

# Spin and Polarization effects at high energies

## 1. Spin in hadronic reactions

**International School “Standard Model, Quantum Chromodynamics Heavy Ion Collisions”, Skeikampen**

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Oleg Teryaev  
JINR, Dubna



# Main Topics

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*Polarization data has often been the graveyard of fashionable theories.  
If theorists had their way, they might just ban such measurements altogether out of self-protection.*

*J.D. Bjorken  
St. Croix, 1987*

*Господь в своей бесконечной милости сделал все  
нужное простым, а все сложное ненужным.*

*Григорий Саввич Сковорода (1722-1794)*

- Spin in hadronic reactions
- Spin  $\frac{1}{2}$  : **Single** and Double Spin Asymmetries; Nucleon Spin Structure
- Spin 1: Vector and **Tensor** polarization; Geometric model and frame independence
- Spin and BSM physics
- Spin in heavy-ion collisions (9.01)
- Axial Anomaly and spin in hadronic and heavy-ion collisions(10.01)
- Gravity and non-inertial effects in HIC (11.01)

# Свободные кварки

- Свободные кварки – матрица плотности ( $\sim$ ЛЛ-IV)

$$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$$

- Упражнение: проверить положительную определенность
- $\text{Tr}(\rho^2) \leq (\text{Tr} \rho)^2$
- Высокие энергии (проверить, учитывая, что  $(pS)=0$ ,  $S^2=-3^2$ ):

$$S \rightarrow \xi p/m$$

$$\rho \rightarrow \frac{1}{2}\hat{p}(1 + \xi\gamma_5)$$



# Кварки и глюоны в адроне

- Продольно поляризованный адрон

$$\langle P, \xi | \psi_\alpha(0) \hat{E}(0, z) \bar{\psi}_\beta(z) | P, \xi \rangle = \int dx e^{i(Pz)x} [q(x) \hat{P} + \Delta q \hat{P} \gamma_5 \xi] + O(M)$$

- $E(0, z)$  – Wilson path-ordered exponential

Physical light-cone gauge  $n^2 = (An) = 0$ .  $g_\perp$ -in the plane transverse to  $P, n$ . Density matrix of circular polarized gluon.

- ГЛЮОНЫ

$$\langle P, \xi | A^\mu(0) \tilde{E}(0, z) A^\nu(z) | P, \xi \rangle = \int dx e^{i(Pz)x} [G(x) g_\perp^{\mu\nu} + i \Delta G(x) \xi \varepsilon^{\mu\nu\rho\sigma} P_\rho n_\sigma]$$

# Правила сумм – сохраняющиеся операторы

- Векторный ток  
(упражнение: проверить)

$$\int_0^1 dx [u(x) - \bar{u}(x)] = 2$$

$$\int_0^1 dx [d(x) - \bar{d}(x)] = 1$$

- Валентные кварки

$$\int_0^1 dx [s(x) - \bar{s}(x)] = 0$$

- Тензор энергии импульса – первое экспериментальное указание на глюоны

$$\int_0^1 dx x (\sum [q(x) + \bar{q}(x)] + G(x)) = 1$$

# Спиновые правила сумм

What about spin-dependent distributions?  $\int dx$ -axial current. Some matrix elements are known from  $\beta$ -decay.  $\langle p | J_5^\mu | n \rangle$ -due to isospin invariance  $\rightarrow \langle p | J_5^\mu | p \rangle - \langle n | J_5^\mu | n \rangle$  - Bjorken sum rule.

$$\int_0^1 dx (\Delta u(x) + \Delta \bar{u}(x) - \Delta d(x) - \Delta \bar{d}(x)) = \frac{1}{6} g_A \quad (8)$$

Total angular momentum conservation

$$\int_0^1 dx (\sum (\Delta q(x) + \Delta \bar{q}(x)) + \Delta G(x) + L_q(x) + L_G(x)) = \frac{1}{2}$$

Another conserved operator - quark-gluon current (due to axial anomaly)

$$\int_0^1 dx (\sum (\Delta q(x) + \Delta \bar{q}(x)) + N_f \frac{\alpha_S}{2\pi} \Delta G(x)) = const$$

Two faces of nucleon spin structure

Spin asymmetries: single vs double.

DIS structure function  $F_1, F_2$  - averaged over spin.

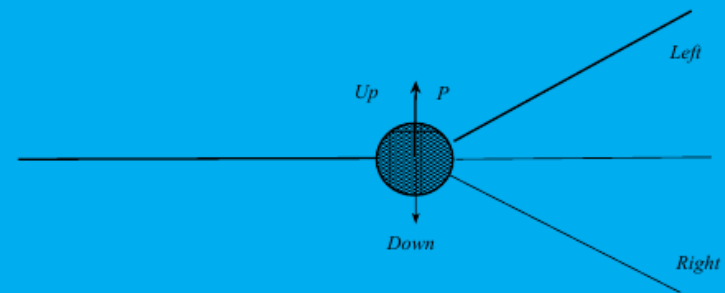
$G_1, G_2$  - for polarised leptons AND nucleons - double spin asymmetries

What about Single Spin Asymmetries (only one particle is polarized)?

Simple experiment - Complicated Theory

# Одиночные спиновые асимметрии (упражнение: проверить )

Simplest example - (non-relativistic) elastic pion-nucleon scattering  $\pi \vec{N} \rightarrow \pi N$



$M = a + ib(\vec{\sigma}\vec{n})$   $\vec{n}$  is the normal to the scattering plane.

Density matrix:  $\rho = \frac{1}{2}(1 + \vec{\sigma}\vec{P})$ ,

Differential cross-section:  $d\sigma \sim 1 + A(\vec{P}\vec{n})$ ,  $A = \frac{2\text{Im}(ab^*)}{|a|^2 + |b|^2}$

The same for the case of initial or final state polarization.  
 Various possibilities to measure the effects: change sign of  $\vec{n}$  or  $\vec{P}$ : left-right or up-down asymmetry.  
 Qualitative features of the asymmetry  
 Transverse momentum required (to have  $\vec{n}$ )  
 Transverse polarization (to maximize  $(\vec{P}\vec{n})$ )  
 Interference of amplitudes  
 IMAGINARY phase between amplitudes - absent in Born approximation



# Relativistic case

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Clearly seen in relativistic approach:

$$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$$

$$\text{Than: } d\sigma \sim \text{Tr}[\gamma_5 \dots] \sim im\varepsilon_{sp_1p_2p_3} \dots$$

Imaginary parts (loop amplitudes) are required to produce real observable.

$\varepsilon_{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$  each index appears once:  $P-$  (compensate  $S$ ) and  $T-$  odd.

However: no real  $T-$  violation: interchange  $|i\rangle \leftrightarrow |f\rangle$  is the nontrivial operation in the case of nonzero phases of  $\langle f|S|i\rangle^* = \langle i|S|f\rangle$ .

SSA - either  $T$ -violation or the phases.

DIS - no phases ( $Q^2 < 0$ )- real  $T$ -violation.





# Одиночные спиновые асимметрии

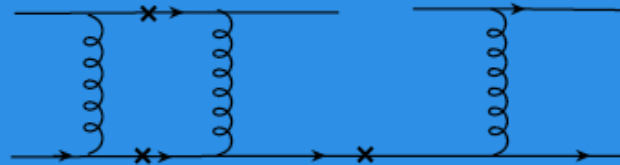
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- Необходимое условие (в Т-инвариантной теории) – интерференция амплитуд с фазовым сдвигом между ними
- КХД факторизация – фазы из жесткой и мягкой частей амплитуды а также из перекрытия этих частей
- (Обобщенная) оптическая теорема – фазы связаны со скачками по положительным кинематическим переменным (инвариантным массам)
- Жесткая часть : Теория возмущений (a la QED: Barut, Fronsdal (1960), обнаружена в JLAB): Kane, Pumplin, Repko (78), А.В. Ефремов (78)

# Пертурбативные фазы в КХД

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like  $q - e$  scattering in DIS):



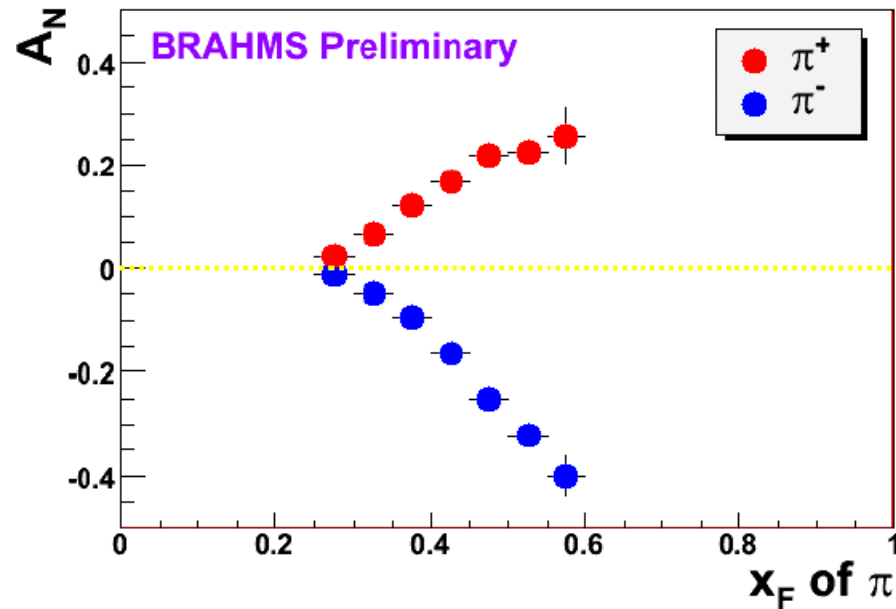
$$A \sim \frac{\alpha_S m_{PT}}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

