PHASE DIAGRAM OF HADRON MATTER IN THE NJL-LIKE MODELS

Oslo winter school 2020, Skeikampen

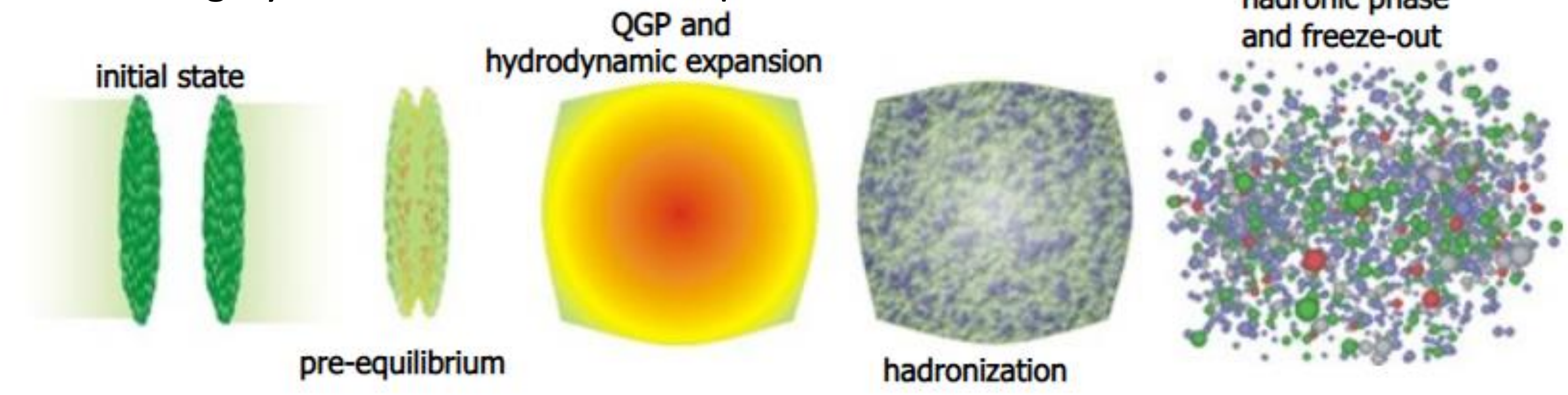
31.12.2019-12.01.2020

HEAVY ION COLLISION: STAGES

Glauber model,
Glasma,
String dynamics

bag model,
QCD motivated models
Lattice QCD+quarks

Statistical models



Lattice-Gauge Theory:
- rigorous calculation of QCD quantities
- works in the infinite size / equilibrium limit

Experiments:
- observe the final state + penetrating probes
- rely on QGP signatures predicted by Theory

Transport-Models & Phenomenology:
- full description of collision dynamics
- connects intermediate state to observables
- provides link between LGT and data

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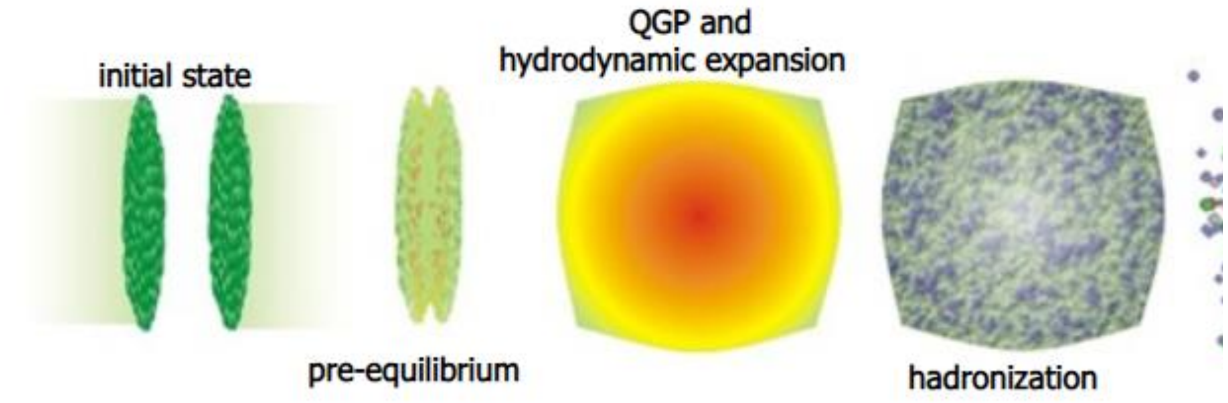

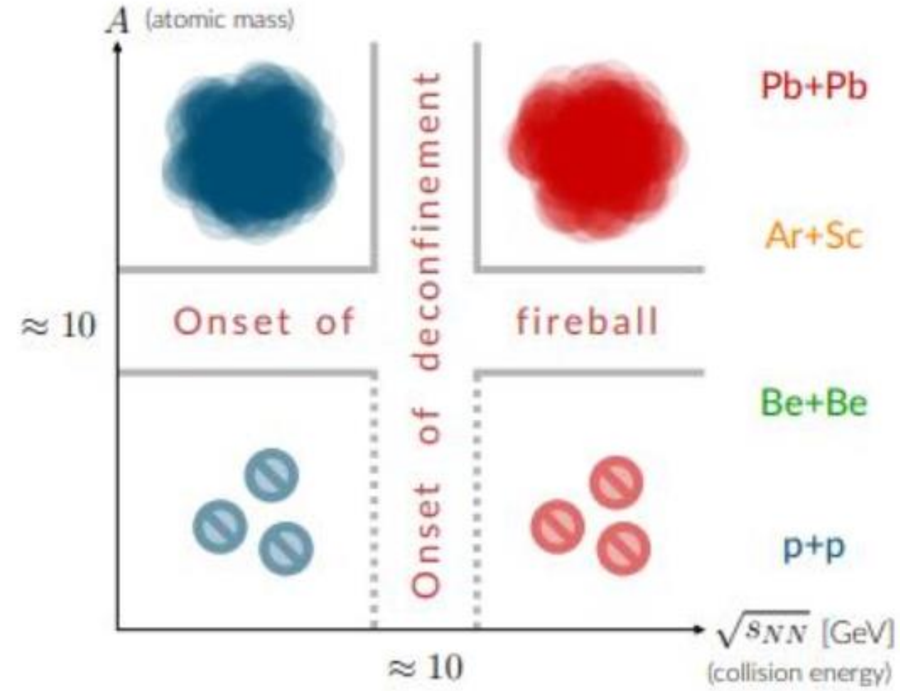

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CONTENT:

- Introduction
- NJL model
  - Lagrangian
  - Mean field approximation
  - Finite temperature NJL model
  - Gap equation
  - Mass spectra

Sleeping time

- Phase diagram
- Conclusion and outlooks (I)
- PNJL model
  - PNJL model with vector interaction
  - Extended PNJL model
- Can we use those models for getting practical results?

MOTIVATION: QUANTUM CHROMODYNAMICS

The QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\gamma_\mu D^\mu - \widehat{m}_q\right) q - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

with interaction term:

$$F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^a_\nu A^b_\nu$$

and covariant derivative

$$D_\mu = \partial_\mu - ig\lambda^a A^a_\mu$$

$A^a_\mu \; (a = 1, 2, \ldots, 8)$ are the gluon fields

QCD is the fundamental theory of the strong interaction, where the quarks and gluons are the basic degrees of freedom

$$(q_\alpha)^A_f \quad \begin{cases} \text{colour} & A = 1,\, 2,\, 3 \\ \text{spin} & \alpha = \uparrow, \downarrow \\ \text{flavour} & f = u,\, d,\, s,\, c,\, b,\, t \end{cases} \qquad A^a_\mu \quad \begin{cases} \text{colour} & a = 1, \ldots, 8 \\ \text{spin} & \varepsilon^\pm_\mu \end{cases}$$

Fundamental constituents of the Standard Model (SM) of particle physics: Quantum Chromodynamics (QCD) & Electroweak (EW) theories.

QCD: ASYMPTOTIC FREEDOM AND CONFINEMENT

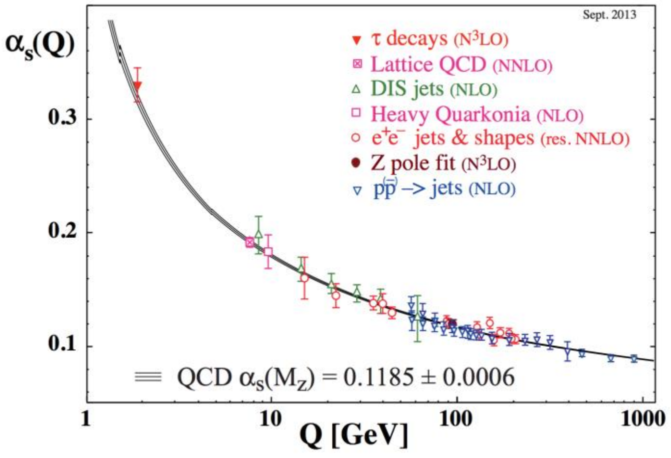
Asymptotic freedom:

Momentum-dependent coupling ⇔ interaction strength between quarks and gluons grows with separation:

$$\alpha_s(k^2) \stackrel{\text{def}}{=} \frac{g_s^2(k^2)}{4\pi} \approx \frac{1}{\beta_0 \ln\left(\frac{k^2}{\Lambda^2}\right)}$$

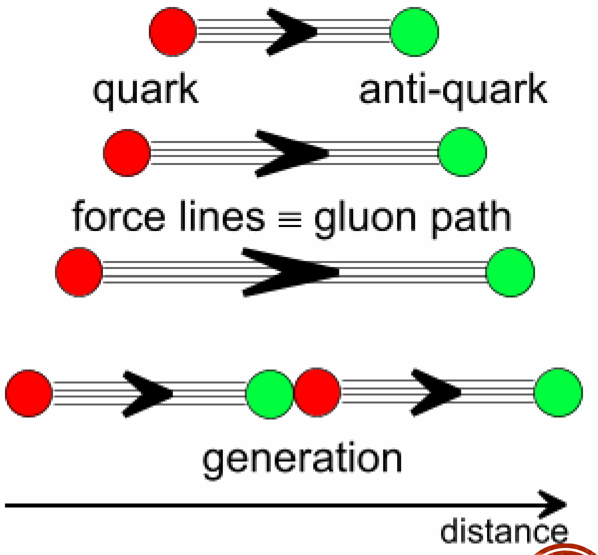
(Wilczek, Gross and Politzer)

$\Lambda_{QCD} \simeq 1\ \text{fm}^{-1}$ – sets scale most important parameter in QCD



Confinement:

- particles that carry the colour charge cannot be isolated and can not be directly observed



Hadron structure @ QCD is characterized by two emergent phenomena (both of those phenomena are not evident from the QCD Lagrangian):

- Confinement: all known hadrons are colour singlets, even though they are composed of coloured quarks and gluons: baryons (qqq) & mesons ($q\bar{q}$)
- dynamical chiral symmetry breaking (DCSB) (QCDs chiral symmetry is explicitly broken by small current quark masses)

QCD: WHY DO WE NEED OTHER APPROACHES?

