Summary Status and Issues for (not only NLO) QED/EW corrections for weak mixing angle and m_W

E. Richter-Was (IF UJ, Krakow)

- EW and QED corrections for \( \sin^2\theta_W \) measurement
  - EW schemes: from LEP to LHC
  - Observables, Pseudo-observables, \( \sin^2\theta_{\text{eff}} \) at LHC
  - QED ISR/IFI
  - Photon induced processes
- EW and QED corrections for m_W measurement

Please note:
NLO EW is insufficient for both \( \sin^2\theta_{\text{eff}} \) and m_W!
We know from LEP times that fixed-order is not necessarily the right way to access corrections for precision measurements sensitive to QED/EW corrections. For m_W, QED FSR beyond state-of-art YFS or Photos calculations may be needed.
The Monte Carlo generator which does „all in one” with required precision does not exist yet. What is needed:

- NNLO QCD + resumation
- Multi-photon QED emission with exponentiation and pair creation
- Genuine and linshape EW corrections with theoretical precision below \(5 \times 10^{-5}\) for \(\sin^2\theta_{\text{eff}}\)

We split modeling into components and make the best of existing tools:

- Powheg\_ew, MCSANC, Dizet form-factors, TauSpinner \(w_{\text{EW}}\), Photos, Winhac
- HORACE, KKMC-hh

For overview of different tools see eg. talks at recent, „Ultimate Precision Workshop”, indico.cern.ch/event/835066
Genuine weak and lineshape corrections

$\omega^{EW}$: TauSpinner + Dizet 6.21, 6.42, 6.45

$\omega^{EW}$: with updated $\alpha(M_Z)$

MC events
EW LO

Arbitrary
EW setup

$\alpha(0) v0$: LO, NLO+HO

$\alpha(0) v1$: LO

$G_\mu$: LO

$\sin^2 eff$: LO

New developments

Powheg_ew: QCD LO, Z

$\alpha(0) v0$: LO

$\alpha(0) v1$: LO, NLO, NLO+HO

$G_\mu$: LO, NLO, NLO+HO

$\sin^2 eff$: LO, NLO, NLO+HO

MCSANC: QCD LO, Z

$\alpha(0) v1$: LO, NLO, NLO+HO

$G_\mu$: LO, NLO, NLO+HO
**EW schemes: input + renorm. counterterms**

- **LEP legacy: (\(\alpha(0), G_\mu, M_Z\))**
  - Inputs are very precisely measured physics quantities
  - \(M_Z, M_W\) are on-shell masses
  - Genuine EW and lineshape corrections in form of (multiplicative) form-factors to LO couplings. Implemented in Dizet library.
  
    \[
    \sin^2 \theta_{eff}^f = Re(K_Z^f) s_W^2 + I_f^2
    \]

- **LHC paradigm: (\(G_\mu, M_Z, M_W\)).**
  - \(M_Z, M_W\) are pole-masses or complex masses.
  - Absorbs most of universal corrections into lowest-order couplings.
  - Higher-order corrections redefine couplings in non-multiplicative manner.

  \[
  s_W^2 \rightarrow \bar{s}_W^2 \equiv s_W^2 + \Delta \rho c_W^2, \quad c_W^2 \rightarrow \bar{c}_W^2 \equiv 1 - \bar{s}_W^2 = (1 - \Delta \rho) c_W^2.
  \]

- **New development: \( (G_\mu, \sin^2 \theta_{eff}, M_Z) \)**
  - The weak mixing angle at Z-pole matches tree-level expression to all orders following LEP definition: real part of ratio of \(g_V\), and \(g_A\) couplings.
  - Ones input \(\sin^2 \theta_{eff} \neq \text{measured value at LEP}\), corrections to \(A_{FB}\) become very small.

  \[
  \sin^2 \theta = \sin^2 \theta_{eff} = \frac{I_3^l}{2Q_1} \left( 1 - \frac{g_V^l}{g_A^l} \right) = \frac{I_3^l}{Q_1} \left( \frac{-g_R^l}{g_L^l - g_R^l} \right).
  \]
SM relation used to calculate EW LO parameters for different schemes. On-shell mass.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LEP-legacy</th>
<th>LHC-paradigm</th>
<th>New scheme</th>
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<td>(M_Z) (GeV)</td>
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<td>91.1876</td>
<td>91.1876</td>
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<td>(\Gamma_Z) (GeV)</td>
<td>2.4952</td>
<td>2.4952</td>
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<tr>
<td>(\Gamma_W) (GeV)</td>
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<td>2.085</td>
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<td>(1/\alpha)</td>
<td>0.007297353</td>
<td>0.1254734 \cdot 10^{-5}</td>
<td>0.007562396</td>
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<tr>
<td>(\alpha)</td>
<td>0.007297353</td>
<td>0.1254734 \cdot 10^{-5}</td>
<td>0.007562396</td>
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<tr>
<td>(G_\mu) (GeV(^{-2}))</td>
<td>1.1663787 \cdot 10^{-5}</td>
<td>1.1254734 \cdot 10^{-5}</td>
<td>1.1663787 \cdot 10^{-5}</td>
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<tr>
<td>(M_W) (GeV)</td>
<td>80.93886</td>
<td>80.385</td>
<td>80.385</td>
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<tr>
<td>(s_W^2)</td>
<td>0.1212157</td>
<td>0.2228972</td>
<td>0.1212157</td>
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<tr>
<td>(G_\mu \cdot M_Z^2 \cdot s_W^2 = 1.0)</td>
<td>(\rightarrow s_W^2, M_W)</td>
<td>(\rightarrow G_\mu, s_W^2)</td>
<td>(\rightarrow \alpha, s_W^2)</td>
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<tr>
<td>(s_W^2 = 1 - m_W^2 / m_Z^2)</td>
<td>0.120178900000</td>
<td>0.120178900000</td>
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</table>

\[ s_W^2 = 1 - m_W^2 / m_Z^2 \]

\[ G_\mu = \frac{\pi \alpha}{\sqrt{2} M_W^2 s_W^2} \]
Pseudo-observables at Z-pole: benchmarks

„Best predictions” in each EW scheme, i.e. EW NLO+HO

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(α(0), Gμ, MZ)</th>
<th>(α(0), MZ, MW)</th>
<th>(Gμ, MZ, MZ)</th>
<th>(α(0), sin²θeff, MZ)</th>
<th>(Gμ, sin²θeff, MZ)</th>
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Experiments measure observables: cross-sections, asymmetries, distributions. We need predictions to interpret these measurements. For now, only Dizet provides predictions for LEP-style pseudo-observables.
**Benchmark for key observable: $\Delta A_{fb}$**

- **$G_\mu$:** $G_\mu$, $M_W$, $M_Z$
- **$\sin^2 \theta_{eff}^\ell$:** $\sin^2 \theta_{eff}^\ell = 0.23147$, $G_\mu$, $M_Z$

Close to the value measured at LEP.

- $M_{ll} = 89$–93 GeV
- 80-100 GeV

- $\Delta A_{fb} = 1.6 \times 10^{-2}$
- $1.6 \times 10^{-2}$

- $\Delta A_{fb} = 2.0 \times 10^{-4}$
- $1.7 \times 10^{-4}$

<table>
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<tr>
<th>$A_{FB}$</th>
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<th>NLO</th>
<th>NLO+HO</th>
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<td>0.046547(30)</td>
<td>0.030043(30)</td>
<td>0.030830(27)</td>
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<td></td>
<td>0.043445(26)</td>
<td>0.026908(27)</td>
<td>0.027702(22)</td>
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<td></td>
<td>0.042135(25)</td>
<td>0.025699(25)</td>
<td>0.026492(22)</td>
</tr>
</tbody>
</table>

| $G_\mu$ | 0.046548(30) | 0.029058(28) | 0.030903(27) |
| | 0.043440(26) | 0.025922(28) | 0.027778(22) |
| | 0.042130(25) | 0.024719(26) | 0.026569(22) |

| $\sin^2 \theta_{eff}^\ell$ $v1$ | 0.030598(15) | 0.030397(15) | 0.030396(15) |
| | 0.027437(13) | 0.027267(13) | 0.027267(13) |
| | 0.026213(13) | 0.026056(13) | 0.026057(22) |
Benchmark for key observable: $\Delta A_{fb}$

Reference:
effective Born with LEP parametrisations


Factor 3-20 (depending on mass window) smaller full corrections with $(G_\mu, \alpha(0), M_Z)$ scheme than predicted with $(G_\mu, \sin^2\text{eff}, M_Z)$ scheme. We need to understand it.
### New best options:

