi_n^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3} \frac{2+v_l}{(1+v_l)^2}\,\overline{T}_1(iQ, Q^2) \text{ with } v_l = \sqrt{1+4m^2/Q^2}$$

- Inelastic contribution for $\nu_s = iQ$ is order of magnitude smaller than for $\nu_s = 0$
- Prospects for future lattice QCD and EFT calculations



FH, V. Pascalutsa, Nucl. Phys. A **1016** (2021) 122323

based on Bosted-Christy parametrization:

$$\Delta E_{2S}^{(\mathrm{inel})}(\nu_s = 0) \simeq -12.3\,\mu\mathrm{eV}$$

$$\Delta E_{2S}^{\prime\,(\mathrm{inel})}(\nu_s = iQ) \simeq 1.6\,\mu\mathrm{eV}$$

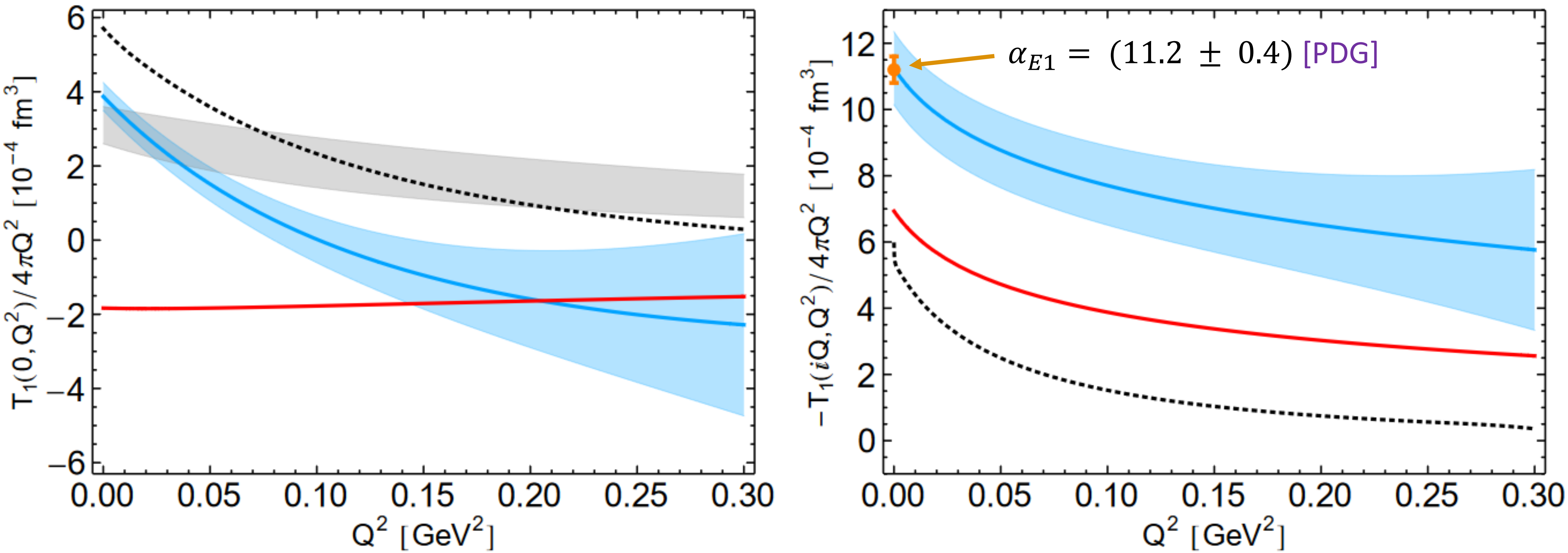
DATA-DRIVEN EVALUATION

2305.08814
see talk by
V. Biloshytskyi

- New integral equations for data-driven evaluation of subtraction functions
- High-quality parametrization of σ_L at $Q \to 0$ needed

$$T_1(0,Q^2) = \frac{2Q^2}{\pi} \int_{\nu_0}^{\infty} \frac{\mathrm{d}\nu}{\nu^2 + Q^2} \left[\sigma_T - \frac{\nu^2}{Q^2}\sigma_L\right](\nu, Q^2)$$

$$T_L(iQ,Q^2) = \frac{2}{\pi} \int_{\nu_0}^{\infty} \mathrm{d}\nu\, \nu^2 \frac{\sigma_L(\nu, Q^2)}{\nu^2 + Q^2}$$



MAID

NLO χPT [Lensky et al., PRC (2014)] [Alarcón et al., PRD (2020)]

LO χPT: πN-loops

HBχPT [Birse and McGovern, EPJA, (2012)]

HYPERFINE SPLITTING IN μH

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}]\, E_F(nS)$$

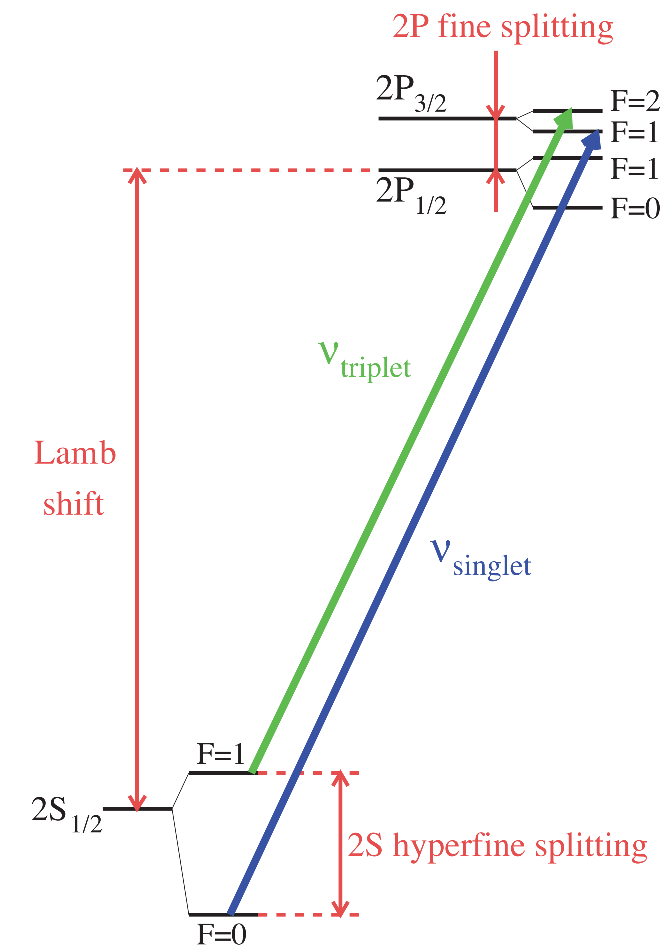
with $\Delta_{\mathrm{structure}} = \boxed{\Delta_Z} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_{\mathrm{Z}}$$

experimental value: $R_Z = 1.082(37)\,\mathrm{fm}$

A. Antognini, et al., Science **339** (2013) 417–420



Measurements of the μH ground-state HFS planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations

- Very precise input for the 2γ effect needed to narrow down frequency search range for experiment
- Zemach radius can help to pin down the magnetic properties of the proton

POLARIZABILITY EFFECT IN THE HFS

- Polarizability effect on the HFS is completely constrained by empirical information

$$\Delta_{\text{pol.}} = \Delta_1 + \Delta_2 = \frac{\alpha m}{2\pi(1+\kappa)M}\left(\delta_1 + \delta_2\right)$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2) + F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4 + \frac{1}{1+v_x} + \frac{1}{v_l+1}\right)\right\}$$

