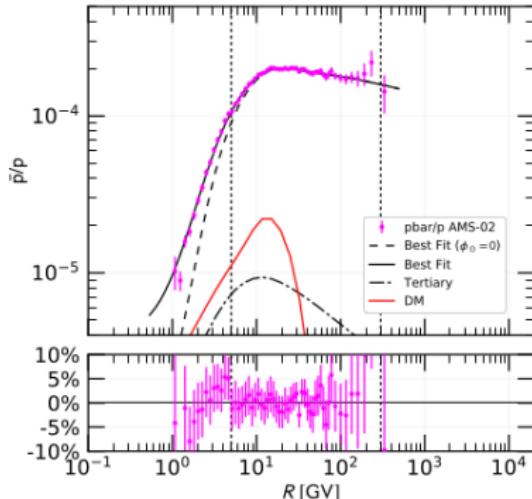


A new charge-sign dependent model of cosmic-ray modulation

Marco Kuhlen, Philipp Mertsch

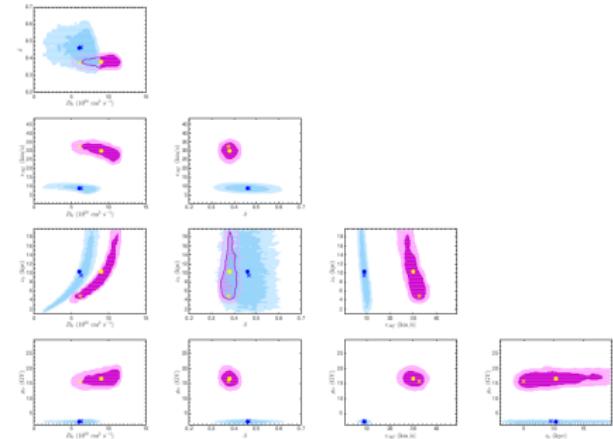
Institute for Theoretical Particle Physics and Cosmology (TTK)

Importance of Solar Modulation



Understand cosmic ray fluxes at low energies to interpret potential dark matter signals.

Cuoco et al. arXiv:1903.01472



Disentangle modulation of galactic cosmic rays from processes in the heliosphere.

Johannesson et al. arXiv:1602.02243

Solving the Transport Equation

computational expense

Force-field

- ✓ fast
- ✗ inaccurate
- ✗ local

Gleeson & Axford 1968

Numerical codes

- ✗ slow
- ✓ accurate
- ✓ global

Aslam et al.,
arXiv:1811.10710,
Boschini et al.,
arXiv:1704.03733,
Vittino et al.,
arXiv:1707.09003,
Kappl, arXiv:1601.02832

Solving the Transport Equation

computational expense



Force-field	Semi-analytical	Numerical codes
✓ fast	✓ fast	✗ slow
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✗ local	✗ local	✓ global

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Kappl, arXiv:1601.02832

Force Field Approximation

Gleeson & Axford, Caballero-Lopez & Moraal

Rewrite the transport equation as

$$\frac{\partial f}{\partial t} + \nabla \cdot (C\mathbf{V}f - \mathbf{K} \cdot \nabla f) + \frac{1}{3p^2} \frac{\partial}{\partial p} (p^3 \mathbf{V} \cdot \nabla f) = Q,$$

with Compton Getting factor $C \equiv -\frac{p}{3} \frac{1}{f} \frac{\partial f}{\partial p}$.

Assumptions:

- ▶ Steady state, $\partial f / \partial t = 0$
- ▶ No sources, $Q = 0$
- ▶ No average momentum loss in lab frame, $\langle \dot{p} \rangle = \frac{1}{3} \mathbf{V} \cdot \nabla f / f = 0$

Zero streaming condition:

$$C\mathbf{V}f - \mathbf{K} \cdot \nabla f = 0.$$

Force Field Solution

Gleeson & Axford, Caballero-Lopez & Moraal

Assuming spherical symmetry:

$$\frac{\partial f}{\partial r} + \frac{Vp}{3\kappa} \frac{\partial f}{\partial p} = 0,$$

Method of characteristics:

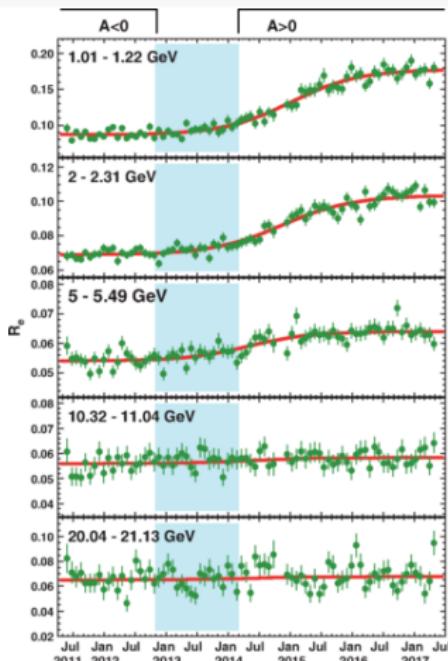
$$\int_{p_{TOA}}^{p_{LIS}} \frac{\beta \kappa_{p'}}{p'} dp' = \int_{r_{TOA}}^{r_{LIS}} \frac{V}{3\kappa_{r'}} dr' \equiv \phi(r),$$

For $\kappa_p \propto p$ and $\beta \approx 1 \rightarrow \phi = p_{LIS} - p_{TOA}$.

Conservation of the phase-space density f leads to:

$$\frac{J_{TOA}}{p_{TOA}^2} = \frac{J_{LIS}}{p_{LIS}^2}.$$

Time-Dependent Experimental Data



AMS Collaboration

Phys. Rev. Lett. 121, 051102

$$R_e = \frac{\Phi_{e^+}}{\Phi_{e^-}}$$

Explanation of current data requires charge sign dependent effects.

⇒ Importance of drifts.

To explain the AMS-02 data we make modifications to the force field model.

