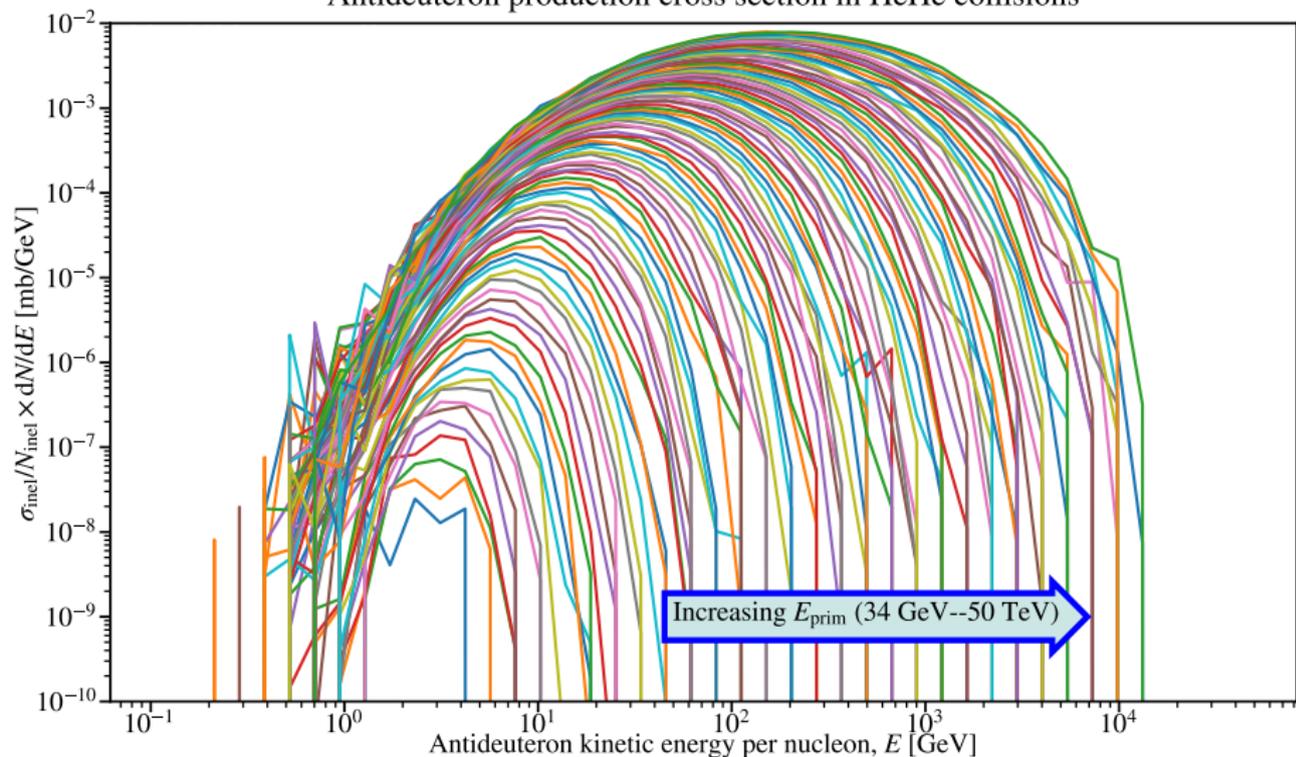


Cross-Sections for Antinuclei – M. Kachelrieß

Antideuteron production cross section in HeHe collisions



Outline of the talk

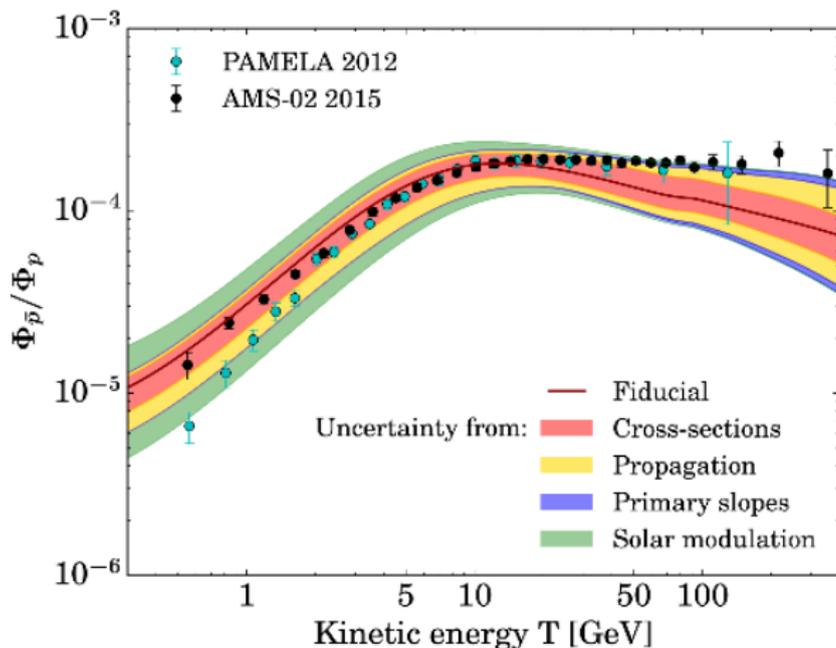
- 1 **Introduction – yesterday**
 - ▶ Physical motivation of different coalescence approaches
- 2 **Describing antiparticle production**
 - ▶ Parametrisations
 - ▶ Monte Carlo simulations
- 3 **Coalescence models and antinuclei production**
 - ▶ Coalescence in momentum space
 - ▶ Coalescence in coordinate space
 - ▶ Coalescence in phase space
- 4 **Conclusions**

Parametrisations

- perfect fit \leftrightarrow physically motivated fit function

Parametrisations

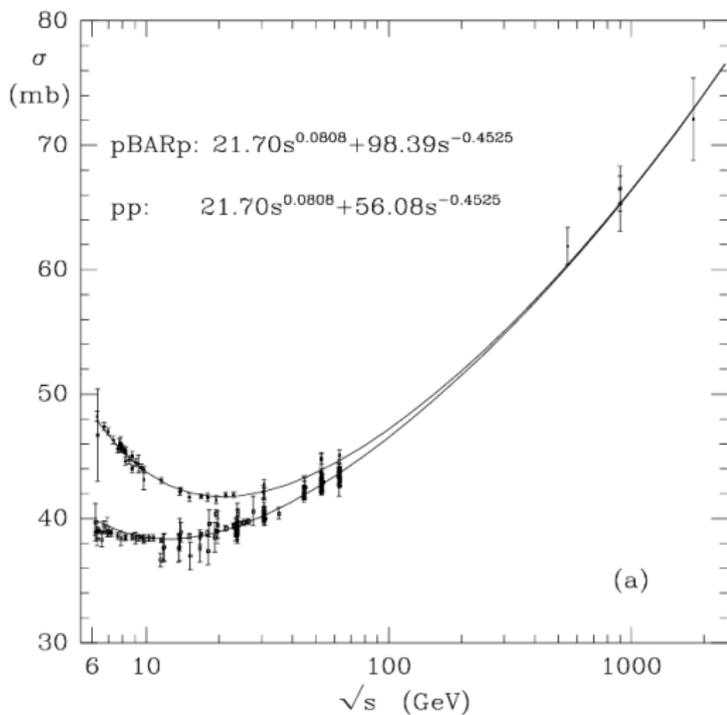
- perfect fit \leftrightarrow physically motivated fit function
- **easy to use, fast:** allows parametric studies



[Giesen et al. [1504.04276]]

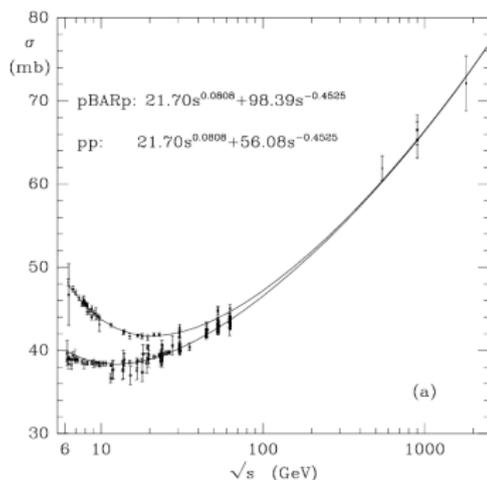
Parametrisations

Ex.: **Donnachie-Landshoff fit of σ_{tot}**



Parametrisations

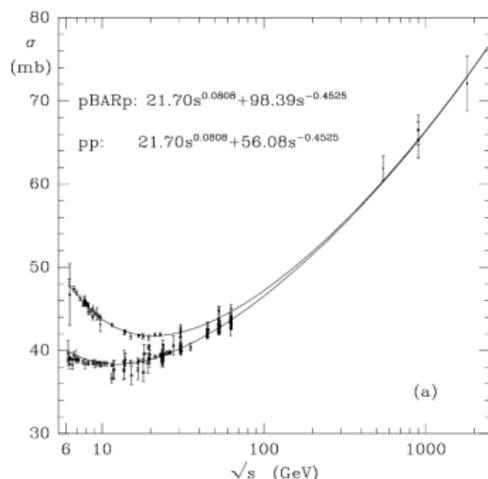
Ex.: Donnachie-Landshoff fit of σ_{tot}



- fit function $\sigma = As^a + Bs^{-b}$: physically well motivated

Parametrisations

Ex.: Donnachie-Landshoff fit of σ_{tot}



- fit function $\sigma = As^a + Bs^{-b}$: physically well motivated

- **violates unitarity**

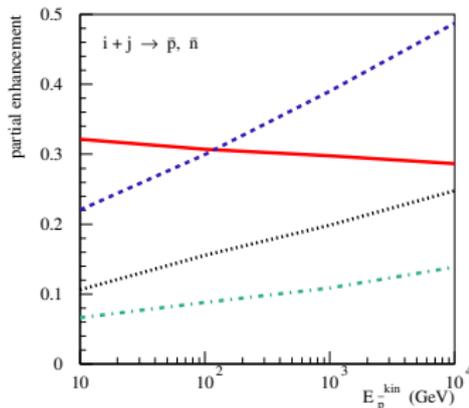
⇒ extrapolation outside $[s_{\text{min}} : s_{\text{max}}]$ dangerous

Parametrisations

- need more exclusive quantities, e.g. $d\sigma(E, E')/dE'$
- ⇒ even for $s \in [s_{\min} : s_{\max}]$: measurements cover only part of d^3p_f
- miss often important part $x_E \rightarrow 1$

Parametrisations

- need more exclusive quantities, e.g. $d\sigma(E, E')/dE'$
- ⇒ even for $s \in [s_{\min} : s_{\max}]$: measurements cover only part of d^3p_f
- miss often important part $x_E \rightarrow 1$
- generalisation from pp to **Ap**, **AA** only via ε factors



Parametrisations

- need more exclusive quantities, e.g. $d\sigma(E, E')/dE'$
- ⇒ even for $s \in [s_{\min} : s_{\max}]$: measurements cover only part of d^3p_f
- miss often important part $x_E \rightarrow 1$
- generalisation from pp to Ap, AA only via ε factors
- provide **no correlations** needed for anti-nuclei

Monte Carlos for accelerator physics

- large number of hard processes
- angular-ordered perturbative QCD cascade

Monte Carlos for accelerator physics

- large number of hard processes
- angular-ordered perturbative QCD cascade
- hadronisation based on string/cluster model:
 - ▶ large number of parameters to adjust yields of mesons&baryons
 - ▶ includes all relevant decay chains

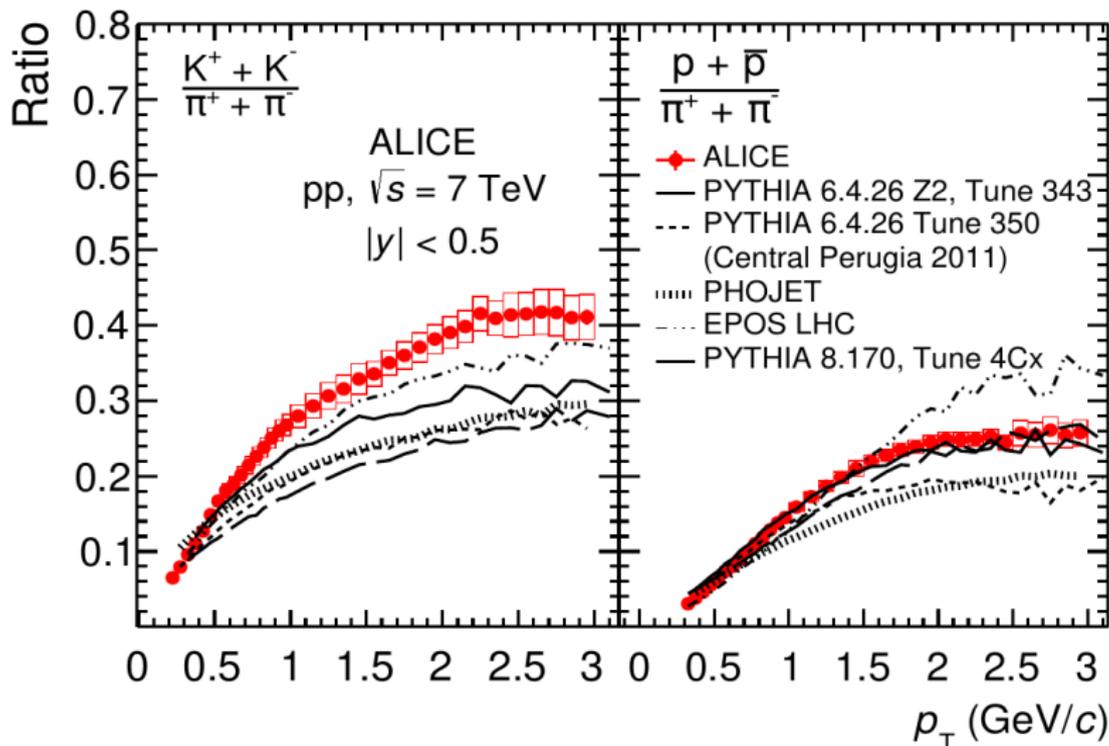
Monte Carlos for accelerator physics

- large number of hard processes
- angular-ordered perturbative QCD cascade
- hadronisation based on string/cluster model:
 - ▶ large number of parameters to adjust yields of mesons&baryons
 - ▶ includes all relevant decay chains
- oversimplified procedure to include multiple parton interactions in pp, even worse for Ap, AA