# Экспериментально – большие одиночные асимметрии

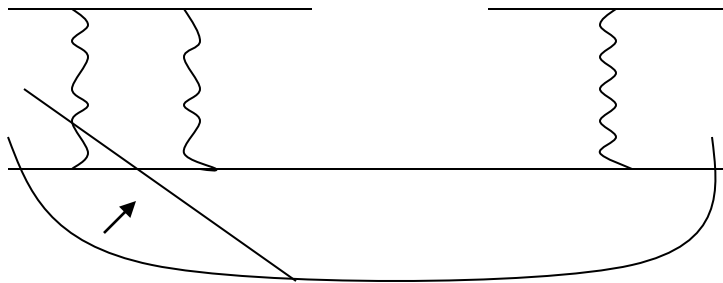
- RHIC

$A_N(\pi)$  at 62 GeV



# Корреляция жесткой и мягкой области – твист 3

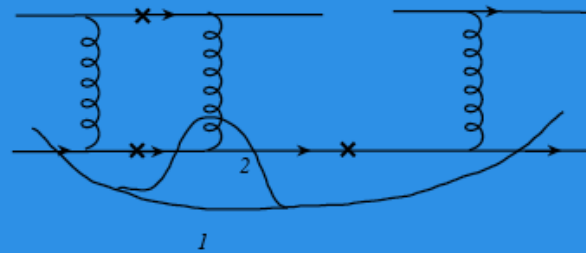
- Кварки – только из адронов
- Разные возможности для факторизации – сдвиг границы между жесткой и мягкой областями



- Новая возможность: Вместо of 1-петлевого твиста 2 - Борновский твист 3: А.В. Ефремов, ОТ (85, Фермионные полюса – мягкие кварки); Qiu, Sterman (91, глюонные полюса – мягкие глюоны)

# Корреляторы

Escape: QCD factorization - possibility to shift the borderline between large and short distances



At short distances - Loop  $\rightarrow$  Born diagram

At Large distances - quark distribution  $\rightarrow$  quark-gluon correlator.

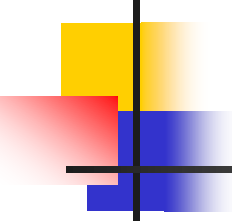
Physically - process proceeds in the external gluon field of the hadron.

Leads to the shift of  $\alpha_S$  to non-perturbative domain AND

”Renormalization” of quark mass in the external field up to an order of hadron’s one

$$\frac{\alpha_S m_{PT}}{p_T^2 + m^2} \rightarrow \frac{M b(x_1, x_2) p_T}{p_T^2 + M^2}$$

Further shift of phases completely to large distances - T-odd fragmentation functions. Leading twist transversity distribution - no hadron mass suppression.



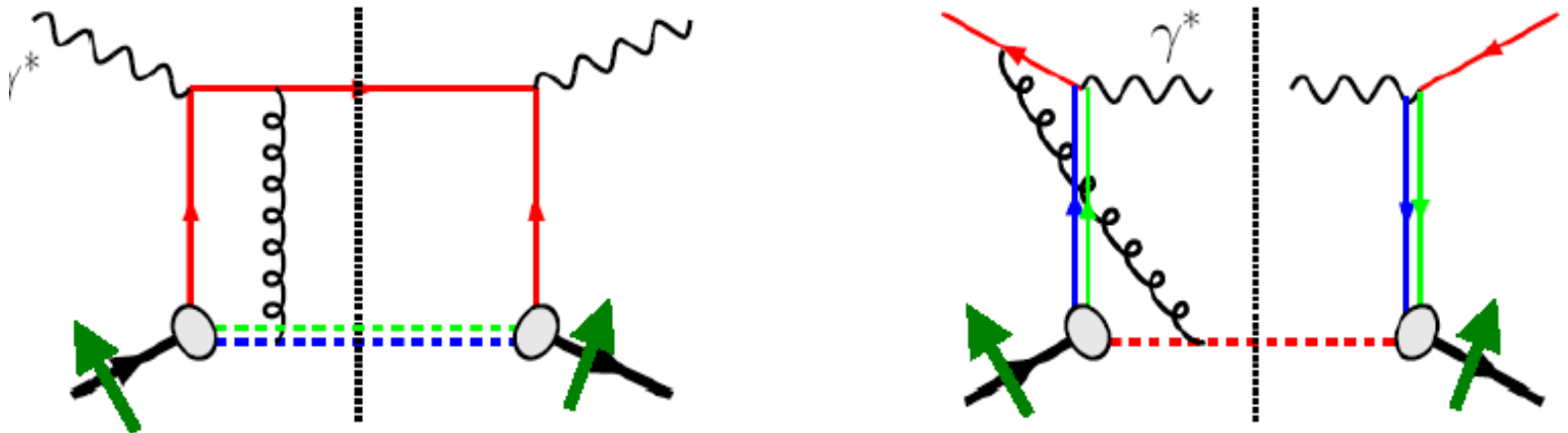
# Одиночные Спиновые Асимметрии из функции распределения

- D. Sivers (90) - корреляция спина протона и поперечного импульса кварка

$$d\Delta\sigma \sim \int d^2k_T dx f_S(x, k_T) \text{Tr}[\gamma_\rho H(xP, k_T)] \epsilon^{\rho s P k_T}$$

- Откуда фаза?! – нет скачка по массе протона (протон стабилен). Но! Вклад твиста 3 (фаза есть) можно записать в виде эффективной функции распределения (Boer, Mulders, OT, 97), оказывающейся функцией Сиверса (Boer, Mulders, Pijlman, 2001) и не подавленной по  $1/Q$  (“твист 2”)

# Простейшая форма эффективности – знак (Collins, 2002)



$$\text{Sivers}|_{\text{DIS}} = -\text{Sivers}|_{\text{DY}}$$

# Функция Сиверса в Полуинклюзивном ГНР

## Sivers asymmetry

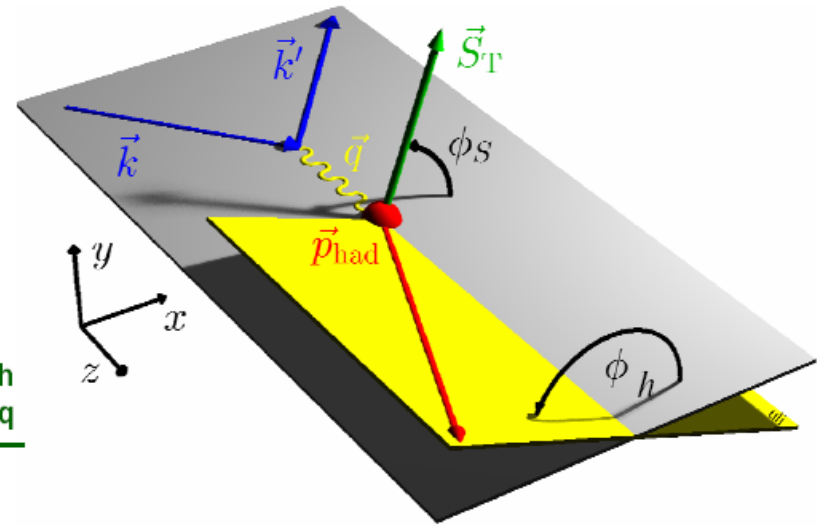
appears in SIDIS as a modulation in the “Sivers angle”  $\Phi_S$

$$\mathbf{N}_h^\pm(\Phi_S) = \mathbf{N}_h^0 \cdot [1 \pm \mathbf{P}_T \cdot \mathbf{A}_{\text{Siv}} \cdot \sin\Phi_S]$$

$$\Phi_S = \phi_h - \phi_S$$

$\phi_h$  azimuthal angle of hadron momentum  
 $\phi_S$  azimuthal angle of the spin of the nucleon

$$\mathbf{A}_{\text{Siv}} \approx \frac{\sum_{\mathbf{q}} \mathbf{e}_q^2 \cdot \Delta_0^T \mathbf{q} \cdot \mathbf{D}_q^h}{\sum_{\mathbf{q}} \mathbf{e}_q^2 \cdot \mathbf{q} \cdot \mathbf{D}_q^h}$$





# Наблюдаемые асимметрии

- proton data**

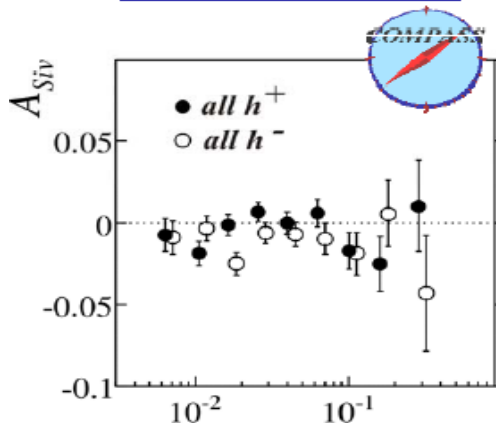
$$A_{Siv}^{p,\pi^+} \simeq \frac{4\Delta_0^T u_v D_1 + \Delta_0^T d_v D_2}{4u_v D_1 + d_v D_2} \quad A_{Siv}^{p,\pi^-} \simeq \frac{4\Delta_0^T u_v D_2 + \Delta_0^T d_v D_1}{4u_v D_2 + d_v D_1}$$