Changes to Force Field Model

Starting again from divergence free streaming

$$\int_S (C\mathbf{V}f - \mathbf{K} \cdot \nabla f) \cdot d\mathbf{S} = 0$$

Solve the transport equation in 2D including gradient curvature drifts.
 Where we introduce angular averages:

$$\begin{aligned}\tilde{f} &= \int_0^{\pi/2} d\theta \sin \theta f & \tilde{K}_{rr} &= \left(\frac{\partial \tilde{f}}{\partial r} \right)^{-1} \int_0^{\pi/2} d\theta \sin \theta K_{rr} \partial_r f \\ \tilde{V} &= \left(\frac{\partial \tilde{f}}{\partial p} \right)^{-1} \int_0^{\pi/2} d\theta \sin \theta V \partial_p f & \tilde{v}_{gc,r} &= \left(\tilde{f} \right)^{-1} \int_0^{\pi/2} d\theta \sin \theta v_{gc,r} f\end{aligned}$$

Changes to Force Field Model

After angular averages this reduces
to

$$\frac{\partial \tilde{f}}{\partial r} + \frac{p \tilde{V}}{3\tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f}$$

With $p_{\text{LIS}}(r, p)$ the solution to the
initial value problem

$$\frac{dp}{dr} = \frac{p \tilde{V}}{3\tilde{K}_{rr}},$$

Can be solved using the method of
characteristics

$$\tilde{f}(r, p) = f_{\text{LIS}}(p_{\text{LIS}})$$

with $p_{\text{LIS}}(R, p) = p$.

$$e^{-\int_0^r dr' \frac{\tilde{v}_{gc,r}(r', p'_{\text{LIS}})}{K_{rr}(r', p'_{\text{LIS}})}}$$

Changes to Force Field Model

After angular averages this reduces to

$$\frac{\partial \tilde{f}}{\partial r} + \frac{p \tilde{V}}{3\tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f}$$

Can be solved using the method of characteristics

$$\tilde{f}(r, p) = f_{\text{LIS}}(p_{\text{LIS}}) e^{-\int_0^r dr' \frac{\tilde{v}_{gc,r}(r', p'_{\text{LIS}})}{\tilde{K}_{rr}(r', p'_{\text{LIS}})}}$$

We parametrize them as

$$\tilde{V} = V_0(1 + \Delta V \theta(p - p_b))$$

$$\tilde{K}_{rr} = K_0 R^a \left(\frac{R^c + R_k^c}{1 + R_k^c} \right)^{(b-a)/c}$$

$$\tilde{v}_{gc,r} = \kappa_0 \frac{\beta p}{3B_0} \frac{10 p^2}{1 + 10 p^2}$$

Changes to Force Field Model

After angular averages this reduces to

$$\frac{\partial \tilde{f}}{\partial r} + \frac{p \tilde{V}}{3\tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f}$$

Can be solved using the method of characteristics

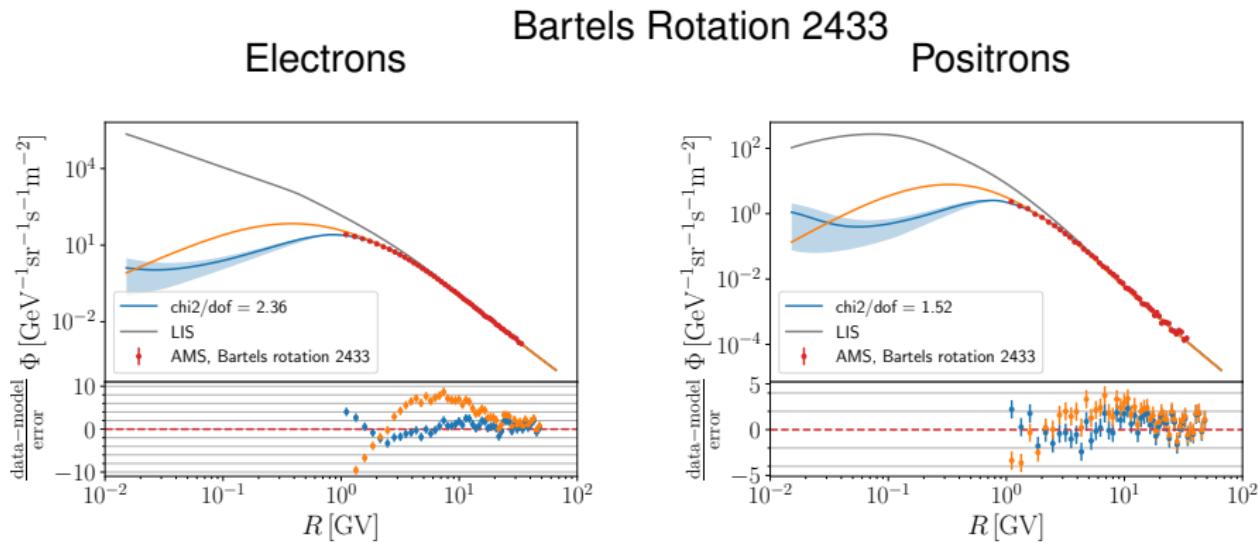
$$\tilde{f}(r, p) = f_{\text{LIS}}(p_{\text{LIS}}) e^{-\int_0^r dr' \frac{\tilde{v}_{gc,r}(r', p'_{\text{LIS}})}{\tilde{K}_{rr}(r', p'_{\text{LIS}})}}$$

In addition we include two scaling factors that will be determined by fitting to data.

$$\frac{p \tilde{V}}{3\tilde{K}_{rr}} \rightarrow g_1 \frac{p \tilde{V}}{3\tilde{K}_{rr}}$$

$$\frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \rightarrow g_2 \frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}}$$

Example: Fit to AMS-02 Data



Can explain data accurately while the conventional force field model fails.