Monte Carlo for accelerator physics

- large number of hard processes
- angular-ordered perturbative QCD cascade
- hadronisation based on string/cluster model:
 - ▶ large number of parameters to adjust yields of mesons&baryons
 - ▶ includes all relevant decay chains
- oversimplified procedure to include multiple parton interactions in pp, even worse for Ap, AA
- various **tunes**

Monte Carlos for accelerator physics



Monte Carlos for cosmic ray physics

- only hard QCD processes
- angular-ordered perturbative QCD cascade

Monte Carlos for cosmic ray physics

- only hard QCD processes
- angular-ordered perturbative QCD cascade
- hadronisation based on string model:
 - ▶ large number of parameters to adjust yields of mesons&baryons
 - ▶ includes **only main decay chains**

Monte Carlos for cosmic ray physics

- only hard QCD processes
- angular-ordered perturbative QCD cascade
- hadronisation based on string model:
 - ▶ large number of parameters to adjust yields of mesons&baryons
 - ▶ includes only main decay chains
- Gribov-Reggeon approach for pp , A_p and AA

Monte Carlos for cosmic ray physics

- only hard QCD processes
- angular-ordered perturbative QCD cascade
- hadronisation based on string model:
 - ▶ large number of parameters to adjust yields of mesons&baryons
 - ▶ includes only main decay chains
- Gribov-Reggeon approach for pp, Ap and AA
- aim to cover **broad energy range**

“Old” coalescence model in momentum space

- all **nucleons** with **cms** momentum difference $\Delta p < p_0$ form a **nucleus**

“Old” coalescence model in momentum space

- all nucleons with cms momentum difference $\Delta p < p_0$ form a nucleus
- for **pp** and **pA collisions**: “fireball” **isotropic distribution** d^3N/d^3p

“Old” coalescence model in momentum space

- all nucleons with cms momentum difference $\Delta p < p_0$ form a nucleus
- for pp and pA collisions: “fireball” isotropic distribution d^3N/d^3p
- antideuterons \sim antiprotons²

$$\frac{dN_{\bar{d}}}{dT} = \frac{p_0^3}{6} \frac{m_d}{m_N^2} \frac{1}{\sqrt{T_d^2 + 2m_d T_d}} \left(\left. \frac{dN_{\bar{N}}}{dT} \right|_{T_d = T_{\bar{N}}/2} \right)^2$$

“Old” coalescence model in momentum space

- all nucleons with cms momentum difference $\Delta p < p_0$ form a nucleus
- for pp and pA collisions: “fireball” isotropic distribution d^3N/d^3p
- antideuterons \sim antiprotons²

$$\frac{dN_{\bar{d}}}{dT} = \frac{p_0^3}{6} \frac{m_d}{m_N^2} \frac{1}{\sqrt{T_d^2 + 2m_d T_d}} \left(\left. \frac{dN_{\bar{N}}}{dT} \right|_{T_d = T_{\bar{N}}/2} \right)^2$$

- consider e.g. **DM annihilation** $XX \rightarrow W^+ W^-$:

$$\frac{dN_{\bar{d}}}{dx} \propto \frac{1}{M_X^2} \frac{dN_{\bar{n}}}{dx} \frac{dN_{\bar{p}}}{dx}$$

“Old” coalescence model in momentum space

- all nucleons with cms momentum difference $\Delta p < p_0$ form a nucleus
- for pp and pA collisions: “fireball” isotropic distribution d^3N/d^3p
- antideuterons \sim antiprotons²

$$\frac{dN_{\bar{d}}}{dT} = \frac{p_0^3}{6} \frac{m_d}{m_N^2} \frac{1}{\sqrt{T_d^2 + 2m_d T_d}} \left(\frac{dN_{\bar{N}}}{dT} \Big|_{T_{\bar{d}}=T_{\bar{N}}/2} \right)^2$$

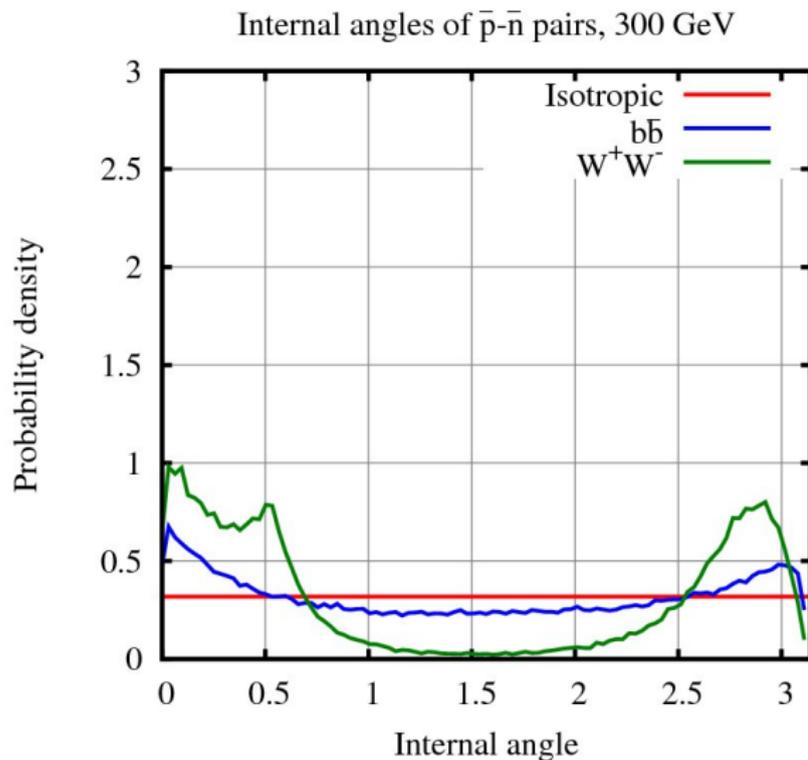
- consider e.g. DM annihilation $XX \rightarrow W^+ W^-$:

$$\frac{dN_{\bar{d}}}{dx} \propto \frac{1}{M_X^2} \frac{dN_{\bar{n}}}{dx} \frac{dN_{\bar{p}}}{dx}$$

- $1/M_X^2$ suppression in **contradiction to Lorentz invariance**:
- **decay products** of W are **boosted** in cone with $\vartheta \sim m_W/m_X$

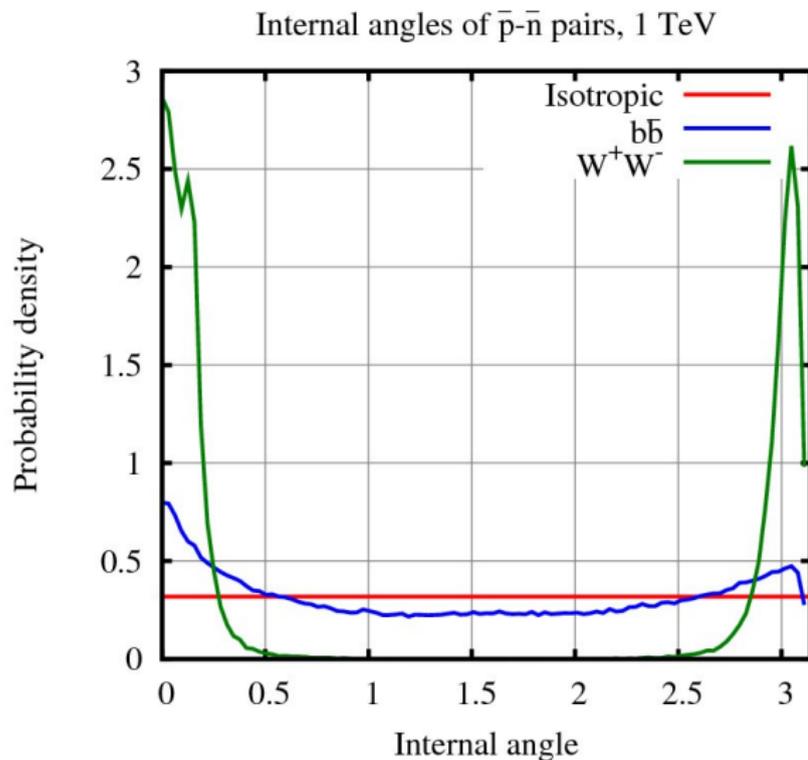
Angular distribution of $\bar{p}\bar{n}$ pairs

[Dal, MK '12]



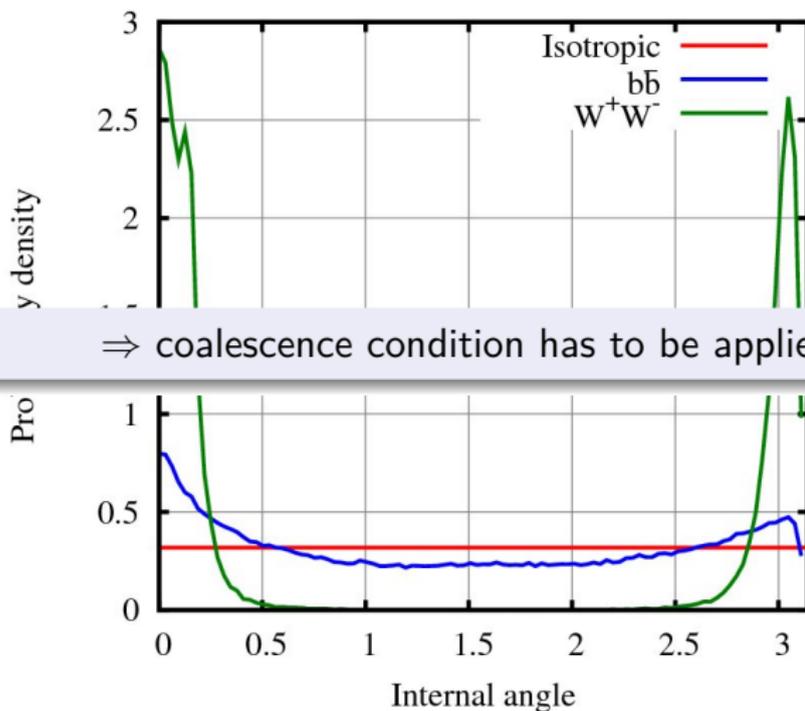
Angular distribution of $\bar{p}\bar{n}$ pairs

[Dal, MK '12]



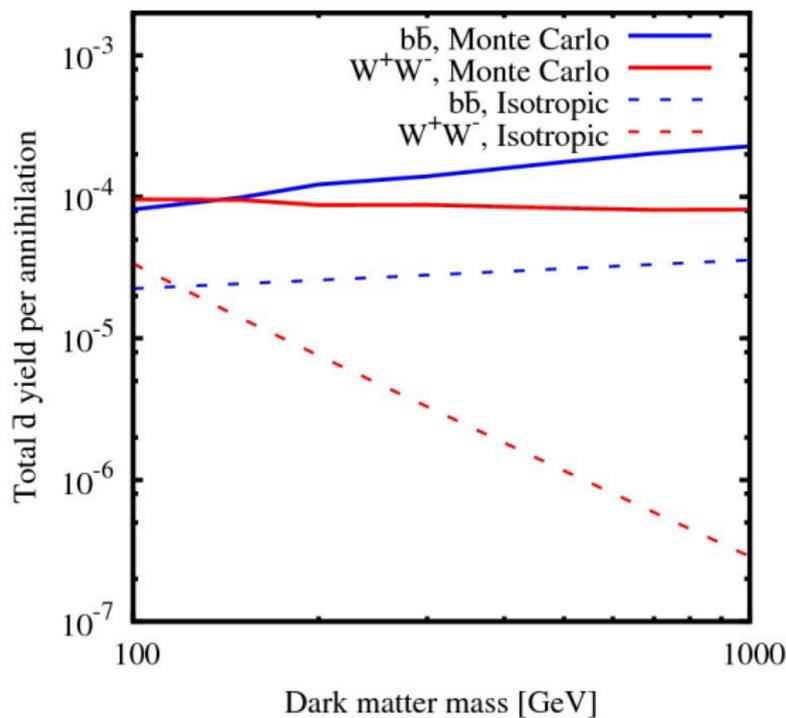
Angular distribution of $\bar{p}\bar{n}$ pairs