$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x} - \frac{1}{v_l+1}\right)$$

with $v_l = \sqrt{1 + \frac{1}{\tau_l}}$, $v_x = \sqrt{1 + x^2\tau^{-1}}$, $\tau_l = \frac{Q^2}{4m^2}$ and $\tau = \frac{Q^2}{4M^2}$

- BChPT calculation puts the reliability of dispersive calculations (and BChPT) to the test

2305.09633

POLARIZABILITY EFFECT IN THE HFS

- Polarizability effect on the HFS is completely constrained by empirical information

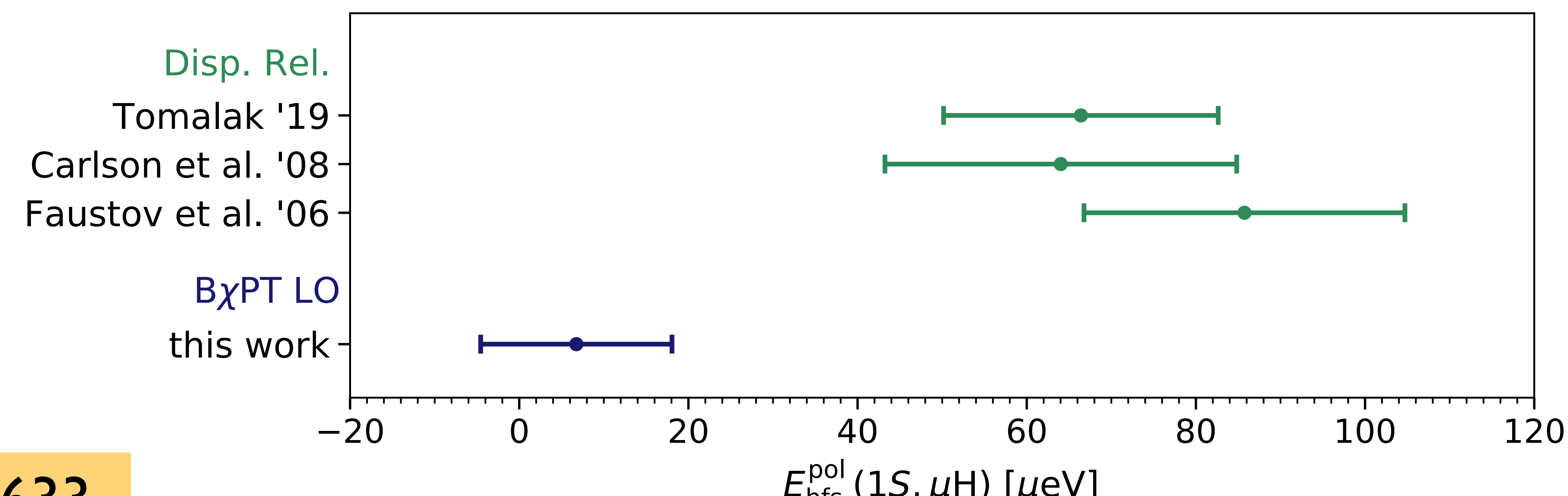
$$\Delta_{\text{pol.}} = \Delta_1 + \Delta_2 = \frac{\alpha m}{2\pi(1+\kappa)M}\left(\delta_1 + \delta_2\right)$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left\{\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2) + F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\frac{1}{(v_l+v_x)(1+v_x)(1+v_l)}\left(4 + \frac{1}{1+v_x} + \frac{1}{v_l+1}\right)\right\}$$

$$\delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left(\frac{1}{v_l+v_x} - \frac{1}{v_l+1}\right)$$

with $v_l = \sqrt{1+\frac{1}{\tau_l}}$, $v_x = \sqrt{1+x^2\tau^{-1}}$, $\tau_l = \frac{Q^2}{4m^2}$ and $\tau = \frac{Q^2}{4M^2}$

- BChPT calculation puts the reliability of dispersive calculations (and BChPT) to the test



2305.09633

2γ EFFECT IN THE μH HFS

see talk by C. Carlson

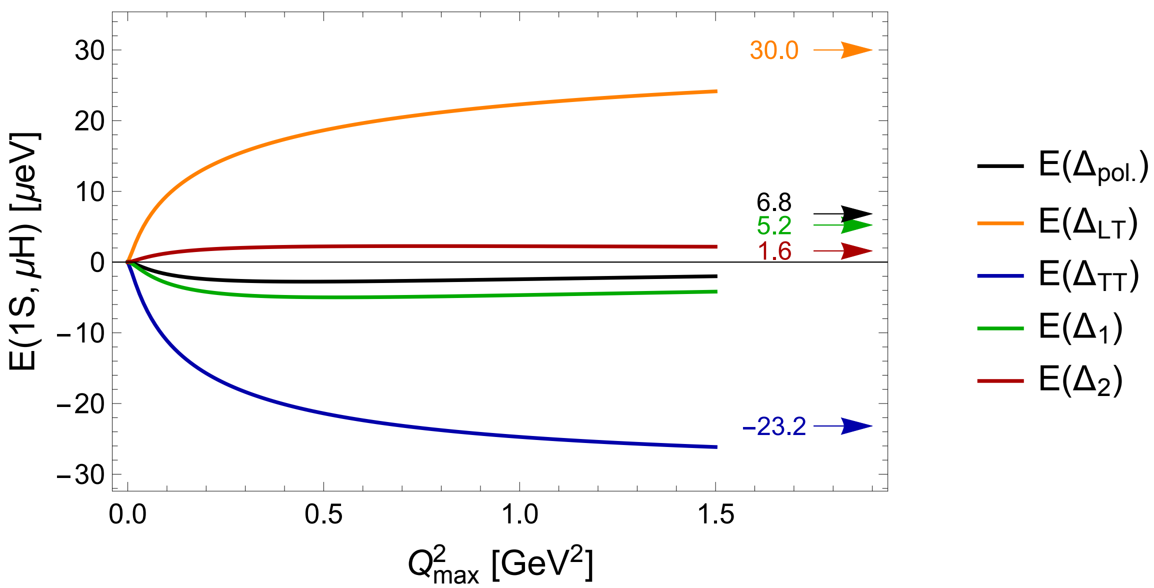
Table 1 Forward 2γ-exchange contribution to the HFS in μH.