IHVP=5

IAMT4=8
Dizet 6.45

**Control printout**

E. Richter-Was, IF JU

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PARAMETER(ALFAI=137.035999139D0, ALFA=1.0D0/ALFAI, CONS=1.0D0)
PARAMETER(ZMASS=91.1876D0, TMASS=173.0D0, HMASS=125.0D0)
PARAMETER(ALFAS=0.120178900000D0)

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6 - old default, 7,8 - new option for SIN2_EFF
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**DIZET input parameters:**

- **ZMASS**: 91.1876000000000000
- **TMASS**: 173.0000000000000000
- **HMASS**: 125.0000000000000000
- **DALSH**: 0.000000000000000000
- **ALFAS**: 0.120178900000000000

**DIZET results:**

- **SIN2_TW**: 0.22340108421408789
- **WMASSsin**: 89.358935565128405
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- **DALSH**: 2.757619313462830E-002
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**CONTROL printout**

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</tbody>
</table>

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A. Arbuzov, L. Kalinovskaya, T. Riemann, S. Riemann, V. Yermolchyk
1) Implementation of the results from paper ’Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy’ of Ievgen Dubovyk, Ayres Freitas, Janusz Gluza, Tord Riemann and Johann Usovitsch (arXiv:1906.08815), which provides complete fitting formula for Z-boson decay widths, branching ratios and cross-section.

2) Implementation of new electroweak schemes: $(G_f, M_Z, M_W)$, $(G_f, \sin \theta_{eff}, M_Z)$. 
Electroweak Pseudo-Observables at LEP: the meeting point between data and theory

Pseudo-observables: cross-sections, ratios, asymmetries, corrected for QED and interpolated to the Z-pole.
\[ \alpha_{\text{QED}}(s) \text{ and } \sin^2 \theta_W(s) \]

\[ \alpha(s) = \frac{\alpha}{1 - \Delta \alpha(s)} ; \quad \Delta \alpha(s) = \Delta \alpha_{\text{lep}}(s) + \Delta \alpha_{\text{had}}^{(5)}(s) + \Delta \alpha_{\text{top}}(s) \]

\[ \sin^2 \Theta_i \cos^2 \Theta_i = \frac{\pi \alpha}{\sqrt{2} G_\mu M_Z^2} \frac{1}{1 - \Delta r_i} \]

\[ \Delta r_i = \Delta r_i(\alpha, G_\mu, M_Z, m_H, m_{f \neq t}, m_t) \]

\[ \Delta \alpha_{\text{hadrons}}^{(5)}(M_Z^2) = 0.027738 \pm 0.000158 [0.027523 \pm 0.000119] \]
Weak mixing angle @ LHC

Using the formalism of Improved Born Approximation (Dizet)

\[ \sin^2 \theta_{\text{eff}}^f = \Re \left( K_Z^f \right) s_W^2 + I_f^2 \]

\[ g_Z^f = \frac{v_f}{a_f} = 1 - 4|q_f|(K_Z^f s_W^2 + I_f^2) \]

\[ s_W^2 = 1 - M_W^2/M_Z^2 \]

\[ I_f^2 = \alpha^2(s) \frac{35}{18} \left[ 1 - \frac{8}{3} \Re (K_Z^f s_W^2) \right] \approx 10^{-4} \]

We can extend definition outside the Z-pole

\[ v_\ell = (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2 \cdot K_\ell(s, t)) / \Delta \]

\[ v_f = (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s, t)) / \Delta \]

\[ a_\ell = (2 \cdot T_3^\ell) / \Delta \]

\[ a_f = (2 \cdot T_3^f) / \Delta \]

\[ \Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)} \]

\[ \sin^2 \theta_W^f (s, t) = K_f(s, t) \cdot s_w^2 = K_i(s, t) / K_Z^f * \sin^2 \theta_{\text{eff}}^f \]

EW form-factors, functions of (s,t)=(m_t, cos\theta)
Calculated with Dizet library.
\( \sin^2 \theta^f_w (s,t) = \text{Re}(K^f(s,t)) \times s^2_w = \text{Re}(K^f(s,t)/K^f_Z) \times \sin^2 \theta^f_{\text{eff}}(M_Z) \)
Electroweak Pseudo-Observables at LHC: the meeting point between data and theory

\[ N^n_{\text{exp}}(A, \sigma, \theta) = \sum_{j=0}^{N_{\text{bins}}} \sigma_j \times L \times \left[ t^n_{\text{8j}}(\beta) + \sum_{i=0}^{7} A_{ij} \times t^n_{ij}(\beta) \right] \times \gamma^n + \sum_{B} T^n_{\beta}(\beta), \]

\[ A_{4,j}(\sin^2 \theta_{\text{eff}}^{\ell}, \theta) = a_j(\theta) \times \sin^2 \theta_{\text{eff}}^{\ell} + b_j(\theta) \]
Electroweak Pseudo-Observables at LHC: the meeting point between data and theory

Pseudo-observables or observables: cross-sections and asymmetries (A_{f4}, A_4), (unfolded to truth level) in different M_{ll}, Y bins.

$\Delta \sin^2_{\text{eff,lep}} \text{(scan)} \rightarrow \Delta A_4(\text{EW, QCD}) \text{ predicted} \rightarrow A_4 \text{ (measured/fitted)} \rightarrow \sin^2_{\text{eff,lep}} \text{ (best)}$
\[ \sin^2 \theta_{\text{eff}} \] scan for \( A_{FB} \) and \( A_4 \)

**Formulas used for this plot, varied \( \delta V \)**

\[
\begin{align*}
\nu_{\ell} &= \frac{(2 \cdot T_3^\ell - 4 \cdot q_{\ell} \cdot (s_W^2 \cdot K_\ell(s, t) + \delta V))}{\Delta} \\
\nu_f &= \frac{(2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 \cdot K_f(s, t) + \delta V))}{\Delta} \\
\nu_{\text{eff}} &= \frac{1}{\nu_{\ell} \cdot \nu_f} [(2 \cdot T_3^\ell)(2 \cdot T_3^f) - 4 \cdot q_{\ell} \cdot (s_W^2 + K_\ell(s, t) + \delta V)(2 \cdot T_3^\ell) - 4 \cdot q_f \cdot (s_W^2 \cdot K_f(s, t) + \delta V)(2 \cdot T_3^f) + (4 \cdot q_{\ell} \cdot s_W^2)(4 \cdot q_f \cdot s_W^2)K_{\ell f}(s, t) + 2 \cdot (4 \cdot q_{\ell})(4 \cdot q_f) \cdot s_W^2 \cdot K_{\ell f}(s, t) \cdot \delta V] \quad \frac{1}{\Delta^2} 
\end{align*}
\]

**ATLAS Simulation Preliminary**

- \( \sqrt{s} = 8 \, \text{TeV}, Z/\gamma^* \) (NLO QCD)
- \( 80 \, \text{GeV} \leq m^2 \leq 100 \, \text{GeV} \)

ATL-COMP-2018-037
$\sin^2 \theta_{\text{eff}}$ scan for $A_4$

$$A_4 = \frac{8}{3} A_{FB}$$

$\sqrt{s} = 8$ TeV, input: $G_\mu$, $\sin^2 \vartheta^\ell_{\text{eff}}$, $M_Z$

80 GeV $\leq m_{\ell\ell} \leq 100$ GeV

- $\sin^2 \vartheta^\ell_{\text{eff}} = 0.228 \rightarrow A^{LO}_{FB} = 0.0338(1), A^{NLO}_{FB} = 0.0337(1)$
- $\sin^2 \vartheta^\ell_{\text{eff}} = 0.231499 \rightarrow A^{LO}_{FB} = 0.02739(1), A^{NLO}_{FB} = 0.02722(1)$
- $\sin^2 \vartheta^\ell_{\text{eff}} = 0.235 \rightarrow A^{LO}_{FB} = 0.02106(1), A^{NLO}_{FB} = 0.02088(1)$
Choice of the EW scheme

• At LEP-times, one EW scheme and at least two independent calculations (Zfitter and TOPAZO). Used/cross-checked by independent experimental groups + theory community brainstorming on every tiny detail.

• For $\sin^2\theta_{\text{eff}}$ measurement at LHC, we continue with LEP-legacy code. Cross-checking/ confronting with new developments. At least three codes and far less people involved in testing/benchmarking.