[Dal, MK '12]

Internal angles of $\bar{p}\bar{n}$ pairs, 1 TeV

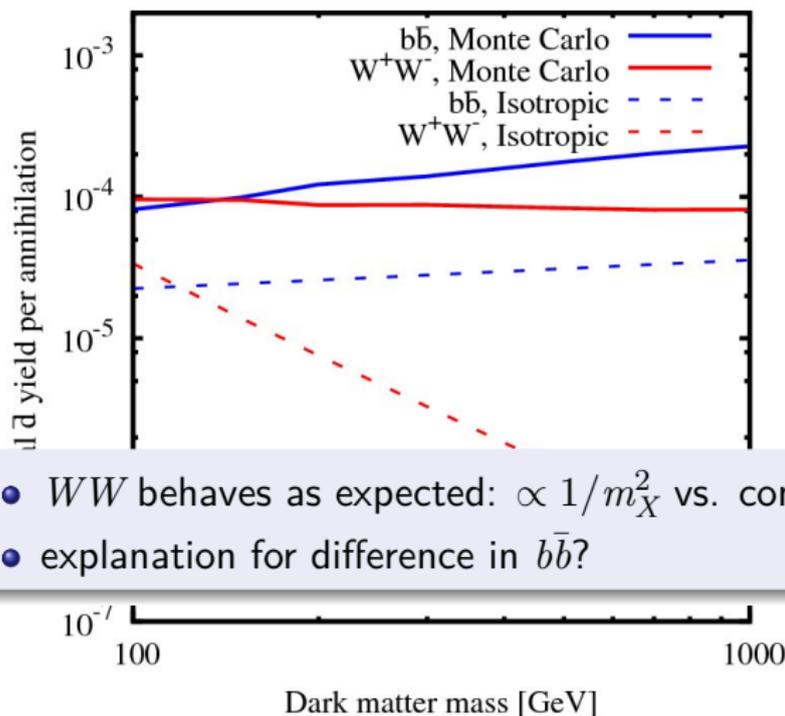
Deuteron yield: “Isotropic” vs. event-by-event:

[Dal, MK '12]

 \bar{d} yield energy dependence

Deuteron yield: “Isotropic” vs. event-by-event:

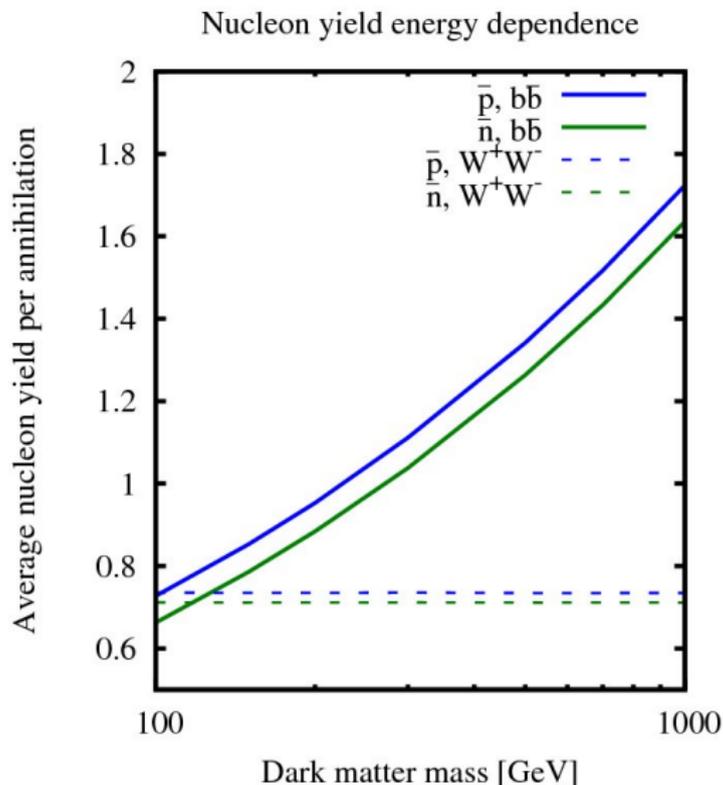
[Dal, MK '12]

 \bar{d} yield energy dependence

- WW behaves as expected: $\propto 1/m_X^2$ vs. const.
- explanation for difference in $b\bar{b}$?

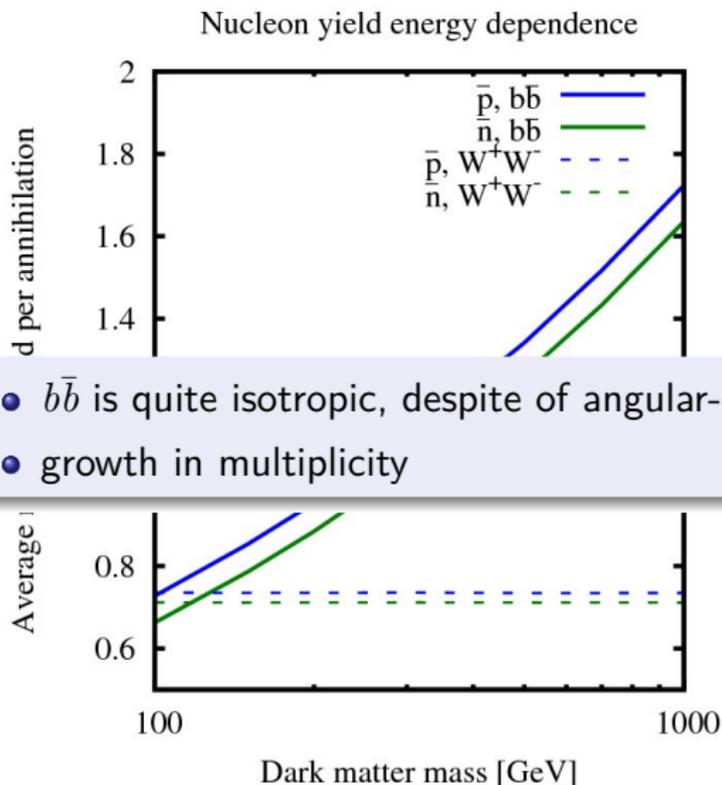
Deuteron yield: “Isotropic” vs. event-by-event:

[Dal, MK '12]



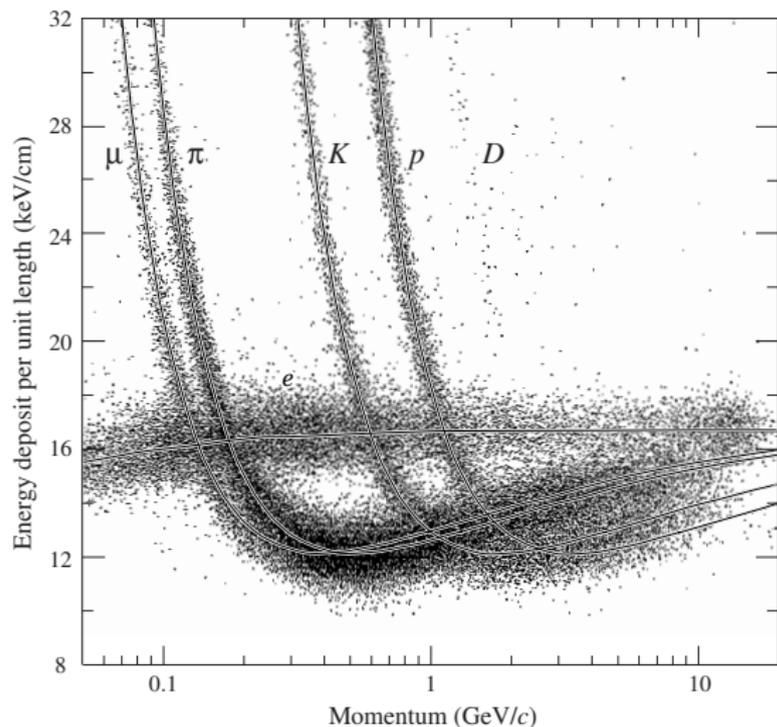
Deuteron yield: “Isotropic” vs. event-by-event:

[Dal, MK '12]



- $b\bar{b}$ is quite isotropic, despite of angular-ordered jets
- growth in multiplicity

Determining p_0 : particle ID in a TCP



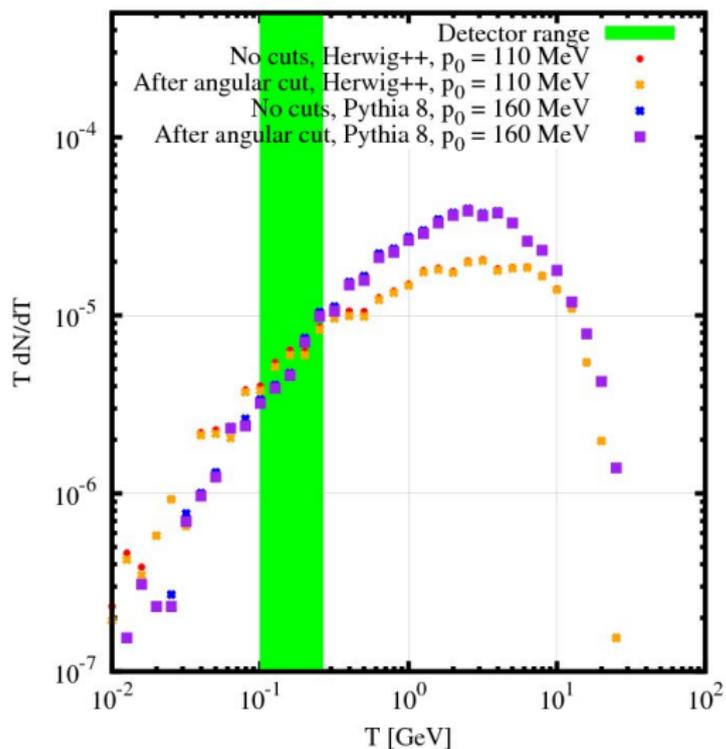
ALEPH: in
momentum range

$$0.62 < p < 1.03 \text{ GeV}$$

$$(5.9 \pm 1.8 \pm 0.5) \times 10^{-6} \bar{d}$$

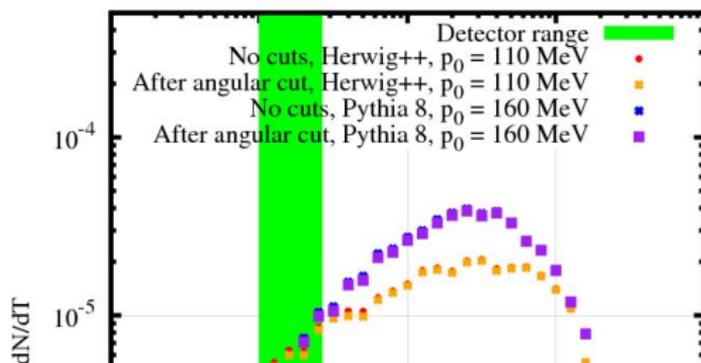
Determining p_0 :

e+e- antideuteron spectrum at Z resonance

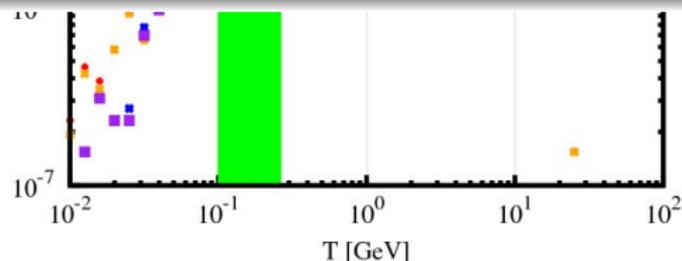


Determining p_0 :

e+e- antideuteron spectrum at Z resonance

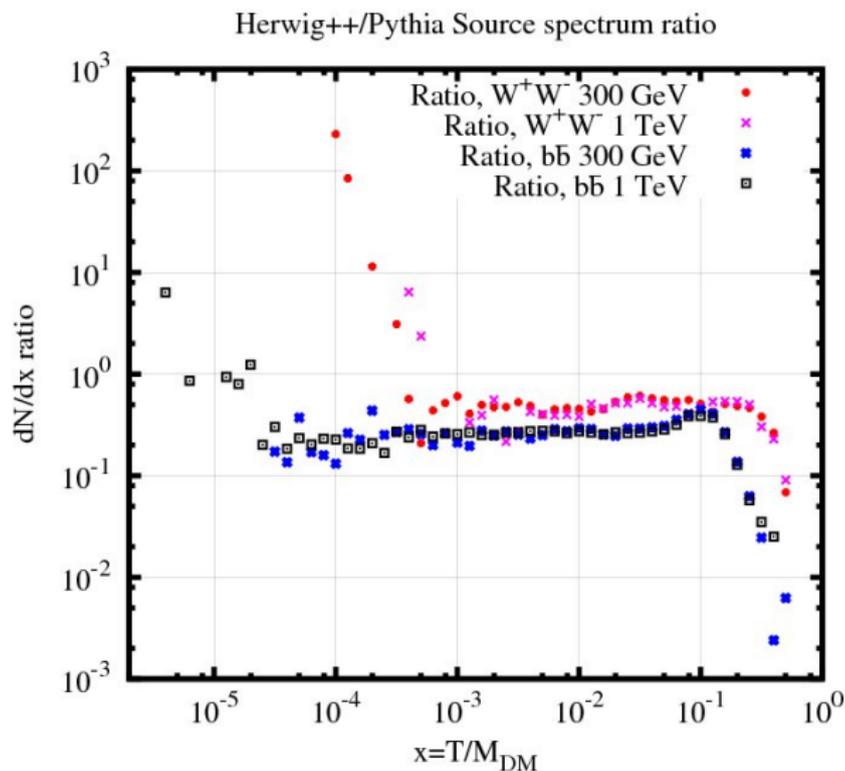


even in calibration reaction factor “few” difference between hadronization models