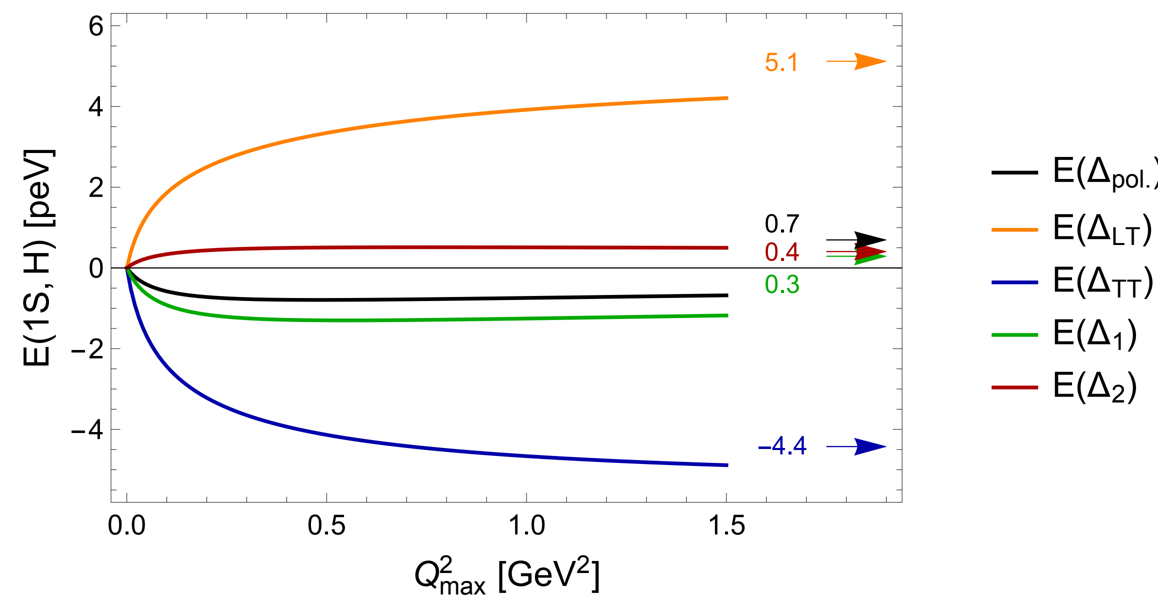
Reference	Δ_Z [ppm]	Δ_{recoil} [ppm]	Δ_{pol} [ppm]	Δ_1 [ppm]	Δ_2 [ppm]	$E^{\langle 2\gamma\rangle}_{1S\text{-hfs}}$ [meV]
DATA-DRIVEN						
Pachucki '96 (1)	−8025	1666	0(658)			−1.160
Faustov et al. '01 (9)[a]	−7180		410(80)	468	−58	
Faustov et al. '06 (10)[b]			470(104)	518	−48	
Carlson et al. '11 (11)[c]	−7703	931	351(114)	370(112)	−19(19)	−1.171(39)
Tomalak '18 (12)[d]	−7333(48)	846(6)	364(89)	429(84)	−65(20)	−1.117(19)
HEAVY-BARYON χPT						
Peset et al. '17 (13)						−1.161(20)
LEADING-ORDER χPT						
Hagelstein et al. '16 (14)			37(95)	29(90)	9(29)	
+ $\Delta(1232)$ EXCIT.						
Hagelstein et al. '18 (15)			−13	84	−97	

[a] Adjusted values: Δ_{pol} and Δ_1 corrected by −46 ppm as described in Ref. 16.
[b] Different convention was used to calculate the Pauli form factor contribution to Δ_1, which is equivalent to the approximate formula in the limit of $m = 0$ used for H in Ref. 11.
[c] Elastic form factors from Ref. 17 and updated error analysis from Ref. 16. Note that this result already includes radiative corrections for the Zemach-radius contribution, $(1+\delta_Z^{rad})\Delta_Z$ with $\delta_Z^{rad} \sim 0.0153$ (18, 19), as well as higher-order recoil corrections with the proton anomalous magnetic moment, cf. (11, Eq. 22) and (18).
[d] Uses r_p from μH (20) as input.

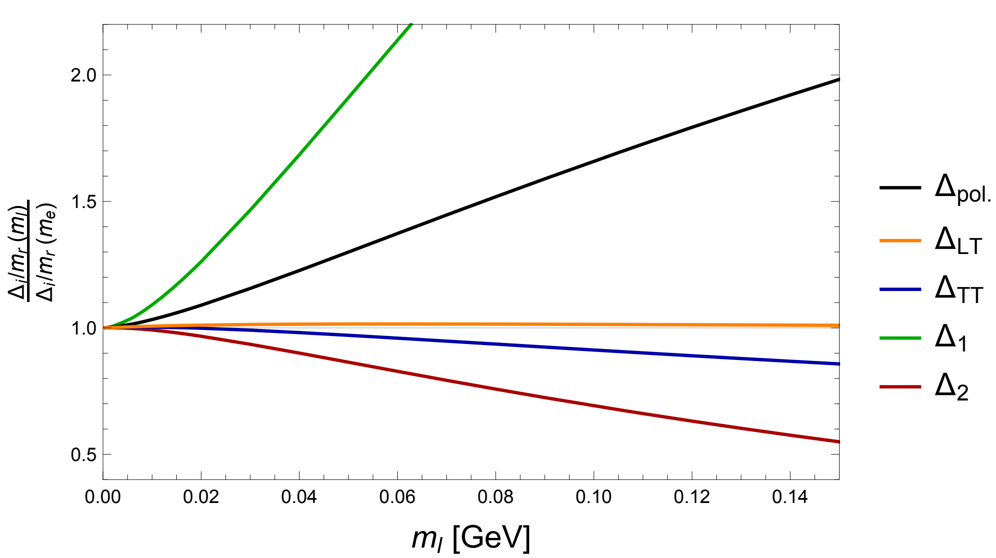
POLARIZABILITY EFFECT FROM BCHPT

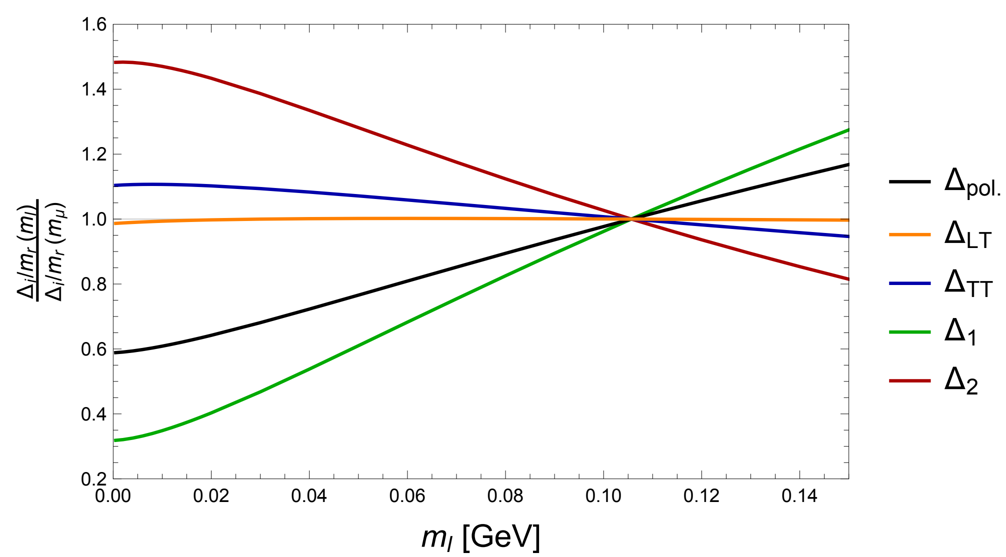
- LO BChPT result is compatible with zero
  - Contributions from σ_{LT} and σ_{TT} are sizeable and largely cancel each other





- Are the data-driven evaluations/uncertainties affected by cancelations?
- Scaling with lepton mass of the lepton-proton bound state