asymmetry for  $\pi^+ > 0$ , asymmetry for  $\pi^- \approx 0$

→ **Sivers DF for d-quark  $\approx -2$  Sivers DF for u-quark**

$$\Delta_0^T d_v \simeq -2 \Delta_0^T u_v$$

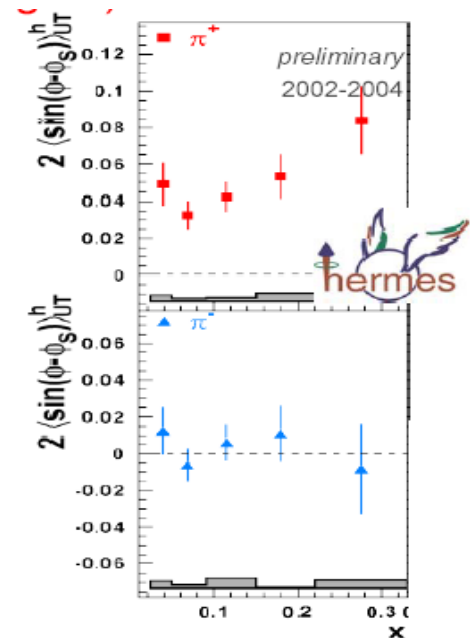
- deuteron data**



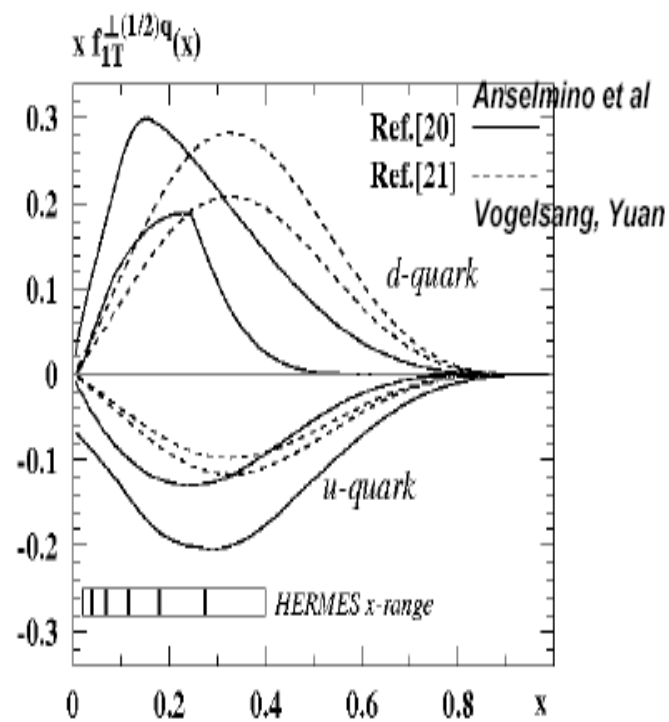
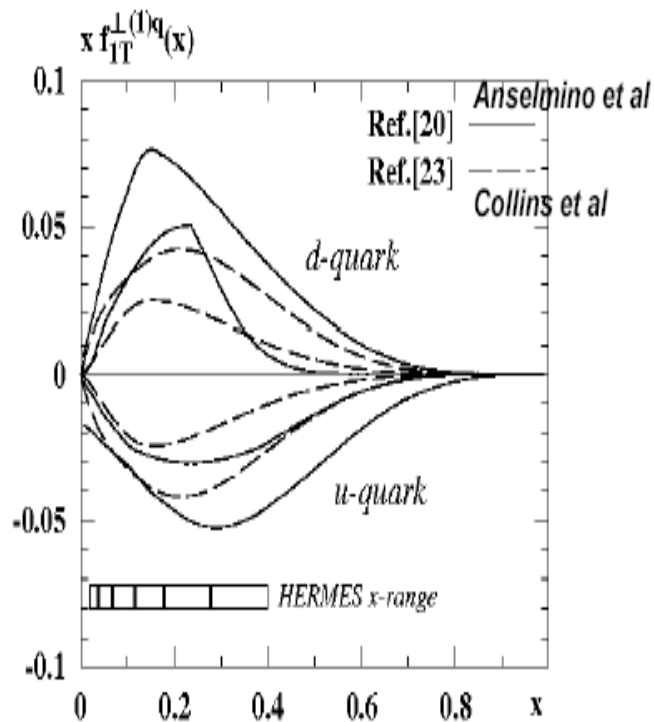
$$A_{Siv}^{d,\pi^+} \simeq A_{Siv}^{d,\pi^-} \simeq \frac{\Delta_0^T u_v + \Delta_0^T d_v}{u_v + d_v}$$

the measured asymmetries compatible with zero suggest

$$\Delta_0^T d_v \simeq -\Delta_0^T u_v$$



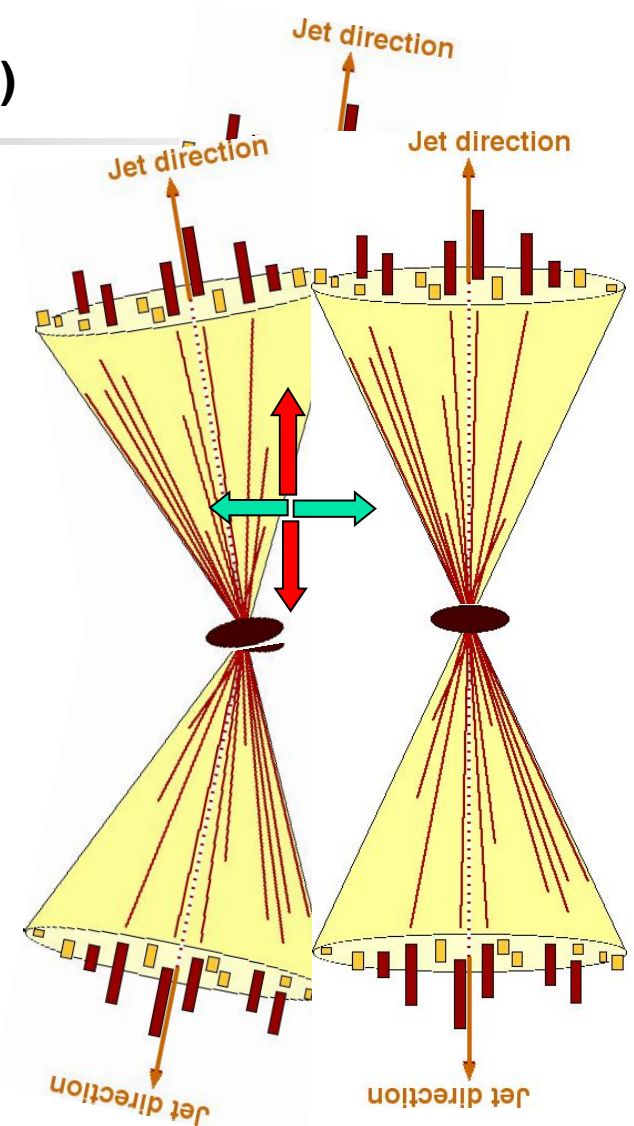
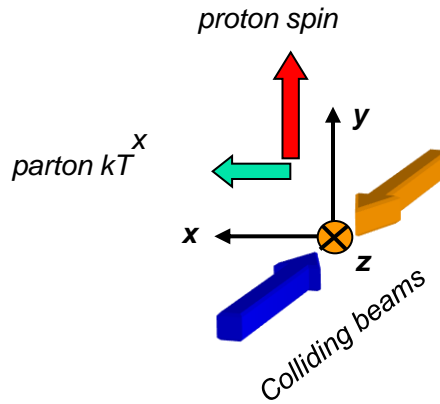
# Извлечение функции Сиверса



Anselmino et al., hep-ph/0511017

# Адронные процессы – двухструйная азимутальная асимметрия – глюонная функция Сиверса

Boer & Vogelsang (PRD 69, 094025 (2004))





# Spin density matrix: photons

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- Expansion of 2(transverse)d matrix to Pauli matrices: coefficients - Stokes parameters:  $\beta_1, \beta_2, \beta_3$
- (Anti)Symmetric part – (Circular)Linear polarization
- Scattering of unpolarized photons results in linear polarization perpendicular to scattering plane (used for gravity waves search)
- Scalar QED:  $\beta_3 = \sin^2\theta / (1 + \cos^2\theta)$
- Spinor QED Compton:  $\beta_3 = \sin^2\theta / (z + 1/z - \sin^2\theta)$
- Same for final photon energy fraction  $z \rightarrow 1$



# Virtual photons density matrix

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- 3 component of wf  $\rightarrow$  8 parameters
- $\rho_{ij} = \epsilon_{ijk} S_k + 2S_{ij}$
- Circular  $\rightarrow$  3 components of **vector** (P-odd)
- Linear  $\rightarrow$  5 components of symmetric traceless **tensor** (P-even)
- Partons collision  $\rightarrow$  tensor polarized virtual photons  $\rightarrow$  angular distributions of final particles
- Annihilation of (massless) quarks to leptons:  
 $d\sigma \sim 1 + \cos^2\theta$

# Positivity for dilepton angular distribution

- Angular distribution

$$d\sigma \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi + \rho \sin 2\theta \sin \phi + \sigma \sin^2 \theta \sin 2\phi$$

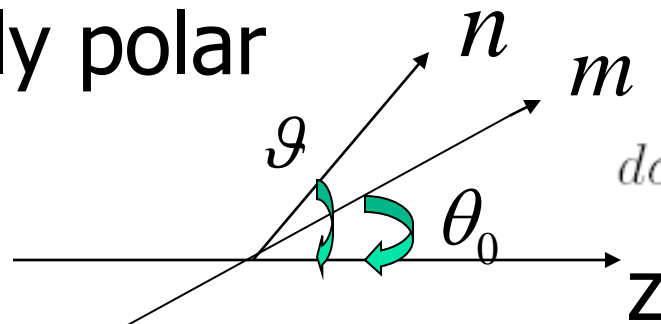
- Positivity of the matrix (= hadronic tensor in dilepton rest frame)

$$\begin{pmatrix} \frac{1-\lambda}{2} & \mu & \rho \\ \mu & \frac{1+\lambda+\nu}{2} & \sigma \\ \rho & \sigma & \frac{1+\lambda-\nu}{2} \end{pmatrix} \quad \begin{aligned} |\lambda| \leq 1, \quad |\nu| \leq 1 + \lambda, \quad \mu^2 &\leq \frac{(1-\lambda)(1+\lambda-\nu)}{4} \\ \rho^2 &\leq \frac{(1-\lambda)(1+\lambda+\nu)}{4}, \quad \sigma^2 \leq \frac{(1-\lambda)^2 - \nu^2}{4} \end{aligned}$$