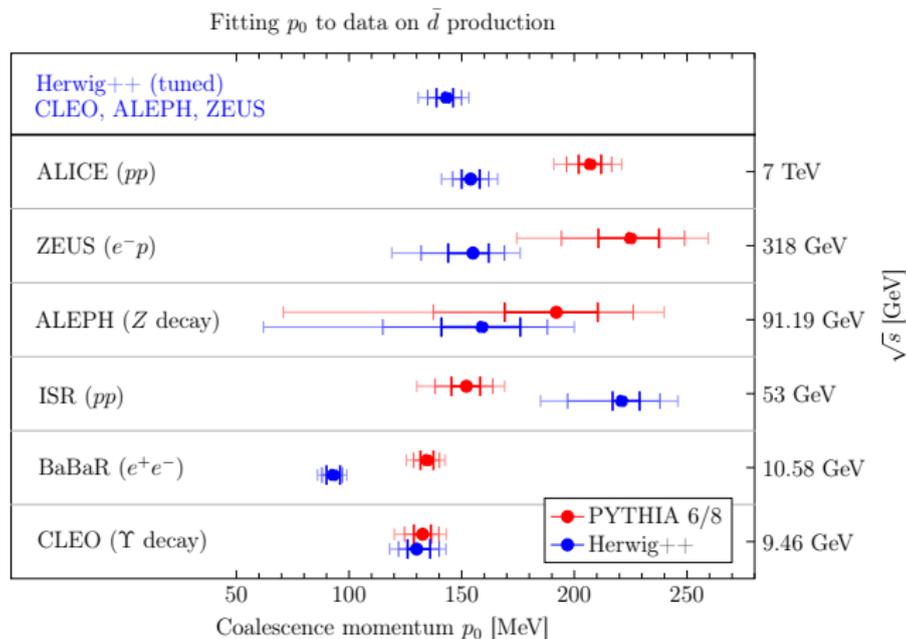
Hadronisation dependence on source spectrum

[Dal, MK '12]



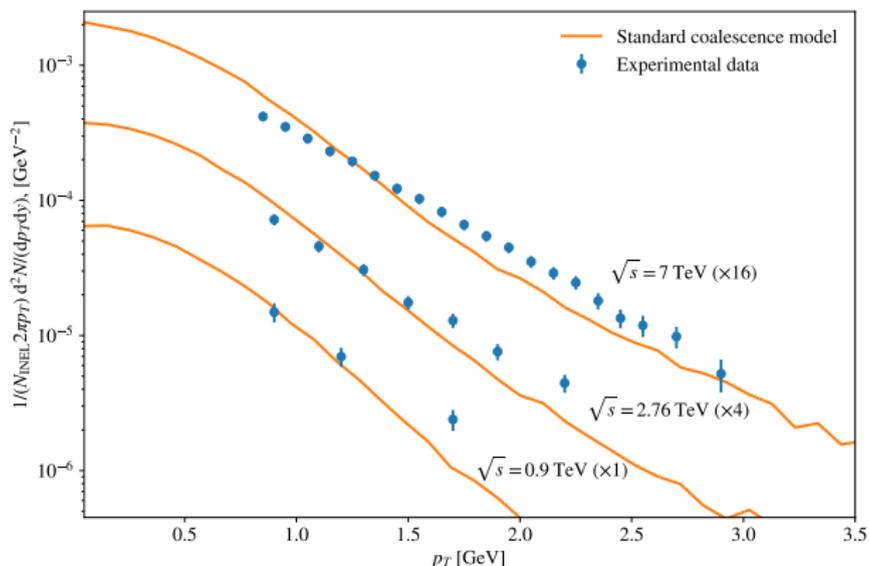
Problems of this approach:

- discrepancies in p_0 between e^+e^- , pp and A_p



Problems of this approach:

- discrepancies in p_0 between e^+e^- , pp and A_p
- **bad fit of ALICE data** at large p_\perp



Modeling $\sigma(NN \rightarrow \bar{d}X)$

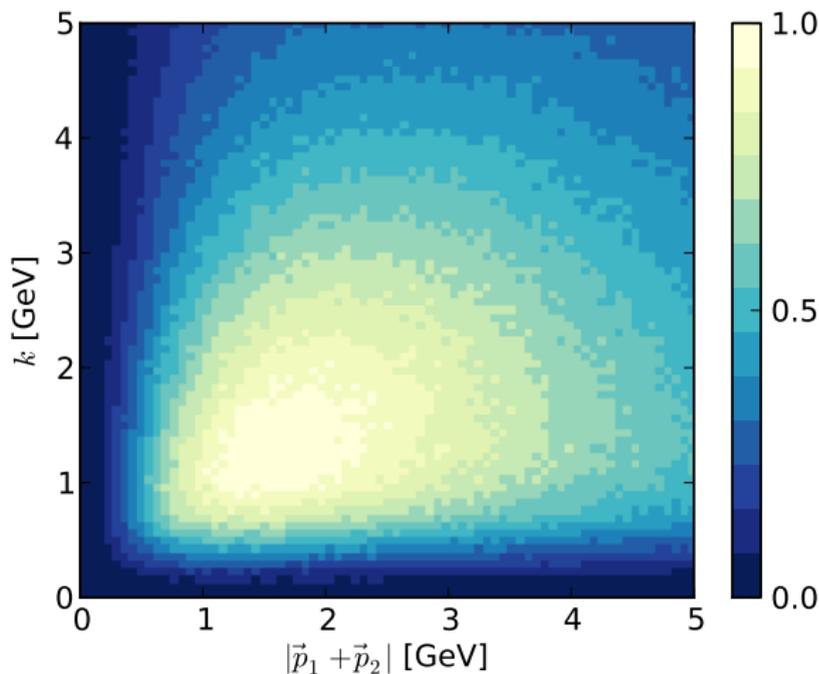
[Dal, Raklev '14]

- assume $P(\bar{p}\bar{n} \rightarrow \bar{d}X | k) \propto \sigma_{\bar{p}\bar{n} \rightarrow \bar{d}X}(k)$

Modeling $\sigma(NN \rightarrow \bar{d}X)$

[Dal, Raklev '14]

- assume $P(\bar{p}\bar{n} \rightarrow \bar{d}X | k) \propto \sigma_{\bar{p}\bar{n} \rightarrow \bar{d}X}(k)$



Modeling $\sigma(NN \rightarrow \bar{d}X)$

[Dal, Raklev '14]

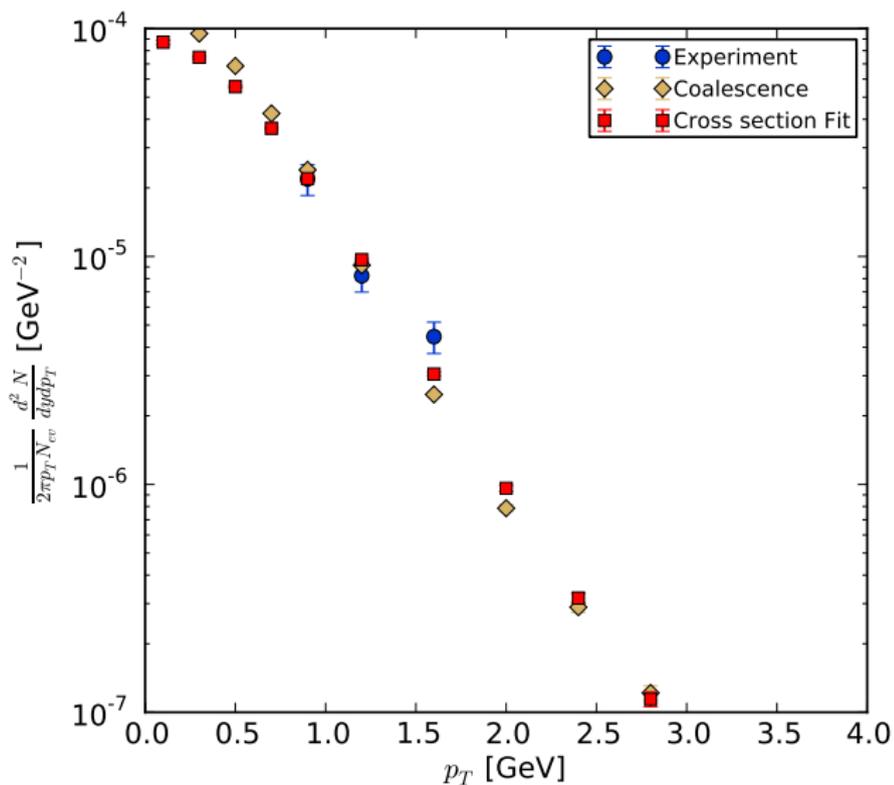
- assume $P(\bar{p}\bar{n} \rightarrow \bar{d}X | k) \propto \sigma_{\bar{p}\bar{n} \rightarrow \bar{d}X}(k)$

⇒ large number of pairs in Δ region

⇒ include reactions above pion threshold

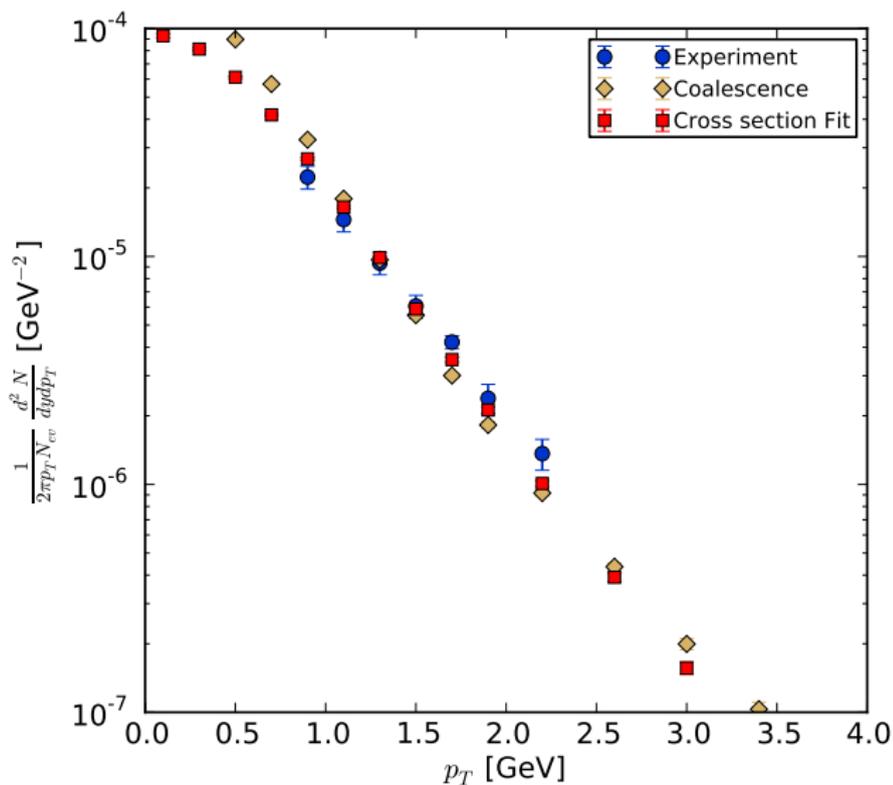
Modeling $\sigma(NN \rightarrow \bar{d}X)$

[Dal, Raklev '14]