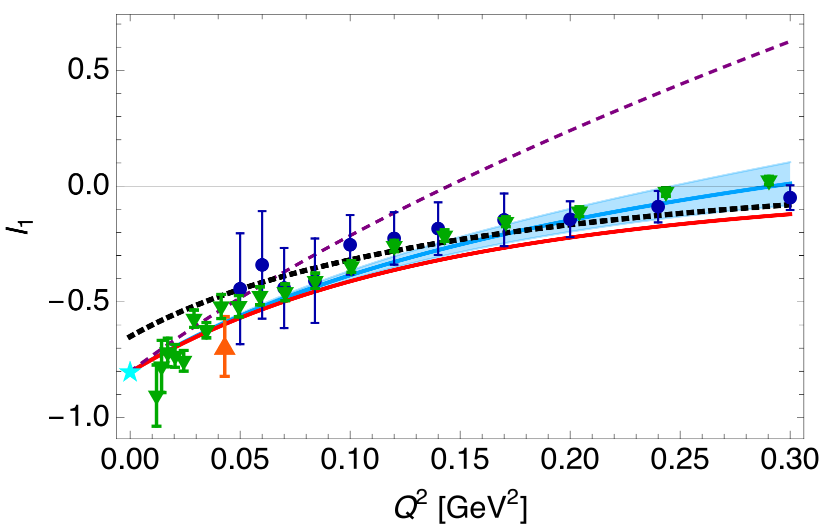


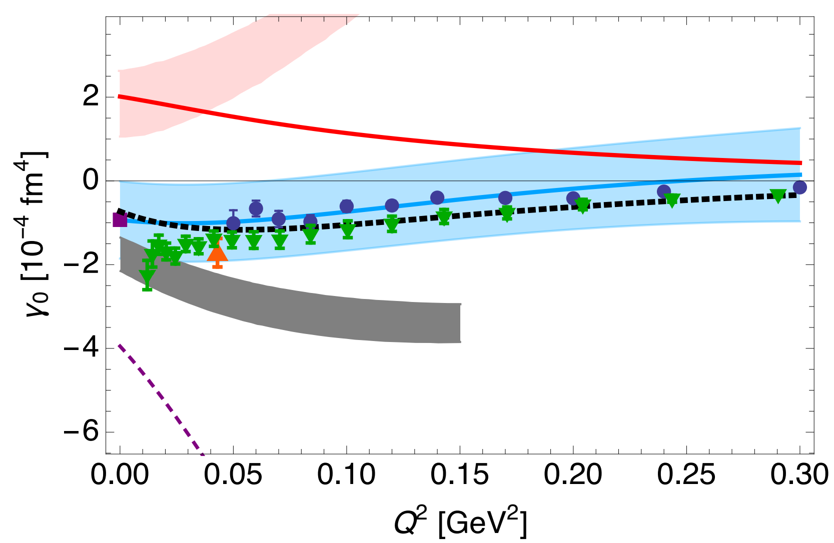


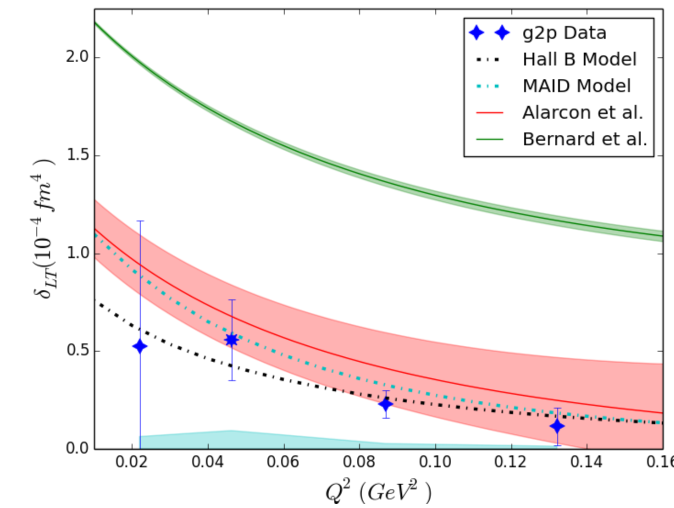
DATA-DRIVEN EVALUATION

see talks by C. Carlson, A. Deur, D. Ruth and K. Slifer

- Empirical information on spin structure functions from JLab Spin Physics Programme



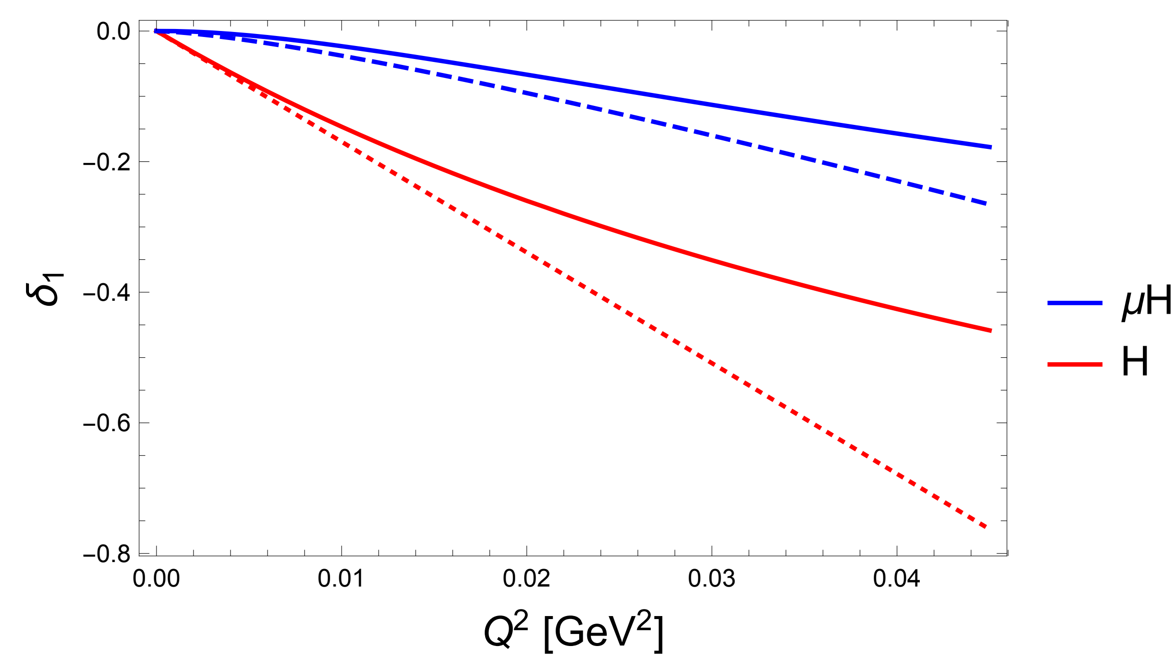




- Low-Q region is very important → cancelation between $I_1(Q^2)$ and $F_2(Q^2)$

$$\delta_1(\mathrm{H}) \sim \left(\underbrace{-\frac{3}{4}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -2.19} + \underbrace{18M^2 c_{1B}}_{\to 3.54} \right) Q^2_{\mathrm{max}} = 1.35(90),$$

$$\delta_1(\mu\mathrm{H}) \sim \left[\underbrace{-\frac{1}{3}\kappa^2 r_{\mathrm{Pauli}}^2}_{\to -1.45} + \underbrace{8M^2 c_1}_{\to 2.13} \underbrace{-\frac{M^2}{3\alpha}\gamma_0}_{\to 0.18} \right] \int_0^{Q^2_{\mathrm{max}}} \mathrm{d}Q^2 \beta_1(\tau_\mu) = 0.86(69)$$