- + cubic – det M > 0

# Kinematic azimuthal asymmetry from polar one (exercise: test!)

Only polar



$$d\sigma \propto 1 + \lambda_0 (\vec{n}\vec{m})^2 = 1 + \lambda_0 \cos^2 \theta_{nm}$$

asymmetry with respect to  $m$ !

$$\cos \theta_{nm} = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos \phi$$

- azimuthal

angle appears with new

$$\lambda = \lambda_0 \frac{2 - 3 \sin^2 \theta_0}{2 + \lambda_0 \sin^2 \theta_0}$$

$$\nu = \lambda_0 \frac{2 \sin^2 \theta_0}{2 + \lambda_0 \sin^2 \theta_0}$$

# Generalized Lam-Tung relation (OT'05)



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- Relation between coefficients (high school math sufficient!)

$$\lambda_0 = \frac{\lambda + \frac{3}{2}\nu}{1 - \frac{1}{2}\nu}$$

- Reduced to standard LT relation for transverse polarization ( $\lambda_0 = 1$ )
- LT - contains two very different inputs: kinematical asymmetry+transverse polarization





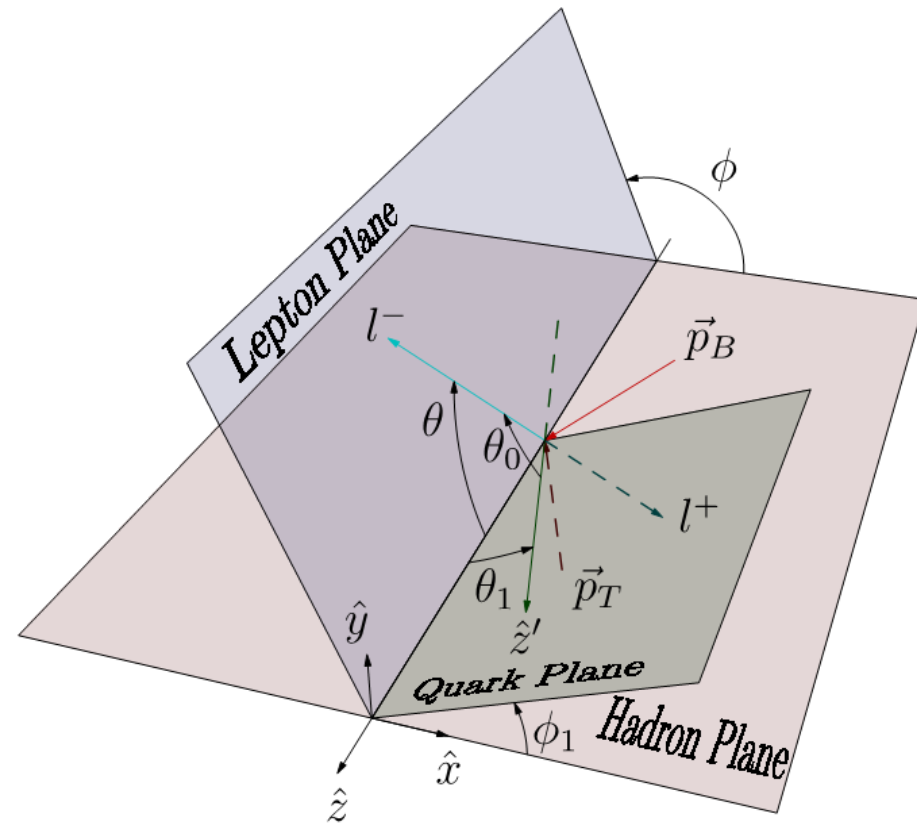
# Angular distributions

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- SM gauge bosons: detailed check of production mechanisms
- Higgs – spin 0 – isotropic distributions
- Gravitons – spin 2 – 4 component density matrix –  $\cos^4\theta$  enters – searched for and not found
- Any s-channel resonance – slow decrease with angle/transverse momentum

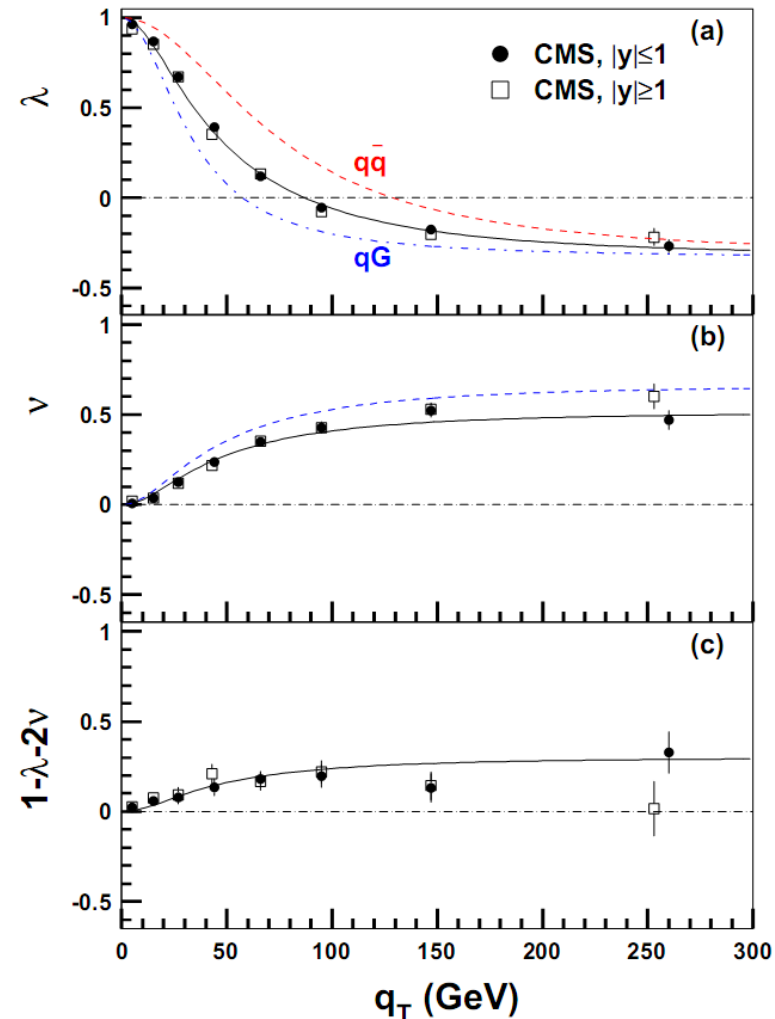
# Detailed tests of SM at LHC

- Interpretation of Angular Distributions of Z-boson Production at Colliders; Jen-Chieh Peng, Wen-Chen Chang, Randall Evan McClellan, and Oleg Teryaev; 1511.09893 and PLB
- Geometrical picture
- Non-coplanarity – disbalance of quark and hadron planes



# CMS (8 TeV) data

- Necessity to account for
- qq - 41.5(1.6)%
- qG - 58.5(1.6)%
- $\langle \cos 2\varphi_1 \rangle = 0.77$





# Summary of lecture 1

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- Spin  $1/2$ : Single Spin asymmetries require imaginary phase and (spin-orbital interference)
- Spin 1 – tensor polarization of produced bosons is sensitive to production mechanism (in particular BSM)
- Lecture 2: Heavy-Ion collisions



# Spin-gravity interactions

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- 1. Dirac equation
- Gauge structure of gravity manifested; limit of classical gravity - FW transformation
- 2. Matrix elements of Energy- Momentum Tensor
- May be studied in non-gravitational experiments/theory
- Simple interpretation in comparison to EM field case



# Gravitational Formfactors

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$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[ A_{q,g}(\Delta^2) \gamma^{(\mu} p^{\nu)} + B_{q,g}(\Delta^2) P^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M \right] u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment :  $\mu_G = J$  (g=2)

$$P_{q,g} = A_{q,g}(0) \quad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

# Generalized Parton Diistributions (related to matrix elements of non local operators ) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x, b) = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q}\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1 + \tau}$$

$$\rho_0^{\text{Gr}}(b) = \frac{1}{2\pi} \int_0^\infty dq q J_0(qb) A(q^2)$$

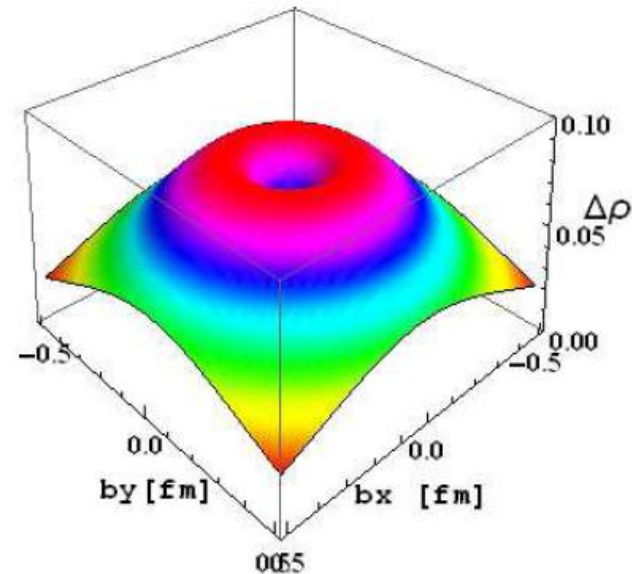


FIG. 17: Difference in the forms of charge density  $F_1^P$  and "matter" density ( $A$ )



# Electromagnetism vs Gravity

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- Interaction – field vs metric deviation

$$M = \langle P' | J_q^\mu | P \rangle A_\mu(q)$$

$$M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P | J_q^\mu | P \rangle = 2e_q P^\mu$$

$$\sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu$$
$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P | J_q^\mu | P \rangle A_\mu = 2e_q M \phi(q)$$

$$M_0 = \frac{1}{2} \sum_{q,G} \langle P | T_i^{\mu\nu} | P \rangle h_{\mu\nu} = 2M \cdot M \phi(q)$$