• The work is ongoing now, but should involve more people to review and pheraphs improve final results.
**Z lineshape**

- Deconvolution of initial-state QED radiation:
  \[ \sigma[e^+e^- \rightarrow f \bar{f}] = R_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s') \]

- Subtraction of \( \gamma \)-exchange, \( \gamma-Z \) interference, box contributions:
  \[ \sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} \ ; \ \sigma_{\text{box}} \]

- **Z-pole contribution:**
  \[ \sigma_Z = \frac{R}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + \sigma_{\text{non-res}} \]

- In experimental analyses: No clear definition of \( \sigma_{\text{non-res}} \) in running width scheme
  \[ \sigma \sim \frac{1}{(s - M_Z^2)^2 + s^2 \Gamma_Z^2 / M_Z^2} \]

**Numerical Values:**
- \( M_Z = 91.1876 \) GeV (mass of Z boson)
- \( \Gamma_Z = 2.495 \) GeV (width of Z boson)
- \( \overline{M}_Z = M_Z / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx M_Z - 34 \) MeV
- \( \overline{\Gamma}_Z = \Gamma_Z / \sqrt{1 + \Gamma_Z^2 / M_Z^2} \approx \Gamma_Z - 0.9 \) MeV
Running $Z$-boson width in the propagator is taking into account resumed fermionic loop corrections to $\Gamma_Z$

- **Fixed width**
  \[ \chi_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z} . \]

- **Running width (LEP legacy)**
  \[ \chi_Z'(s) = \frac{1}{s(1 + i \cdot \Gamma_Z/M_Z) - M_Z^2} \]
  \[ = \frac{1}{s(1 + \Gamma_Z^2/M_Z^2) - M_Z^2(1 - i \cdot \Gamma_Z/M_Z)} \]
  \[ = \frac{1}{(1 + \Gamma_Z^2/M_Z^2) s - \frac{M_Z^2}{1+\Gamma_Z^2/M_Z^2} + i \cdot \Gamma_Z M_Z} \]
  \[ = N_Z \frac{1}{s - M_Z^2 + i \Gamma_Z M_Z} \]

Both equivalent if redefined parameters $m_Z$, $\Gamma_Z$, $N_Z$ (normalization).

Keeping $N_Z$ is important for getting right interference term with photon contributions outside $Z$–pole, impacts $A_{FB}$ and is numerically relevant for $\sin^2\theta_{\text{eff}}$ measurement.
Discussed since fall last year, problem in nutshell.

- LEP1 legacy (Dizet+Zfitter, experiments):
  - use running width in the Born propagator
  - form-factors calculated with pole-mass/fixed width (internally converted), applied to Born with on-shell mass/running width
  - see references: hep-ex/0509008, hep-ph/9908433
- LEP2, LHC standard
  - use complex-mass scheme, pole masses, fixed width propagator
- Zfitter+Dizet v6.42, v6.45, FCCee standard
  - stayed with LEP1 convention

Is that a concern for $\sin^2\theta_{\text{eff}}$ measurement at LHC?

For now we took pragmatic approach: use defaults of each code:

- Powheg_ew and MCSANC: pole-mass and fixed width propagator
- wt$^{\text{EW}}$: calculated with on-shell masses and running width propagator, as it is standard used by Zfitter+Dizet

We should keep it in mind, that ones we reach precision of the comparisons which might be sensitive to the effect of $\chi(s)$ implementation. It should be discussed as component of theoretical uncertainties of the predictions.
Theoretical and parametric uncertainties

**ALDO+ SLD + Tevatron, arXiv:1012.2367**

- The remaining theoretical uncertainties were estimated to be 4 MeV on $m_W$ and $0.000049$ on $\sin^2\theta_{\text{eff}}^\text{lep}$. We can use this estimate @ Z-pole.

- The parametric uncertainties were dominated by $\Delta\alpha_{\text{had}}(M_Z^2)$. The uncertainty of $\pm0.00035$ caused an error of $\pm0.00013$ on $\sin^2\theta_{\text{eff}}^\text{lep}$.

**EW $\alpha(0)$ scheme:**

- The same code as ALDO for calculating EW genuine corrections. We can use this estimate.

  **Theoretical uncertainties (EW, @Z-pole):** $5 \times 10^{-5}$ on $\sin^2\theta_{\text{eff}}^\text{lep}$

- **Parametric uncertainties:** $4 \times 10^{-5}$ [35 $10^{-6}$ (from $\Delta\alpha_{\text{had}}$) and 16 $10^{-6}$ (from $m_t$)]
Parametric uncertainties: $\Delta \alpha_{\text{had}}, m_t$

@ Z-pole, $\alpha(0)$ scheme:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Delta \alpha_{\text{h}}^{(5)}(M_Z^2) - 0.0001$</th>
<th>$\Delta \alpha_{\text{h}}^{(5)}(M_Z^2) = 0.0275762$</th>
<th>$\Delta \alpha_{\text{h}}^{(5)}(M_Z^2) + 0.0001$</th>
<th>$\Delta/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha(M_Z^2)$</td>
<td>0.0077540999</td>
<td>0.0077549240</td>
<td>0.0077557482</td>
<td>0.000035</td>
</tr>
<tr>
<td>$1/\alpha(M_Z^2)$</td>
<td>128.9640328306</td>
<td>128.9503292550</td>
<td>128.9366256793</td>
<td></td>
</tr>
<tr>
<td>$s_W^2$</td>
<td>0.22340146</td>
<td>0.22343647</td>
<td>0.22347148</td>
<td>0.000035</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{\text{eff}}(M_Z^2)$ (electron, muon)</td>
<td>0.23150412</td>
<td>0.23153917</td>
<td>0.23157421</td>
<td>0.000035</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{\text{eff}}(M_Z^2)$ (up-quark)</td>
<td>0.23139759</td>
<td>0.23143261</td>
<td>0.23146763</td>
<td>0.000035</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{\text{eff}}(M_Z^2)$ (down-quark)</td>
<td>0.23127052</td>
<td>0.23130551</td>
<td>0.23134049</td>
<td>0.000035</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.35892 GeV</td>
<td>80.35710 GeV</td>
<td>80.35529 GeV</td>
<td>1.8 MeV</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>0.03625683</td>
<td>0.036609</td>
<td>0.03647535</td>
<td></td>
</tr>
<tr>
<td>$\Delta r_{\text{rem}}$</td>
<td>0.01170310</td>
<td>0.01170287</td>
<td>0.01170264</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m_t - 0.5 \text{ GeV}$</th>
<th>$m_t = 173.2 \text{ GeV}$</th>
<th>$m_t + 0.5 \text{ GeV}$</th>
<th>$\Delta/2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha(M_Z^2)$</td>
<td>0.0077549205</td>
<td>0.0077549240</td>
<td>0.0077549274</td>
<td>0.000058</td>
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<tr>
<td>$1/\alpha(M_Z^2)$</td>
<td>128.950387392</td>
<td>128.9503292550</td>
<td>128.9502716590</td>
<td></td>
</tr>
<tr>
<td>$s_W^2$</td>
<td>0.22349450</td>
<td>0.22343647</td>
<td>0.22337836</td>
<td>0.000016</td>
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<tr>
<td>$\sin^2\theta_W^{\text{eff}}(M_Z^2)$ (electron, muon)</td>
<td>0.23155486</td>
<td>0.23153917</td>
<td>0.23152344</td>
<td>0.000016</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{\text{eff}}(M_Z^2)$ (up-quark)</td>
<td>0.23144830</td>
<td>0.23143261</td>
<td>0.23141688</td>
<td>0.000016</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{\text{eff}}(M_Z^2)$ (down-quark)</td>
<td>0.23132119</td>
<td>0.23130551</td>
<td>0.23128979</td>
<td>0.000016</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.354102 GeV</td>
<td>80.35710 GeV</td>
<td>80.360111 GeV</td>
<td>3 MeV</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>0.03654697</td>
<td>0.036609</td>
<td>0.03618477</td>
<td></td>
</tr>
<tr>
<td>$\Delta r_{\text{rem}}$</td>
<td>0.01169343</td>
<td>0.01170287</td>
<td>0.01171229</td>
<td></td>
</tr>
</tbody>
</table>

Parametric uncertainties: 4 10-5 [35 10-6 (from $\Delta \alpha_{\text{had}}$ ) and 16 10-6 ( from $m_t$)]
Parametric uncertainties: $m_W, m_t$

$m_t^2$ dependence from $\Delta \rho$

EW $(G_\mu, \sin^2 \theta_{\text{eff}}, M_Z)$ scheme:
- Very small uncertainties from $m_t$
- No parametric uncertainties from $m_W$

EW $(G_\mu, M_W, M_Z)$ scheme:
- Large uncertainties from $m_t$
- Large parametric uncertainties from $m_W$

@ Z-pole, $\sin^2 \theta_{\text{eff}}$ scheme:
Parametric uncertainties from $m_t$ only, well below $1 \times 10^{-5}$. 