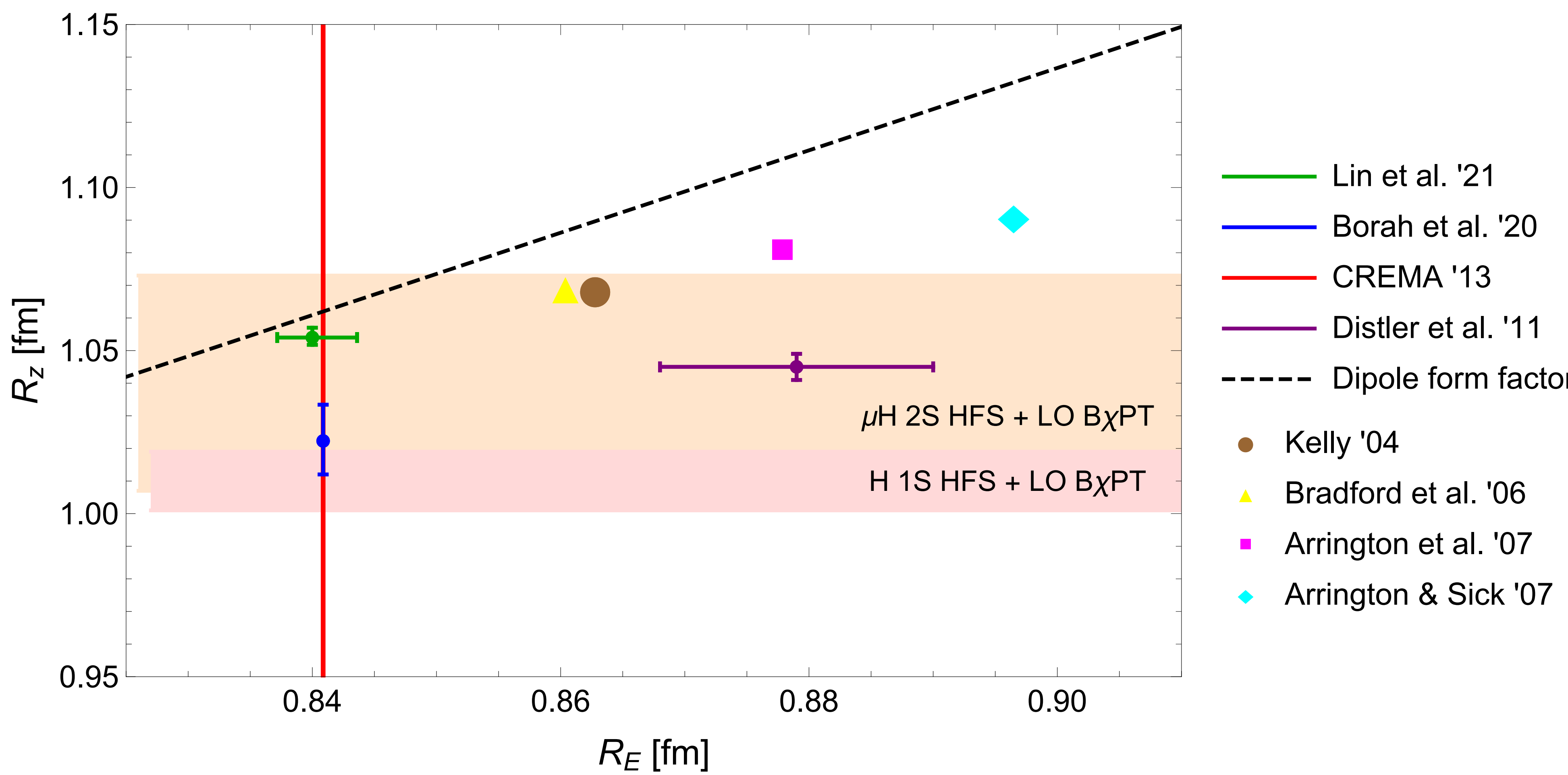


PROTON ZEMACH RADIUS

- BChPT polarizability contribution implies smaller Zemach radius (smaller, just like r_p)

TABLE I. Determinations of the proton Zemach radius R_Z, in units of fm.

ep scattering		μH 2*S* hfs		H 1*S* hfs	
Lin *et al.* '21	Borah *et al.* '20	Antognini *et al.* '13	LO BχPT	Volotka *et al.* '04	LO BχPT
$1.054^{+0.003}_{-0.002}$	1.0227(107)	1.082(37)	1.040(33)	1.045(16)	1.010(9)



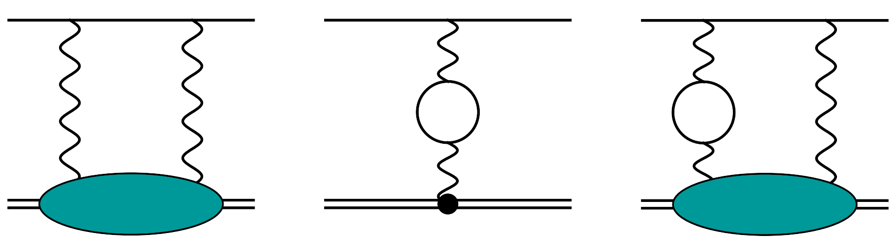
THEORY OF HYPERFINE SPLITTING

A. Antognini, FH, V. Pascalutsa, Ann. Rev. Nucl. Part. **72** (2022) 389-418

The hyperfine splitting of μH (theory update):

$$E_{1S\text{-hfs}} = \Big[\underbrace{182.443}_{E_\mathrm{F}}\ \underbrace{+1.350(7)}_{\text{QED+weak}}\ \underbrace{+0.004}_{\text{hVP}}\ \underbrace{-1.30653(17)\left(\frac{r_{Zp}}{\text{fm}}\right)+E_\mathrm{F}\left(1.01656(4)\,\Delta_\text{recoil}+1.00402\,\Delta_\text{pol}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{meV}$$

- 2γ + radiative corrections $\Longrightarrow$ differ for H vs. μH and 1S vs. 2S



+ ...

The hyperfine splitting of H (theory update):

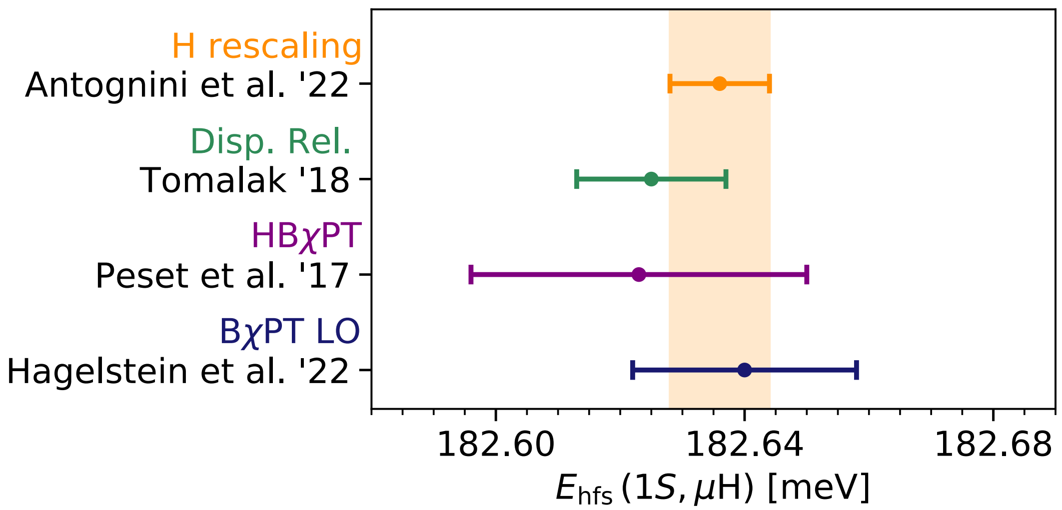
$$E_{1S\text{-hfs}}(\text{H}) = \Big[\underbrace{1\,418\,840.082(9)}_{E_\mathrm{F}}\ \underbrace{+1\,612.673(3)}_{\text{QED+weak}}\ \underbrace{+0.274}_{\mu\text{VP}}\ \underbrace{+0.077}_{\text{hVP}}$$
$$\underbrace{-54.430(7)\left(\frac{r_{Zp}}{\text{fm}}\right)+E_\mathrm{F}\left(0.99807(13)\,\Delta_\text{recoil}+1.00002\,\Delta_\text{pol}\right)}_{2\gamma\text{ incl. radiative corr.}}\Big]\,\text{kHz}$$