- Mass as charge – equivalence principle





# Gravitomagnetism

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- Gravitomagnetic field (weak, except in gravity waves) – action on spin from  $M = \frac{1}{2} \sum_{q,G} \langle P' | T_{q,G}^{\mu\nu} | P \rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} \text{rot} \vec{g}; \quad \vec{g}_i \equiv g_{0i}$$

spin dragging twice  
smaller than EM

- Lorentz force – similar to EM case: factor  $1/2$  cancelled with 2 from frequency same as EM

$$h_{00} = 2\phi(x) \quad \text{Larmor}$$

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L \quad \vec{H}_L = \text{rot} \vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle



# Equivalence principle

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- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’ ); rederived from conservation laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment is ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- - not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko, OT’07)



# Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW  
LETTERS

VOLUME 68

13 JANUARY 1992

NUMBER 2

Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

Physics Department, FM-15, University of Washington, Seattle, Washington 98195

(Received 25 September 1991)

- If (CP-odd!)  $G_{EDM}=0 \rightarrow$  constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \mathbf{B} \cdot \mathbf{S} - \zeta \hbar \boldsymbol{\omega} \cdot \mathbf{S}, \quad \zeta = 1 + \chi$$

$$|\chi(^{201}\text{Hg}) + 0.369\chi(^{199}\text{Hg})| < 0.042 \quad (95\% \text{C.L.})$$

# Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics

$$h_{zz} = h_{xx} = h_{yy} = h_{00}$$

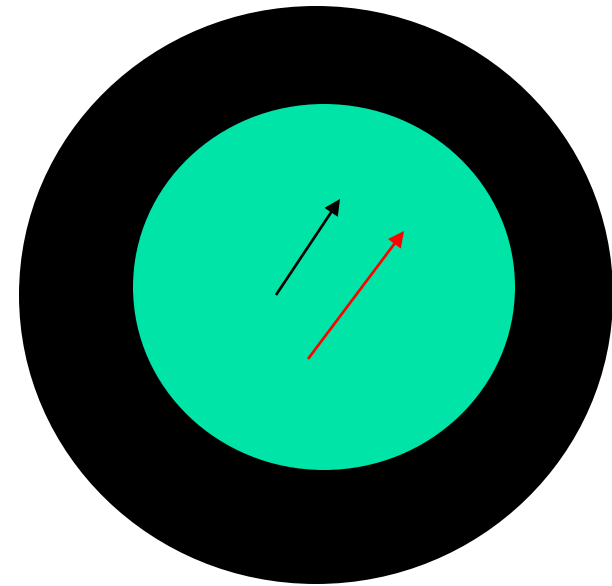
- Matrix elements DIFFER

$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \quad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$

- Ratio of accelerations:  $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$  - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09, '11, '13

# Cosmological implications of PNEP

- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and **quantum** rotators – PNEP!
- More elaborate models - Tests for cosmology ?!





# Manifestation of equivalence principle (cf with EM)

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- Classical and quantum rotators rotate with the same frequency (EM: spin  $\frac{1}{2}$  – twice faster); Dirac eq. analysis (Obukhov, Silenko, OT) – for strong fields
- Velocity rotates twice faster than classical rotator- **helicity changes** (EM – helicity of Dirac fermion conserved – used for AMM measurement) –BUT conserved in the rotating comoving frame

# Dirac Eq and Foldy - Wouthausen transformation

- Metric of the type

$$ds^2 = V^2 c^2 dt^2 - \delta_{\hat{a}\hat{b}} W^{\hat{a}}{}_c W^{\hat{b}}{}_d (dx^c - K^c c dt)(dx^d - K^d c dt).$$

- Tetrads in Schwinger gauge

$$e_{\hat{0}}^0 = V \delta_{\hat{0}}^0, \quad e_{\hat{0}}^{\hat{a}} = W^{\hat{a}}{}_b (\delta_{\hat{0}}^b - c K^b \delta_{\hat{0}}^0),$$

$$e_{\hat{a}}^0 = \frac{1}{V} (\delta_{\hat{a}}^0 + \delta_{\hat{a}}^i c K^i), \quad e_{\hat{a}}^i = \delta_{\hat{a}}^i W^b{}_{\hat{a}}, \quad a = 1, 2, 3,$$

- Dirac eq  $(i\hbar \gamma^\alpha D_\alpha - mc)\Psi = 0, \quad \alpha = 0, 1, 2, 3.$

$$D_\alpha = e_\alpha^i D_i, \quad D_i = \partial_i + \frac{iq}{\hbar} A_i + \frac{i}{4} \sigma^{\alpha\beta} \Gamma_{i\alpha\beta}.$$

# Dirac hamiltonian

## ■ Connection

$$\Gamma_{ia\hat{0}} = \frac{c^2}{V} W^b_{\hat{a}} \partial_b V e_i^{\hat{0}} - \frac{c}{V} Q_{(a\hat{b})} e_i^{\hat{b}},$$

$$\Gamma_{ia\hat{b}} = \frac{c}{V} Q_{[a\hat{b}]} e_i^{\hat{0}} + (C_{a\hat{b}\hat{c}} + C_{a\hat{c}\hat{b}} + C_{\hat{c}\hat{b}a}) e_i^{\hat{c}}.$$

$$Q_{a\hat{b}} = g_{a\hat{c}} W^d_{\hat{b}} \left( \frac{1}{c} \dot{W}^{\hat{c}}_d + K^e \partial_e W^{\hat{c}}_d + W^{\hat{c}}_e \partial_d K^e \right),$$

$$C_{a\hat{b}}^{\hat{c}} = W^d_{\hat{a}} W^e_{\hat{b}} \partial_{[d} W^{\hat{c}}_{e]}, \quad C_{a\hat{b}\hat{c}} = g_{\hat{c}\hat{d}} C_{a\hat{b}}^{\hat{d}}.$$

## ■ Hermitian Hamiltonian

$$i\hbar \frac{\partial \psi}{\partial t} = \mathcal{H} \psi, \quad \psi = (\sqrt{-g} e_0^0)^{\frac{1}{2}} \Psi.$$

$$\begin{aligned} \mathcal{H} = & \beta mc^2 V + q\Phi + \frac{c}{2} (\pi_b \mathcal{F}^b_a \alpha^a + \alpha^a \mathcal{F}^b_a \pi_b) \\ & + \frac{c}{2} (\mathbf{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \mathbf{K}) + \frac{\hbar c}{4} (\boldsymbol{\Xi} \cdot \boldsymbol{\Sigma} - Y \gamma_5). \end{aligned}$$

$$Y = V \epsilon^{\hat{a}\hat{b}\hat{c}} \Gamma_{a\hat{b}\hat{c}} = -V \epsilon^{\hat{a}\hat{b}\hat{c}} C_{a\hat{b}\hat{c}},$$

$$\Xi_a = \frac{V}{c} \epsilon_{a\hat{b}\hat{c}} \Gamma_0^{\hat{b}\hat{c}} = \epsilon_{a\hat{b}\hat{c}} Q^{\hat{b}\hat{c}}.$$



# Foldy-Wouthuysen transformation

- Even and odd parts  $\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \quad \beta\mathcal{M} = \mathcal{M}\beta,$   
 $\beta\mathcal{E} = \mathcal{E}\beta, \quad \beta\mathcal{O} = -\mathcal{O}\beta.$

- FW transformation (Silenko '08)

$$U = \frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}\beta, \quad \psi_{\text{FW}} = U\psi, \quad \mathcal{H}_{\text{FW}} = U\mathcal{H}U^{-1} - i\hbar U\partial_t U^{-1},$$

$$U^{-1} = \beta \frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}, \quad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}.$$

$$\mathcal{H}' = \beta\epsilon + \mathcal{E} + \frac{1}{2T}([T, [T, (\beta\epsilon + Z)]) + \beta[\mathcal{O}, [\mathcal{O}, \mathcal{M}]] - [\mathcal{O}, [\mathcal{O}, Z]])$$

$$T = \sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2} - [(\epsilon + \mathcal{M}), [(\epsilon + \mathcal{M}), Z]] - [(\epsilon + \mathcal{M}), [\mathcal{M}, \mathcal{O}]]$$

$$Z = \mathcal{E} - i\hbar \frac{\partial}{\partial t} - \beta\{\mathcal{O}, [(\epsilon + \mathcal{M}), Z]\} + \beta\{(\epsilon + \mathcal{M}), [\mathcal{O}, Z]\} \frac{1}{T},$$

$$\mathcal{H}_{\text{FW}} = \beta\epsilon + \mathcal{E}' + \frac{1}{4}\beta\left\{\mathcal{O}^2, \frac{1}{\epsilon}\right\}.$$

# FW for arbitrary gravitational field

## ■ Result

$$\mathcal{H}_{\text{FW}} = \mathcal{H}_{\text{FW}}^{(1)} + \mathcal{H}_{\text{FW}}^{(2)}$$

$$\epsilon' = \sqrt{m^2 c^4 V^2 + \frac{c^2}{4} \delta^{ac} \{p_b, \mathcal{F}^b{}_a\} \{p_d, \mathcal{F}^d{}_c\}},$$

$$\mathcal{T} = 2\epsilon'^2 + \{\epsilon', mc^2 V\}.$$

$$\mathcal{M} = mc^2 V,$$

$$\mathcal{E} = q\Phi + \frac{c}{2}(\mathbf{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \mathbf{K}) + \frac{\hbar c}{4} \boldsymbol{\Xi} \cdot \boldsymbol{\Sigma},$$

$$\mathcal{O} = \frac{c}{2}(\boldsymbol{\pi}_b \mathcal{F}^b{}_a \alpha^a + \alpha^a \mathcal{F}^b{}_a \boldsymbol{\pi}_b) - \frac{\hbar c}{4} Y \gamma_5.$$

$$\begin{aligned} \mathcal{H}_{\text{FW}}^{(1)} = & \beta \epsilon' + \frac{\hbar c^2}{16} \left\{ \frac{1}{\epsilon'}, (2\epsilon^{cae} \Pi_e \{p_b, \mathcal{F}^d{}_c \partial_d \mathcal{F}^b{}_a\} \right. \\ & \left. + \Pi^a \{p_b, \mathcal{F}^b{}_a Y\}) \right\} \\ & + \frac{\hbar m c^4}{4} \epsilon^{cae} \Pi_e \left\{ \frac{1}{\mathcal{T}}, \{p_d, \mathcal{F}^d{}_c \mathcal{F}^b{}_a \partial_b V\} \right\}, \end{aligned}$$

$$\begin{aligned} \mathcal{H}_{\text{FW}}^{(2)} = & \frac{c}{2} (K^a p_a + p_a K^a) + \frac{\hbar c}{4} \Sigma_a \Xi^a \\ & + \frac{\hbar c^2}{16} \left\{ \frac{1}{\mathcal{T}}, \left\{ \Sigma_a \{p_e, \mathcal{F}^e{}_b\}, \left\{ p_f, \left[ \epsilon^{abc} \left( \frac{1}{c} \dot{\mathcal{F}}^f{}_c \right. \right. \right. \right. \right. \right. \\ & \left. \left. \left. \left. \left. - \mathcal{F}^d{}_c \partial_d K^f + K^d \partial_d \mathcal{F}^f{}_c \right) \right. \right. \right. \right. \\ & \left. \left. \left. \left. \left. - \frac{1}{2} \mathcal{F}^f{}_d (\delta^{db} \Xi^a - \delta^{da} \Xi^b) \right] \right] \right\} \right\}, \end{aligned}$$