LIS from Vittino et al. arXiv:1904.05899

Numerical Model

$$\frac{\partial f}{\partial t} = \underbrace{\nabla \cdot (K \cdot \nabla f)}_{\text{Diffusion}} - \underbrace{\mathbf{V} \cdot \nabla f}_{\text{Advection}} + \underbrace{\frac{1}{3}(\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p}}_{\text{Adiabatic Losses}} + Q$$

$$K = \begin{bmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp\theta} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp r} \end{bmatrix} \quad \kappa_{\parallel} = \kappa_{\parallel}^0 \beta \left(\frac{1 \text{nT}}{B} \right) \left(\frac{P}{P_0} \right)^a \left[\frac{\left(\frac{P}{P_0} \right)^c + \left(\frac{P_k}{P_0} \right)^c}{1 + \left(\frac{P_k}{P_0} \right)^c} \right]^{\frac{b-a}{c}}$$

Numerical Model

$$\frac{\partial f}{\partial t} = \underbrace{\nabla \cdot (K \cdot \nabla f)}_{\text{Diffusion}} - \underbrace{\mathbf{V} \cdot \nabla f}_{\text{Advection}} + \underbrace{\frac{1}{3}(\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p}}_{\text{Adiabatic Losses}} + Q$$

$$\mathbf{B} = B_0 \left(\frac{r_e}{r} \right)^2 \left(1 - 2 \Theta(\theta - \theta') \right) (\mathbf{e}_r + \left(\frac{r \delta(\theta, \phi)}{r_\odot} \right) \mathbf{e}_\theta - \tan \psi \mathbf{e}_\phi)$$

$$\nabla \cdot \mathbf{B} = 0 \implies \delta(\theta, \phi) = \frac{\delta_m}{\sin \theta}$$

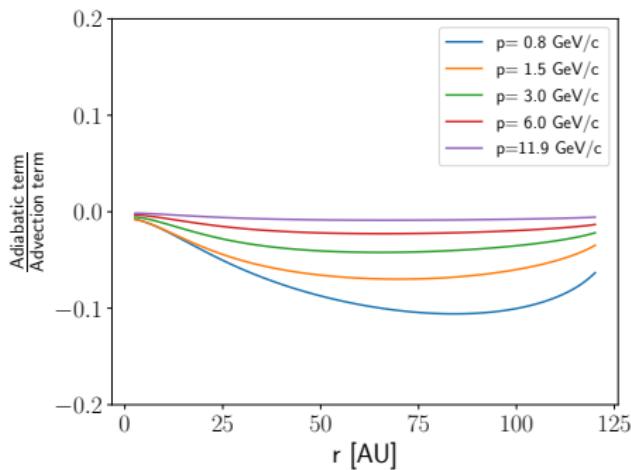
with $\delta_m = 8.7 \times 10^{-5}$. Langner 2004

Numerical Model

$$\frac{\partial f}{\partial t} = \underbrace{\nabla \cdot (K \cdot \nabla f)}_{\text{Diffusion}} - \underbrace{\mathbf{V} \cdot \nabla f}_{\text{Advection}} + \underbrace{\frac{1}{3}(\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p}}_{\text{Adiabatic Losses}} + Q$$

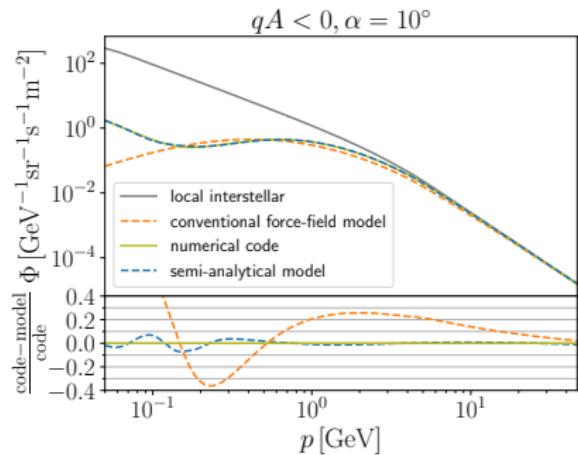
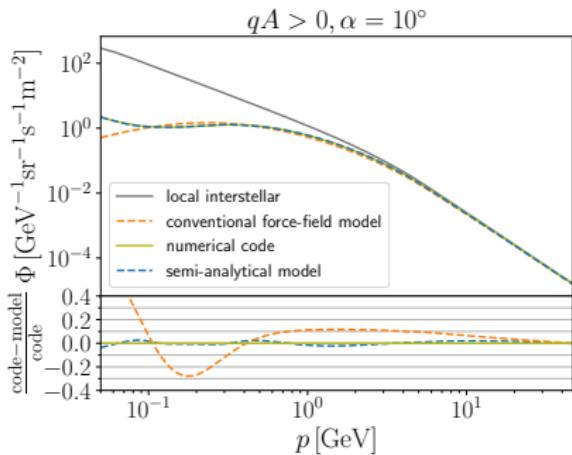
$$K = \begin{bmatrix} \kappa_{\parallel} & 0 & 0 \\ 0 & \kappa_{\perp\theta} & \kappa_A \\ 0 & -\kappa_A & \kappa_{\perp r} \end{bmatrix} \quad \mathbf{v_d} = \nabla \times (\kappa_A \mathbf{e_B})$$
$$\kappa_A = \kappa_A^0 \frac{v(R/R_0)}{3cB} \left(\frac{10(R/R_0)^2}{1 + 10(R/R_0)^2} \right)$$

Checking the Adiabatic Term



- ▶ Adiabatic term small in the inner heliosphere
- ▶ Becomes important only at very low energies
- ▶ Adiabatic term never contributes more than 10%