Further analysis of the content of the theory is complicated. Various techniques have been developed to work with QCD.

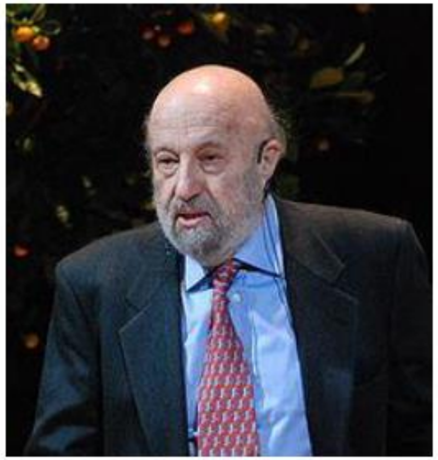
The tools available are:

- Lattice QCD
- Chiral perturbation theory
- 1/*N* expansion (also known as the "large *N*" expansion)
- QCD inspired models (linear sigma model, local/nonlocal NJL model)

We will review the model of Nambu and Jona-Lasinio (NJL model)

NAMBU – JONA-LASINIO MODEL

Giovanni Jona-Lasinio
1932



Yoichiro Nambu
1921-2015

Nobel Prize in Physics (2008)

The Nambu–Jona-Lasinio (NJL) Model was invented in 1961 by Yoichiro Nambu and Giovanni Jona-Lasinio while at The University of Chicago [**Y. Nambu, G. Jona-Lasinio,(April 1961). "Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. I".** ***Physical Review*****122: 345**]

- was inspired by the BCS theory of superconductivity
- was originally a theory of elementary nucleons
- rediscovered in the 80s as an effective quark theory

It is a relativistic quantum field theory, that is relatively easy to work with, and is very successful in the description of hadrons, nuclear matter, the spontaneous breaking of the chiral symmetry etc.

Nambu won half the 2008 Nobel prize in physics in part for the NJL model: “for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics” [Nobel Committee]

SU(2) NJL MODEL: LAGRANGIAN

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\mathrm{q}}\,(\mathrm{i}\partial\!\!\!/ - \hat{\mathrm{m}}_0 - \gamma_0\mu)\,\mathrm{q} + \mathrm{G_s}\left[(\bar{\mathrm{q}}\mathrm{q})^2 + (\bar{\mathrm{q}}\mathrm{i}\gamma_5\vec{\tau}\mathrm{q})^2\right]$$

G_s the effective coupling strength,
q, $\bar{q}$ - quark fields
$\widehat{m}_0 = \; diag\;(m_{0u},\; m_{0d}),\;\; m_{0u} = m_{0d}$ _ the current quark masses, -
τ – Pauli matrices SU(2).
[M. K. Volkov, Ann. Phys. 157,282 (1989); Sov. J. Part and Nuclei 17, 433 (1986) S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992)]

We can:
- explain the spontaneous chiral symmetry breaking;
- describe the light quarks and mesons properties,
- describe the phase transitions.

MEAN FIELD APPROXIMATION

The partition function in the path integral formalism (imaginary time τ = it) is given by

$$\mathcal{Z}[\bar{q}, q] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{\int_0^\beta d\tau \int_v d^3x \, [\mathcal{L}_{\text{NJL}}]\right\}$$

$$Z[T, V, \mu] = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{\int_0^\beta d\tau \int_V d^3x \left[\bar{q}\left(i\gamma^\mu\partial_\mu - m_0 - \gamma^0\mu\right) q + G_S\left(\bar{q}\Gamma_{\sigma'}q\right)^2 + G_S\left(\bar{q}\vec{\Gamma}_{\pi'}q\right)^2\right]\right\},$$

We can use Hubbard-Stratonovich transformation formula:

$$\boxed{\exp\left[\int_0^\beta d\tau \int d^3x \, G\left(\bar{\psi}\mathcal{O}\psi\right)^2\right] = N' \int \mathcal{D}\phi \exp\left[-\int_0^\beta d\tau \int d^3x \left(\frac{\phi^2}{4G} \pm \phi\bar{\psi}\mathcal{O}\psi\right)\right]}$$

and obtain:

$$Z[T, V, \mu] = \int \mathcal{D}\sigma'\mathcal{D}\vec{\pi}'\mathcal{D}\bar{q}\mathcal{D}q \exp\left\{\int_0^\beta d\tau \int_V d^3x \, [\bar{q}\underbrace{\left(i\gamma^\mu\partial_\mu - m_0 - \sigma'\Gamma_{\sigma'} - \vec{\pi}'\vec{\Gamma}_{\pi'} - \gamma^0\mu\right)}_{S^{-1}[\sigma', \vec{\pi}']} q - \frac{\sigma'^2 + \vec{\pi}'^2}{4G_S}]\right\}.$$

MEAN FIELD APPROXIMATION

$$Z[T,V,\mu] = \int \mathcal{D}\sigma' \mathcal{D}\vec{\pi}' \mathcal{D}\bar{q} \mathcal{D}q \exp\left\{ \int_0^\beta d\tau \int_V d^3x \left[\bar{q} S^{-1}[\sigma', \vec{\pi}'] q - \frac{\sigma'^2 + \vec{\pi}'^2}{4G_S} \right] \right\}$$

Now we use:

1. $\int \mathcal{D}\bar{q} \mathcal{D}q \exp\left\{ \int_0^\beta d\tau \int_V d^3x \left[\bar{q} S^{-1}[\sigma', \vec{\pi}'] q \right] \right\} = \mathrm{det S}^{-1} = \exp\left(\ln \mathrm{det S}^{-1}\right) = \exp\left(\mathrm{Tr} \ln \mathrm{S}^{-1}\right)$

2. $\sigma' = \sigma_{MF} + \sigma, \qquad \vec{\pi}' = \vec{\pi}_{MF} + \vec{\pi} = \vec{\pi}$

3. $\mathrm{Tr} \ln S^{-1}[\sigma', \vec{\pi}'] = \mathrm{Tr} \ln S^{-1}_{MF}[m] + \mathrm{Tr} \ln \left[1 - S_{MF}[m] \left(\sigma + i\gamma_5 \vec{\tau}\vec{\pi} \right) \right]$

As the mean sigma field has no space-time dependence, we can perform the integrations over dτ and d^3x obtaning:

$$\boxed{Z_{MF}[T,V,\mu] = \exp\left\{ -\frac{V}{T} \frac{\sigma^2_{MF}}{4G_S} + \mathrm{Tr} \ln S^{-1}_{MF}[m] \right\}}$$

THE GRAND POTENTIAL

$$Z_{MF}[T,V,\mu] = \exp\left\{-\frac{V}{T}\frac{\sigma_{MF}^2}{4G_S} + \mathrm{Tr}\ln S_{MF}^{-1}[m]\right\}$$

The thermodynamic potential in mean field approximation:

$$\Omega_{MF}(T,\mu) = -\frac{T}{V}\ln Z_{MF}[T,V,\mu]$$

$$\Omega_{MF}(T,\mu) = -\frac{T}{V}\ln\left(\exp\left\{-\frac{V}{T}\frac{\sigma_{MF}^2}{4G_S} + \mathrm{Tr}\ln S_{MF}^{-1}[m]\right\}\right) = \frac{\sigma_{MF}^2}{4G_S} - \frac{T}{V}\mathrm{Tr}\ln S_{MF}^{-1}[m]$$

$$\ln(\det(A)) = \mathrm{tr}(\log(A))$$

Finally: $\Omega_{MF}(T,\mu) = \dfrac{\sigma_{MF}^2}{4G_S} + \Omega_q$

$$\Omega_q = -2N_cN_f\int\frac{d^3p}{(2\pi)^3}E_\mathrm{p} - 2N_cN_fT\int\frac{d^3p}{(2\pi)^3}\left[\ln N^+(E_\mathrm{p}) + \ln N^-(E_\mathrm{p})\right]$$

THE CHIRAL CONDENSATE

Under which conditions we can really have chiral symmetry breaking? When we have maxima with respect to our order parameter:

$$\frac{\partial \Omega_{MF}}{\partial \sigma_{MF}} = 0, \quad \frac{\partial^2 \Omega}{\partial \sigma_{MF}^2} > 0$$

and taking the derivative:

$$= -1$$

$$\frac{\partial \Omega_{MF}}{\partial \sigma_{MF}} = \frac{\sigma_{MF}}{2G_s} - \frac{T}{V} \mathrm{Tr}\, S_{MF}[m] \frac{\partial S_{MF}^{-1}[m]}{\partial \sigma} = \frac{\sigma_{MF}}{2G_s} + \frac{T}{V} \mathrm{Tr}\, S_{MF}[m] = 0$$

In the momentum representation: $S_{MF}^{-1} = \gamma^\mu p_\mu - m - \gamma^0 \mu = \gamma^0 (i\omega_n - \mu) - \vec{\gamma} \cdot \vec{p} - m$

and from this, keeping in mind that Tr[γ_μ] = 0, Tr[1] = 4 and performing the trick named Matsubara summation(for fermions $\omega_n = (2n + 1)\, T$), we have:

$$\lim_{V\to\infty} \frac{1}{\beta V} \sum_k = \lim_{V\to\infty} \frac{T}{V} \sum_k = T \sum_n \int \frac{d^3p}{(2\pi)^3} \quad \Rightarrow \quad \lim_{V\to\infty} \sum_k = V \sum_n \int \frac{d^3p}{(2\pi)^2}$$

$$\frac{\sigma_{MF}}{2G_s} = -T N_c N_f \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{4m}{(i\omega_n - \mu)^2 - E_p^2}$$