# Operator EOM

---

- Polarization operator  $\mathbf{\Pi} = \beta \mathbf{\Sigma}$

$$\frac{d\mathbf{\Pi}}{dt} = \frac{i}{\hbar} [\mathcal{H}_{\text{FW}}, \mathbf{\Pi}] = \mathbf{\Omega}_{(1)} \times \mathbf{\Sigma} + \mathbf{\Omega}_{(2)} \times \mathbf{\Pi}.$$

- Angular velocities

$$\begin{aligned} \Omega_{(1)}^a = & \frac{mc^4}{2} \left\{ \frac{1}{\mathcal{T}}, \{p_e, \epsilon^{abc} \mathcal{F}_b^e \mathcal{F}_c^d \partial_d V\} \right\} \\ & + \frac{c^2}{8} \left\{ \frac{1}{\epsilon^f}, \{p_e, (2\epsilon^{abc} \mathcal{F}_b^d \partial_d \mathcal{F}_c^e + \delta^{ab} \mathcal{F}_b^e Y)\} \right\}, \end{aligned}$$

$$\begin{aligned} \Omega_{(2)}^a = & \frac{\hbar c^2}{8} \left\{ \frac{1}{\mathcal{T}}, \left\{ \{p_e, \mathcal{F}_b^e\}, \left\{ p_f, \left[ \epsilon^{abc} \left( \frac{1}{c} \dot{\mathcal{F}}_c^f \right. \right. \right. \right. \right. \right. \\ & \left. \left. \left. \left. \left. - \mathcal{F}_c^d \partial_d K^f + K^d \partial_d \mathcal{F}_c^f \right) \right. \right. \right. \right. \\ & \left. \left. \left. \left. \left. - \frac{1}{2} \mathcal{F}_d^f (\delta^{db} \Xi^a - \delta^{da} \Xi^b) \right] \right] \right] \right\} \right\} + \frac{c}{2} \Xi^a. \end{aligned}$$



# Semi-classical limit

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- Average spin

$$\frac{ds}{dt} = \mathbf{\Omega} \times s = (\mathbf{\Omega}_{(1)} + \mathbf{\Omega}_{(2)}) \times s,$$

$$\Omega_{(1)}^a = \frac{c^2}{\epsilon'} \mathcal{F}^d {}_c P_d \left( \frac{1}{2} Y \delta^{ac} - \epsilon^{aef} V C_{ef}{}^c + \frac{\epsilon'}{\epsilon' + mc^2 V} \epsilon^{abc} W^e {}_b \partial_e V \right),$$

$$\Omega_{(2)}^a = \frac{c}{2} \Xi^a - \frac{c^3}{\epsilon'(\epsilon' + mc^2 V)} \epsilon^{abc} Q_{(bd)} \delta^{dn} \mathcal{F}^k {}_n P_k \mathcal{F}^l {}_c P_l,$$

# Application to anisotropic universe (Kamenshchik, OT)

- Bianchi-1 Universe

$$ds^2 = dt^2 - a^2(t)(dx^1)^2 - b^2(t)(dx^2)^2 - c^2(t)(dx^3)^2.$$

- Particular case  $W_1^1 = a(t), W_2^2 = b(t), W_3^3 = c(t).$

$$W_1^1 = \frac{1}{a(t)}, W_2^2 = \frac{1}{b(t)}, W_3^3 = \frac{1}{c(t)}.$$

- No anholonomy  $\Upsilon = 0$

$$\Omega_{(2)}^i = \frac{\gamma}{\gamma + 1} v_2 v_3 \left( \frac{\dot{b}}{b} - \frac{\dot{c}}{c} \right).$$
$$Q_{ii} = -\frac{\dot{a}}{a}, Q_{22} = -\frac{\dot{b}}{b}, Q_{33} = -\frac{\dot{c}}{c}.$$



# Kasner solution

---

- t-dependence

$$a(t) = a_0 t^{p_1}, \quad b(t) = b_0 t^{p_2}, \quad c(t) = c_0 t^{p_3},$$

$$p_1 + p_2 + p_3 = 1, \quad p_1^2 + p_2^2 + p_3^2 = 1.$$

- Euler-type expressions

$$\Omega_{(2)}^i = \frac{\gamma}{\gamma + 1} v_2 v_3 \left( \frac{p_2 - p_3}{t} \right)$$



# Heckmann-Schucking solution

---

- Dust admixture

$$a(t) = a_0 t^{p_1} (t_0 + t)^{\frac{2}{3} - p_1}, \quad b(t) = b_0 t^{p_2} (t_0 + t)^{\frac{2}{3} - p_2}, \\ c(t) = c_0 t^{p_3} (t_0 + t)^{\frac{2}{3} - p_3}.$$

- Modification:

$$\Omega_{(2)}^i = \frac{\gamma}{\gamma + 1} v_2 v_3 \frac{(p_2 - p_3)t_0}{t(t_0 + t)} \\ = \frac{\gamma}{\gamma + 1} v_2 v_3 \frac{(p_2 - p_3)t_0}{t^2} \left( 1 + o\left(\frac{t_0}{t}\right) \right)$$



# Biancki-IX Universe

---

- **Metric**  $W_a^{\hat{b}} = \begin{pmatrix} -a \sin x^3 & a \sin x^1 \cos x^3 & 0 \\ b \cos x^3 & b \sin x^1 \sin x^3 & 0 \\ 0 & c \cos x^1 & c \end{pmatrix}$   $W_{\hat{b}}^c = \begin{pmatrix} -\frac{1}{a} \sin x^3 & \frac{1}{b} \cos x^3 & 0 \\ \frac{1}{a} \cos x^3 & \frac{1}{b} \sin x^3 & 0 \\ -\frac{1}{a} \frac{\cos x^1 \cos x^3}{\sin x^1} & -\frac{1}{b} \frac{\sin x^3 \cos x^1}{\sin x^1} & \frac{1}{c} \end{pmatrix}$

- **Anholonomy coefficients**

- $C_{\hat{1}\hat{2}}^{\hat{3}} = \frac{c}{ab}$  + cyclic permutations

- -> non-zero  $\Upsilon = 2 \left( \frac{c}{ab} + \frac{b}{ac} + \frac{a}{bc} \right)$

$$\Omega_{(1)}^{\hat{1}} = v^{\hat{1}} \left( \frac{c}{ab} + \frac{b}{ac} - \frac{a}{bc} \right)$$





# Approach to singularity

---

- Chaotic oscillations – sequence of Kasner regimes

$$p_1 = -\frac{u}{1+u+u^2}, \quad p_2 = \frac{1+u}{1+u+u^2}, \quad p_3 = \frac{u(1+u)}{1+u+u^2}$$

- If Lifshitz-Khalatnikov parameter  $u > 1$  – “epochs”

$$p'_1 = p_2(u-1), \quad p'_2 = p_1(u-1), \quad p'_3 = p_3(u-1)$$

- If  $u < 1$  – “eras”

$$p'_1 = p_1 \left( \frac{1}{u} \right), \quad p'_2 = p_3 \left( \frac{1}{u} \right), \quad p'_3 = p_2 \left( \frac{1}{u} \right)$$

- Change of eras – chaotic mapping of  $[0,1]$  interval

$$Tx = \left\{ \frac{1}{x} \right\}, \quad x_{s+1} = \left\{ \frac{1}{x_s} \right\}$$



# Angular velocities

---

- New epoch:  $u \rightarrow -u$
- New era – changed sign

$$\Omega_{(2)}^{\hat{1}} = \frac{\gamma}{(\gamma + 1)t} v_2 v_3 \cdot \frac{1 - u^2}{1 + u + u^2},$$

$$\Omega_{(2)}^{\hat{2}} = \frac{\gamma}{(\gamma + 1)t} v_1 v_3 \cdot \frac{2u + u^2}{1 + u + u^2},$$

$$\Omega_{(2)}^{\hat{3}} = -\frac{\gamma}{(\gamma + 1)t} v_1 v_2 \cdot \frac{1 + 2u}{1 + u + u^2}.$$

- Odd velocity

$$\Omega_{(1)}^{\hat{1}} \sim -v^{\hat{1}}(t) \left( -1 - \frac{2u}{1 + u + u^2} \right),$$

$$\Omega_{(1)}^{\hat{b}} \sim v^{\hat{b}}(t) \left( -1 - \frac{2u}{1 + u + u^2} \right), \quad b = 2, 3.$$

$$\Omega_{(1)}^{\hat{2}} \sim -v^{\hat{2}}(t) \left( -1 - \frac{2u - 2}{1 - u + u^2} \right),$$

$$\Omega_{(1)}^{\hat{a}} \sim v^{\hat{a}}(t) \left( -1 - \frac{2u - 2}{1 - u + u^2} \right), \quad a = 1, 3.$$

- New epoch
- New era - preserved



# Possible applications

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- Anisotropy (c.f. crystals)  $\sim$  magnetic field
- Spin precession + equivalence principle = helicity flip ( $\sim$ AMM effect)
- Dirac neutrino – transformed to sterile component in early (bounced) Universe
- Angular velocity  $\sim 1/t$   $\rightarrow$  amount of decoupled  $\sim 1$
- Possible new candidate for dark matter?!
- Other fields AFTER inflation?