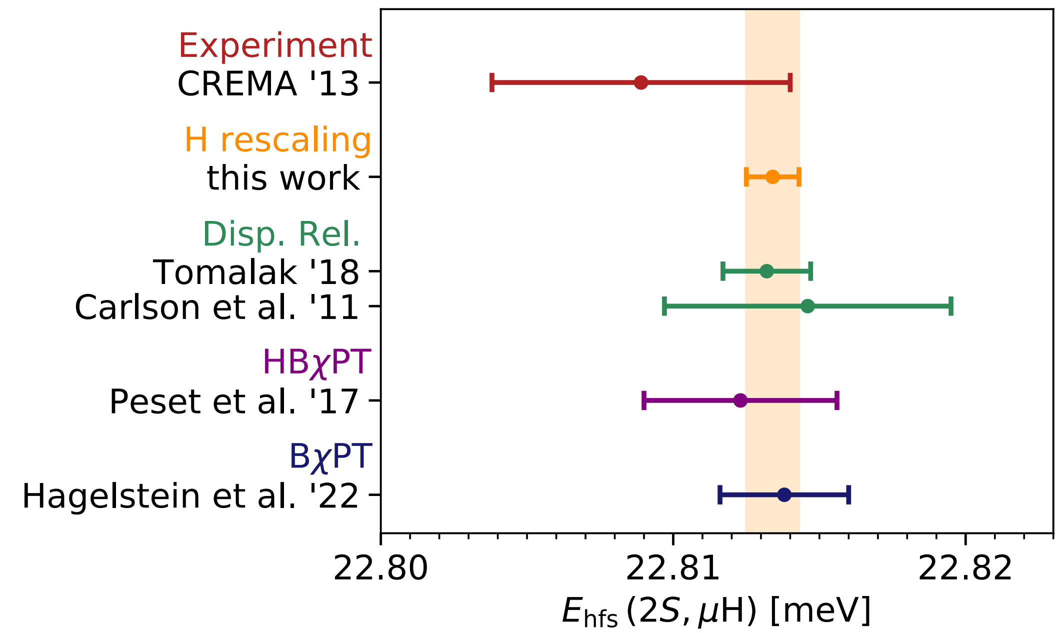
High-precision measurement of the "21cm line" in H:

$$\delta\left(E^\text{exp.}_{1S-\text{hfs}}(\text{H})\right) = 10\times10^{-13}$$

Hellwig et al., 1970

IMPACT OF H 1S HFS





- Leverage radiative corrections $E^{\text{Z+pol}}_{1S-\text{hfs}}(\text{H}) = E_\text{F}(\text{H})\left[b_{1S}(\text{H})\,\Delta_\text{Z}(\text{H}) + c_{1S}(\text{H})\,\Delta_\text{pol}(\text{H})\right] = -\,54.900(71)\,\text{kHz}$ and assume the non-recoil $\mathcal{O}(\alpha^5)$ effects have simple scaling $\dfrac{\Delta_i(\text{H})}{m_r(\text{H})} = \dfrac{\Delta_i(\mu\text{H})}{m_r(\mu\text{H})}, \quad i = \text{Z, pol}$

1. Prediction for μH HFS from empirical 1S HFS in H

$$E^{\text{Z+pol}}_{nS-\text{hfs}}(\mu\text{H}) = \frac{E_\text{F}(\mu\text{H})\,m_r(\mu\text{H})\,b_{nS}(\mu\text{H})}{n^3 E_\text{F}(\text{H})\,m_r(\text{H})\,b_{1S}(\text{H})}E^{\text{Z+pol}}_{1S-\text{hfs}}(\text{H}) - \frac{E_\text{F}(\mu\text{H})}{n^3}\,\Delta_\text{pol}(\mu\text{H})\quad \underbrace{\left[c_{1S}(\text{H})\frac{b_{nS}(\mu\text{H})}{b_{1S}(\text{H})} - c_{nS}(\mu\text{H})\right]}_{=\,-\,6\times 10^{-5}\text{ for n = 1}\quad =\,-\,5\times 10^{-5}\text{ for n = 2}}$$

2. Disentangle Zemach radius and polarizability contribution
3. Testing the theory

HYPERFINE SPLITTING

Theory: QED, ChPT, data-driven dispersion relations, ab-initio few-nucleon theories

Experiment: HFS in μH, μHe$^+$, …

Guiding the exp.

find narrow 1S HFS transitions with the help of full theory predictions: QED, weak, finite size, polarizability

Interpreting the exp.

extract E^{TPE}, $E^{\mathrm{pol.}}$ or R_Z

Input for data-driven evaluations

form factors, structure functions, polarizabilities

Electron and Compton Scattering

Testing the theory

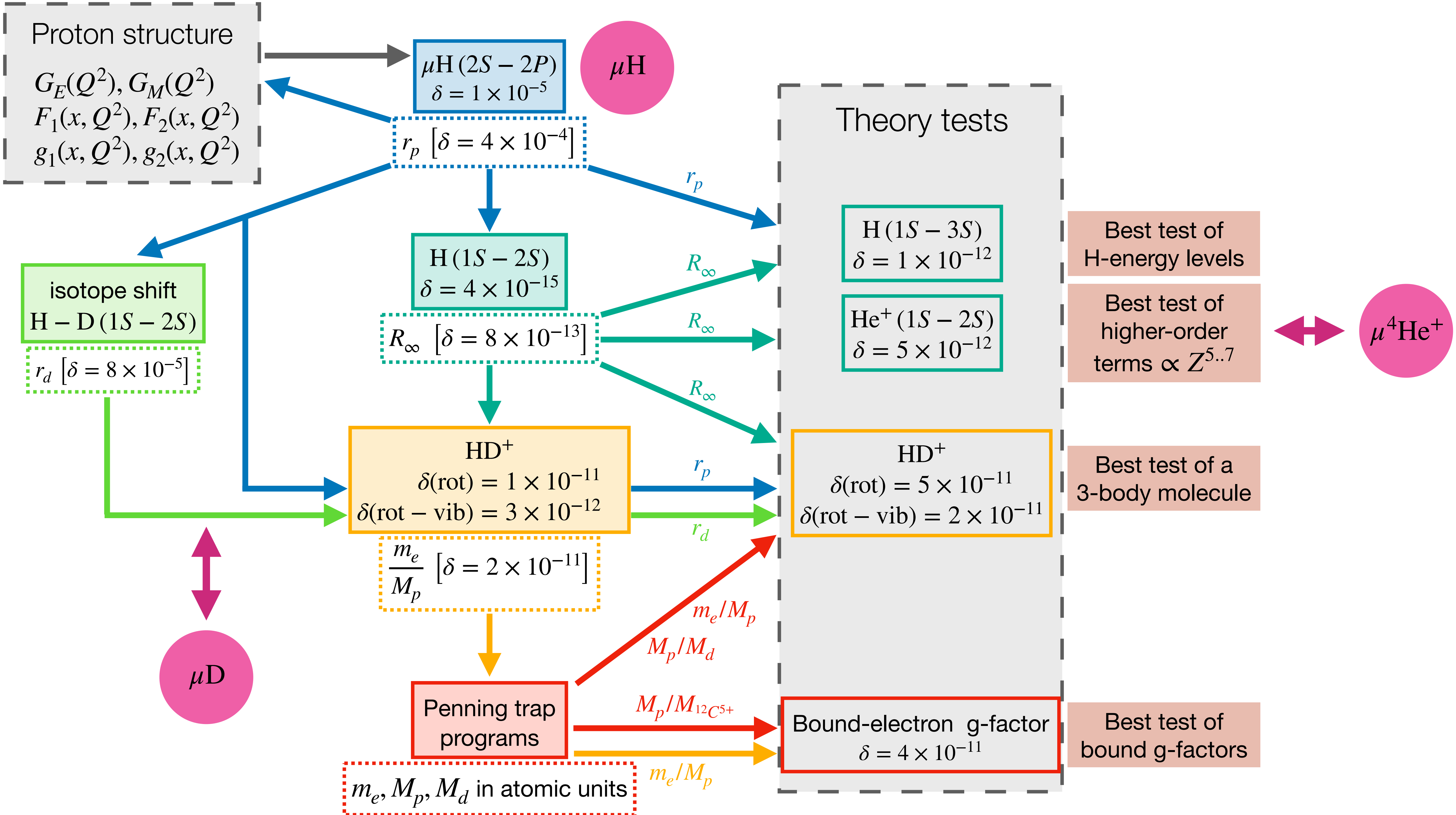
- discriminate between theory predictions for polarizability effect
    - disentangle R_Z & polarizability effect by combining HFS in H & μH
- test HFS theory
    - combining HFS in H & μH with theory prediction for polarizability effect
- test nuclear theories