NJL AT FINITE TEMPERATURE

$$\frac{1}{(i\omega_n-\mu)^2-E_p^2}=-\frac{1}{2E_p}\left(\frac{1}{(i\omega_n-\mu)+E_p}-\frac{1}{(i\omega_n-\mu)-E_p}\right)$$

Performing the summation, we introduce the Fermi distribution functions:

$$T\sum_n\frac{1}{(i\omega_n-\mu)^2-E_p^2}=-\frac{1}{2E_p}T\sum_n\left(\frac{1}{(i\omega_n-\mu)+E_p}-\frac{1}{(i\omega_n-\mu)-E_p}\right)=$$

$$=-\frac{1}{2E_p}T\sum_n\left(\frac{1}{i\omega_n+(E_p-\mu)}-\frac{1}{i\omega_n-(E_p+\mu)}\right)=-\frac{1}{2E_p}\Big(f\big(-(E_p-\mu)\big)-f\left(E_p+\mu\right)\Big)=$$

$$=-\frac{1}{2E_p}\Big(1-f\left(E_p-\mu\right)-f\left(E_p+\mu\right)\Big)$$

where in the last equality we have used the basic property of the Fermi distribution function: $f(-x) = 1 - f(x)$. In the end, what we get is

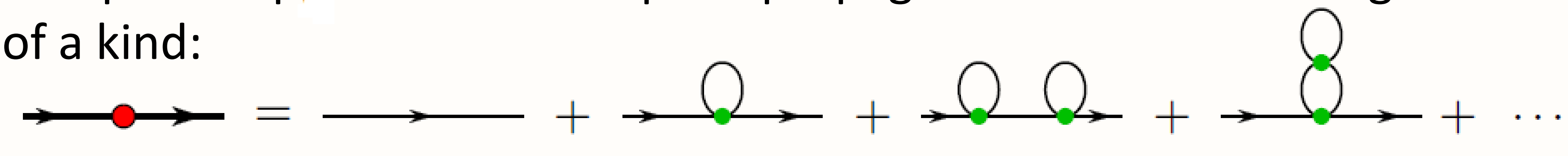
$$\frac{\sigma_{MF}}{2G_s}=N_cN_f\int\frac{d^3p}{(2\pi)^3}\frac{2m}{E_p}\Big(1-f\left(E_p-\mu\right)-f\left(E_p+\mu\right)\Big)$$

which, with Nc = 3 and Nf = 2, is equivalent to the gap equation:

$$\sigma_{MF}=m-m_0=\ 4G_s\,N_cN_f\int\frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\Big(1-f\left(E_p-\mu\right)-f\left(E_p+\mu\right)\Big)$$

GAP EQUATION II (DYSON-SCHWINGER EQUATION)

Complete expression for the quark propagator contained all diagrams of a kind:



All these diagrams can be summed:

And the gap equation has a form:

$$m = m_0 + 2iG_s \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} S(p)$$

Where $S^{-1}(p) = (\hat{p} - m)$ is a propagator of dressed quark. And finally, inrodicing the quark condensate,

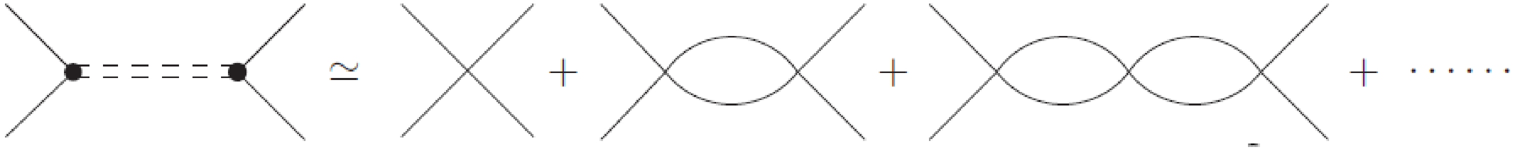
$$< \bar{q}q >= \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr} S(p) = N_c N_f \int \frac{d^4p}{(2\pi)^4} \frac{m}{p^2 - m^2}$$

we obtain the gap equation:

$$m = m_0 + 2G_s < \bar{q}q >$$

BETHE-SALPETER EQUATION FOR MESONS

The effective interaction resulting from the exchange of a π meson can be obtained as an infinite sum of loops in the random-phase approximation (RPA):



It leads to appearing of the T-matrix $$T_M(k^2) = \frac{2iG_s}{1 - 2G_s\Pi_M(k^2)}$$

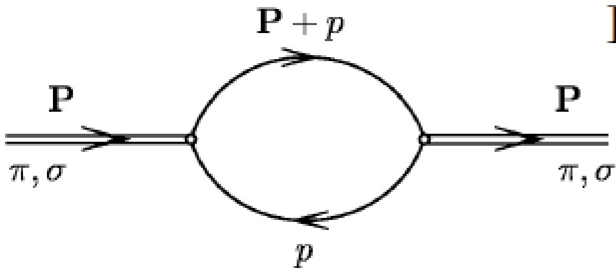
and can be find that the mass of mesons is related to the pole of the matrix, which is the solution of the following equation:

$$1 - 2G_s\, \Pi_M(k^2)|_{k^2=M^2} = 0$$

Where all information about mesons is concluded into the polarization operator ($\Gamma_\sigma = 1, \Gamma_\pi = i\gamma_5$)

$$\Pi_M(k^2) = i\int \frac{d^4p}{(2\pi)^4}\mathrm{Tr}\ [\Gamma_M S(p+k)\Gamma_M S(p)]$$

BETHE-SALPETER EQUATION FOR MESONS



$$\Pi_{ab}^{PP}(P^2) = \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\,[i\gamma_5\tau^a S(p+P) i\gamma_5 \tau^b S(p)]$$

$$\Pi^{SS}(P^2) = \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\,[S(p+P)S(p)]\,,$$

$$\Pi_{ps}(k^2) = 4N_cN_fI_1 - 2N_cN_fk^2I_2(k^2)$$

$$\Pi_s(k^2) = 4N_cN_fI_1 - 2N_cN_f(k^2-4m^2)I_2(k^2)$$

$$I_1 = \int \frac{d^4p}{(2\pi)^4}\frac{1}{p^2-m^2}$$

$$I_2(k^2) = \int \frac{dp}{(2\pi)^4}\frac{1}{(p^2-m^2)((p-k)^2-m^2)}$$

$$g_{Mqq}^{-2} = \frac{\partial \Pi_M(k^2)}{\partial k^2}\Big|_{k^2=M^2}$$

PARAMETERS AND REGULARIZATION SCHEMES

The coupling constants G_S and Λ_S can be determined using the Gell-Mann– Oakes–Renner relation:

$$\boxed{m_\pi^2 f_\pi^2 = -(m_u^0 + m_d^0)\langle \bar{u}u \rangle}$$

- The pion decay constant $f_\pi = 0.092$ GeV,
- The pion mass $M_\pi = 0.139$ GeV
- The quark condensate $< \bar{q}q >= (-0.25 \text{ GeV})^3$

Regularization schemes

3D cutoff

$$\int \frac{p^2 dp}{4\pi^2} \to \int_0^\Lambda \frac{p^2 dp}{4\pi^2}$$

The Pauli-Villars regularization

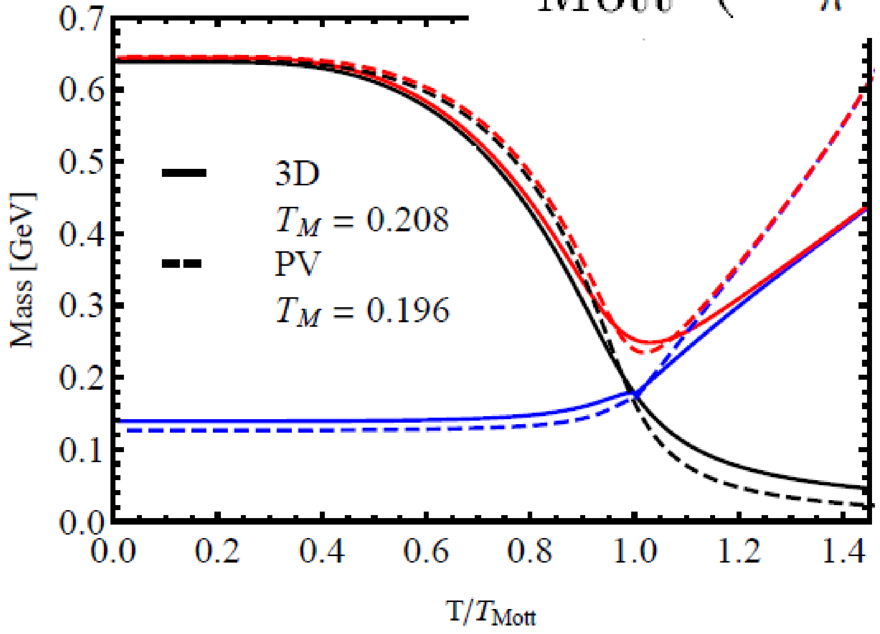
$$f^{reg}(m,p) \to \sum_a C_a f(M_a, p)$$

$$M_a^2 = m^2 + \alpha_a \Lambda^2 \quad \alpha_a = (0,2,1), \quad C_a = (1,1,-2)$$