# CONCLUSIONS

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- Polarization – extra sensitive tests
- Gravity leads to spin effects related to Kobzarev-Okun equivalence principle
- Bianchi universe – spin precession and neutrino helicity flip

- 
- 
- **BACKUP SLIDES**



# Semi-classical limit

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- Average spin precession

$$\frac{d\vec{s}}{dt} = \vec{\Omega} \times \vec{s} = (\vec{\Omega}_{(1)} + \vec{\Omega}_{(2)}) \times \vec{s}.$$

- Angular velocity contributions

$$\Omega_{(1)}^{\hat{a}} = \frac{1}{\varepsilon'} W_{\hat{c}}^d P_d \left( \frac{1}{2} \Upsilon \delta^{\hat{a}\hat{c}} - \varepsilon^{\hat{a}\hat{e}\hat{f}} C_{\hat{e}\hat{f}}^{\hat{c}} \right),$$
$$\Omega_{(2)}^{\hat{a}} = \frac{1}{2} \Xi^{\hat{a}} - \frac{1}{\varepsilon'(\varepsilon' + m)} \varepsilon^{\hat{a}\hat{b}\hat{c}} Q_{(\hat{b}\hat{d})} \delta^{\hat{d}\hat{n}} W_{\hat{n}}^k P_k W_{\hat{c}}^l P_l.$$

# Torsion – acts only on spin

Dirac eq+FW transformation-Obukhov,Silenko,OT

## ■ Hermitian Dirac Hamiltonian

$$e_i^{\hat{0}} = V \delta_i^0, \quad e_i^{\hat{a}} = W^{\hat{a}}_b (\delta_i^b - cK^b \delta_i^0) \quad \mathcal{H} = \beta mc^2 V + q\Phi + \frac{c}{2} (\pi_b \mathcal{F}^b_a \alpha^a + \alpha^a \mathcal{F}^b_a \pi_b)$$

$$ds^2 = V^2 c^2 dt^2 - \delta_{\hat{a}\hat{b}} W^{\hat{a}}_c W^{\hat{b}}_d (dx^c - K^c c dt) (dx^d - K^d c dt) \quad + \frac{c}{2} (\mathbf{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \mathbf{K}) + \frac{\hbar c}{4} (\boldsymbol{\Xi} \cdot \boldsymbol{\Sigma} - \Upsilon \gamma_5),$$

$$\mathcal{F}^b_a = V W^b_{\hat{a}}, \quad \Upsilon = V \epsilon^{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{a}\hat{b}\hat{c}}, \quad \Xi^a = \frac{V}{c} \epsilon^{\hat{a}\hat{b}\hat{c}} (\Gamma_{\hat{0}\hat{b}\hat{c}} + \Gamma_{\hat{b}\hat{c}\hat{0}} + \Gamma_{\hat{c}\hat{0}\hat{b}})$$

## ■ Spin-torsion coupling

$$- \frac{\hbar c V}{4} (\boldsymbol{\Sigma} \cdot \check{\mathbf{T}} + c \gamma_5 \check{T}^{\hat{0}})$$

$$\check{T}^\alpha = - \frac{1}{2} \eta^{\alpha\mu\nu\lambda} T_{\mu\nu\lambda}$$

## ■ FW – semiclassical limit - precession

$$\Omega^{(T)} = - \frac{c}{2} \check{\mathbf{T}} + \beta \frac{c^3}{8} \left\{ \frac{1}{\epsilon'}, \{p, \check{T}^{\hat{0}}\} \right\} + \frac{c}{8} \left\{ \frac{c^2}{\epsilon'(\epsilon' + mc^2)}, (\{p^2, \check{\mathbf{T}}\} - \{p, (p \cdot \check{\mathbf{T}})\}) \right\}$$

# Experimental bounds for torsion

- Magnetic field+rotation+torsion

$$H = -g_N \frac{\mu_N}{\hbar} \mathbf{B} \cdot \mathbf{s} - \boldsymbol{\omega} \cdot \mathbf{s} - \frac{c}{2} \tilde{\mathbf{T}} \cdot \mathbf{s},$$

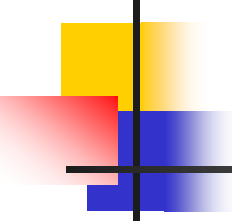
- Same '92 EDM experiment

$$\frac{\hbar c}{4} |\tilde{\mathbf{T}}| \cdot |\cos \Theta| < 2.2 \times 10^{-21} \text{ eV}, \quad |\tilde{\mathbf{T}}| \cdot |\cos \Theta| < 4.3 \times 10^{-14} \text{ m}^{-1}$$

- New(based on Gemmel et al '10)

$$\frac{\hbar c}{2} |\tilde{\mathbf{T}}| \cdot |(1 - \mathcal{G}) \cos \Theta| < 4.1 \times 10^{-22} \text{ eV}, \quad |\tilde{\mathbf{T}}| \cdot |\cos \Theta| < 2.4 \times 10^{-15} \text{ m}^{-1},$$
$$\mathcal{G} = g_{He}/g_{Xe}$$



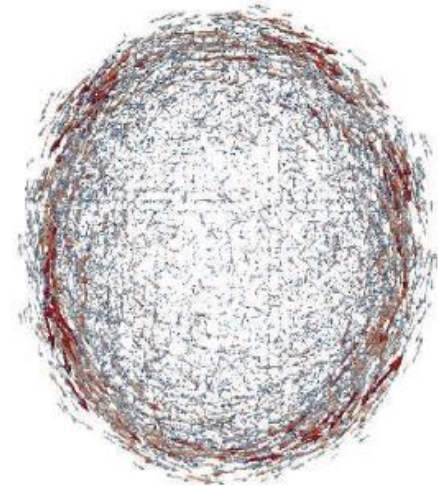
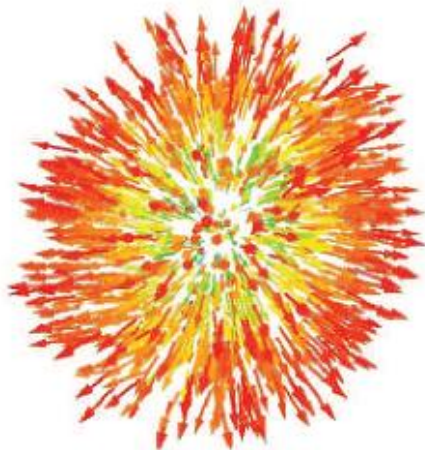
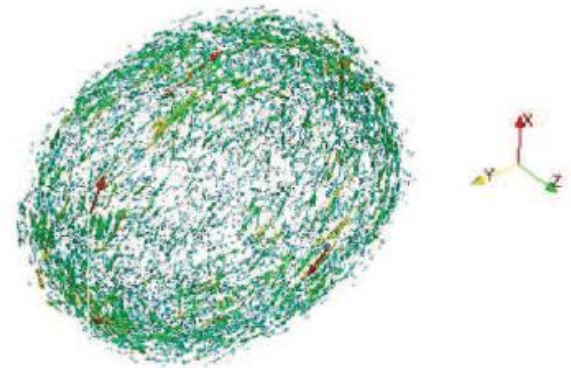
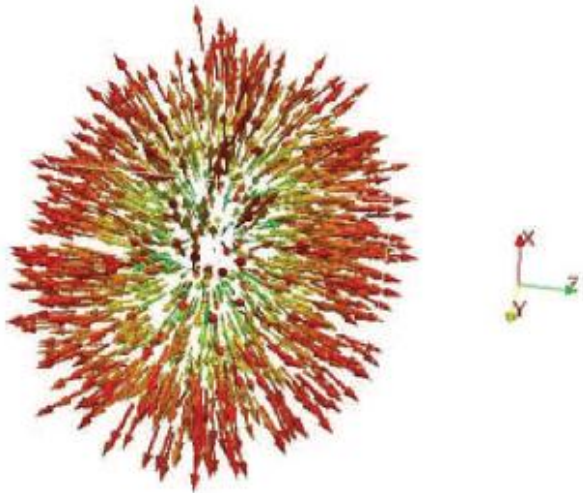