Spectroscopy of ordinary atoms (H, He$^+$)

Determine fundamental constants

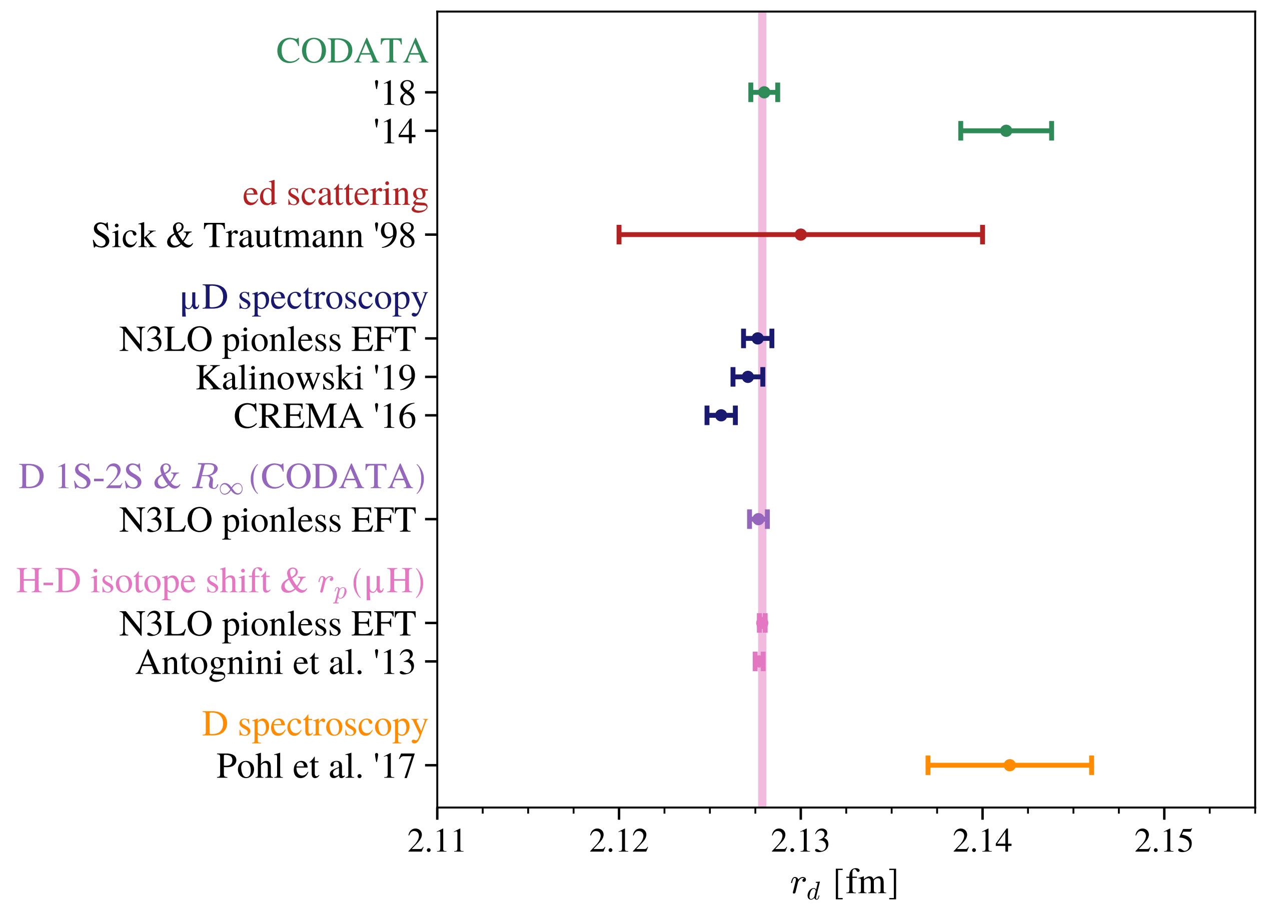
Zemach radius R_Z

IMPACT MUONIC ATOMS



A. Antognini, FH, V. Pascalutsa, 2205.10076, accepted for publication in Ann. Rev. Nucl. Part. **72** (2022)

DEUTERON CHARGE RADIUS



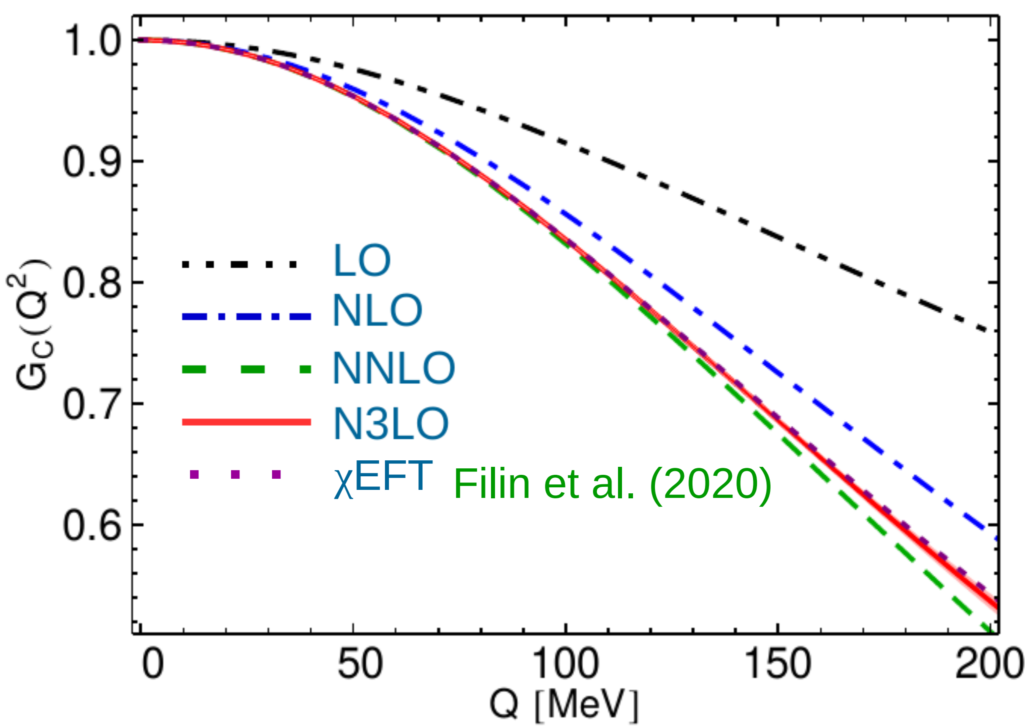
- Precise deuteron radius from H-D 1S-2S isotope shift and μH Lamb shift
- Higher-order contributions to μD Lamb shift are important:

$$E_{2P-2S}(\mu\mathrm{D}) = \left[228.77408(38) - 6.10801(28)\left(\frac{r_d}{\mathrm{fm}}\right)^2 - \boxed{E_{2S}^{2\gamma}} + \boxed{0.00219(92)}\right]\,\mathrm{meV}$$