The integral is replaced with the set: $f(m,p) = (\sqrt{p^2 + m^2})^{-1}$

$$\int \frac{d^4p}{(2\pi)^4} f(m,p) \to \int \frac{d^4p}{(2\pi)^4} (f(m,p) + f(\sqrt{m^2 + 2\Lambda^2}, p) - 2f(\sqrt{m^2 + \Lambda^2}, p))$$

$$T_{Mott}\ (M_\pi = 2m_q\ )$$



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$$\Pi_s(k^2) = 4N_cN_fI_1 - 2N_cN_f(k^2 - 4m^2)I_2(k^2)$$

$$P_0 = M_M - \frac{1}{2}i\Gamma_M$$

$$I_2(k^2) = \int \frac{dk}{(2\pi)^3}\frac{1}{(k_0^2 - E_1^2)((k_0 - p_0)^2 - E_2^2)}$$

with $p_0 = i\omega_n = i2n\pi T$ – bozonic frequency

$k_0 = i\nu_m = i(2m+1)\pi T$ – fermionic frequency

$$\lim_{\epsilon\to 0}\frac{f(x)}{x - x_0 \pm i\epsilon} = \frac{\mathcal{P}}{x - x_0} \mp i\pi f(x)\delta(x - x_0)$$

$$I_2(k^2) = \mathrm{Re}I_2(k^2) + i\ \mathrm{Im}I_2(k^2)$$

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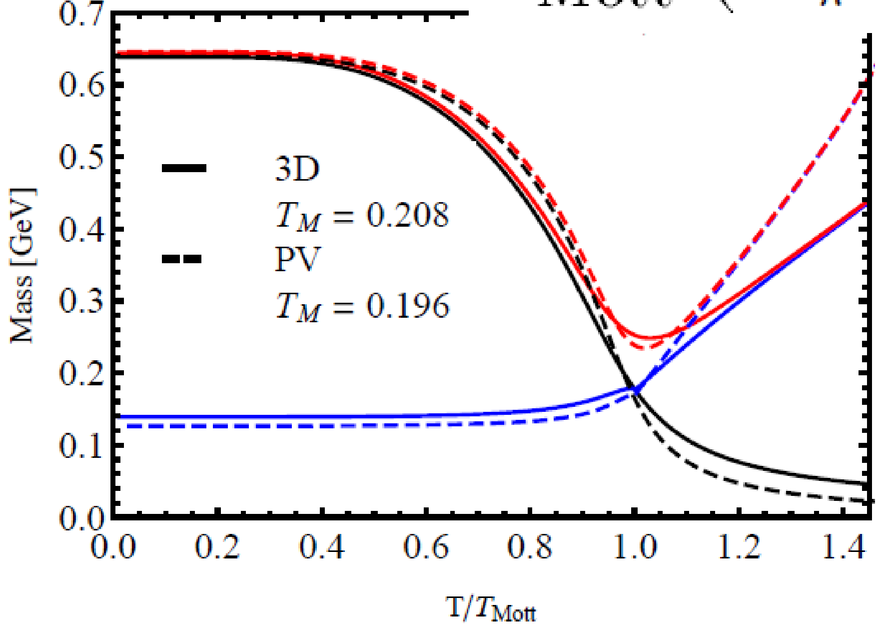

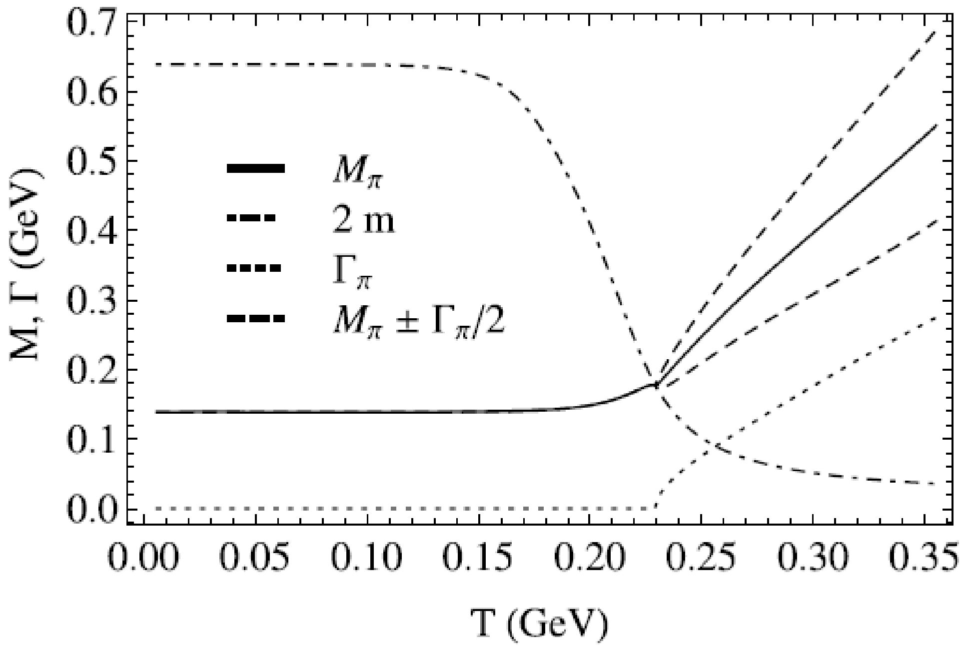

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$$I_2(k^2) = \mathrm{Re} I_2(k^2) + i\ \mathrm{Im} I_2(k^2)$$

CLASSIFICATION OF PHASE TRANSITIONS

order parameter ψ:

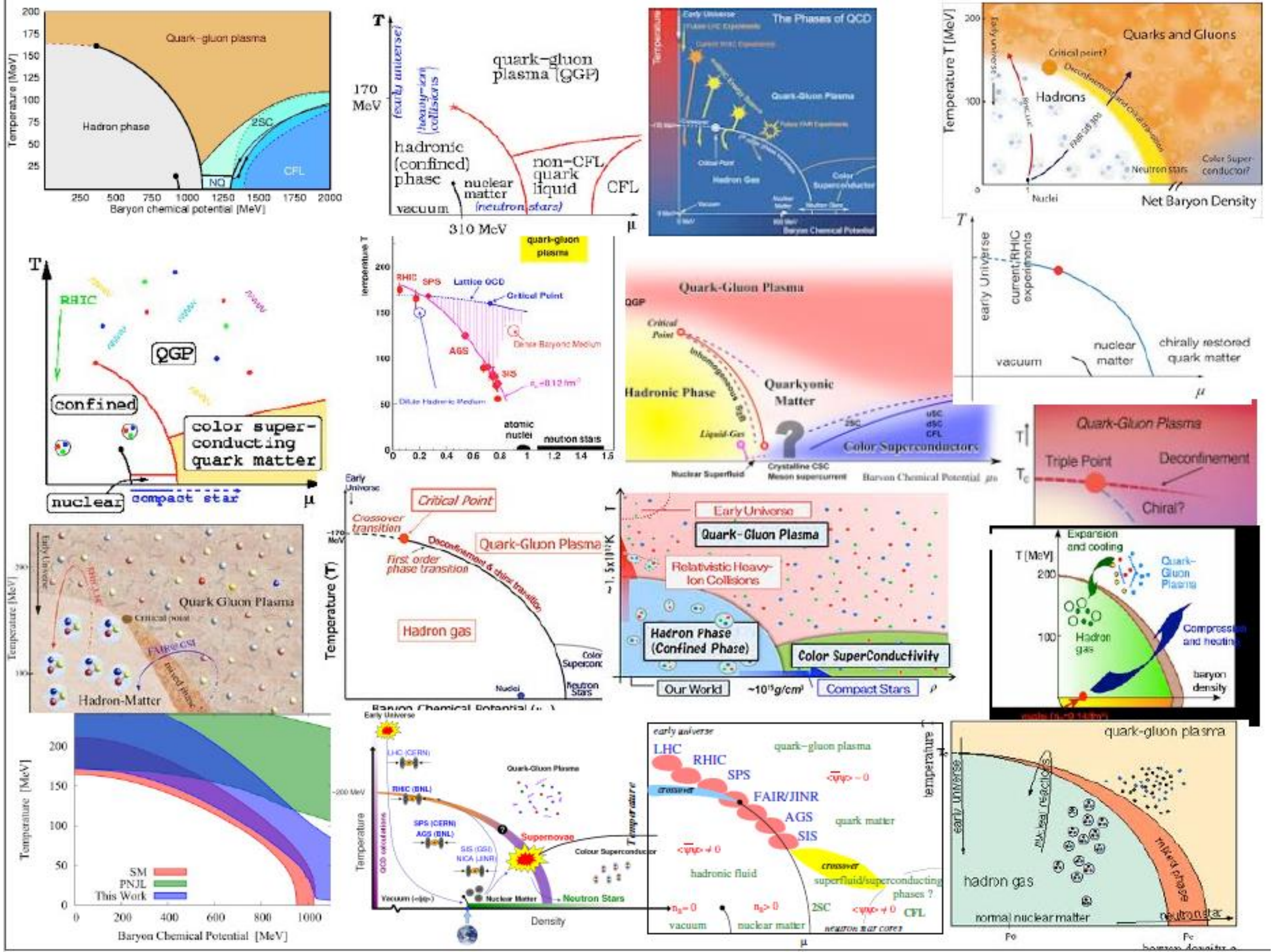
is a (macroscopic) quantity that changes characteristically at a phase transition, e.g. M/M_0 (magnetization) for paramagnetic <-> ferromagnetic or $(n-n_c)/n_c$ for liquid <-> gas; it is zero in one phase (usually above the critical point), and non-zero in the other.

order of phase transitions - modern classification:

- ψ changes discontinuously: first-order
- ψ changes continuously: second-order

order of phase transitions - Ehrenfest classification:

- $\partial\psi/\partial y$ changes discontinuously: first-order
- $\partial\psi/\partial y$ is continuous but $\partial^2 \psi/\partial y^2$ discontinuous: second-order