Fitting to Numerical Result



Can reproduce numerical results

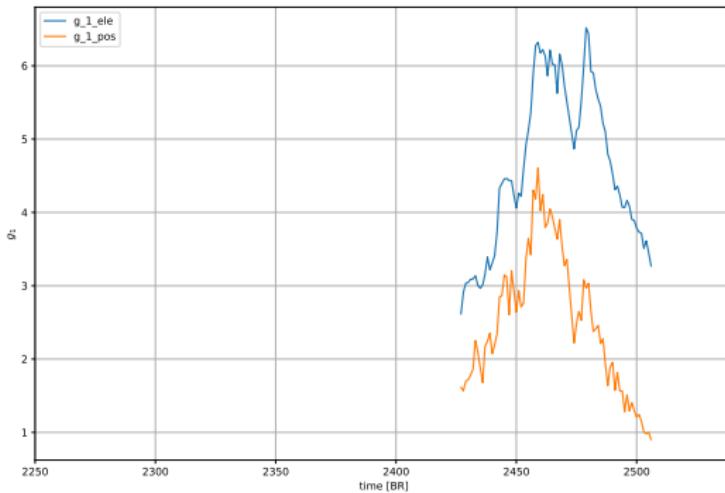
Making the model predictive

$$\frac{\partial \tilde{f}}{\partial r} + \textcolor{red}{g_1} \frac{p \tilde{V}}{3\tilde{K}_{rr}} \frac{\partial \tilde{f}}{\partial p} = -\textcolor{red}{g_2} \frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}} \tilde{f}$$

Express the scaling factors $\textcolor{red}{g_1}$ and $\textcolor{red}{g_2}$ as functions of the solar wind parameters.

Correlation with Solar Wind Parameters

We find a correlation between the tilt angle and the diffusion coefficient.



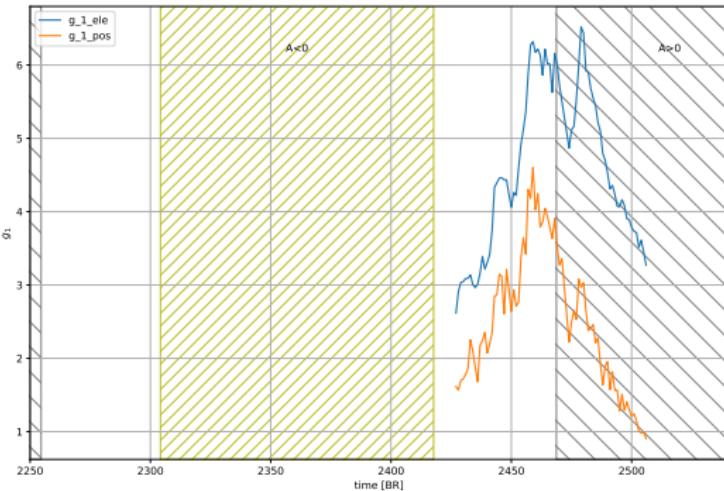
$$g_1^- \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_1^+ \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

Tilt angle from <http://wso.stanford.edu/>

Correlation with Solar Wind Parameters

We find a correlation between the tilt angle and the diffusion coefficient.



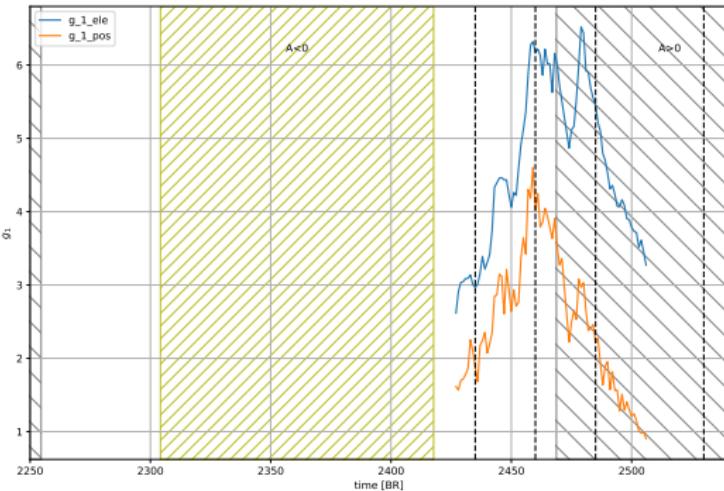
$$g_{\perp}^{+} \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_{\perp}^{-} \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

Tilt angle from <http://wso.stanford.edu/>

Correlation with Solar Wind Parameters

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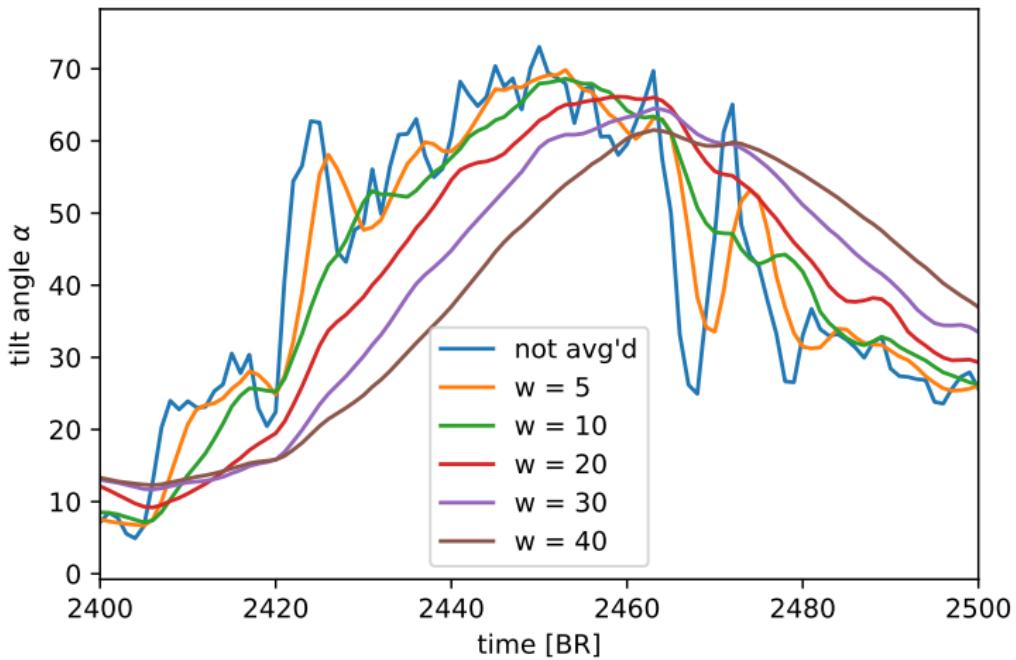


$$g_1^- \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_1^+ \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