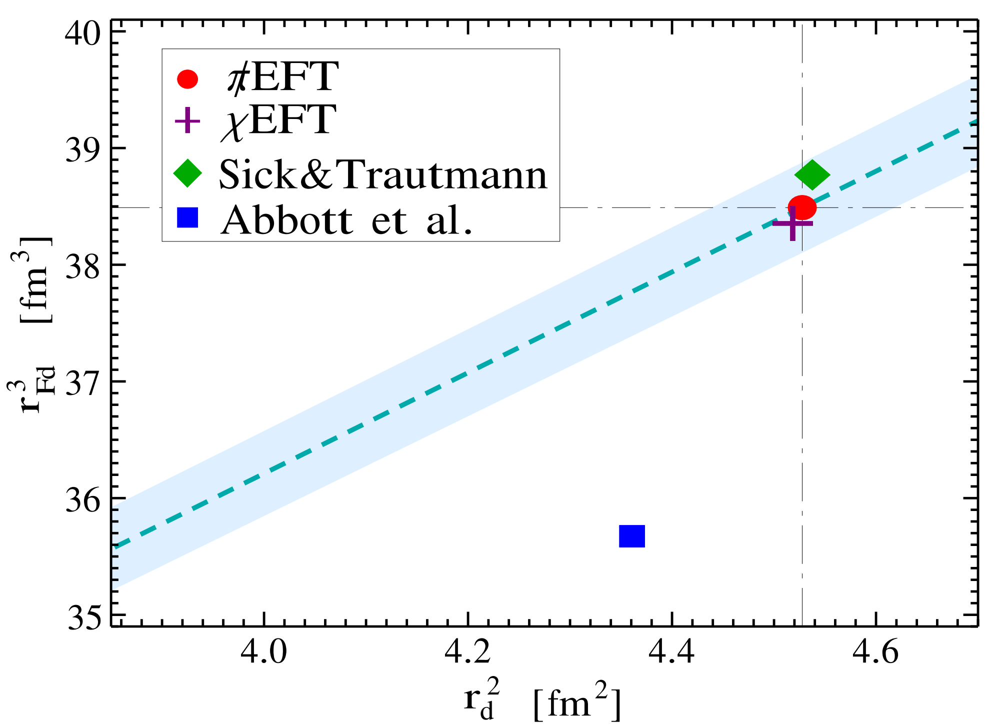
  - Coulomb (non-forward) distortion (starting $\alpha^6 \log \alpha$): $\boxed{E_{2S}^{\mathrm{Coulomb}} = 0.2625(15)\,\mathrm{meV}}$
  - $\boxed{2\gamma \text{ incl. eVP}}$ and $\boxed{3\gamma}$ contributions starting α^6 [Kalinowski, Phys. Rev. A **99** (2019) 030501]

D FORM FACTOR IN PIONLESS EFT

V. Lensky, A. Hiller Blin, V. Pascalutsa, Phys. Rev. C **104** (2021) 054003

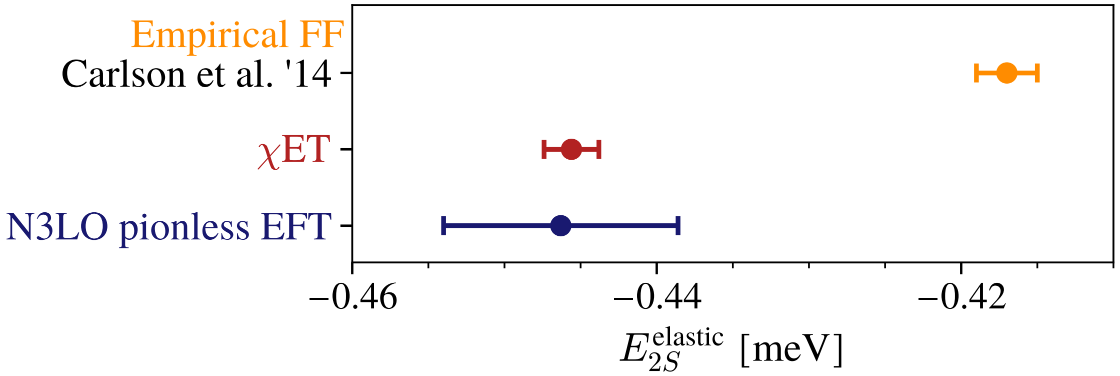


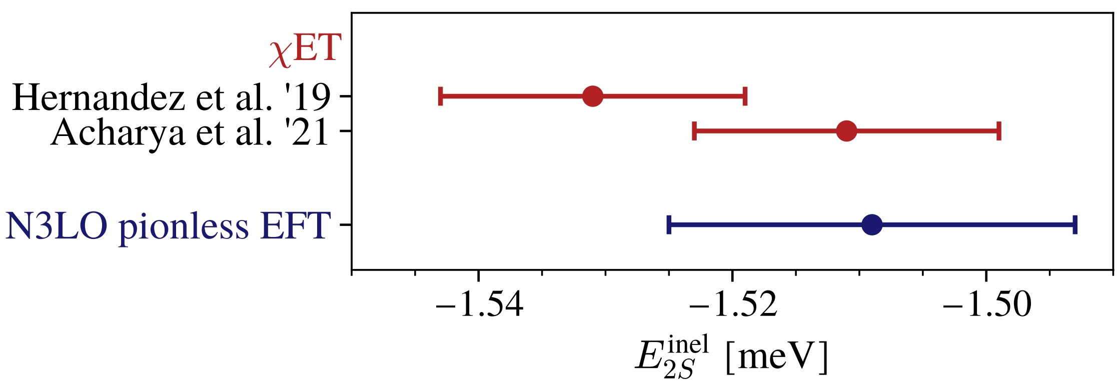
- Only one unknown low-energy constant l_1 of a longitudinal photon coupling to two nucleons
- Agreement of chiral EFT and pionless EFT



- Use r_d and r_{Fd} correlation to test low-Q properties of form factor parametrisations
- Abbott parametrisation gives different radii

2γ EFFECT IN μD LAMB SHIFT





	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	$-1.7096(200)$
Kalinowski '19 [6, Eq. (6) + (19)]	$-1.740(21)$
$\not\pi$EFT (this work)	$-1.752(20)$
Empirical (μH + iso)	
Pohl et al. '16 [3]	$-1.7638(68)$
This work	$-1.7585(56)$

V. Lensky, A. Hiller Blin, FH, V. Pascalutsa, 2203.13030
V. Lensky, FH, V. Pascalutsa, in preparation

- N3LO pionless EFT + higher-order single-nucleon effects:

$$E_{2S}^{\mathrm{elastic}} = -0.446(8)\,\mathrm{meV}$$
$$E_{2S}^{\mathrm{inel},L} = -1.509(16)\,\mathrm{meV}$$
$$E_{2S}^{\mathrm{inel},T} = -0.005\,\mathrm{meV}$$
$$E_{2S}^{\mathrm{hadr}} = -0.032(6)\,\mathrm{meV}$$
$$E_{2S}^{\mathrm{eVP}} = -0.027\,\mathrm{meV}$$

- Elastic 2γ several standard deviations larger
- Inelastic 2γ consistent with other results
- Agreement with precise empirical value for the 2γ effect extracted with $r_d(\mu\mathrm{H} + \mathrm{iso})$

Thank you for your attention!