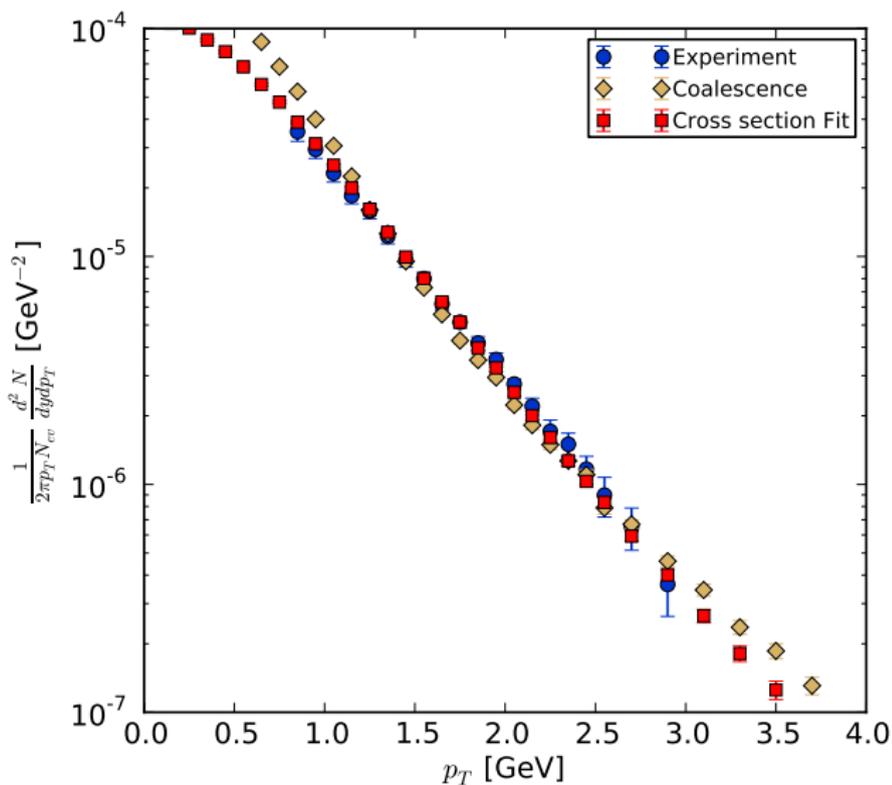
Modeling $\sigma(NN \rightarrow \bar{d}X)$

[Dal, Raklev '14]



Modeling $\sigma(NN \rightarrow \bar{d}X)$

[Dal, Raklev '14]



Statistical approaches

- Hagedorn '60-'65:

- ▶ pp collision creates fireball with $V \sim 4\pi/4m_\pi^3$
- ▶ probability for formation of cluster with quantum numbers X :

$$p(X \cap V) \times p(X)$$

Statistical approaches

- Hagedorn '60-'65:

- ▶ pp collision creates fireball with $V \sim 4\pi/4m_\pi^3$
- ▶ probability for formation of cluster with quantum numbers X :

$$p(X \cap V) \times p(X)$$

- modern version: **generalised Cooper-Frye formula**

$$E \frac{d^3 N_A}{d^3 P} = \frac{2J_A + 1}{(2\pi)^3} \int_{\Sigma_f} P \cdot d^3 \sigma(R) f_p^Z(R, P/A) f_n^N(R, P/A), \quad (1)$$

with $\mu_A(R)$ and $T_A(R)$ as free parameters

Wigner function based deuteron formation model: basis

- standard QM using **density matrices**

$$\frac{d^3 N_d}{dP_d^3} = \text{tr}\{\rho_d \rho_{\text{nucl}}\}$$

Wigner function based deuteron formation model: basis

- standard QM using density matrices

$$\frac{d^3 N_d}{dP_d^3} = \text{tr}\{\rho_d \rho_{\text{nucl}}\}$$

with

- ▶ deuteron density matrix

$$\rho_d = |\varphi_d\rangle \langle \varphi_d|$$

- ▶ two-nucleon density matrix

$$\rho_{\text{nucl}} = |\psi_p \psi_n\rangle \langle \psi_n \psi_p|$$

Wigner function based deuteron formation model: basis

- standard QM using density matrices

$$\frac{d^3 N_d}{dP_d^3} = \text{tr}\{\rho_d \rho_{\text{nucl}}\}$$

with

- ▶ deuteron density matrix

$$\rho_d = |\varphi_d\rangle \langle \varphi_d|$$

- ▶ two-nucleon density matrix

$$\rho_{\text{nucl}} = |\psi_p \psi_n\rangle \langle \psi_n \psi_p|$$

- **evaluate trace in coordinate space** using $1 = \int d^3 x |\mathbf{x}\rangle \langle \mathbf{x}|$

$$\begin{aligned} \frac{d^3 N_d}{dP_d^3} &= S \int d^3 x_1 d^3 x_2 d^3 x'_1 d^3 x'_2 \varphi_d^*(\mathbf{x}_1, \mathbf{x}_2) \varphi_d(\mathbf{x}'_1, \mathbf{x}'_2) \\ &\quad \times \left\langle \psi_n^\dagger(\mathbf{x}'_2) \psi_p^\dagger(\mathbf{x}'_1) \psi_p(\mathbf{x}_1) \psi_n(\mathbf{x}_2) \right\rangle \end{aligned}$$

Wigner function based deuteron formation model: ingredients

- factorize $\varphi_d(\mathbf{x}_1, \mathbf{x}_2)$ into
 - ▶ plane wave describing CoM motion with momentum \mathbf{P}_d
 - ▶ an internal wave function φ_d

Wigner function based deuteron formation model: ingredients

- factorize $\varphi_d(\mathbf{x}_1, \mathbf{x}_2)$ into
 - ▶ plane wave describing CoM motion with momentum \mathbf{P}_d
 - ▶ an internal wave function φ_d

⇒

$$\varphi_d(\mathbf{x}_1, \mathbf{x}_2) = (2\pi)^{-3/2} \exp\{i\mathbf{P}_d \cdot (\mathbf{x}_1 + \mathbf{x}_2)/2\} \varphi_d(\mathbf{x}_1 - \mathbf{x}_2).$$

Wigner function based deuteron formation model: ingredients

- factorize $\varphi_d(\mathbf{x}_1, \mathbf{x}_2)$ into
 - ▶ plane wave describing CoM motion with momentum \mathbf{P}_d
 - ▶ an internal wave function φ_d

⇒

$$\varphi_d(\mathbf{x}_1, \mathbf{x}_2) = (2\pi)^{-3/2} \exp\{i\mathbf{P}_d \cdot (\mathbf{x}_1 + \mathbf{x}_2)/2\} \varphi_d(\mathbf{x}_1 - \mathbf{x}_2).$$

- **replace the two-nucleon density matrix** by its two-body **Wigner function**

Reminder: Wigner functions

- $W(x, p)$ contains **full quantum mechanical information** of a system

Reminder: Wigner functions

- $W(x, p)$ contains full quantum mechanical information of a system
- obtained by a **Weyl transformation**

$$W(x, p) = \int d\xi \exp(-ip\xi) \psi(x + \xi/2) \psi^*(x - \xi/2)$$

Reminder: Wigner functions

- $W(x, p)$ contains full quantum mechanical information of a system
- obtained by a Weyl transformation

$$W(x, p) = \int d\xi \exp(-ip\xi) \psi(x + \xi/2) \psi^*(x - \xi/2)$$

- normalised as $\int \frac{dp}{2\pi} dx W(x, p) = 1$

Reminder: Wigner functions

- $W(x, p)$ contains full quantum mechanical information of a system
- obtained by a Weyl transformation

$$W(x, p) = \int d\xi \exp(-ip\xi) \psi(x + \xi/2) \psi^*(x - \xi/2)$$

- normalised as $\int \frac{dp}{2\pi} dx W(x, p) = 1$
- **probability distributions**

$$\int dx W(x, p) = \psi^*(p) \psi(p), \quad \int \frac{dp}{2\pi} W(x, p) = \varphi^*(x) \varphi(x)$$

Reminder: Wigner functions

- $W(x, p)$ contains full quantum mechanical information of a system
- obtained by a Weyl transformation

$$W(x, p) = \int d\xi \exp(-ip\xi) \psi(x + \xi/2) \psi^*(x - \xi/2)$$

- normalised as $\int \frac{dp}{2\pi} dx W(x, p) = 1$
- probability distributions

$$\int dx W(x, p) = \psi^*(p) \psi(p), \quad \int \frac{dp}{2\pi} W(x, p) = \varphi^*(x) \varphi(x)$$

- connection to density matrix

$$\langle \psi(\mathbf{x})^\dagger \psi(\mathbf{x}') \rangle = \int \frac{dp}{2\pi} W\left(\mathbf{p}, \frac{\mathbf{x} + \mathbf{x}'}{2}\right) \exp[i\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')]$$

Reminder: Wigner functions

- $W(x, p)$ contains full quantum mechanical information of a system
- obtained by a Weyl transformation

$$W(x, p) = \int d\xi \exp(-ip\xi) \psi(x + \xi/2) \psi^*(x - \xi/2)$$

- normalised as $\int \frac{dp}{2\pi} dx W(x, p) = 1$
- probability distributions

$$\int dx W(x, p) = \psi^*(p) \psi(p), \quad \int \frac{dp}{2\pi} W(x, p) = \varphi^*(x) \varphi(x)$$

- connection to density matrix

$$\langle \psi(\mathbf{x})^\dagger \psi(\mathbf{x}') \rangle = \int \frac{dp}{2\pi} W\left(\mathbf{p}, \frac{\mathbf{x} + \mathbf{x}'}{2}\right) \exp[i\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')]$$

- For our ansatz $W(x, p) = h(x)g(p)$, it follows that $h(x)$ describes the

Wigner function based deuteron formation model

- two-nucleon density matrix \Rightarrow two-body Wigner function:

$$\left\langle \psi_n(\mathbf{x}'_2)^\dagger \psi_p(\mathbf{x}'_1)^\dagger \psi_p(\mathbf{x}_1) \psi_n(\mathbf{x}_2) \right\rangle = \int \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} W_{np} \left(\mathbf{p}_n, \mathbf{p}_p, \frac{\mathbf{x}_2 + \mathbf{x}'_2}{2}, \frac{\mathbf{x}_1 + \mathbf{x}'_1}{2} \right) e^{i\mathbf{p}_n \cdot (\mathbf{x}_2 - \mathbf{x}'_2)} e^{i\mathbf{p}_p \cdot (\mathbf{x}_1 - \mathbf{x}'_1)}.$$

Wigner function based deuteron formation model

- two-nucleon density matrix \Rightarrow two-body Wigner function:

$$\left\langle \psi_n(\mathbf{x}'_2)^\dagger \psi_p(\mathbf{x}'_1)^\dagger \psi_p(\mathbf{x}_1) \psi_n(\mathbf{x}_2) \right\rangle = \int \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} W_{np} \left(\mathbf{p}_n, \mathbf{p}_p, \frac{\mathbf{x}_2 + \mathbf{x}'_2}{2}, \frac{\mathbf{x}_1 + \mathbf{x}'_1}{2} \right) e^{i\mathbf{p}_n \cdot (\mathbf{x}_2 - \mathbf{x}'_2)} e^{i\mathbf{p}_p \cdot (\mathbf{x}_1 - \mathbf{x}'_1)}.$$

- separate CMS movement

$$\frac{d^3 N_d}{dP_d^3} = \frac{S}{(2\pi)^6} \int d^3 q d^3 r_p d^3 r_n \mathcal{D}(\mathbf{r}, \mathbf{q}) W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p)$$

where $\mathbf{q} = (\mathbf{p}_n - \mathbf{p}_p)/2$, $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$, and

$$\mathcal{D}(\mathbf{r}, \mathbf{q}) = \int d^3 \xi e^{-i\mathbf{q} \cdot \xi} \varphi_d(\mathbf{r} + \xi/2) \varphi_d^*(\mathbf{r} - \xi/2)$$

Wigner function based deuteron formation model

- two-nucleon density matrix \Rightarrow two-body Wigner function:

$$\left\langle \psi_n(\mathbf{x}'_2)^\dagger \psi_p(\mathbf{x}'_1)^\dagger \psi_p(\mathbf{x}_1) \psi_n(\mathbf{x}_2) \right\rangle = \int \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} W_{np} \left(\mathbf{p}_n, \mathbf{p}_p, \frac{\mathbf{x}_2 + \mathbf{x}'_2}{2}, \frac{\mathbf{x}_1 + \mathbf{x}'_1}{2} \right) e^{i\mathbf{p}_n \cdot (\mathbf{x}_2 - \mathbf{x}'_2)} e^{i\mathbf{p}_p \cdot (\mathbf{x}_1 - \mathbf{x}'_1)}.$$

- separate CMS movement

$$\frac{d^3 N_d}{dP_d^3} = \frac{S}{(2\pi)^6} \int d^3 q d^3 r_p d^3 r_n \mathcal{D}(\mathbf{r}, \mathbf{q}) W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p)$$

where $\mathbf{q} = (\mathbf{p}_n - \mathbf{p}_p)/2$, $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$, and

$$\mathcal{D}(\mathbf{r}, \mathbf{q}) = \int d^3 \xi e^{-i\mathbf{q} \cdot \xi} \varphi_d(\mathbf{r} + \xi/2) \varphi_d^*(\mathbf{r} - \xi/2)$$

Wigner function based deuteron formation model

- two-nucleon density matrix \Rightarrow two-body Wigner function:

$$\left\langle \psi_n(\mathbf{x}'_2)^\dagger \psi_p(\mathbf{x}'_1)^\dagger \psi_p(\mathbf{x}_1) \psi_n(\mathbf{x}_2) \right\rangle = \int \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} W_{np} \left(\mathbf{p}_n, \mathbf{p}_p, \frac{\mathbf{x}_2 + \mathbf{x}'_2}{2}, \frac{\mathbf{x}_1 + \mathbf{x}'_1}{2} \right) e^{i\mathbf{p}_n \cdot (\mathbf{x}_2 - \mathbf{x}'_2)} e^{i\mathbf{p}_p \cdot (\mathbf{x}_1 - \mathbf{x}'_1)}.$$