M.Chiesa, F. Piccinini, A. Vicini
arXiv:1906.11569 (PRD100, 071302 (2019))
QED corrections: LEP legacy

**Precision**: calculations are not done order-by-order

\[
\begin{align*}
(a) & \quad 0.5\% \\
& \quad \frac{1}{\alpha L} - \frac{\alpha}{\alpha^2 L^2} - \frac{\alpha^2}{\alpha^3 L^3} - \frac{\alpha^3}{\alpha^4 L^4} - \frac{\alpha^4}{\alpha^5 L^5} - \cdots \\
\end{align*}
\]

\[
\begin{align*}
(b) & \quad 0.02\% \\
& \quad \frac{1}{\alpha L} - \frac{\alpha}{\alpha^2 L^2} - \frac{\alpha^2}{\alpha^3 L^3} - \frac{\alpha^3}{\alpha^4 L^4} - \frac{\alpha^4}{\alpha^5 L^5} - \cdots \\
\end{align*}
\]

\[
\begin{align*}
(c) & \quad 0.001\% \\
& \quad \frac{1}{\alpha L} - \frac{\alpha}{\alpha^2 L^2} - \frac{\alpha^2}{\alpha^3 L^3} - \frac{\alpha^3}{\alpha^4 L^4} - \frac{\alpha^4}{\alpha^5 L^5} - \cdots \\
\end{align*}
\]

\[L_f = \ln\left(\frac{s}{m_f^2}\right)\]

Figure 2: QED perturbative leading and subleading corrections. Rows represent corrections in consecutive perturbative orders; the first row is the Born contribution. The first column represents the leading logarithmic (LO) approximation and the second column depicts the next-to-leading (NLO) approximation. In the figure, terms selected for the same precision level (a) \(5 \cdot 10^{-2}\) (b) \(2 \cdot 10^{-3}\) and (c) \(1 \cdot 10^{-5}\) are limited with the help of an additional line.
• **QED FSR:**
  – Simulated with PHOTOS in the experimental analysis, now also available option of including pair-creation.
  – Tests performed in the past with KKMC, Powheg-ew or HORACE confirmed its adequateness for LHC precision physics.

• **QED IFI and ISR:**
  – HORACE, Powheg\_ew, MCSANC, KKMC-hh.
  – Good progress on convergence between different codes. As expected effect small and will be accounted for as systematics of the $\sin^2\theta_{\text{eff}}$ measurement.
  – Present estimates for IFI is an effect of $10^{-4}$ on $A_4$ and $10^{-5}$ on $A_{fb}$. 
**KKMC-hh**

- KKMC-hh is an event generator for $pp \to f \bar{f} + n\gamma, f = e, \mu, \tau$, based on KKMC, which was used at LEP with a precision tag of 0.2% (LEP2).
- ISR and FSR $\gamma$ emission are included to $O(\alpha^2 L)$ including interference (IFI).
- The MC structure is based on CEEX (Coherent Exclusive Exponentiation), which is similar to YFS exponentiation but implemented at the level of spinor amplitudes.

- The following tests are based on runs generating $5.7 \times 10^9$ muon events at 8 TeV, using NNPDF3.1 NLO PDFs ($\alpha_s(M_Z) = 0.12018$). **The QCD shower is off in these results.**
- All results include a dilepton mass cut $60$ GeV $< M_{\ell\ell} < 116$ GeV.
- Uncut / Without cuts means there are no additional cuts.
- Cuts / With cuts means there is a cut $P_T > 25$ GeV, $|\eta| < 2.5$ on the individual muon.
- Levels of photonic corrections:
  1. FSR only using KKMC-hh with non-QED NNPDF3.1 NLO
  2. FSR + ISR using KKMC-hh with non-QED NNPDF3.1 NLO
  3. FSR + ISR + IFI using non-QED NNPDF3.1 NLO (KKMC-hh best result)
  4. FSR + LuxQED using KKMC-hh with NNPDF3.1 NLO + QED

All KKMC-hh photonic corrections are calculated using CEEX exponentiation with exact $O(\alpha^2 L)$ residuals.
For more details/results see talk by Scott Yost, EW subgroup meeting
https://indico.cern.ch/event/83765
Integrated $A_{FB}$ and difference $\Delta A = [A_{FB} - A_{FB}(LO)]$

<table>
<thead>
<tr>
<th>$m_{ll}$</th>
<th>60-120</th>
<th>89-93</th>
</tr>
</thead>
<tbody>
<tr>
<td>[LO+ISR]</td>
<td>$A_{FB}$</td>
<td>0.03678(1)</td>
</tr>
<tr>
<td>[LO+ISR]-[LO]</td>
<td>$\Delta A$</td>
<td>-0.00004(1)</td>
</tr>
<tr>
<td>[LO+IFI]</td>
<td>$A_{FB}$</td>
<td>0.03652(1)</td>
</tr>
<tr>
<td>[LO+IFI]-[LO]</td>
<td>$\Delta A$</td>
<td>-0.00031(1)</td>
</tr>
</tbody>
</table>

For more details/results see talk by S. Bondarenko, EW subgroup meeting https://indico.cern.ch/event/83765
Configuration and caveats:
- LO QCD, reference is LO EW with LUXQED PDF from NNPDF3.1
- Narrow mass window for MC-SANC (89-93 GeV)
- Intermediate mass window for Powheg-EW (80-102 GeV), low stats
- Wide mass window for KKMC-hh (60-116 GeV), virtual corrections incl.

<table>
<thead>
<tr>
<th>Variable mass window around Z pole</th>
<th>$A_{FB}(\text{LO})$</th>
<th>$\Delta A(\text{ISR})$ ($10^{-4}$)</th>
<th>$\Delta A(\text{IFI})$ ($10^{-4}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No lepton cuts</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC-SANC</td>
<td>0.0047</td>
<td>-0.3 $\pm$ 0.1</td>
<td>-1.3 $\pm$ 0.1</td>
</tr>
<tr>
<td>Powheg EW</td>
<td>0.0448</td>
<td>0.0 $\pm$ 0.6</td>
<td>1.3 $\pm$ 0.8</td>
</tr>
<tr>
<td>KKMC-hh</td>
<td>0.0241</td>
<td>-1.5 $\pm$ 0.2</td>
<td>1.5 $\pm$ 0.1</td>
</tr>
</tbody>
</table>

A shift of $1 \times 10^{-5}$ on $A_{FB}$ corresponds to shift of $3 \times 10^{-5}$ on $\sin^2 \theta_{\text{eff}}$.\(^1\)
**Open issues: Photon induced processes**

**AFB distribution: photon-induced contributions**

A. Vicini, L. Kalinovskaya, S. Bondarenko

Simulation with $\gamma$-induced: NNPDF31_nlo_as_0118_luxqed and $\gamma$-induced subprocesses

Simulation without $\gamma$-induced: NNPDF31_nlo_as_0118 and NO $\gamma$-induced subprocesses

Preliminary results for $\gamma$-ind. contribution: effect on the asymmetry large

Will be discussed in talk by A. Vicini, Wednesday afternoon.