Quark-gluon plasma
Hadron phase
2SC
NQ
CFL
Temperature [MeV]
Baryon chemical potential [MeV]
quark–gluon plasma (QGP)
early universe
heavy-ion collisions
170 MeV
hadronic (confined) phase
nuclear matter
non-CFL quark liquid
CFL
vacuum
neutron stars
310 MeV
The Phases of QCD
Early Universe
Quark-Gluon Plasma
Critical Point
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Color Superconductor
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T
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SIS
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Dilute Hadronic Medium
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Liquid-Gas
Nuclear Superfluid
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Meson supercurrent
Baryon Chemical Potential
early Universe
current RHIC experiments
vacuum
nuclear matter
chirally restored quark matter
μ
Quark-Gluon Plasma
Triple Point
Deconfinement
Chiral?
T
T_c
Early Universe
Critical Point
Crossover transition
Quark-Gluon Plasma
Deconfinement & chiral transition
First order phase transition
Hadron gas
Temperature (T)
Nuclei
Neutron Stars
Baryon Chemical Potential
Early Universe
Quark Gluon Plasma
Critical point
Hadron-Matter
Temperature [MeV]
Early Universe
Quark-Gluon Plasma
Relativistic Heavy-Ion Collisions
Hadron Phase (Confined Phase)
Color SuperConductivity
Our World
Compact Stars
ρ
Expansion and cooling
T [MeV]
Quark-Gluon Plasma
Hadron gas
Compression and heating
baryon density
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PNJL
This Work
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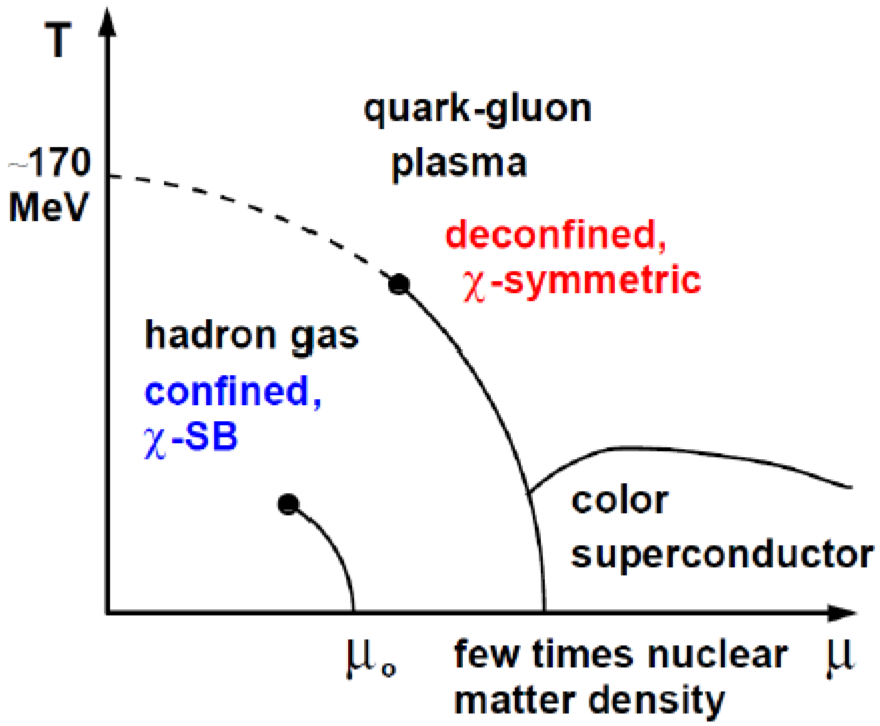
AGS (BNL)
SIS (GSI)
NICA (JINR)
Supernovae
Colour Superconductor
Nuclear Matter
Neutron Stars
Temperature
Density
early universe
LHC
RHIC
SPS
FAIR/JINR
AGS
SIS
quark–gluon plasma
crossover
quark matter
hadronic fluid
superfluid/superconducting phases ?
2SC
CFL
vacuum
nuclear matter
neutron star cores
μ
Temperature
quark-gluon plasma
early universe
nuclear reactions
mixed phase
hadron gas
normal nuclear matter
neutron star

THE QCD PHASE DIAGRAM FROM THEORY - STANDARD PHASE DIAGRAM

hadron gas:
- moderate temperatures and densities
- quarks and gluons are confined
- chiral symmetry is spontaneously broken

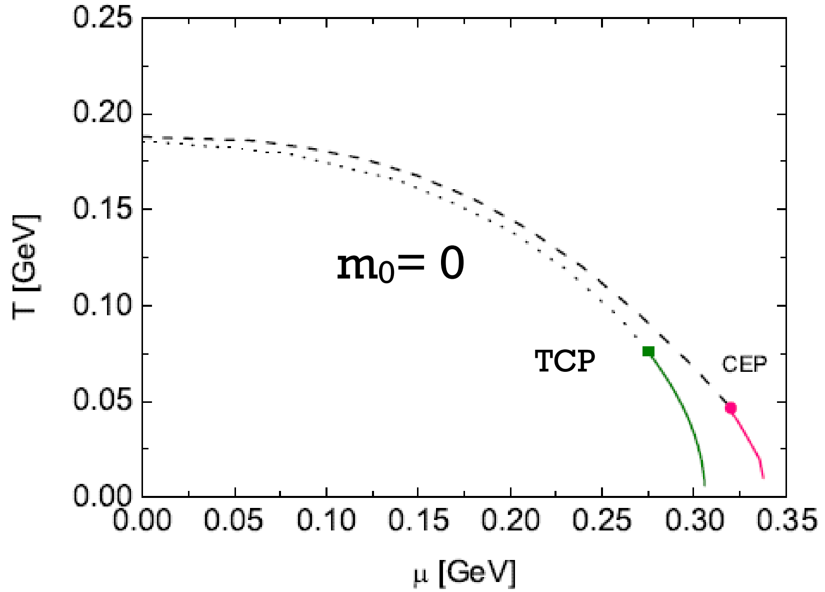
quark-gluon plasma:
- very high temperatures and densities
- deconfined quarks and gluons
- chiral symmetry is restored

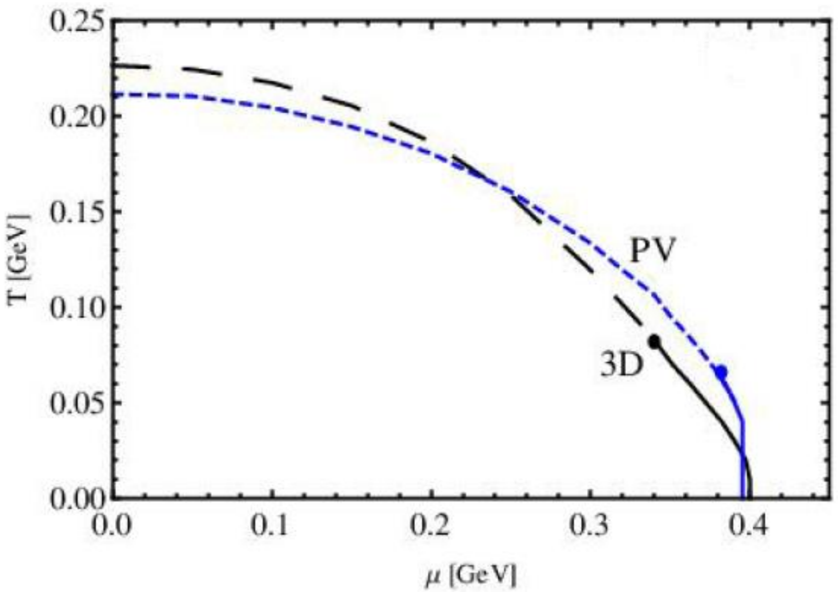
color superconductor:
- T < 100 MeV and very high densities
- quarks form bosonic pairs in analogy to BCS theory



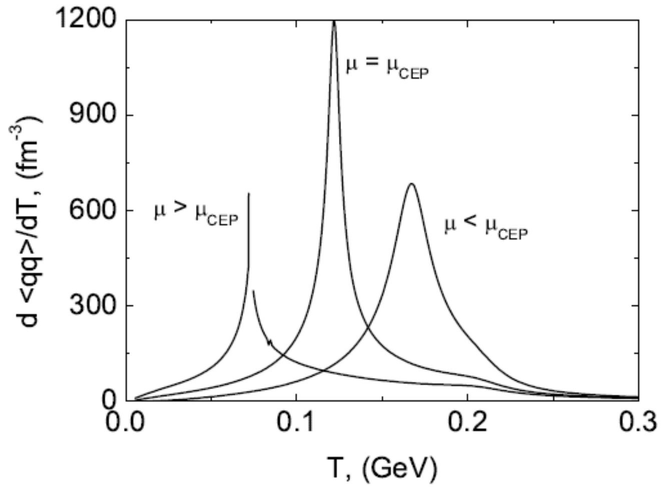
[CBM physics book (2011)]

$$\left[\frac{1}{4G}+\frac{\partial\Omega_q}{\partial m^2}\right]_{m=0}=0$$

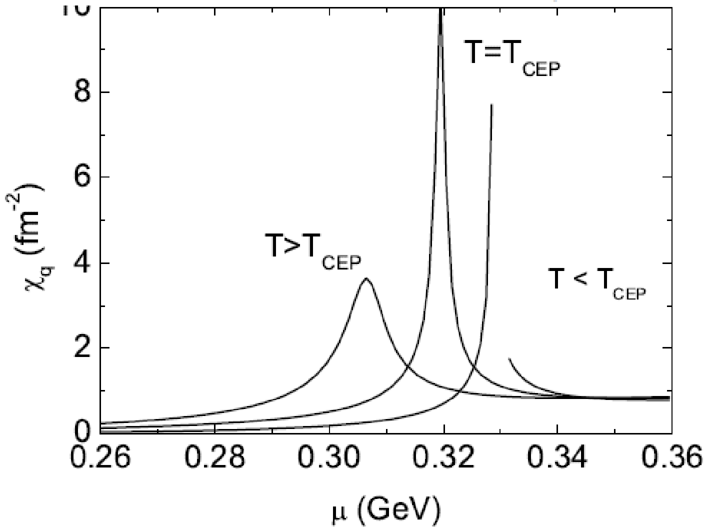




$$T_c \text{ - crossover line } \max \frac{\partial < q\bar{q} >}{\partial T}$$



$$1^{st}\text{-order transition - } \max \frac{\partial^2\Omega}{\partial\mu^2}\Big|_{T=\text{const}}$$