- separate CMS movement

$$\frac{d^3 N_d}{dP_d^3} = \frac{S}{(2\pi)^6} \int d^3 q d^3 r_p d^3 r_n \mathcal{D}(\mathbf{r}, \mathbf{q}) W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p)$$

where $\mathbf{q} = (\mathbf{p}_n - \mathbf{p}_p)/2$, $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$, and

$$\mathcal{D}(\mathbf{r}, \mathbf{q}) = \int d^3 \xi e^{-i\mathbf{q} \cdot \xi} \varphi_d(\mathbf{r} + \xi/2) \varphi_d^*(\mathbf{r} - \xi/2)$$

Wigner function based deuteron formation model

- two-nucleon density matrix \Rightarrow two-body Wigner function:

$$\left\langle \psi_n(\mathbf{x}'_2)^\dagger \psi_p(\mathbf{x}'_1)^\dagger \psi_p(\mathbf{x}_1) \psi_n(\mathbf{x}_2) \right\rangle = \int \frac{d^3 p_n}{(2\pi)^3} \frac{d^3 p_p}{(2\pi)^3} W_{np} \left(\mathbf{p}_n, \mathbf{p}_p, \frac{\mathbf{x}_2 + \mathbf{x}'_2}{2}, \frac{\mathbf{x}_1 + \mathbf{x}'_1}{2} \right) e^{i\mathbf{p}_n \cdot (\mathbf{x}_2 - \mathbf{x}'_2)} e^{i\mathbf{p}_p \cdot (\mathbf{x}_1 - \mathbf{x}'_1)}.$$

- separate CMS movement

$$\frac{d^3 N_d}{dP_d^3} = \frac{S}{(2\pi)^6} \int d^3 q d^3 r_p d^3 r_n \mathcal{D}(\mathbf{r}, \mathbf{q}) W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p)$$

where $\mathbf{q} = (\mathbf{p}_n - \mathbf{p}_p)/2$, $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_p$, and

$$\mathcal{D}(\mathbf{r}, \mathbf{q}) = \int d^3 \xi e^{-i\mathbf{q} \cdot \xi} \varphi_d(\mathbf{r} + \xi/2) \varphi_d^*(\mathbf{r} - \xi/2)$$

= Wigner function of internal deuteron wave function φ_d

Deuteron wave-function

- best description: Hulthen wave-function

$$\varphi_d(\mathbf{r}) = \sqrt{\frac{ab(a+b)}{2\pi(a-b)^2}} \frac{e^{-ar} - e^{-br}}{r}$$

Deuteron wave-function

- best description: Hulthen wave-function

$$\varphi_d(\mathbf{r}) = \sqrt{\frac{ab(a+b)}{2\pi(a-b)^2}} \frac{e^{-ar} - e^{-br}}{r}$$

- simpler: Gaussian

$$\varphi_d(\mathbf{r}) = (\pi d^2)^{-3/4} \exp\left(-\frac{r^2}{2d^2}\right),$$

with d fitted to deuteron's rms charge radius $r_{\text{rms}} = 2.14$ fm

Deuteron wave-function

- best description: Hulthen wave-function

$$\varphi_d(\mathbf{r}) = \sqrt{\frac{ab(a+b)}{2\pi(a-b)^2}} \frac{e^{-ar} - e^{-br}}{r}$$

- simpler: Gaussian

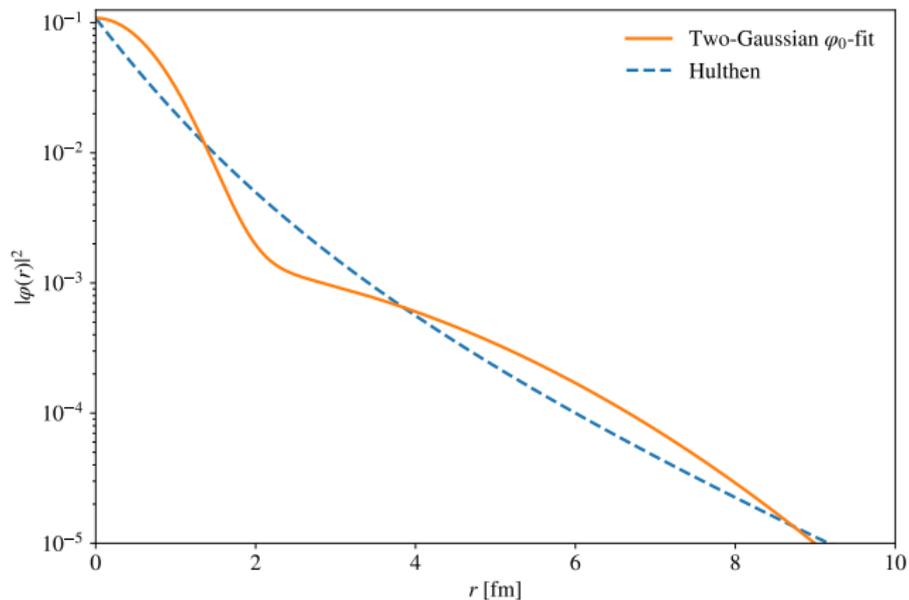
$$\varphi_d(\mathbf{r}) = (\pi d^2)^{-3/4} \exp\left(-\frac{r^2}{2d^2}\right),$$

with d fitted to deuteron's rms charge radius $r_{\text{rms}} = 2.14 \text{ fm}$

⇒ Wigner function

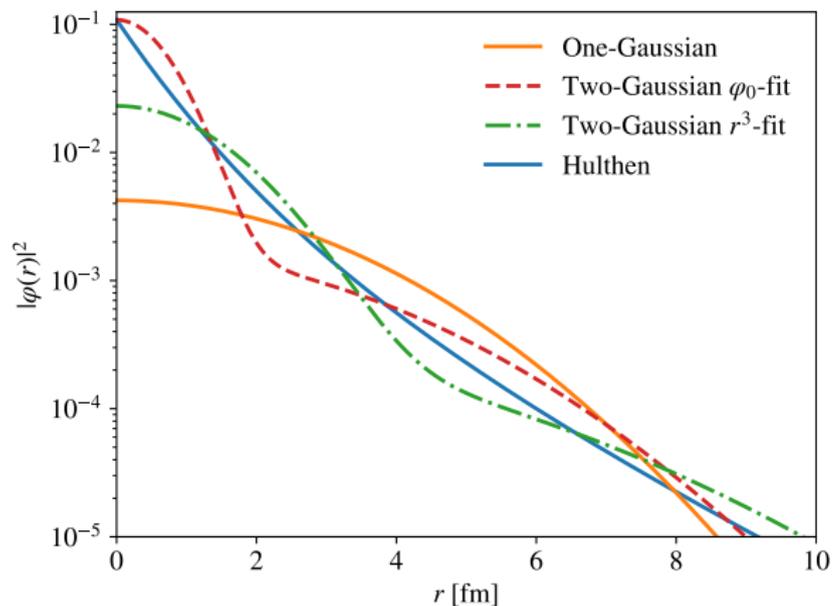
$$\mathcal{D}(\mathbf{r}, \mathbf{q}) = 8e^{-r^2/d^2} e^{-q^2 d^2}$$

Deuteron wave-function



Deuteron wave-function

- better: superposition of two Gaussians



Choice for Wigner function

- “fireball” motivates thermal equilibrium state
 - + exact Wigner function

Choice for Wigner function

- “fireball” motivates thermal equilibrium state
 - + exact Wigner function
 - neglects two-particle correlations
 - holds (if at all) only for central heavy-ion collision

Choice for Wigner function

- “fireball” motivates thermal equilibrium state
 - + exact Wigner function
 - neglects two-particle correlations
 - holds (if at all) only for central heavy-ion collision
- **Monte Carlos** provide **momentum distributions**,
 - + including **momentum correlations**

Choice for Wigner function

- “fireball” motivates thermal equilibrium state
 - + exact Wigner function
 - neglects two-particle correlations
 - holds (if at all) only for central heavy-ion collision
- Monte Carlos provide momentum distributions,
 - + including momentum correlations
- both cases: spatial information has to be added

Fireball and Wigner functions

[Scheibl, Heinz '98]

- deuterons formed during **freeze-out** from **expanding fireball**

Fireball and Wigner functions

[Scheibl, Heinz '98]

- deuterons formed during freeze-out from expanding fireball
- particle momenta are correlated within fireball

Fireball and Wigner functions

[Scheibl, Heinz '98]

- deuterons formed during freeze-out from expanding fireball
- particle momenta are correlated within fireball
- within a “homogeneity volume” $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ similar momenta, able to produce deuterons

Fireball and Wigner functions

[Scheibl, Heinz '98]

- deuterons formed during freeze-out from expanding fireball
- particle momenta are correlated within fireball
- within a “homogeneity volume” $\mathcal{R}_{\perp}^2 \mathcal{R}_{\parallel}$ similar momenta, able to produce deuterons
- $\mathcal{R}_{\perp}^2 \mathcal{R}_{\parallel}$ determined from HBT measurements via 2-particle correlations

Fireball and Wigner functions

[Scheibl, Heinz '98]

- deuterons formed during freeze-out from expanding fireball
- particle momenta are correlated within fireball
- within a “homogeneity volume” $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ similar momenta, able to produce deuterons
- $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ determined from HBT measurements via 2-particle correlations
- after some calculations and approximations:

$$B_2 = \frac{3 \pi^{3/2} \langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)} e^{2(m_t - m) \left(\frac{1}{T_p^*} - \frac{1}{T_d^*} \right)}$$

Fireball and Wigner functions

[Scheibl, Heinz '98]

- deuterons formed during freeze-out from expanding fireball
- particle momenta are correlated within fireball
- within a “homogeneity volume” $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ similar momenta, able to produce deuterons
- $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ determined from HBT measurements via 2-particle correlations
- after some calculations and approximations:

$$B_2 = \frac{3 \pi^{3/2} \langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)} e^{2(m_t - m) \left(\frac{1}{T_p^*} - \frac{1}{T_d^*} \right)}$$

- Scheibl, Heinz '98: formalism covers 4% of all Pb+Pb collisions
- bad fit of p_\perp spectra

Application to Ap and pp collisions

[Blum et al. '17]

- coalescence factor

$$B_2 = \frac{3 \pi^{3/2} \langle \mathcal{C}_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)} e^{2(m_t - m) \left(\frac{1}{T_p^*} - \frac{1}{T_d^*} \right)}$$

Application to Ap and pp collisions

[Blum et al. '17]

- coalescence factor **simplifies for a Gaussian profile**

$$B_2 = \frac{3 \pi^{3/2} \langle C_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)}$$

- add simple estimate for $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ as function of A :

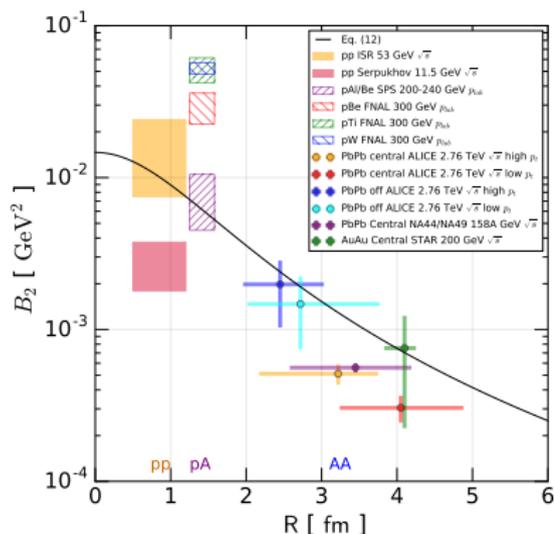
Application to Ap and pp collisions

[Blum et al. '17]

- coalescence factor simplifies for a Gaussian profile

$$B_2 = \frac{3 \pi^{3/2} \langle C_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)}$$

- add simple estimate for $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ as function of A :



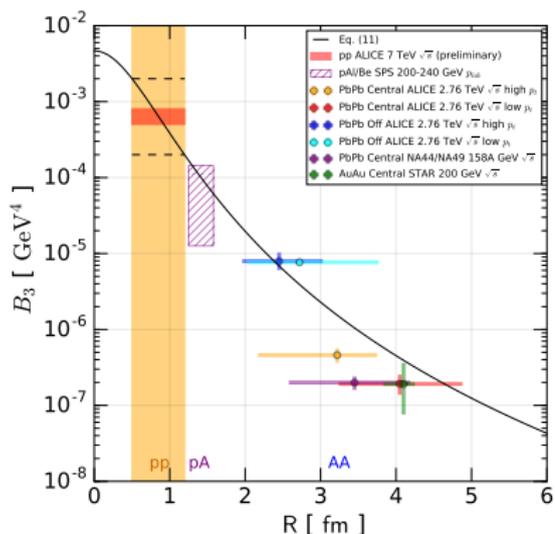
Application to Ap and pp collisions

[Blum et al. '17]

- coalescence factor simplifies for a Gaussian profile

$$B_2 = \frac{3 \pi^{3/2} \langle C_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)}$$

- add simple estimate for $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ as function of A :



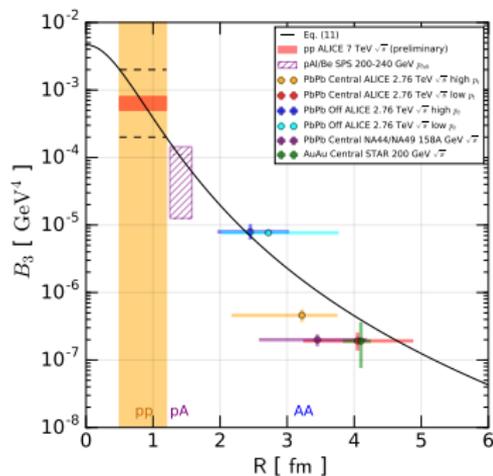
Application to Ap and pp collisions

[Blum et al. '17]

- coalescence factor simplifies for a Gaussian profile

$$B_2 = \frac{3 \pi^{3/2} \langle C_d \rangle}{2m_t \mathcal{R}_\perp^2(m_t) \mathcal{R}_\parallel(m_t)}$$

- add simple estimate for $\mathcal{R}_\perp^2 \mathcal{R}_\parallel$ as function of A :



- no check of **distributions** from LHC (?)