E. Richter-Was, IF JU

CERN, LHC EW precision, 17.12.2019
m_w measurement: physics modeling

- The QCD aspects: discussed in talk by V. Bertone
- The QED/EW aspects:
  - No activity in the EW precision WG
  - Uncertainties published by ATLAS 7 TeV analysis, within the budget of total error.
The ATLAS result equals in precision the previous single-experiment best measurement of CDF

\[ M_W = 80369.5 \pm 18.5 \text{ MeV} \]

\[ M_W = 80369.5 \pm 6.8 \text{ (stat)} \pm 10.6 \text{ (exp.syst.)} \pm 13.6 \text{ (model.syst.)} \text{ MeV} \]

The dominant uncertainty is due to the physics modelling

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_T-p_T, W^\pm, e-\mu )</td>
<td>80369.5</td>
<td>6.8</td>
<td>6.6</td>
<td>6.4</td>
<td>2.9</td>
<td>4.5</td>
<td>8.3</td>
<td>5.5</td>
<td>9.2</td>
<td>18.5</td>
<td>29/27</td>
</tr>
</tbody>
</table>

and the largest contributions are from QCD/PDF
QED/EW corrections for mW

- QED FSR: dominant correction, included in the simulation with PHOTOS or others MC
- Other NLO electroweak corrections are usually estimated independently from QCD corrections, and applied as uncertainty

<table>
<thead>
<tr>
<th>Decay channel</th>
<th>$W \rightarrow e\nu$</th>
<th>$W \rightarrow \mu\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kinematic distribution</td>
<td>$p_T^e$</td>
<td>$m_T$</td>
</tr>
<tr>
<td>$\delta m_W$ [MeV]</td>
<td>$&lt; 0.1$</td>
<td>$&lt; 0.1$</td>
</tr>
<tr>
<td>FSR (real)</td>
<td>3.3</td>
<td>2.5</td>
</tr>
<tr>
<td>Pure weak and IFI corrections</td>
<td>3.6</td>
<td>0.8</td>
</tr>
<tr>
<td>FSR (pair production)</td>
<td>4.9</td>
<td>2.6</td>
</tr>
</tbody>
</table>

- Many recent developments in higher order corrections, and benchmarking between different codes presented in the LPCC EW working group
- Main challenge for the $m_W$ analyses: include electroweak corrections in the analyses, coherently combined with QCD corrections. Available tools are Powheg-EW, DIZET form factors, WINHAC, KKMC
- Open point: is the running-width propagator still the best definition for $m_W$, in view of higher order EW corrections?
• Recent work with Sherpa YFS formalism to reach NNLO QED + NLO EW accuracy. Impact is not large but not yet quantified at the level required for W mass measurement.

\[
2m \cdot \Gamma = \sum_{n_R}^{n_R} \frac{1}{n_R!} \int d\Phi_{p_f} d\Phi_{k_f} (2\pi)^4 \delta^4 \left( q - \sum_f p_f - \sum_{i=0}^{n_R} k_i \right) \times e^{Y(\Omega, \{q\})} \prod_{i=1}^{n_R} \tilde{S}(k_i, \{q\}) \Theta(k_i, \Omega) \tilde{\beta}_0^0(\{q\}) C(\{p\}, \{q\}) J(\{p\}, \{q\})
\]

Improving the YFS formalisms

YFS form-factor (resummed)  
Born ME  
Jacobian  
Correction factor

\[
C = 1 + \frac{1}{\beta_0^0} \left( \tilde{\beta}_0^1 + \sum_{i=1}^{n_f} \tilde{\beta}_1^1(k_i) \right) + \frac{1}{\beta_0^0} \left( \tilde{\beta}_0^2 + \sum_{i=1}^{n_f} \tilde{\beta}_1^2(k_i) + \sum_{i,j=1}^{n_f} \tilde{\beta}_2^2(k_i, k_j) \right) + \frac{1}{\beta_0^0} \mathcal{O}(\alpha^3)
\]

J. Lindert, F. Krauss, R. Linten, M. Schonherr
Durham WS, March 2019
Progress on QED FSR relevant for W mass measurement

- Example of lepton $p_T$ spectrum shown below for W decays

J. Lindert, F. Krauss, R. Linten, M. Schonherr
Durham WS, Mach2019
Summary

• We are on track for preparing precise theoretical predictions with their uncertainties for $\sin^2 \theta_{\text{eff}}$ measurement at LHC.
  – Reviving LEP-legacy codes
  – Integrating new developments (eg. EW schemes)
• Main areas of work discussed here:
  – Genuine weak and lineshape corrections
  – QED ISR/IFI/FSR
  – Photon induced processes
• Less effort at present in topics specific to mW measurement.
• Seems now realistic to converge with publication next year.
SPARES slides
EW schemes: input parameters

SM fundamental relation used to calculate EW LO parameters for different schemes (pole mass).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$(\alpha(0), G_\mu, M_Z)$ $\alpha(0)$ v0</th>
<th>$(\alpha(0), M_W, M_Z)$ $\alpha(0)$ v1</th>
<th>$(G_\mu, M_Z, M_W)$ $G_\mu$</th>
<th>$(\alpha(0), s_W^2, M_Z)$ $\sin^2_{\text{eff}}$ v1</th>
<th>$(G_\mu, s_W^2, M_Z)$ $\sin^2_{\text{eff}}$ v2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_Z$ (GeV)</td>
<td>91.15348</td>
<td>91.15348</td>
<td>91.15348</td>
<td>91.15348</td>
<td>91.15348</td>
</tr>
<tr>
<td>$\Gamma_Z$ (GeV)</td>
<td>2.494266</td>
<td>2.494266</td>
<td>2.494266</td>
<td>2.494266</td>
<td>2.494266</td>
</tr>
<tr>
<td>$\Gamma_W$ (GeV)</td>
<td>2.085</td>
<td>2.085</td>
<td>2.085</td>
<td>2.085</td>
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<tr>
<td>$1/\alpha$</td>
<td>137.035999139</td>
<td>137.035999139</td>
<td>132.3572336357709</td>
<td>137.035999139</td>
<td>128.84133952</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.007297353</td>
<td>0.007297353</td>
<td>0.007555311</td>
<td>0.007297353</td>
<td>0.007761484</td>
</tr>
<tr>
<td>$G_\mu$ (GeV$^{-2}$)</td>
<td>1.1663787$\cdot 10^{-5}$</td>
<td>1.126555497$\cdot 10^{-5}$</td>
<td>1.1663787$\cdot 10^{-5}$</td>
<td>1.1663787$\cdot 10^{-5}$</td>
<td>1.1663787$\cdot 10^{-5}$</td>
</tr>
<tr>
<td>$M_W$ (GeV)</td>
<td>80.91191</td>
<td>80.35797</td>
<td>80.35797</td>
<td>79.90895881</td>
<td>79.90895881</td>
</tr>
<tr>
<td>$s_W^2$</td>
<td>0.2120868</td>
<td>0.22283820939</td>
<td>0.22283820939</td>
<td>0.231499</td>
<td>0.231499</td>
</tr>
<tr>
<td>$G_\mu$, $M_Z$, $s_W^2$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s_W^2 = 1 - m_W^2/m_Z^2$</td>
<td>$G_\mu = \frac{\pi \alpha}{\sqrt{2} M_W^2 s_W^2}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Completed since last meeting!
Effective Born and LEP param.

- TauSpinner + $w^E_W$: LEP and LEP imp.  

$E.\text{Richter}$ - Was, IF JU CERN, LHC EW precision, 11.09.2019

\begin{align*}
\text{EW LO} & \quad \text{Effective Born} \\
\alpha(0) \quad \text{scheme} & \quad \text{LEP} \\
\alpha = 1/137.3599 & \quad \alpha = 1/128.8667 \\
\frac{s_W^2}{2} = 0.21215 & \quad \frac{s_W^2}{2} = 0.23152 \\
\rho_{\epsilon_f} = 1.0 & \quad \rho_{\epsilon_f} = 1.005
\end{align*}

<table>
<thead>
<tr>
<th>Corrections to cross-section</th>
<th>$89 &lt; m_{ee} &lt; 93 \text{ GeV}$</th>
<th>$80 &lt; m_{ee} &lt; 100 \text{ GeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\text{EW corr. to } m_W)/\sigma(\text{EW LO } \alpha(0))$</td>
<td>0.97114</td>
<td>0.97162</td>
</tr>
<tr>
<td>$\sigma(\text{EW corr. to } \chi(Z, \gamma))/\sigma(\text{EW LO } \alpha(0))$</td>
<td>0.98246</td>
<td>0.98346</td>
</tr>
<tr>
<td>$\sigma(\text{EW/QCD FF no boxes})/\sigma(\text{EW LO } \alpha(0))$</td>
<td>0.96469</td>
<td>0.96602</td>
</tr>
<tr>
<td>$\sigma(\text{EW/QCD FF with boxes})/\sigma(\text{EW LO } \alpha(0))$</td>
<td>0.96473</td>
<td>0.96607</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Corrections to $A_{FB}$</th>
<th>$89 &lt; m_{ee} &lt; 93 \text{ GeV}$</th>
<th>$80 &lt; m_{ee} &lt; 100 \text{ GeV}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{FB}(\text{EW corr. } m_W) - A_{FB}(\text{EW LO } \alpha(0))$</td>
<td>-0.02097</td>
<td>-0.02103</td>
</tr>
<tr>
<td>$A_{FB}(\text{EW corr. prop. } \chi(Z, \gamma)) - A_{FB}(\text{EW LO } \alpha(0))$</td>
<td>-0.02066</td>
<td>-0.02098</td>
</tr>
<tr>
<td>$A_{FB}(\text{EW/QCD FF no boxes}) - A_{FB}(\text{EW LO } \alpha(0))$</td>
<td>-0.03535</td>
<td>-0.03569</td>
</tr>
<tr>
<td>$A_{FB}(\text{EW/QCD FF with boxes}) - A_{FB}(\text{EW LO } \alpha(0))$</td>
<td>-0.03534</td>
<td>-0.03567</td>
</tr>
</tbody>
</table>