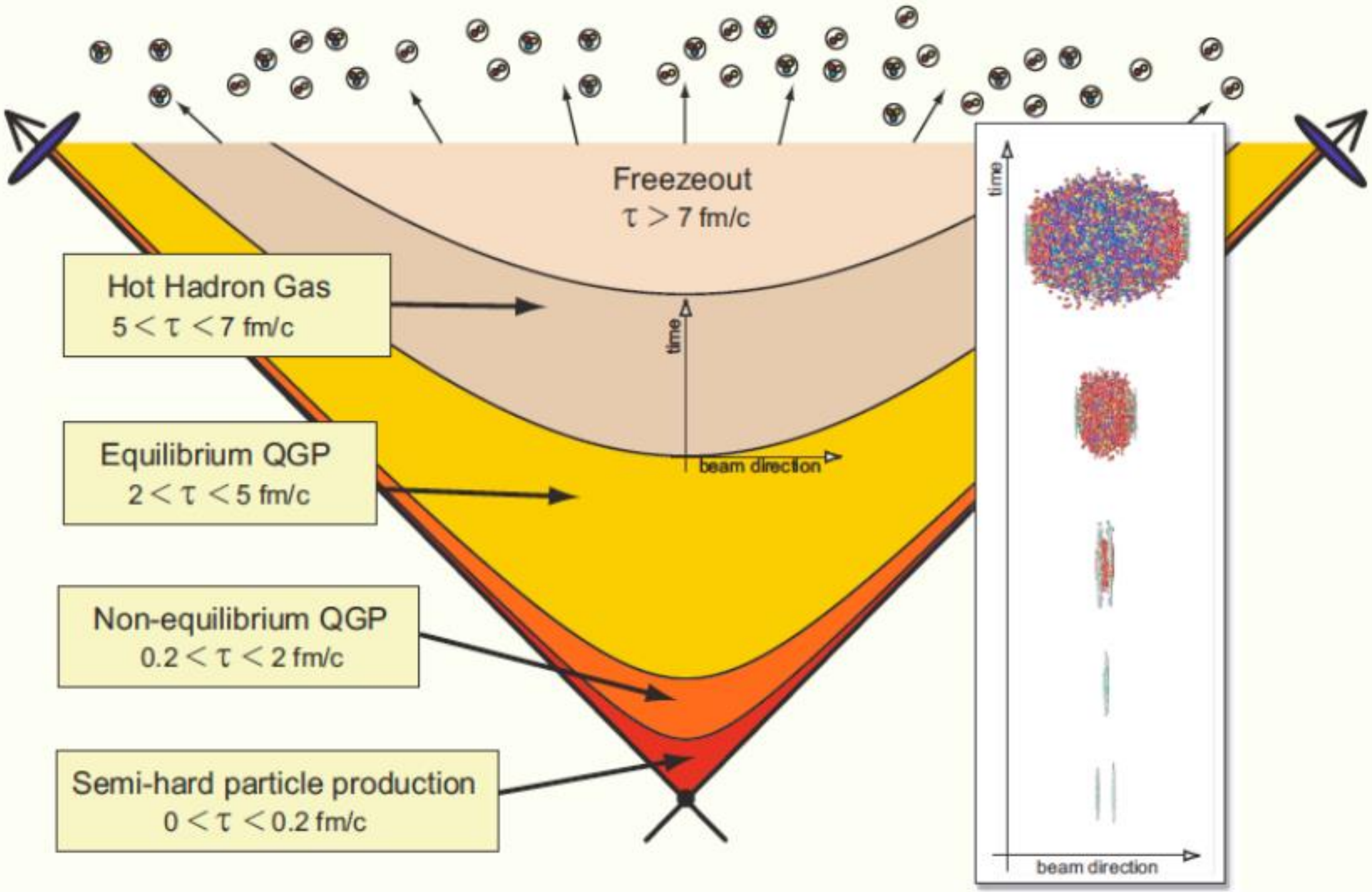
SINCE WE HAVE QCD, WHY SHOULD WE CARE ABOUT A MODEL?

Often NJL-model calculations are much simpler than QCD calculations. But can we trust the results?

- non-renormalizable → results depend on cutoff parameters and the employed regularization scheme, and there are usually cutoff artifacts
- no confinement
- many possible interaction terms, many parameters
- temperature and density dependence of the effective couplings unknown and usually neglected

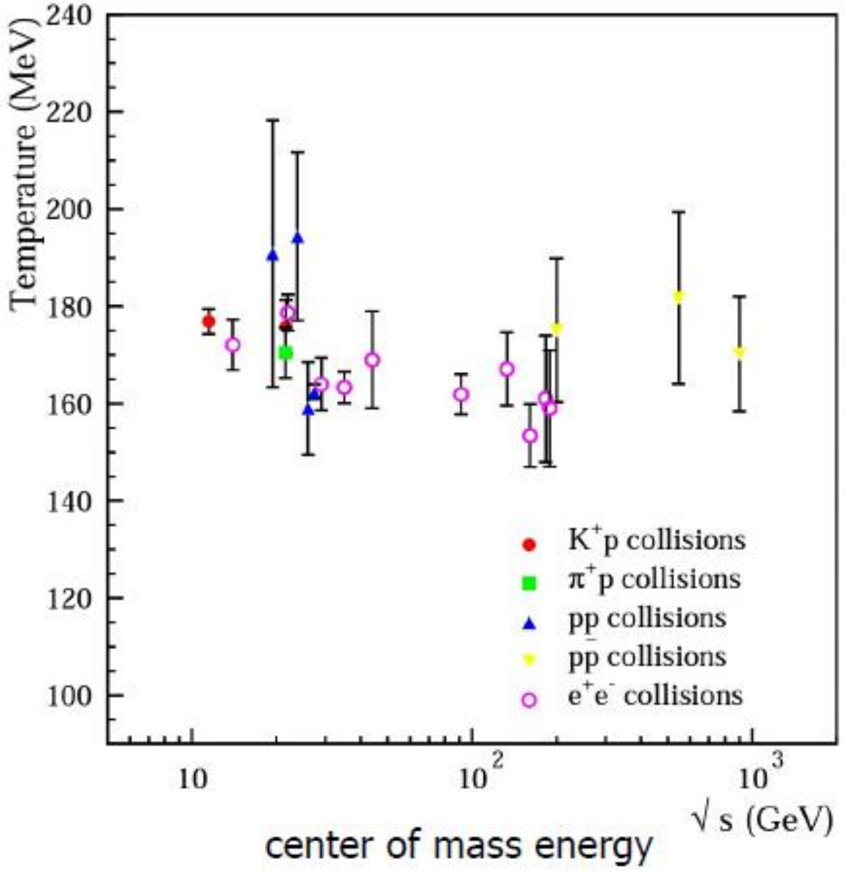
So what can we learn from the NJL model about dense matter? The NJL model is a nice theoretical tool to get new ideas and insights about the QCD phase diagram and the dense-matter equation of state, but it shouldn't be trusted quantitatively

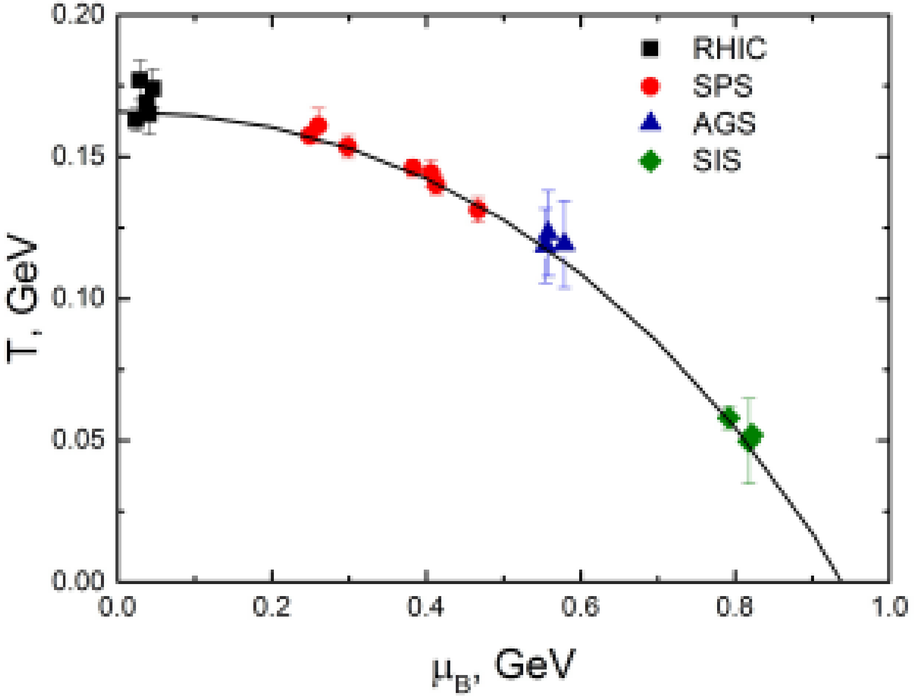
Heavy-ion collision timescales and "epochs" @ RHIC



*1 fm/c $\simeq 3 \times 10^{-24}$ seconds

THE QCD PHASE DIAGRAM FROM EXPERIMENT





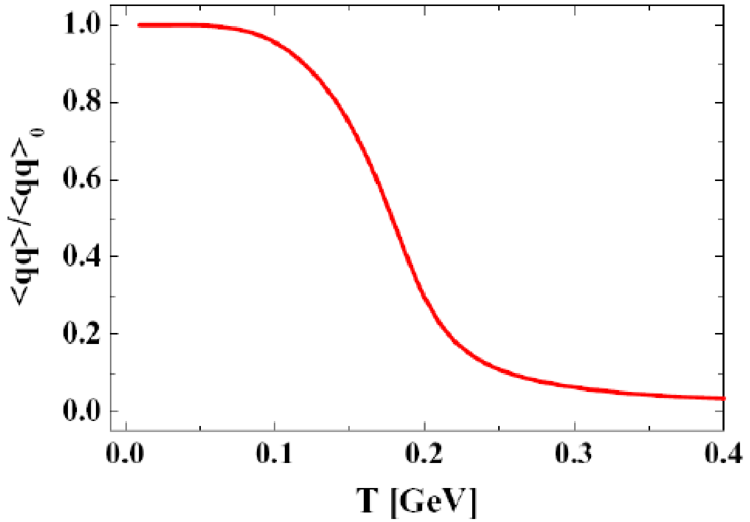
From fitting the data on hadron multiplicities within statistical model, Becattini arXiv:0901.3643 [hep-ph]

The QCD Phase Diagram from Theory II

Phases of QCD matter

Massive quark <-> massless quarks

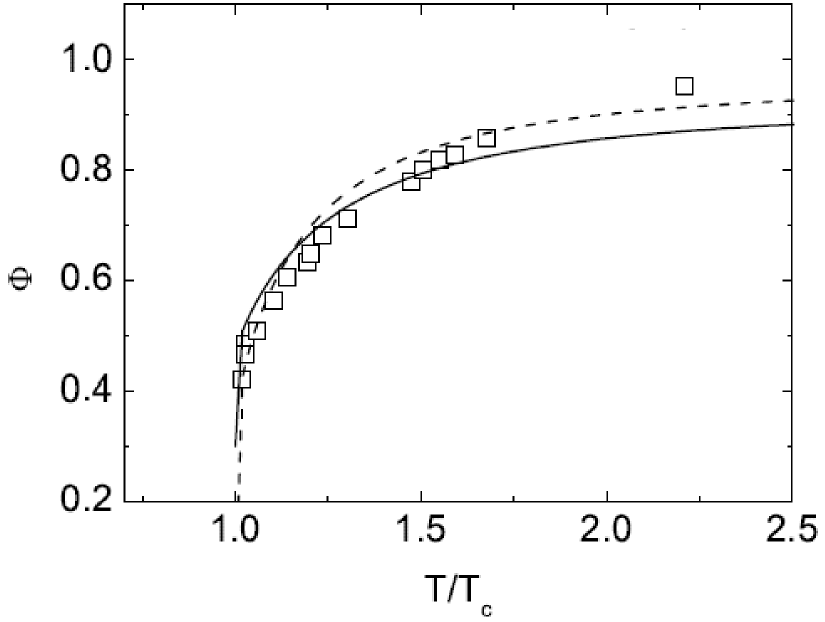
The order parameter – quark condensate σ
Chiral symmetry broken σ≠ 0
Chiral symmetry restoration
σ→ 0