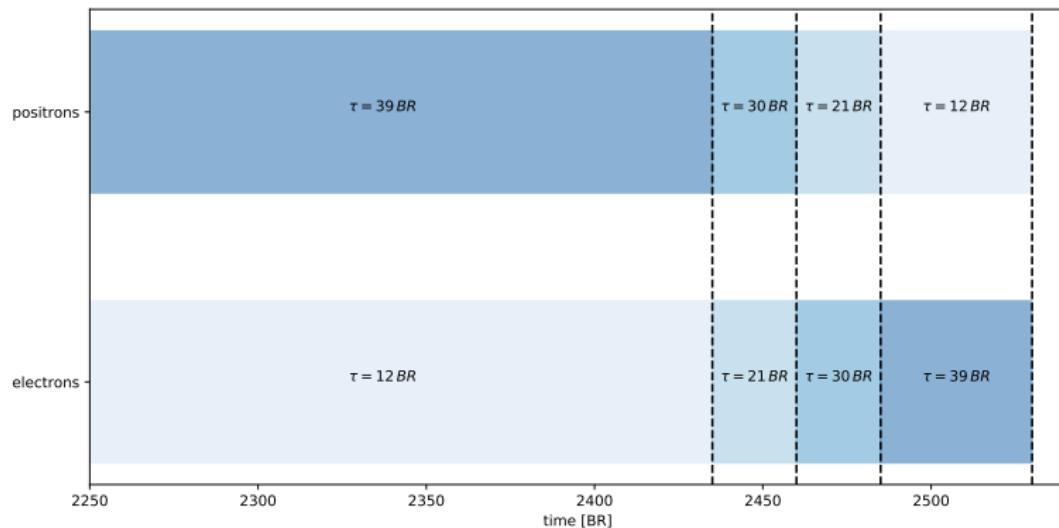
Tilt angle from <http://wso.stanford.edu/>

Moving Average of the Tilt Angle



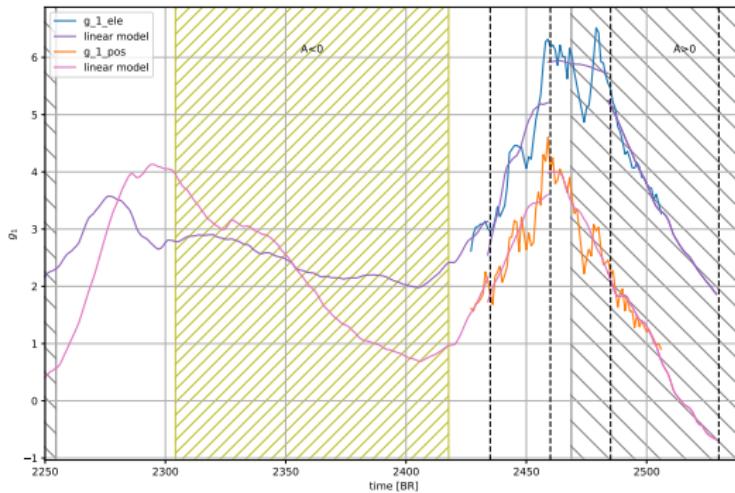
Correlation with Solar Wind Parameters

$$\langle \alpha \rangle_\tau$$



Correlation with Solar Wind Parameters

We find a correlation between the tilt angle and the diffusion coefficient.



$$g_1^- \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_1^- = a^- \langle \alpha \rangle_{\Delta t} + b^-$$

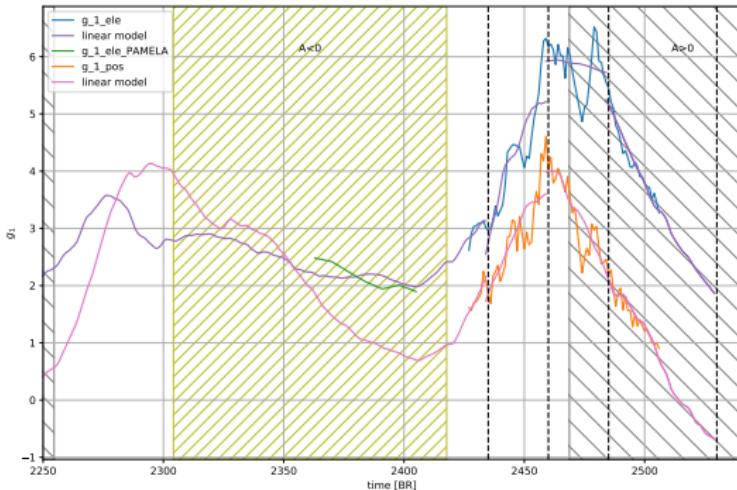
$$g_1^+ \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_1^+ = a^+ \langle \alpha \rangle_{\Delta t} + b^+$$

Tilt angle from <http://wso.stanford.edu/>

Correlation with Solar Wind Parameters

We find a correlation between the tilt angle and the diffusion coefficient.



$$g_1^- \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_1^- = a^- \langle \alpha \rangle_{\Delta t} + b^-$$