Size of corrections anticipated for $\sin^2\theta_{\text{eff}}$ EW scheme.
TauSpinner $w^\text{EW}$: updates

- Updated Dizet 6.21 -> Dizet 6.42 -> Dizet 6.45
  - Updated parametrisation of $\Delta \alpha_{\text{had}}^{(5)}(s)$
    Jegerlehner 2017 (arXiv:1711.06089)
  - AMT4=6: Complete two-loop corrections to $M_w$ and fermionic two-loop corrections to $\sin^2 \theta_{\text{eff}}^{\text{lep}}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m_\ell = 89$-93 GeV</th>
<th>$m_\ell = 89$-93 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha(0)$ scheme</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ NLO+HO/LO (no boxes)</td>
<td>0.96536</td>
<td>0.96503</td>
</tr>
<tr>
<td>$\sigma$ NLO+HO/LO (with boxes)</td>
<td>0.96536</td>
<td>0.96508</td>
</tr>
<tr>
<td>$A_{fb}$ NLO+HO - LO (no boxes)</td>
<td>-0.03529</td>
<td>-0.03496</td>
</tr>
<tr>
<td>$A_{fb}$ NLO+HO - LO (with boxes)</td>
<td>-0.03526</td>
<td>-0.03495</td>
</tr>
</tbody>
</table>

| $G_u$ scheme | | |
| $\sigma$ NLO+HO/LO (no boxes) | 0.99243 | 0.99204 |
| $\sigma$ NLO+HO/LO (with boxes) | 0.99244 | 0.99209 |
| $A_{fb}$ NLO+HO - LO (no boxes) | -0.01553 | -0.01514 |
| $A_{fb}$ NLO+HO - LO (with boxes) | -0.01550 | -0.01512 |

- About + 0.00010 on $\sin^2 \theta_{\text{eff}}$
QED corrections: LEP legacy

**QED Initial-final state Interference (IFI)**

- Suppressed by the $\Gamma_Z/M_Z$ factor, does not contain mass logarithms. Additional factor 10 of suppression comes from partial cancellation of contributions from different quark flavours.
- Codes used at LHC should reproduce LEP benchmarks around Z-pole.

\[
\frac{\sigma^{\text{IFI}}}{\sigma} \sim Q_e Q_f \frac{\alpha}{\pi} \text{Max}\left\{ \frac{\Gamma_Z}{M_Z} ; \frac{s - M_Z^2}{M_Z^2} \right\}
\]


- 0.02% corrections to cross-section $\sigma_{\text{had}}$ at the Z peak
- 0.15 MeV for the Z mass shift at $M_Z$

---

E. Richter-Was, IF JU
Orsay W/Z workshop, 6.02.2019
LEP legacy: QED (radiative) corrections

NOT discussed here.

QED FSR can be simulated by PHOTOS (exponentiated multi-photon emission) implemented as after-burner step on already generated event.

**Real emission + pairs creation**

**Vertex corrections**

**γγ and γZ box diagrams**

It is QED gauge-invariant set of diagrams (D. Bardin, hep-ph/9908433) which can be factorised out and/or convoluted with QCD corrections.

Calculated with fixed value of $\alpha_{\text{QED}}$

$\alpha_{\text{QED}} = 1./137.0359895$
LEP legacy: Genuine EW and lineshape corrections

Also gauge-invariant set of diagrams. Calculated as form-factor corrections to couplings, propagators and masses. Eg. running $\alpha_{\text{QED}}(s), \alpha_{\text{QED}}(M_Z) = 1/128.86674175$

**Zff and $\gamma$ ff vertices**

1. 
2. 
3. 
4. 
5. 
6. 
7. 

**Bosonic self-energies**

1. 
2. 
3. 
4. 
5. 

**WW, ZZ boxes** (shown only WW diagrams)

1. 
2. 

**Fermionic self-energies**

1. 
2. 
3. 
4.
Zfighter is a semi-analytical program for calculating total cross-sections and pseudo-observables (eg. $A_{fb}$, $\sin^2\theta_W^{\text{eff}}$), used by LEP1, and to a lesser degree by LEP2.

DIZET is a library for calculating form-factors and some other corrections. Provides complete EW $O(\alpha)$ weak-loop corrections supplemented with selected higher order terms (eg. vacuum polarisation, $\alpha_{\text{QED}}(Q^2)$).

For analyses at LEP1, LEP2 used always in parallel with MC generators (KoralZ, KoralW) eg. to evaluate systematics of simplified cuts used in analysis integration.

$$A_Z^{\text{OLA}}(s, t) = i\sqrt{2}G_\mu I_e^{(3)} I_f^{(3)} M_Z^2 \chi_Z (s) \rho_{ef}(s, t) \{ \gamma_\mu (1 + \gamma_5) \otimes \gamma_\mu (1 + \gamma_5)$$

$$-4|Q_e| s_w^2 \kappa_e(s, t) \gamma_\mu \otimes \gamma_\mu (1 + \gamma_5) - 4|Q_f| s_w^2 \kappa_f(s, t) \gamma_\mu (1 + \gamma_5) \otimes \gamma_\mu$$

$$+16|Q_e Q_f| s_w^4 \kappa_{ef}(s, t) \gamma_\mu \otimes \gamma_\mu \}.$$  

From Zfighter/Dizet documentation

**Vacuum polarisation corrections**

$$A_{\gamma}^{\text{OLA}} = i\chi_{\gamma}(s) \alpha(s) \gamma_\mu \otimes \gamma_\mu.$$  

Dyson summation leads to the change of $\alpha$ into $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta \alpha^\text{fer}(s)} = \frac{\alpha(0)}{1 - \Delta \alpha^{(5)}(s) - \Delta \alpha^t(s) - \Delta \alpha^{\alpha\alpha s}(s)}.$$
After some trivial algebra one derives the final expressions:

$$\rho_{ef} = 1 + \frac{g^2}{16\pi^2} \left\{ -\Delta \rho^{F}_{\gamma} + D_{\gamma}^{F}(s) + \frac{5}{3} B_{0}^{F}(-s; M_{W}, M_{W}) - \frac{9 c_{W}^{2}}{4 s_{W}^{2}} \ln s_{W}^{2} - 6 \right. $$

$$+ \frac{5 c_{W}^{2}}{8} \left( 1 + c_{W}^{2} \right) + \frac{1}{4 c_{W}^{2}} \left( 3 v_{e}^{2} + a_{e}^{2} + 3 v_{f}^{2} + a_{f}^{2} \right) F_{Z}(s) + F_{W}(s) + F_{W}(s) $$

$$- \frac{1}{4} \left[ B_{0}^{F}(-s; M_{W}, M_{W}) + 1 \right] - c_{W}^{2} \left( R_{Z} - 1 \right) s \tilde{B}_{WW}^{F}(s, t) \right\}, \quad (A.4.80)$$

$$\kappa_{e} = 1 + \frac{g^2}{16\pi^2} \left\{ -\Delta \rho^{F}_{\gamma} - \Pi^{F}_{Z}(s) - \frac{1}{6} B_{0}^{F}(-s; M_{W}, M_{W}) - \frac{1}{9} \frac{v_{e} \sigma_{e}}{2 c_{W}^{2}} F_{Z}(s) $$

$$- \frac{1}{2} \left[ Q_{e}^{2} \left( 1 - 4 |Q_{e}| s_{W}^{2} \right) F_{Z}(s) + c_{W}^{2} \left[ \tilde{F}_{WW}(s) \right. \right. $$

$$- \left. \left. |Q_{e}| \left[ F_{WW}(s) + s \tilde{B}_{WW}^{F}(s, t) \right] \right\} \right. $$

$$ (A.4.81)$$

$$\kappa_{f} = 1 + \frac{g^2}{16\pi^2} \left\{ -\Delta \rho^{F}_{\gamma} - 2 \Pi^{F}_{Z}(s) - \frac{1}{3} B_{0}^{F}(-s; M_{W}, M_{W}) - \frac{2}{9} \right. $$