# Microworld: where is the fastest possible rotation?

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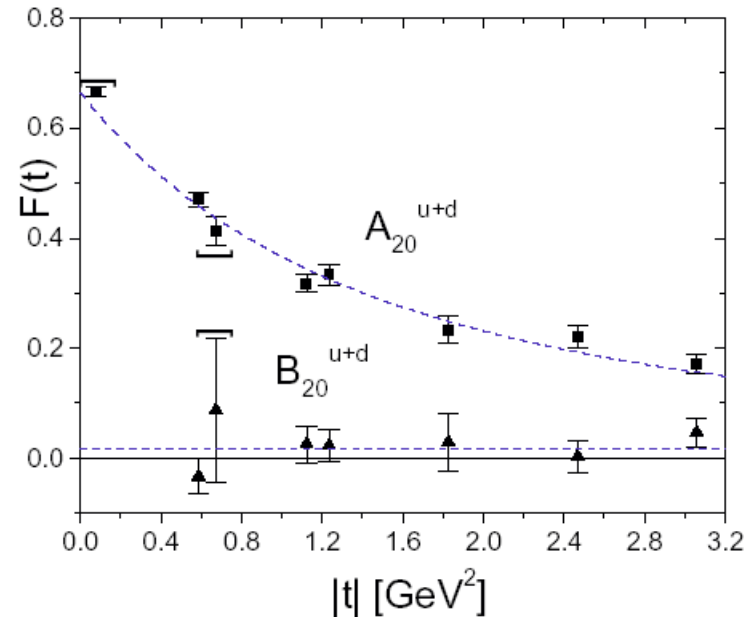
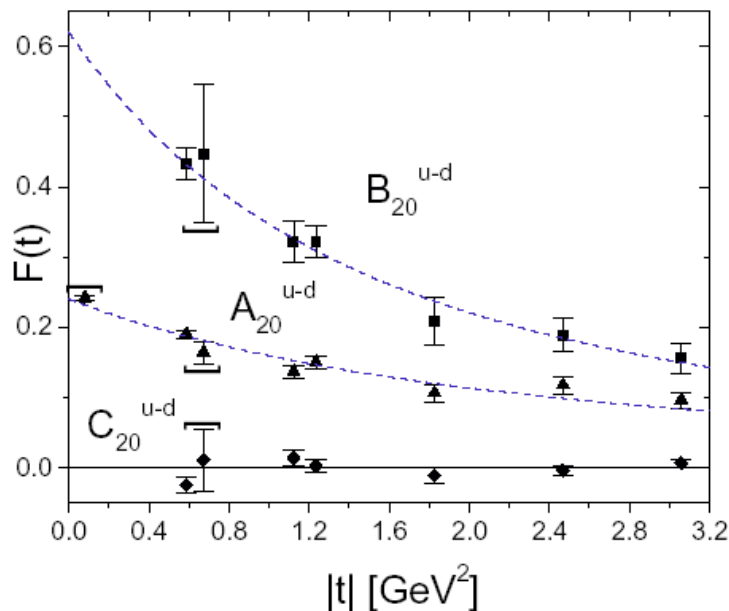
- Non-central heavy ion collisions ( $\sim c/\text{Compton wavelength}$ ) – “small Bang”
- Differential rotation – vorticity
- Leads to hyperons polarization – should be larger at small energy – predicted in 2010 (Rogachevsky, Sorin, OT) now found by STAR@RHIC
- Calculation in quark - gluon string model (Baznat, Gudima, Sorin, OT, PRC'13)

# Structure of velocity and vorticity fields (NICA@JINR-5 GeV/c)



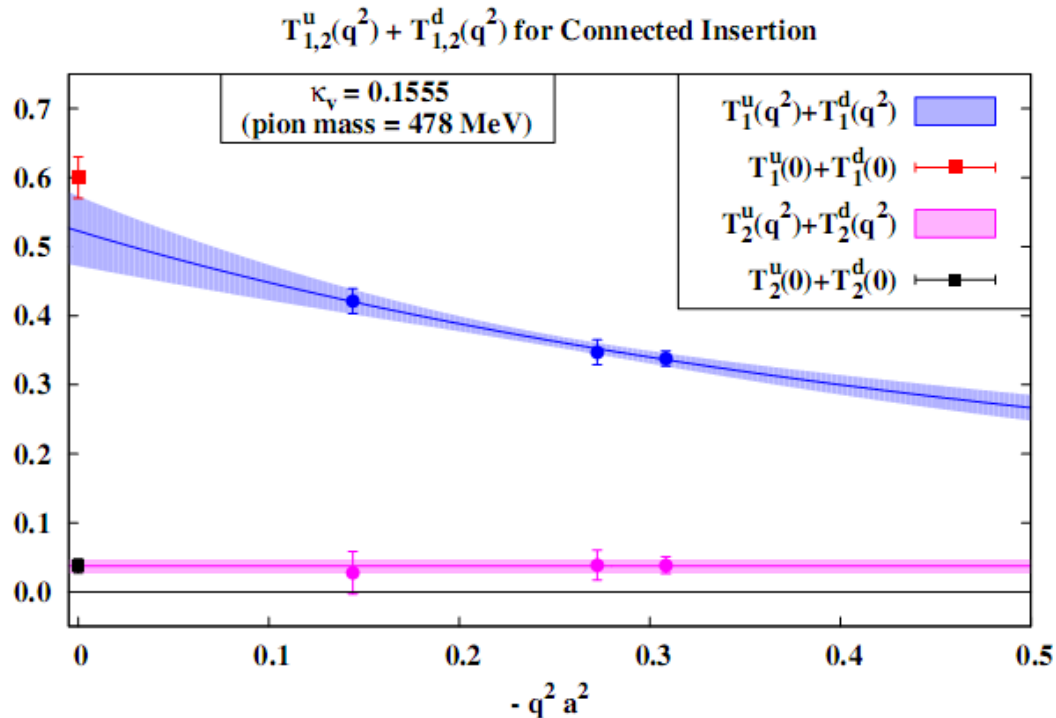
# Generalization of Equivalence principle

- Various arguments:  $AGM \approx 0$  separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)



# Recent lattice study (M. Deka et al. [arXiv:1312.4816](https://arxiv.org/abs/1312.4816); cf plenary talk of K.F. Liu)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



# Extended Equivalence

## Principle=Exact EquiPartition

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- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Supported by generic smallness of  $E$  (isoscalar AMM)



# Sum rules for EMT (and OAM)

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- First (seminal) example: X. Ji's sum rule ('96). Gravity counterpart – OT'99
- Burkardt sum rule – looks similar: can it be derived from EMT?
- Yes, if provide correct prescription to gluonic pole (OT'14)

# Pole prescription and Burkardt SR

- Pole prescription (dynamics!) provides ("T-odd") symmetric part!

- SR:  $\sum \int dx T(x, x) = 0$  (but relation of gluon Sivers to twist 3 still not found – prediction!)  $\sum \int \int dx_1 dx_2 \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon} = 0$

- Can it be valid separately for each quark flavour: nodes (related to "sign problem")?
- Valid if structures forbidden for TOTAL EMT do not appear for each flavour
- Structure contains besides S gauge vector n: If GI separation of EMT – forbidden: SR valid separately!

# Another manifestation of post-Newtonian (E)EP for spin 1 hadrons

- Tensor polarization - coupling of gravity to spin in forward matrix elements - inclusive processes
- Second moments of tensor distributions should sum to zero

$$\langle P, S | \bar{\psi}(0) \gamma^\nu D^{\nu_1} \dots D^{\nu_n} \psi(0) | P, S \rangle_{\mu^2} = i^{-n} M^2 S^{\nu\nu_1} P^{\nu_2} \dots P^{\nu_n} \int_0^1 C_q^T(x) x^n dx$$

$$\sum_q \langle P, S | T_i^{\mu\nu} | P, S \rangle_{\mu^2} = 2P^\mu P^\nu (1 - \delta(\mu^2)) + 2M^2 S^{\mu\nu} \delta_1(\mu^2)$$

$$\langle P, S | T_g^{\mu\nu} | P, S \rangle_{\mu^2} = 2P^\mu P^\nu \delta(\mu^2) - 2M^2 S^{\mu\nu} \delta_1(\mu^2)$$

$$\sum_q \int_0^1 C_i^T(x) x dx = \delta_1(\mu^2) = 0 \text{ for ExEP}$$

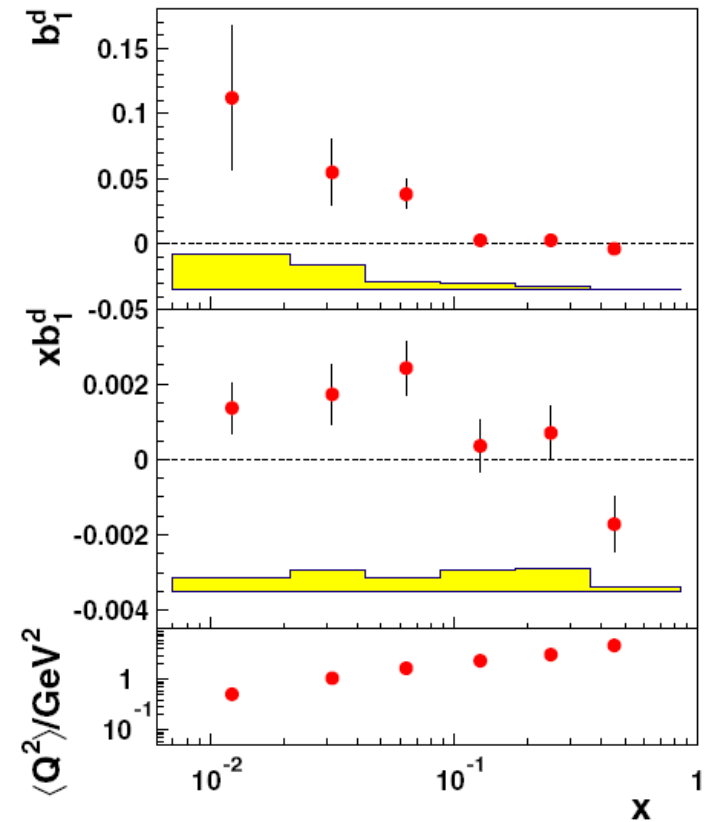


# HERMES – data on tensor spin structure function

PRL 95, 242001 (2005)

- Isoscalar target – proportional to the sum of u and d quarks – combination required by EEP
- Second moments – compatible to zero better than the first one (collective glue  $\ll$  sea) – for valence:

$$\int_0^1 C_i^T(x) dx = 0.$$





# Are more accurate data possible?

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- HERMES – unlikely
- JLab may provide information about collective sea and glue in deuteron and indirect new test of Equivalence Principle



# CONCLUSIONS

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- Spin-gravity interactions may be probed directly in gravitational (inertial) experiments and indirectly – studying EMT matrix element
- Torsion and EP are tested in EDM experiments
- SR's for deuteron tensor polarization- indirectly probe EP and its extension separately for quarks and gluons



# EEP and AdS/QCD

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- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides  $g=2$  identically!
- Experimental test at time –like region possible