Confined quarks <-> deconfined quarks

The order parameter – Polyakov loop field Φ:

Confinement Φ ~ 0
Deconfinement Φ -> 1



The QCD Phase Diagram from Theory II

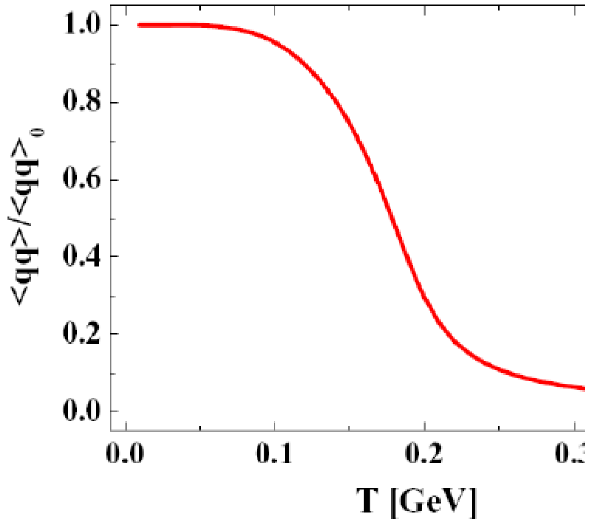
Massive quark <-> ma

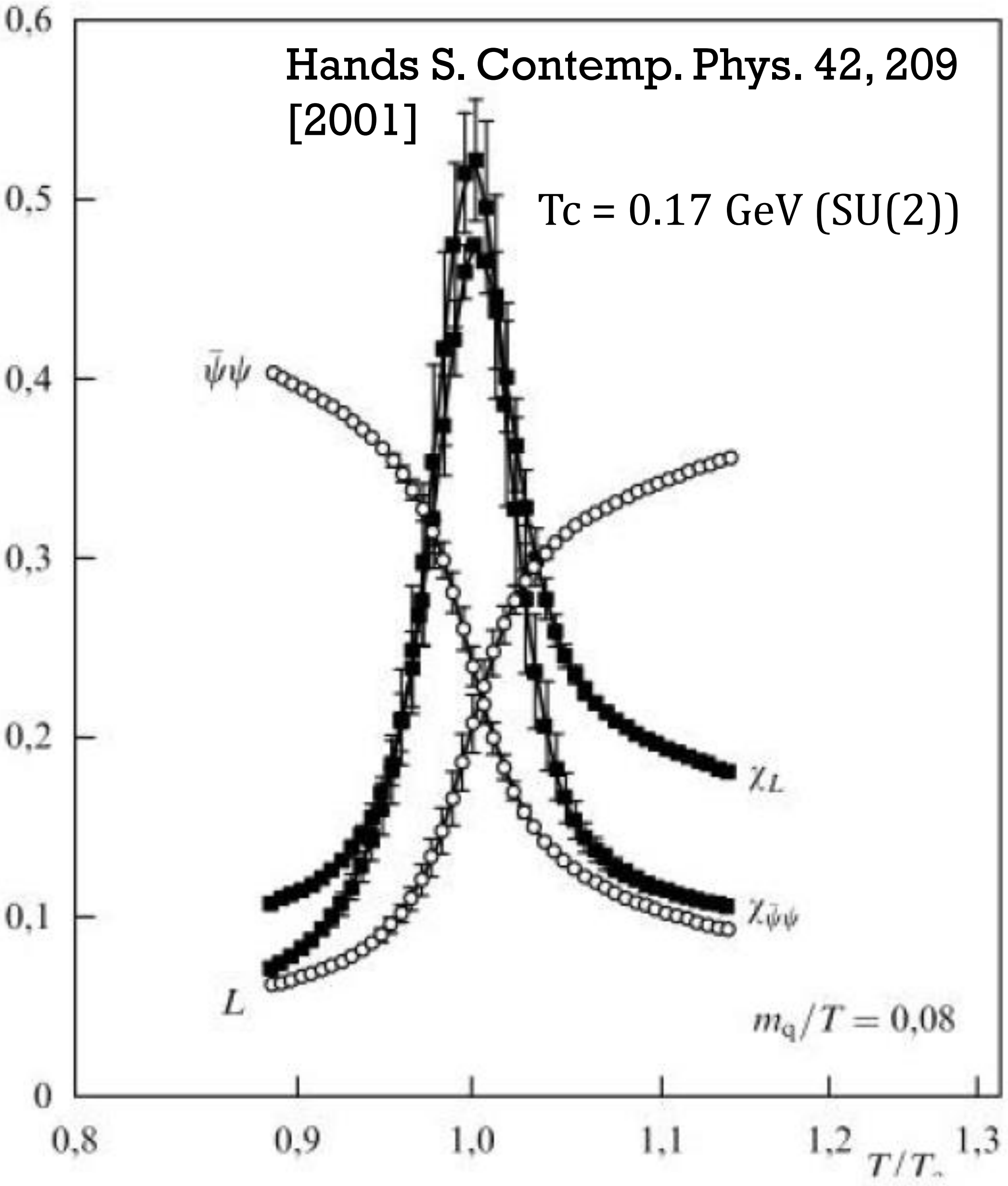
s <-> deconfined quarks

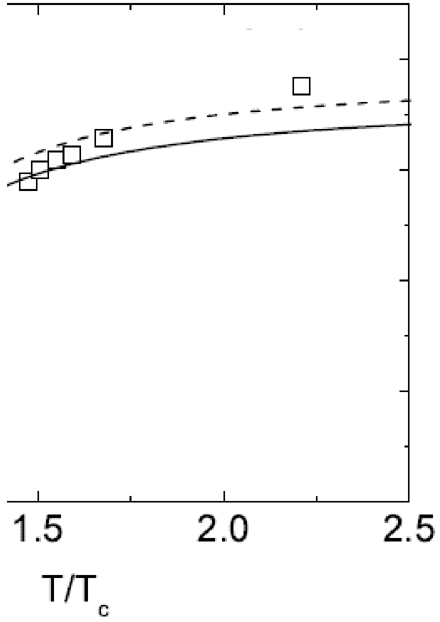
The order parameter
condensate σ
Chiral symmetry brol
Chiral symmetry rest
σ→ 0

er – Polyakov loop field Φ:

-> 1







THE POLYAKOV-LOOP EXTENDED NAMBU-JONA-LASINIO MODEL

$$\mathcal{L}_{\mathrm{PNJL}} = \bar{\mathrm{q}}\left(\mathrm{i}\gamma_\mu \mathrm{D}^\mu - \hat{\mathrm{m}}_0 - \gamma_0 \mu\right)\mathrm{q} + \mathrm{G_s}\left[(\bar{\mathrm{q}}\mathrm{q})^2 + (\bar{\mathrm{q}}\mathrm{i}\gamma_5\vec{\tau}\mathrm{q})^2\right] - \mathcal{U}\left(\Phi, \bar{\Phi}; \mathrm{T}\right)$$

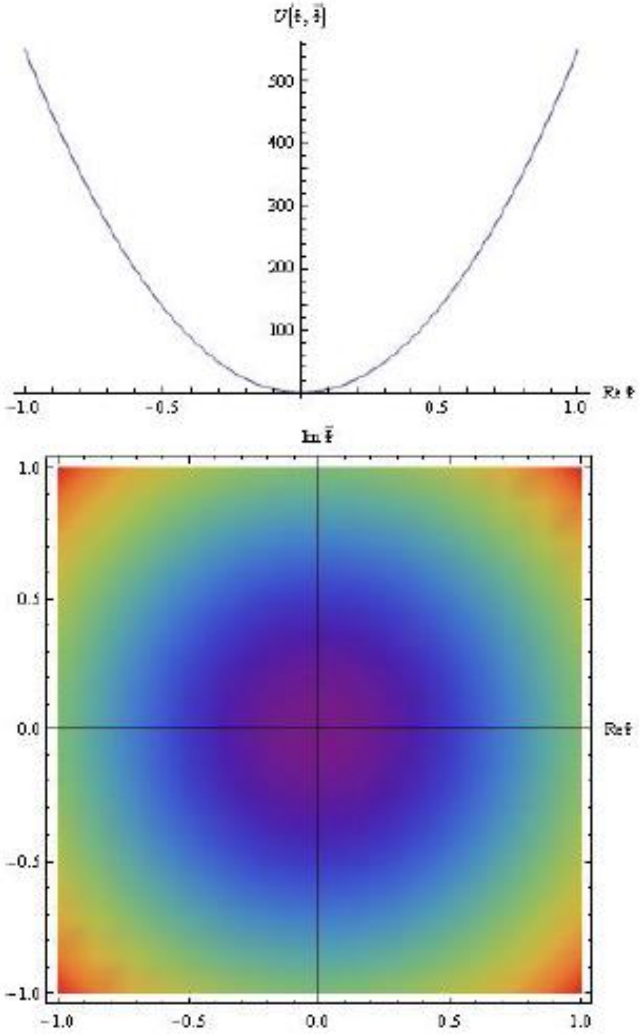
$\mathrm{q} = (\mathrm{q_u}, \mathrm{q_d})^{\mathrm{T}}$ quark fields, $\hat{\mathrm{m}}_0 = \mathrm{diag}\,(\mathrm{m_u^0}, \mathrm{m_d^0})$-current quark masses,
$\mathrm{m_u^0} = \mathrm{m_d^0} = \mathrm{m_0}$
$\mathrm{D}^\mu = \partial^\mu - \mathrm{i}\mathrm{A}^\mu$ - covariant derivative
$\mathrm{A}^\mu(\mathrm{x}) = \mathrm{g}\mathcal{A}_\mathrm{a}^\mu \frac{\lambda_\mathrm{a}}{2}$, $\mathcal{A}_\mathrm{a}^\mu$ the gauge field SU(3),
$\mathrm{A}^\mu = \delta_0^\mu \mathrm{A}^0 = -\mathrm{i}\delta_4^\mu \mathrm{A}_4$,
λ_a - Gell-Mann matrices,
$\mathrm{G_s}$ - coupling strength.