$$- \frac{1}{4 c_{W}^{2}} \left[ \delta_{Z}^{2} + \delta_{f}^{2} \right] \left( R_{W} - 1 \right) + 3 v_{e}^{2} + a_{e}^{2} + 3 v_{f}^{2} + a_{f}^{2} \right] F_{Z}(s) $$

$$- \tilde{F}_{W}(s) - \tilde{F}_{W}(s) - \frac{1}{4} \left[ B_{0}^{F}(-s; M_{W}, M_{W}) + 1 \right] $$

$$+ c_{W}^{2} \left( R_{Z} - 1 \right) \left[ \frac{2}{3} - \tilde{N}_{W}^{F}(s) + s \tilde{B}_{WW}^{F}(s, t) \right\}. \quad (A.4.82)$$

$$\kappa_{ef} = 1 + \frac{g^2}{16\pi^2} \left\{ -\Delta \rho^{F}_{\gamma} - 2 \Pi^{F}_{Z}(s) - \frac{1}{3} B_{0}^{F}(-s; M_{W}, M_{W}) - \frac{2}{9} \right. $$

$$- \frac{1}{4 c_{W}^{2}} \left[ \delta_{Z}^{2} + \delta_{f}^{2} \right] \left( R_{W} - 1 \right) + 3 v_{e}^{2} + a_{e}^{2} + 3 v_{f}^{2} + a_{f}^{2} \right] F_{Z}(s) $$

$$- \tilde{F}_{W}(s) - \tilde{F}_{W}(s) - \frac{1}{4} \left[ B_{0}^{F}(-s; M_{W}, M_{W}) + 1 \right] $$

$$+ c_{W}^{2} \left( R_{Z} - 1 \right) \left[ \frac{2}{3} - \tilde{N}_{W}^{F}(s) + s \tilde{B}_{WW}^{F}(s, t) \right\}. \quad (A.4.83)$$

"Fermionic loops in γ propagator"
Constructing $\text{wt}^{\text{EW}}$: EW Improved Born (IBA)

\[
\mathcal{A}^{\text{Born+EW}} = \frac{\alpha}{s} \left\{ \begin{array}{l}
\bar{u} \gamma^\mu v g_{\mu \nu} \bar{\nu} \gamma^\nu u \cdot (q_\ell \cdot q_f) \\
\Gamma_{\Pi} \cdot \chi_\gamma(s)
\end{array} \right\} \\
+ \left[ \bar{u} \gamma^\mu v g_{\mu \nu} \bar{\nu} \gamma^\nu u \cdot (v_\ell \cdot v_f \cdot \nu u_{\ell f}) + \bar{u} \gamma^\mu v g_{\mu \nu} \bar{\nu} \gamma^\nu \gamma^5 u \cdot (v_\ell \cdot a_f) \right.
\left. + \bar{u} \gamma^\mu \gamma^5 v g_{\mu \nu} \bar{\nu} \gamma^\nu \gamma^5 u \cdot (a_\ell \cdot v_f) + \bar{u} \gamma^\mu \gamma^5 v g_{\mu \nu} \bar{\nu} \gamma^\nu \gamma^5 u \cdot (a_\ell \cdot a_f) \right]\cdot \\
\left. Z_{\Pi} \cdot \chi_Z(s) \right\}
\]

\[\chi_\gamma(s) = 1\]

\[\chi_Z(s) = \frac{G_\mu \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z}\]

\[Z_{\Pi} = \rho_{e,f}(s,t)\]

\[\Gamma_{\Pi} = \frac{1}{2 - (1 + \Pi_{\gamma\gamma}(s))}\]

EW form-factors, functions of $(s,t) = (m_\Pi, \cos\theta)$
Calculated with Dizet 6.21 library.

\[v_\ell = (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2 \cdot \frac{K_\ell(s,t)}{\Delta}\]

\[v_f = (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot \frac{K_f(s,t)}{\Delta}\]

\[a_\ell = (2 \cdot T_3^\ell) / \Delta\]

\[a_f = (2 \cdot T_3^f) / \Delta\]

\[\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}\]

\[w_{\ell f} = \frac{1}{v_\ell \cdot v_f} \left[ (2 \cdot T_3^\ell)(2 \cdot T_3^f) - 4 \cdot q_\ell \cdot s_W^2 \cdot K_f(s,t) \right] \cdot 2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_\ell(s,t) \cdot 2 \cdot T_3^f \]

Vacuum polarisation corrections, used low-energy experiment input.
Warning: problem for analytic continuation.

ERW and Z.Was, arXiv: 1808.08616

LHC EWWG meeting, 17.12.2019
EW schemes: details

EW schemes: come with „on-shell“ or „pole“ definitions!

Table 44: The EW parameters used at tree-level EW, with on-mass-shell definition (LEP convention).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\alpha(0)) v0</th>
<th>(\alpha(0)) v1</th>
<th>(G_\mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_Z)</td>
<td>91.1876 GeV</td>
<td>91.1876 GeV</td>
<td>91.1876 GeV</td>
</tr>
<tr>
<td>(\Gamma_Z)</td>
<td>2.4952 GeV</td>
<td>2.4952 GeV</td>
<td>2.4952 GeV</td>
</tr>
<tr>
<td>(\Gamma_W)</td>
<td>2.085 GeV</td>
<td>2.085 GeV</td>
<td>2.085 GeV</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(\frac{1}{137.03599})</td>
<td>(\frac{1}{137.03599})</td>
<td>(\frac{1}{132.23323})</td>
</tr>
<tr>
<td>(G_\mu)</td>
<td>(1.1663787 \cdot 10^{-5} \text{ GeV}^{-2})</td>
<td>(1.1254734 \cdot 10^{-5} \text{ GeV}^{-2})</td>
<td>(1.1663787 \cdot 10^{-5} \text{ GeV}^{-2})</td>
</tr>
</tbody>
</table>

| \(M_W\)         | 80.93886 GeV         | 80.385 GeV           | 80.385 GeV                |
| \(s_W^2\)       | 0.2121517            | 0.2228972            | 0.2228972                 |
| \(G_\mu M_Z^2 \Delta^2 \sqrt{28 \pi \alpha}\) | 1.0                  | 1.0                  | 1.0                       |

Table 45: The EW parameters used at tree-level EW, with pole definition of the Z, W masses.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\alpha(0)) v0</th>
<th>(\alpha(0)) v1</th>
<th>(G_\mu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M_Z)</td>
<td>91.15348 GeV</td>
<td>91.15348 GeV</td>
<td>91.15348 GeV</td>
</tr>
<tr>
<td>(\Gamma_Z)</td>
<td>2.494266 GeV</td>
<td>2.494266 GeV</td>
<td>2.494266 GeV</td>
</tr>
<tr>
<td>(\Gamma_W)</td>
<td>2.085 GeV</td>
<td>2.085 GeV</td>
<td>2.085 GeV</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(\frac{1}{137.03599})</td>
<td>(\frac{1}{137.03599})</td>
<td>(\frac{1}{132.3572336357709})</td>
</tr>
<tr>
<td>(G_\mu)</td>
<td>(1.1663787 \cdot 10^{-5} \text{ GeV}^{-2})</td>
<td>(1.126555497 \cdot 10^{-5} \text{ GeV}^{-2})</td>
<td>(1.1663787 \cdot 10^{-5} \text{ GeV}^{-2})</td>
</tr>
</tbody>
</table>

| \(M_W\)         | 80.91191 GeV         | 80.35797 GeV         | 80.35797 GeV              |
| \(s_W^2\)       | 0.21208680           | 0.22283820939        | 0.22283820939             |
| \(G_\mu M_Z^2 \Delta^2 \sqrt{28 \pi \alpha}\) | 1.0                  | 1.0                  | 1.0                       |

- **Shift:**
  - -30 MeV for \(M_Z\)
  - change on \(\Gamma_Z\)
  - -0.00006 for \(s^2_w\)

- **Scaling**
  - 0.99906 for \(\alpha\)

- **Fixed \(\Gamma_Z\) in Z-propagator**

- **Running \(\Gamma_Z\) in Z-propagator**

---

E. Richter-Was, IF JU

LHC EWWG meeting, 17.12.2019

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What we have so far ..... 