Using Monte Carlo correlations

[MK, Ostapchenko, Tjemsland '19]

- provides **momentum distribution** $G_{np}(\mathbf{p}_n, \mathbf{p}_p)$

Using Monte Carlo correlations

[MK, Ostapchenko, Tjemsland '19]

- provides momentum distribution $G_{np}(\mathbf{p}_n, \mathbf{p}_p)$
- connection to Wigner function

$$\int d^3 r_p d^3 r_n W_{np}(\mathbf{p}_n, \mathbf{p}_p, \mathbf{r}_n, \mathbf{r}_p) = N_p N_n |\psi_{np}(\mathbf{p}_n, \mathbf{p}_p)|^2 \equiv G_{np}(\mathbf{p}_n, \mathbf{p}_p),$$

Using Monte Carlo correlations

[MK, Ostapchenko, Tjemsland '19]

- provides momentum distribution $G_{np}(\mathbf{p}_n, \mathbf{p}_p)$
- connection to Wigner function

$$\int d^3 r_p d^3 r_n W_{np}(\mathbf{p}_n, \mathbf{p}_p, \mathbf{r}_n, \mathbf{r}_p) = N_p N_n |\psi_{np}(\mathbf{p}_n, \mathbf{p}_p)|^2 \equiv G_{np}(\mathbf{p}_n, \mathbf{p}_p),$$

- **assume factorization** of momentum and coordinates,

$$W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p) = H_{np}(\mathbf{r}_n, \mathbf{r}_p) G_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q})$$

Using Monte Carlo correlations

[MK, Ostapchenko, Tjemsland '19]

- provides momentum distribution $G_{np}(\mathbf{p}_n, \mathbf{p}_p)$
- connection to Wigner function

$$\int d^3 r_p d^3 r_n W_{np}(\mathbf{p}_n, \mathbf{p}_p, \mathbf{r}_n, \mathbf{r}_p) = N_p N_n |\psi_{np}(\mathbf{p}_n, \mathbf{p}_p)|^2 \equiv G_{np}(\mathbf{p}_n, \mathbf{p}_p),$$

- assume factorization of momentum and coordinates,

$$W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p) = H_{np}(\mathbf{r}_n, \mathbf{r}_p) G_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q})$$

- **neglect spatial correlations** between the proton and the neutron,
 $H_{np}(\mathbf{r}_n, \mathbf{r}_p) = h(\mathbf{r}_n) h(\mathbf{r}_p)$

Using Monte Carlo correlations

[MK, Ostapchenko, Tjemsland '19]

- provides momentum distribution $G_{np}(\mathbf{p}_n, \mathbf{p}_p)$
- connection to Wigner function

$$\int d^3 r_p d^3 r_n W_{np}(\mathbf{p}_n, \mathbf{p}_p, \mathbf{r}_n, \mathbf{r}_p) = N_p N_n |\psi_{np}(\mathbf{p}_n, \mathbf{p}_p)|^2 \equiv G_{np}(\mathbf{p}_n, \mathbf{p}_p),$$

- assume factorization of momentum and coordinates,

$$W_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q}, \mathbf{r}_n, \mathbf{r}_p) = H_{np}(\mathbf{r}_n, \mathbf{r}_p) G_{np}(\mathbf{P}_d/2 + \mathbf{q}, \mathbf{P}_d/2 - \mathbf{q})$$

- neglect spatial correlations between the proton and the neutron,
 $H_{np}(\mathbf{r}_n, \mathbf{r}_p) = h(\mathbf{r}_n) h(\mathbf{r}_p)$
- choose a **Gaussian ansatz for $h(\mathbf{r})$,**

$$h(\mathbf{r}) = (2\pi\sigma^2)^{-3/2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Using Monte Carlo correlations

- few simple steps later:

$$\frac{d^3 N_d}{dP_d^3} = \frac{3\zeta}{(2\pi)^6} \int d^3 q e^{-q^2 d^2} G_{np}(+\mathbf{q}, -\mathbf{q}),$$

with

$$\zeta \equiv \left(\frac{d^2}{d^2 + 4\sigma^2} \right)^{3/2} \leq 1$$

Using Monte Carlo correlations

- few simple steps later:

$$\frac{d^3 N_d}{dP_d^3} = \frac{3\zeta}{(2\pi)^6} \int d^3 q e^{-q^2 d^2} G_{np}(+\mathbf{q}, -\mathbf{q}),$$

with

$$\zeta \equiv \left(\frac{d^2}{d^2 + 4\sigma^2} \right)^{3/2} \leq 1$$

- “usual MC approach” is recovered for
 - ▶ $d \gg \sigma \Rightarrow \zeta \rightarrow 1$
 - ▶ $e^{-q^2 d^2} \rightarrow \vartheta(p - p_{\max})$

Using Monte Carlo correlations

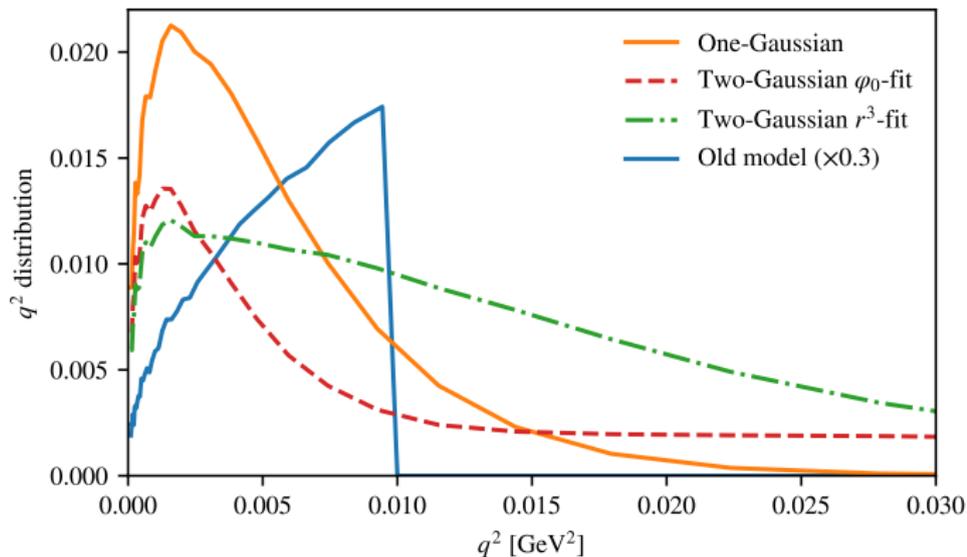
- few simple steps later:

$$\frac{d^3 N_d}{dP_d^3} = \frac{3\zeta}{(2\pi)^6} \int d^3 q e^{-q^2 d^2} G_{np}(+\mathbf{q}, -\mathbf{q}),$$

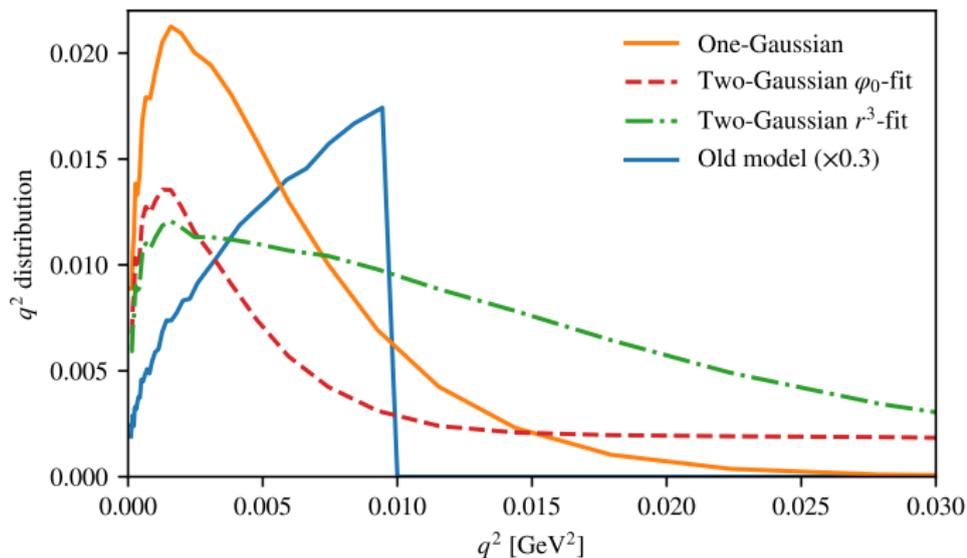
with

$$\zeta \equiv \left(\frac{d^2}{d^2 + 4\sigma^2} \right)^{3/2} \leq 1$$

- “usual MC approach” is recovered for
 - ▶ $d \gg \sigma \Rightarrow \zeta \rightarrow 1$
 - ▶ $e^{-q^2 d^2} \rightarrow \vartheta(p - p_{\max})$
- **fraction $\bar{d}/(\bar{p} + \bar{n})$ is bounded**

Contribution of different q^2 to \bar{d} yield:

Contribution of different q^2 to \bar{d} yield:



- contribution of large q^2 enhanced

Parameters σ_i and frame dependence

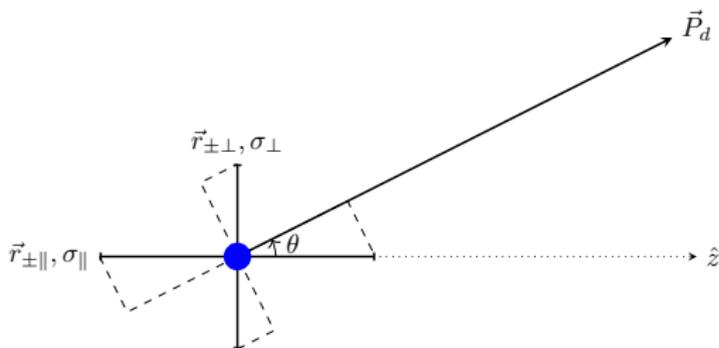
- Recall from **yesterday**: in e^+e^- in cms frame
 - ▶ $\sigma_{\parallel} \simeq \gamma R_p$
 - ▶ $\sigma_{\perp} \simeq \Lambda_{\text{QCD}} \simeq R_p$

Parameters σ_i and frame dependence

- Recall from yesterday: in e^+e^- in cms frame
 - ▶ $\sigma_{\parallel} \simeq \gamma R_p$
 - ▶ $\sigma_{\perp} \simeq \Lambda_{\text{QCD}} \simeq R_p$
- include effect of **Lorentz transformations**:
 - ▶ trivial factor $1/\gamma$, since we derive $G_{np}(+\mathbf{q}, -\mathbf{q})$ in cms frame of pair

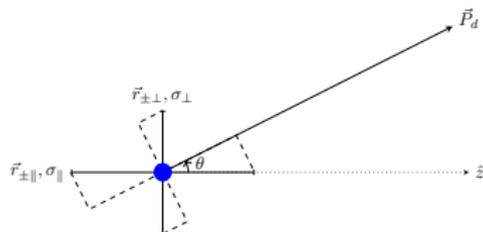
Parameters σ_i and frame dependence

- Recall from yesterday: in e^+e^- in cms frame
 - $\sigma_{\parallel} \simeq \gamma R_p$
 - $\sigma_{\perp} \simeq \Lambda_{\text{QCD}} \simeq R_p$
- include effect of Lorentz transformations:
 - trivial factor $1/\gamma$, since we derive $G_{np}(+\mathbf{q}, -\mathbf{q})$ in cms frame of pair
 - split** σ_{\perp} and σ_{\parallel} on **beam (z -axis)**:



Parameters σ_i and frame dependence

- Recall from yesterday: in e^+e^- in cms frame
 - $\sigma_{\parallel} \simeq \gamma R_p$
 - $\sigma_{\perp} \simeq \Lambda_{\text{QCD}} \simeq R_p$
- include effect of Lorentz transformations:
 - trivial factor $1/\gamma$, since we derive $G_{np}(+\mathbf{q}, -\mathbf{q})$ in cms frame of pair



- split σ_{\perp} and σ_{\parallel} on beam (z -axis):

$$\zeta = \frac{d^2}{d^2 + 4\tilde{\sigma}_{\perp}^2} \sqrt{\frac{d^2}{d^2 + 4\sigma_{\parallel}^2}}$$

with

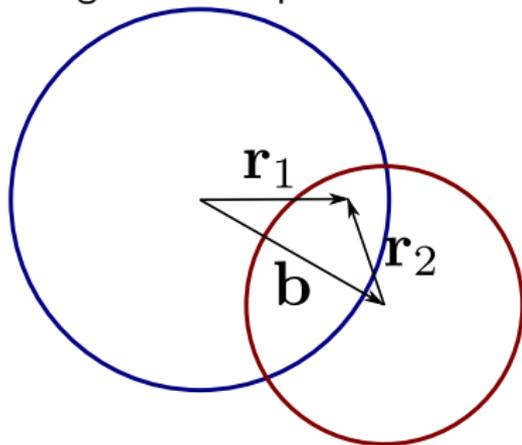
$$\tilde{\sigma}_{\perp} = \frac{\sigma_{\perp}}{\sqrt{\cos^2 \vartheta + \gamma^2 \sin^2 \vartheta}}$$

Generalising to Ap and AA collisions

- hadronic collisions:
 - ▶ **parton cloud** distributed within R_p or R_A
 - ▶ **multiple parton interactions**
 - ▶ cluster can form from different parton interactions

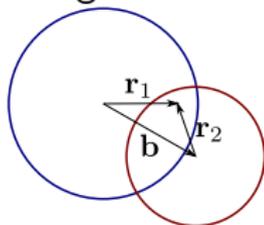
Generalising to A_p and AA collisions

- hadronic collisions:
 - ▶ parton cloud distributed within R_p or R_A
 - ▶ multiple parton interactions
 - ▶ cluster can form from different parton interactions
- using Gaussian profiles



Generalising to Ap and AA collisions

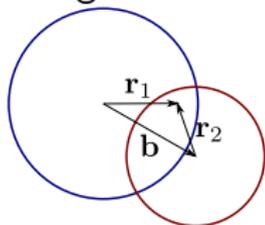
- hadronic collisions:
 - ▶ parton cloud distributed within R_p or R_A
 - ▶ multiple parton interactions
 - ▶ cluster can form from different parton interactions
- using Gaussian profiles



- ▶ pp: $\sigma^{pp} = \sqrt{2}\sigma^{e^+e^-}$

Generalising to Ap and AA collisions

- hadronic collisions:
 - ▶ parton cloud distributed within R_p or R_A
 - ▶ multiple parton interactions
 - ▶ cluster can form from different parton interactions
- using Gaussian profiles

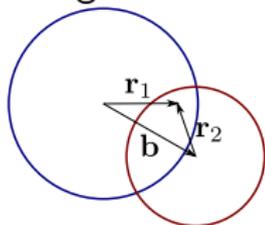


- ▶ pp: $\sigma^{pp} = \sqrt{2}\sigma^{e^+e^-}$
- ▶ AA:

$$\sigma_{\parallel}^2 = (\sigma_{\parallel}^{e^+e^-})^2 + R_A^2, \quad \sigma_{\perp}^2 = (\sigma_{\perp}^{e^+e^-})^2 + \frac{2R_A^2 R_p^2}{R_A^2 + R_p^2}$$

Generalising to Ap and AA collisions

- hadronic collisions:
 - ▶ parton cloud distributed within R_p or R_A
 - ▶ multiple parton interactions
 - ▶ cluster can form from different parton interactions
- using Gaussian profiles

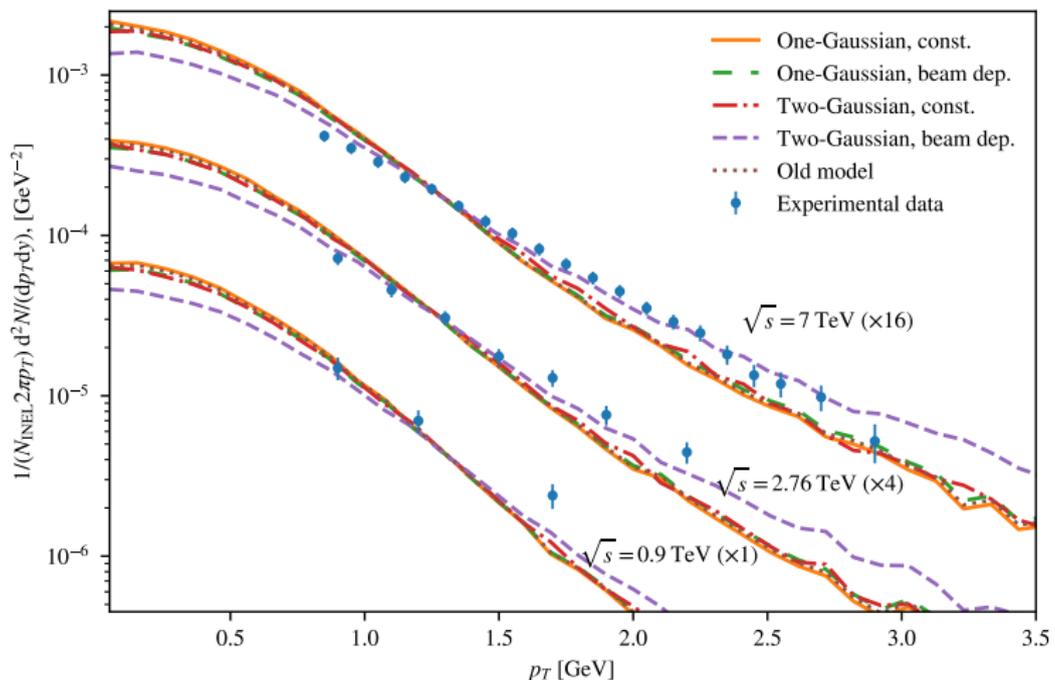


- ▶ pp: $\sigma^{pp} = \sqrt{2}\sigma^{e^+e^-}$
- ▶ AA:

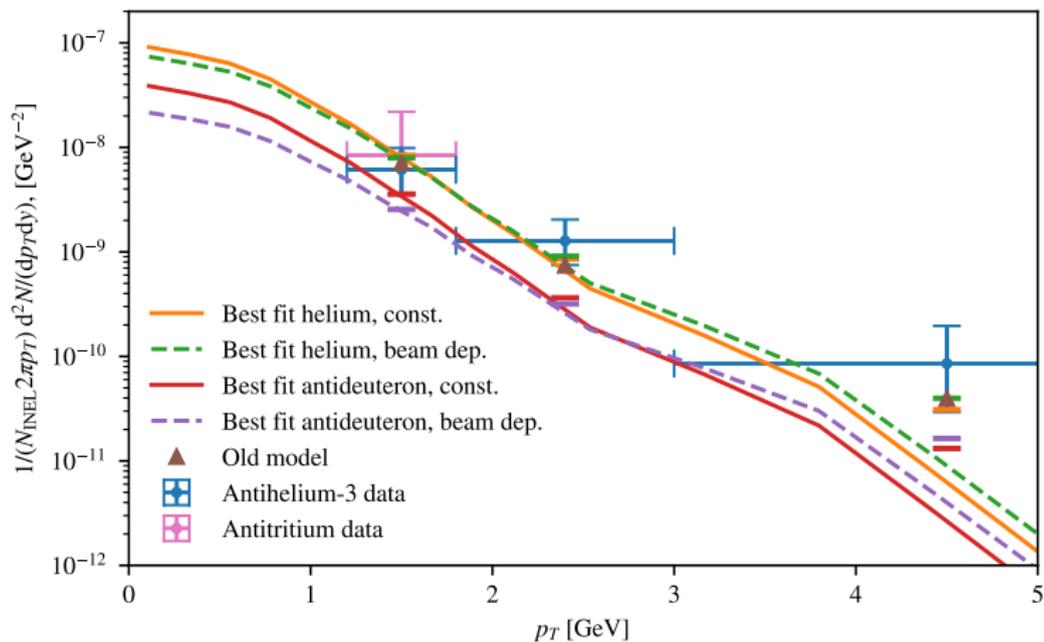
$$\sigma_{\parallel}^2 = (\sigma_{\parallel}^{e^+e^-})^2 + R_A^2, \quad \sigma_{\perp}^2 = (\sigma_{\perp}^{e^+e^-})^2 + \frac{2R_A^2 R_p^2}{R_A^2 + R_p^2}$$

- expectation: $\sigma^{e^+e^-} \simeq 5/\text{GeV}$ and $\sigma^{pp} \simeq 7/\text{GeV}$

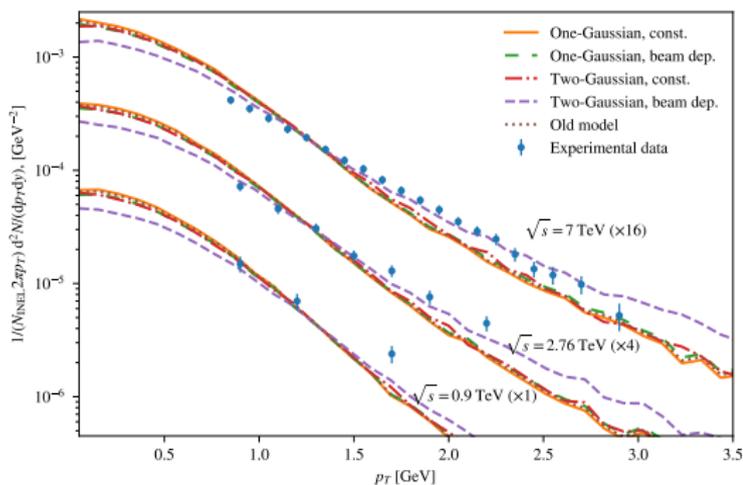
Comparison with ALICE data



Comparison with ALICE data



Comparison with ALICE and LEP data



Best fit values: (using PYTHIA)

- $\sigma^{pp} = (7.6 \pm 0.1) / \text{GeV}$
- $\sigma^{e^+e^-} = (5.3^{+1.0}_{-0.6}) / \text{GeV}$

Comparison with experimental data on pp and Ap:

- assume $R_A \simeq a_0 A^{1/3}$ with a_0 as fit parameter

Comparison with experimental data on pp and Ap:

- assume $R_A \simeq a_0 A^{1/3}$ with a_0 as fit parameter

Best fit values: (using QGSJET-IIm)

pp	0.9 TeV:	$a_0 = 1.2$ fm
pp	2.76 TeV:	$a_0 = 1.3$ fm
pp	7.0 TeV:	$a_0 = 1.1$ fm
pBe	200 GeV:	$a_0 = 1.1$ fm
pAl	200 GeV:	$a_0 = 1.0$ fm

Comparison with experimental data on pp and Ap:

- assume $R_A \simeq a_0 A^{1/3}$ with a_0 as fit parameter

Best fit values: (using QGSJET-IIm)

pp	0.9 TeV:	$a_0 = 1.2$ fm
pp	2.76 TeV:	$a_0 = 1.3$ fm
pp	7.0 TeV:	$a_0 = 1.1$ fm
pBe	200 GeV:	$a_0 = 1.1$ fm
pAl	200 GeV:	$a_0 = 1.0$ fm

- good agreement with expectation $a_0 \sim 1$ fm

Conclusions

- 1 Antideuteron formation is interesting in itself:
 - ▶ inclusion of **two-particle correlations** necessary
 - ▶ coalescence on **event-by-event** basis
 - ▶ how to deal with **spatial correlations**?
 - ▶ **interactions** [$P \sim \sigma(NN \rightarrow \bar{d}X)$] **important**?
- 2 Uncertainties in models using coalescence in phasespace quite reduced
- 3 Antinuclei are a useful tool searching for new astro- or particle physics

Conclusions

- 1 Antideuteron formation is interesting in itself:
 - ▶ inclusion of two-particle correlations necessary
 - ▶ coalescence on event-by-event basis
 - ▶ how to deal with spatial correlations?
 - ▶ interactions [$P \sim \sigma(NN \rightarrow \bar{d}X)$] important?
- 2 **Uncertainties** in models using coalescence in phasespace quite **reduced**
- 3 Antinuclei are a useful tool searching for new astro- or particle physics

Conclusions

- 1 Antideuteron formation is interesting in itself:
 - ▶ inclusion of two-particle correlations necessary
 - ▶ coalescence on event-by-event basis
 - ▶ how to deal with spatial correlations?
 - ▶ interactions [$P \sim \sigma(NN \rightarrow \bar{d}X)$] important?
- 2 Uncertainties in models using coalescence in phasespace quite reduced
- 3 **Antinuclei** are a useful tool searching for **new astro- or particle physics**