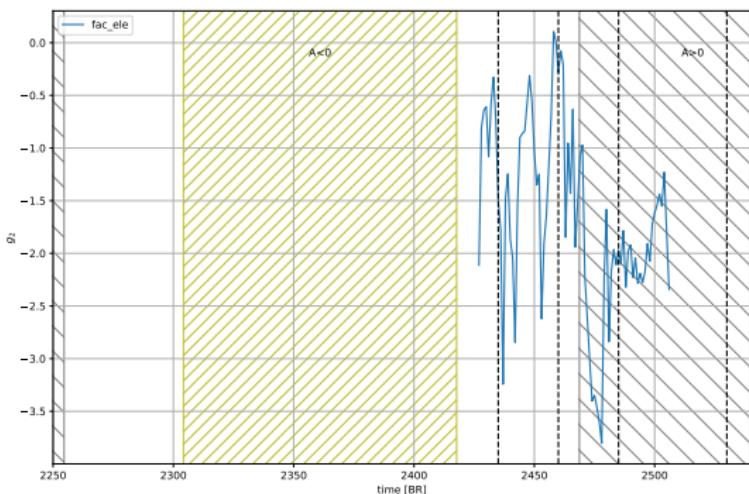
$$g_1^+ \frac{p \tilde{V}}{3 \tilde{K}_{rr}}$$

$$g_1^+ = a^+ \langle \alpha \rangle_{\Delta t} + b^+$$

Tilt angle from <http://wso.stanford.edu/>

Correlation with Solar Wind Parameters

We find a weaker correlation between the magnetic field strength and the normalization of the drift coefficient.

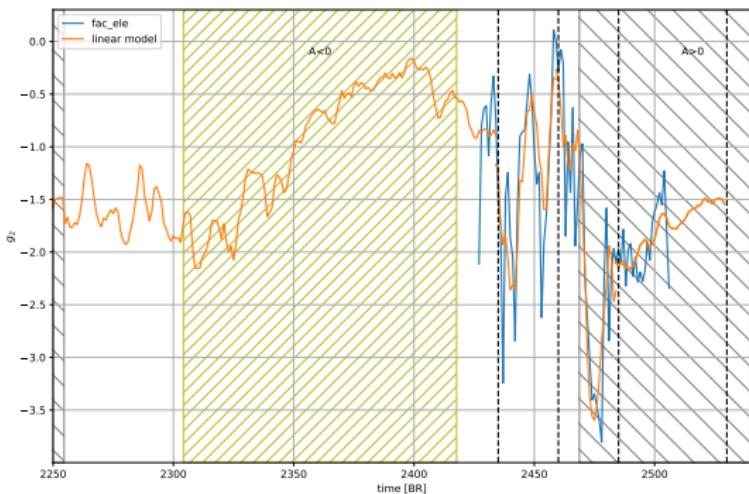


$$g_2 \frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}}$$

Magnetic field strength from <http://www.srl.caltech.edu/ACE/>

Correlation with Solar Wind Parameters

We find a weaker correlation between the magnetic field strength and the normalization of the drift coefficient.