**PowhegZj:** QCD NLO, Z+j  
**wt**\textsuperscript{EW}: **TauSpinner** + **Dizet** 6.21

\begin{align*}
\text{EW LO} & \rightarrow \alpha(0) \ v0: \ LO, \ NLO+HO \\
& \rightarrow \alpha(0) \ v1: \ LO \\
& \rightarrow G_\mu: \ LO \\
\end{align*}

**DYTURBO:** QCD LO, NLO, Z  
\begin{align*}
\alpha(0) \ 0: \ LO \\
\alpha(0) \ v1: \ LO \\
G_\mu: \ LO \\
\end{align*}

**Powheg_\text{ew}:** QCD LO, Z  
\begin{align*}
\alpha(0) \ v0: \ LO \\
\alpha(0) \ v1: \ LO, \ NLO, \ NLO+HO \\
G_\mu: \ LO, \ NLO, \ NLO+HO \\
\end{align*}

**MCSANC:** QCD LO, Z  
\begin{align*}
\alpha(0) \ v1: \ LO, \ NLO, \ HO \\
G_\mu: \ LO, \ NLO, \ HO \\
\end{align*}

**Arbitrary EW setup**
Define per event electroweak weight

\[ \text{wt}^{\text{EW}} = \frac{\sigma_{\text{Born}}^{\text{new}}}{\sigma_{\text{Born}}^{\text{old}}} \]

\[ \text{wt}^{\text{EW}} = \frac{d\sigma_{\text{Born}+\text{EW}}(x_1, x_2, \hat{s}, \cos\theta, s_W^2)}{d\sigma_{\text{Born}}(x_1, x_2, \hat{s}, \cos\theta, s_W^2)} \]

\[ d\sigma_{\text{Born}}(x_1, x_2, \hat{s}, \cos\theta, s_W^2) = \sum_{q_f, \bar{q}_f} [f_{q_f}(x_1, \ldots) f_{\bar{q}_f}(x_2, \ldots) d\sigma_{\text{Born}}^{q_f \bar{q}_f}(\hat{s}, \cos\theta, s_W^2) + f_{q_f}(x_2, \ldots) f_{\bar{q}_f}(x_1, \ldots) d\sigma_{\text{Born}}^{q_f \bar{q}_f}(\hat{s}, -\cos\theta, s_W^2)] \]

Approach developed in TauSpinner, arXiv:1802.05459

\( x_1, x_2, \cos\theta \) (symmetrised) calculated using 4-momenta of outgoing leptons; asymmetry in sign of \( \cos\theta \) from weighted average over PDFs

Allows to reweight MC event generated between different EW LO scheme and to Improved Born Approximation in EW scheme used for form-factors calculation.
## Impact of $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$

**Predictions from Dizet 6.21 library**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Delta \alpha_{\text{h}}^{(5)}(M_Z^2) = 0.0280398$</th>
<th>$\Delta \alpha_{\text{h}}^{(5)}(M_Z^2) = 0.02753$</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha(M_Z^2)$</td>
<td>0.00775884</td>
<td>0.00775463</td>
<td></td>
</tr>
<tr>
<td>$1/\alpha(M_Z^2)$</td>
<td>128.885224</td>
<td>128.95522</td>
<td>0.99932</td>
</tr>
<tr>
<td>$s_W^2$</td>
<td>0.22351946</td>
<td>0.22331458</td>
<td>1.00092</td>
</tr>
<tr>
<td>$\sin^2 \theta^{\text{eff}}_W (M_Z^2)$ (electron, muon)</td>
<td>0.23175990</td>
<td>0.23157062</td>
<td>1.00082</td>
</tr>
<tr>
<td>$\sin^2 \theta^{\text{eff}}_W (M_Z^2)$ (up-quark)</td>
<td>0.23164930</td>
<td>0.23146141</td>
<td>1.00080</td>
</tr>
<tr>
<td>$\sin^2 \theta^{\text{eff}}_W (M_Z^2)$ (down-quark)</td>
<td>0.23152214</td>
<td>0.23133715</td>
<td>1.00080</td>
</tr>
<tr>
<td>$M_W$</td>
<td>80.35281 GeV</td>
<td>80.36341 GeV</td>
<td>1.00013</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>0.03694272</td>
<td>0.03631342</td>
<td>1.01733</td>
</tr>
<tr>
<td>$\Delta r_{\text{rem}}$</td>
<td>0.01169749</td>
<td>0.01170244</td>
<td>0.99958</td>
</tr>
<tr>
<td>$\rho_{\text{eu}}$</td>
<td>1.005408</td>
<td>1.005426</td>
<td>0.99998</td>
</tr>
<tr>
<td>$K_e$</td>
<td>1.036649</td>
<td>1.036770</td>
<td>0.99988</td>
</tr>
<tr>
<td>$K_u$</td>
<td>1.036172</td>
<td>1.036293</td>
<td>0.99988</td>
</tr>
<tr>
<td>$K_{\text{eu}}$</td>
<td>1.074146</td>
<td>1.074397</td>
<td>0.99977</td>
</tr>
<tr>
<td>$\rho_{\text{ed}}$</td>
<td>1.005894</td>
<td>1.005906</td>
<td>0.99999</td>
</tr>
<tr>
<td>$K_e$</td>
<td>1.036649</td>
<td>1.036699</td>
<td>0.99995</td>
</tr>
<tr>
<td>$K_d$</td>
<td>1.035603</td>
<td>1.035719</td>
<td>0.99989</td>
</tr>
<tr>
<td>$K_{\text{ed}}$</td>
<td>1.073556</td>
<td>1.073859</td>
<td>0.99972</td>
</tr>
</tbody>
</table>

- **Shift of about** $-0.00020$ due to corrections to $M_W$
- **Shift by** $+11$ MeV

**ATLAS measurement**

$M_W = 80370 \pm 19$ MeV

\[
M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1 - \Delta r)}}}
\]

\[
\Delta r = \Delta \alpha(M_Z^2) + \Delta r_{\text{EW}}
\]

\[
A_0 = \sqrt{\frac{\pi \alpha(0)}{\sqrt{2} G_\mu}}
\]
# Impact of $m_t$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$m_t = 171$ GeV</th>
<th>$m_t = 173$ GeV</th>
<th>$m_t = 175$ GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha(M_Z^2)$</td>
<td>0.00775882</td>
<td>0.00775884</td>
<td>0.00775885</td>
</tr>
<tr>
<td>$1/\alpha(M_Z^2)$</td>
<td>128.888558</td>
<td>128.885224</td>
<td>128.885079</td>
</tr>
<tr>
<td>$s_W^2$</td>
<td>0.22375411</td>
<td>0.22351946</td>
<td>0.22328310</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)</td>
<td>0.23181756</td>
<td>0.23175990</td>
<td>0.23169368</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)</td>
<td>0.23171096</td>
<td>0.23164930</td>
<td>0.23169368</td>
</tr>
<tr>
<td>$\sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)</td>
<td>0.23158377</td>
<td>0.23152214</td>
<td>0.23145996</td>
</tr>
<tr>
<td>$\Delta r$</td>
<td>0.03766186</td>
<td>0.03694272</td>
<td>0.03621664</td>
</tr>
<tr>
<td>$\Delta r_{rem}$</td>
<td>0.01165959</td>
<td>0.01169749</td>
<td>0.01173500</td>
</tr>
<tr>
<td>$\rho_{eu}$</td>
<td>1.005229</td>
<td>1.005408</td>
<td>1.005589</td>
</tr>
<tr>
<td>$K_e$</td>
<td>1.035837</td>
<td>1.036649</td>
<td>1.037467</td>
</tr>
<tr>
<td>$K_u$</td>
<td>1.035361</td>
<td>1.036172</td>
<td>1.036990</td>
</tr>
<tr>
<td>$K_{eu}$</td>
<td>1.072465</td>
<td>1.074146</td>
<td>1.075843</td>
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<tr>
<td>$\rho_{ed}$</td>
<td>1.005714</td>
<td>1.005894</td>
<td>1.006075</td>
</tr>
<tr>
<td>$K_e$</td>
<td>1.035837</td>
<td>1.036649</td>
<td>1.037467</td>
</tr>
<tr>
<td>$K_d$</td>
<td>1.034792</td>
<td>1.035603</td>
<td>1.036420</td>
</tr>
<tr>
<td>$K_{ed}$</td>
<td>1.071876</td>
<td>1.073556</td>
<td>1.075252</td>
</tr>
</tbody>
</table>

$\pm 2$ GeV shift in $m_t$ corresponds to $\pm 0.00005$ shift in $\sin^2_{\text{eff}}^{\text{lep}}$