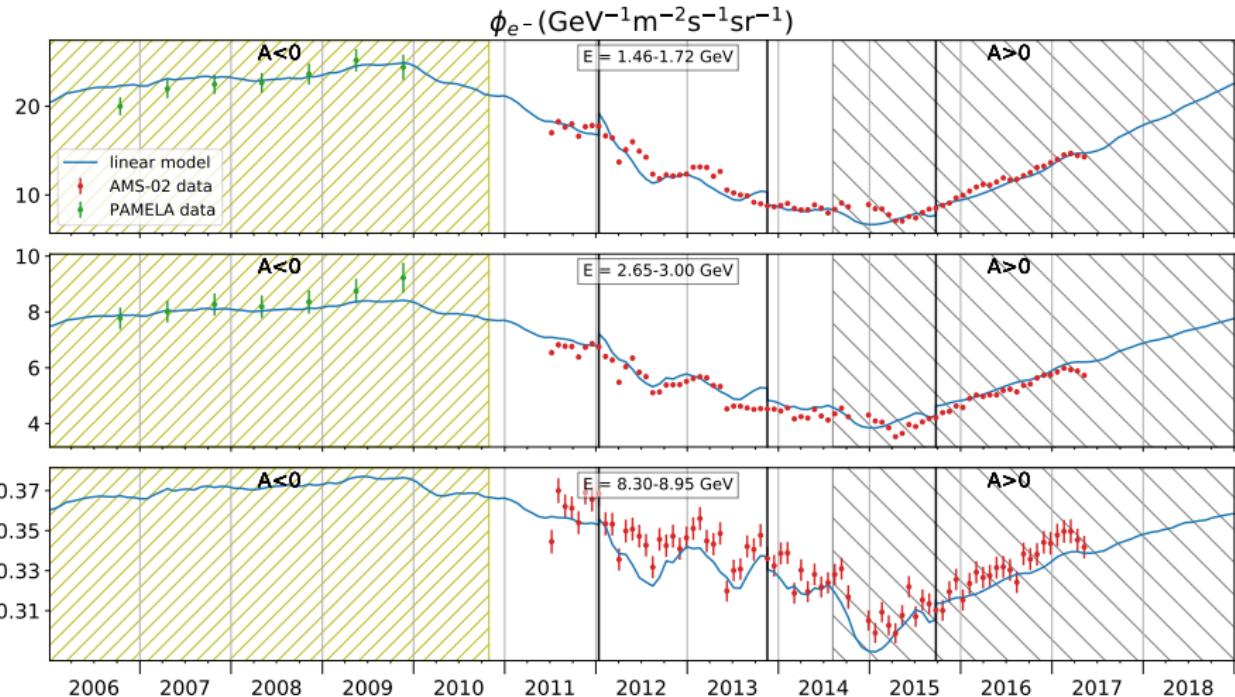


$$g_2 \frac{\tilde{v}_{gc,r}}{\tilde{K}_{rr}}$$

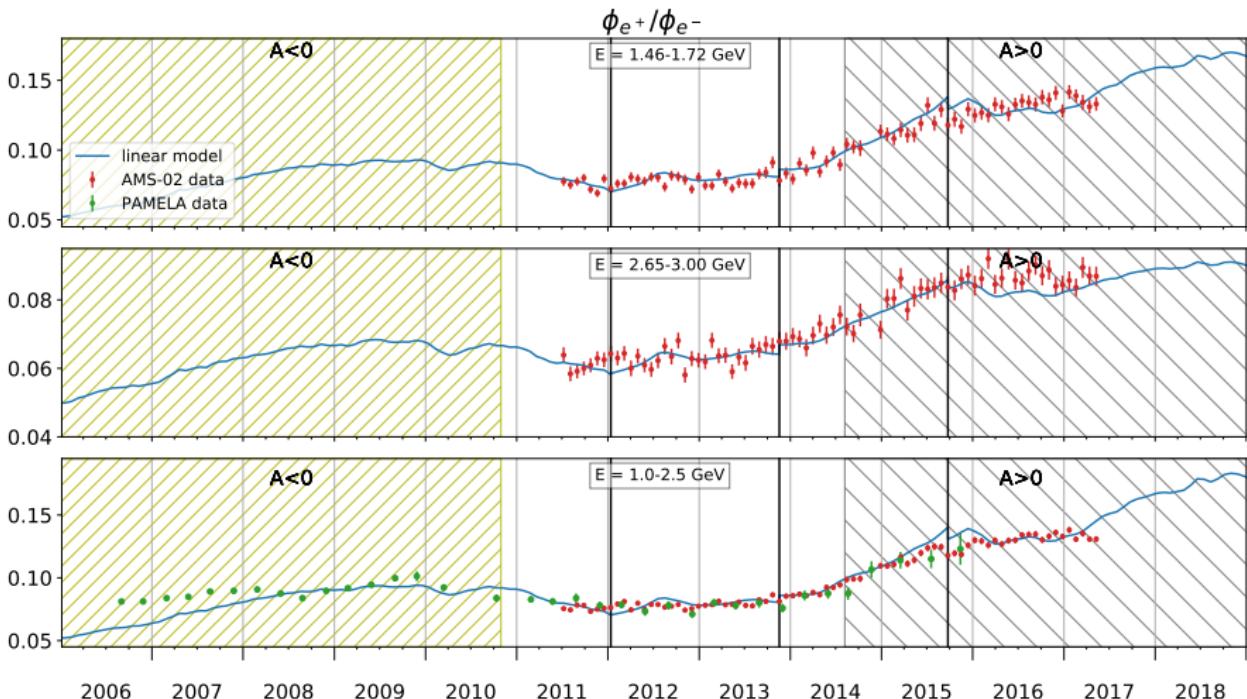
$$g_2 = c_0^+ \langle B_0 \rangle_{\Delta t} + d_0^+$$

Magnetic field strength from <http://www.srl.caltech.edu/ACE/>

Prediction of Electron Flux



Prediction of Positron Ratio



Conclusion

- ▶ We have developed a semi analytical method to solve the 2D transport equation.
- ▶ Our method runs significantly faster than fully numerical model (~ 20 ms).
- ▶ We are able to reproduce AMS-02 electron and positron fluxes.

Thank you for your attention!

Download an example script at
<https://git.rwth-aachen.de/kuhlenmarco/effmod-code>

Transport Equation

Transport equation for the heliosphere derived by Parker:

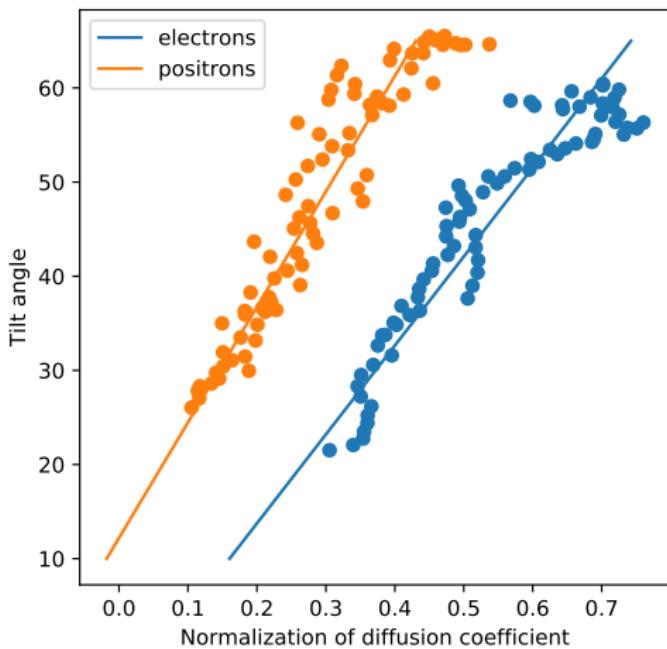
$$\frac{\partial f}{\partial t} = \underbrace{\nabla \cdot (K \cdot \nabla f)}_{\text{Diffusion}} - \underbrace{\mathbf{V} \cdot \nabla f}_{\text{Advection}} + \underbrace{\frac{1}{3}(\nabla \cdot \mathbf{V}) \frac{\partial f}{\partial \ln p}}_{\text{Adiabatic Losses}} + \underbrace{\frac{1}{p^2} \frac{\partial}{\partial p} \left(p^2 D_{pp} \frac{\partial f}{\partial p} \right)}_{\text{2nd order Fermi Acceleration}} + Q$$

f : cosmic ray phase space density

K : diffusion tensor

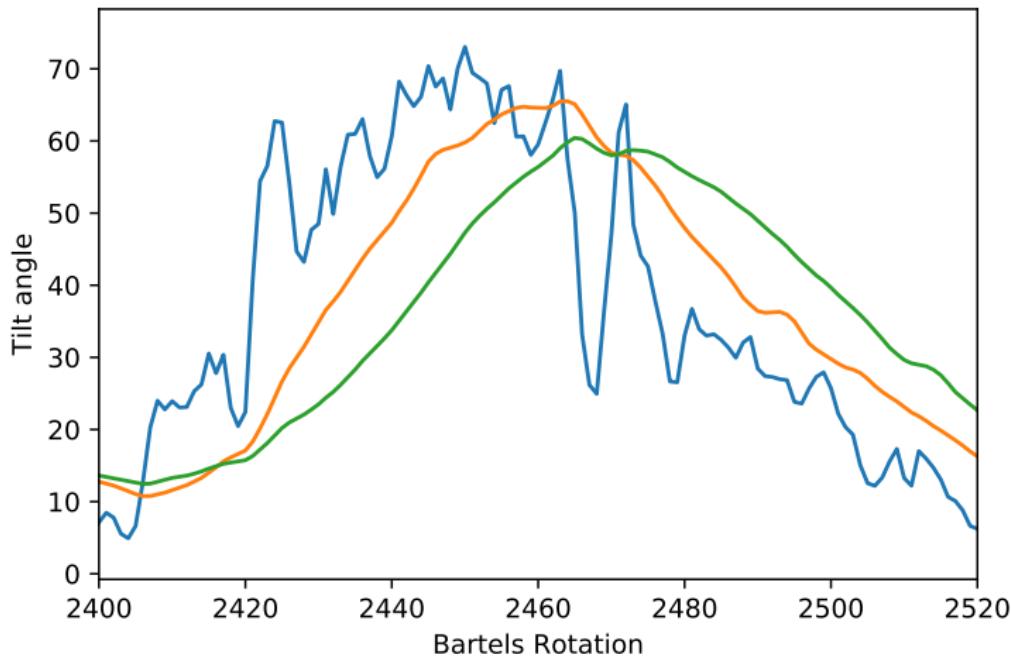
\mathbf{V} : solar wind velocity

Can be solved **numerically** or **analytically** making some simplifying assumptions.

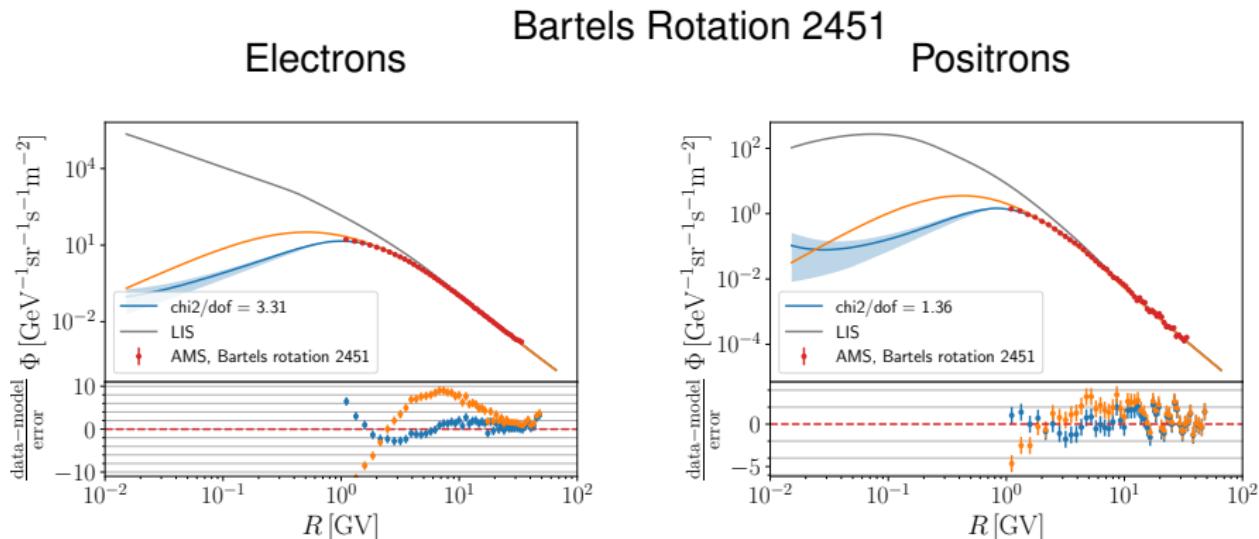
Correlation between Tilt and g_1 

- ▶ Pearson correlation coefficient for both electrons and positrons ~ 0.9
- ▶ Correlation maximal for $\Delta t = 45$ BR for electrons and $\Delta t = 25$ BR for positrons.

Tilt angle



Example: Fit to AMS-02 Data



Can explain data accurately while the conventional force field model fails.

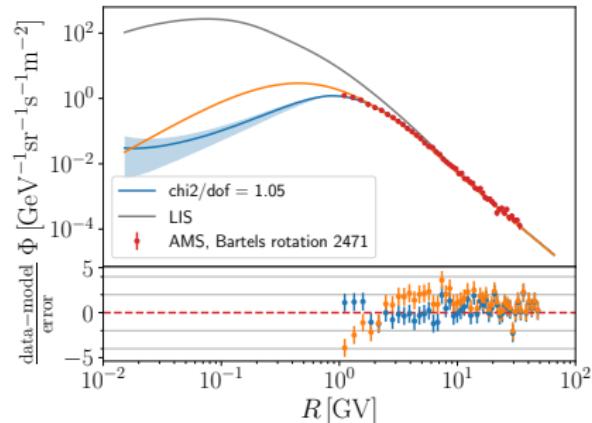
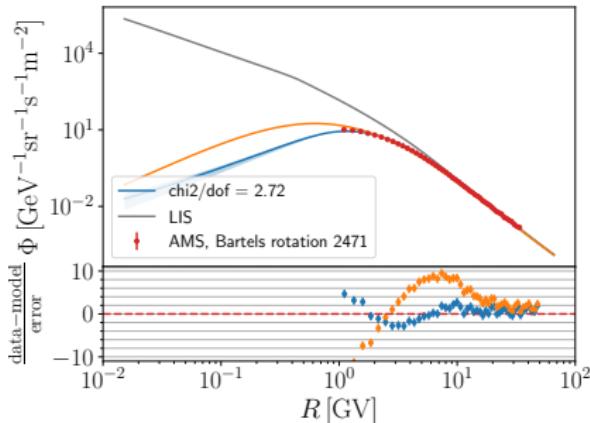
LIS from Vittino et al. arXiv:1904.05899

Example: Fit to AMS-02 Data

Electrons

Bartels Rotation 2471

Positrons



Can explain data accurately while the conventional force field model fails.

LIS from Vittino et al. arXiv:1904.05899