The Plyakov field Φ is determined as: $\Phi[\mathrm{A}] = \frac{1}{\mathrm{N_c}}\mathrm{Tr_c}\mathrm{L}(\vec{\mathrm{x}})$, $\mathrm{L}(\vec{\mathrm{x}}) = \mathcal{P}\exp\left[\mathrm{i}\int_0^\beta \mathrm{d}\tau \mathrm{A}_4(\vec{\mathrm{x}}, \tau)\right]$, $\beta = 1/\mathrm{T}$

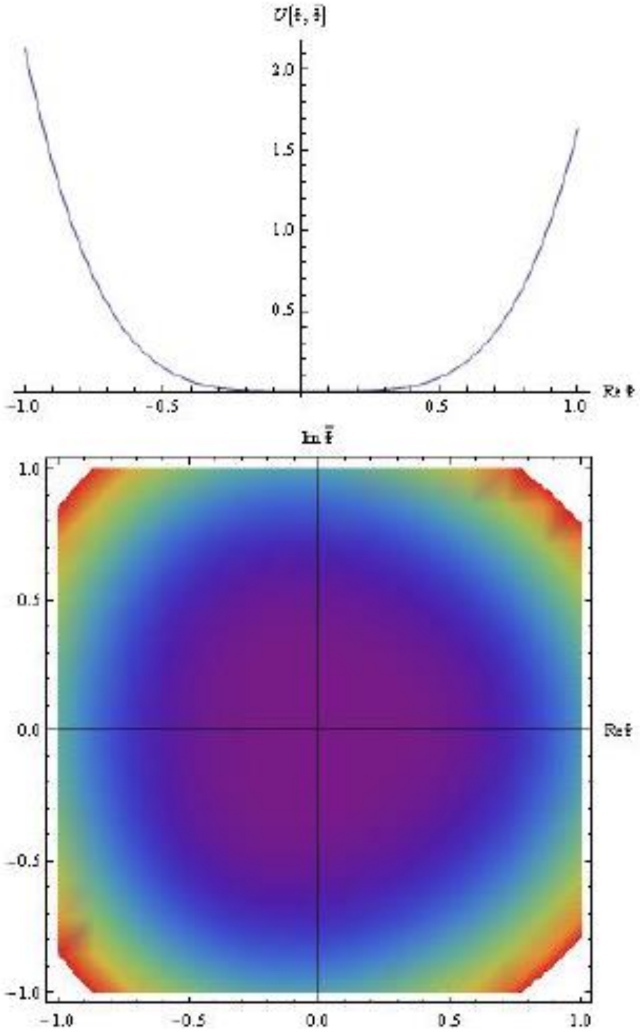
$\langle \mathrm{l}(\vec{\mathrm{x}})\rangle = \mathrm{e}^{-\beta\Delta\mathrm{F_Q}(\vec{\mathrm{x}})}$.

C. Ratti, M. A. Thaler, and W. Weise, PRD 73, 014019 2006.

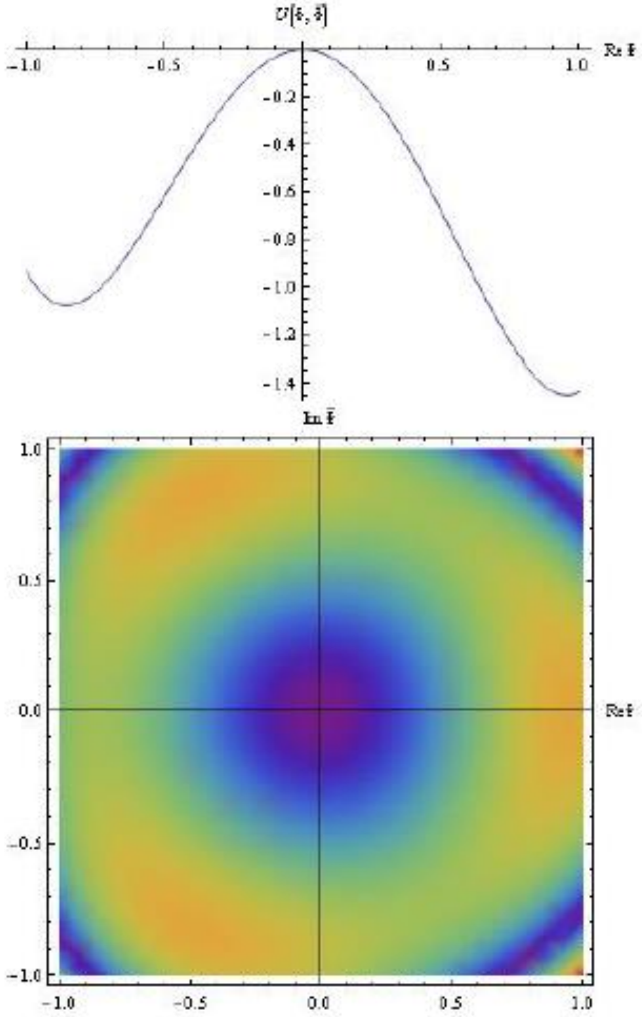
$T = 0.05$ GeV



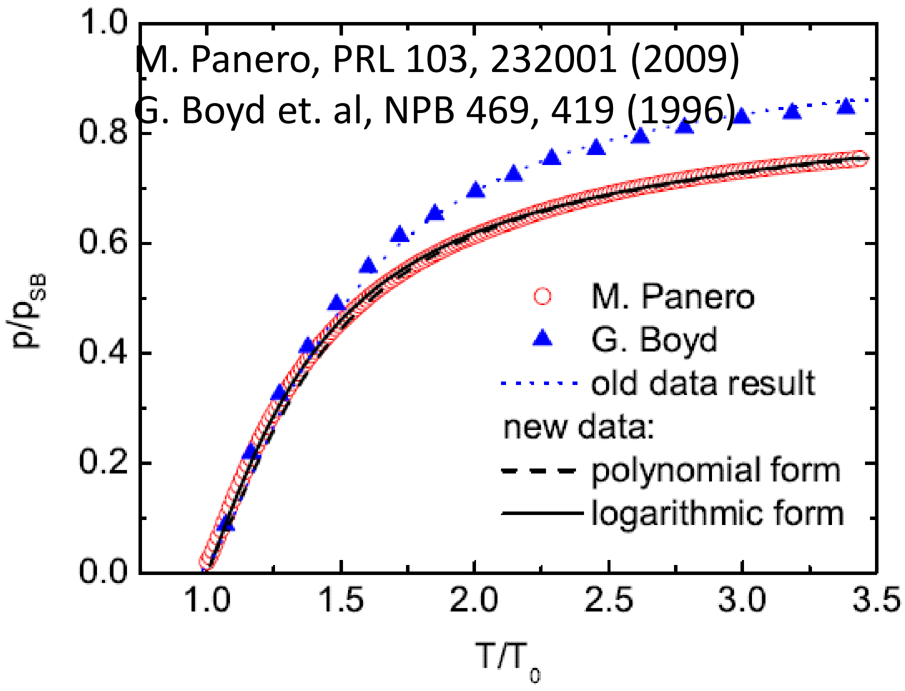
$T = T_0 = 0.27$ GeV



$T = 3T_0 = 0.81$ GeV



THE EFFECTIVE POTENTIAL PARAMETRIZATION



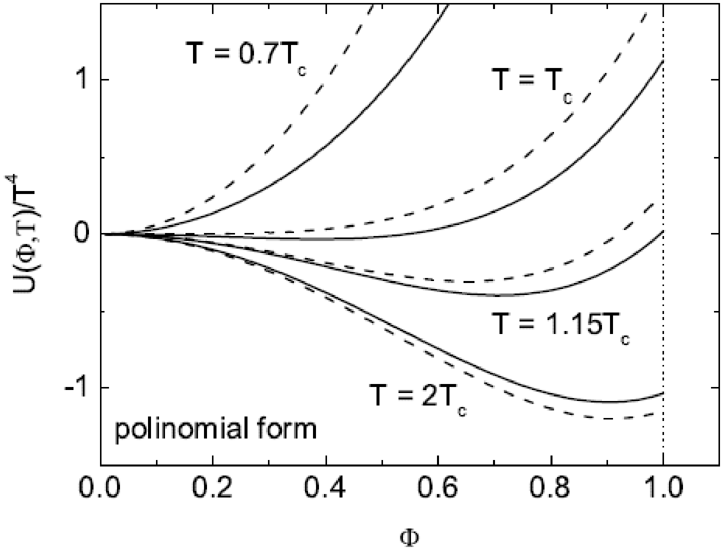
$$\frac{\mathcal{U}_{\mathrm{RTW05}}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,$$

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};\mathrm{T}\right)}{\mathrm{T}^4} = -\frac{1}{2}\mathrm{a}\left(\mathrm{T}\right)\bar{\Phi}\Phi + \mathrm{b}\left(\mathrm{T}\right)\ln\left[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2\right]$$

$$\mathrm{a}\left(\mathrm{T}\right) = \tilde{\mathrm{a}}_0 + \tilde{\mathrm{a}}_1\left(\frac{\mathrm{T}_0}{\mathrm{T}}\right) + \tilde{\mathrm{a}}_2\left(\frac{\mathrm{T}_0}{\mathrm{T}}\right)^2, \mathrm{b}\left(\mathrm{T}\right) = \tilde{\mathrm{b}}_3\left(\frac{\mathrm{T}_0}{\mathrm{T}}\right)^3 .$$

- $\Phi \rightarrow 1$, $\mathrm{p}/\mathrm{T}^4 \rightarrow 1.75$, where $\mathrm{T} \rightarrow \infty$
- $\frac{\partial\mathcal{U}(\Phi,\bar{\Phi},\mathrm{T})}{\partial\Phi}|\mu = 0$ ($\Phi = \bar{\Phi}$ at $\mu = 0$) $\Rightarrow$ the mean square method $\Rightarrow \mathrm{a_i}, \mathrm{b_i}$



THE MEAN FIELD APPROXIMATION

The mean field approximation procedure is almost the same as for NJL model and leads to the partition function:

$$\mathcal{Z}_{\mathrm{MF}} = \exp\left\{\int d^4x \left[-\frac{\sigma'^2_{\mathrm{MF}}}{4G_s}\right] + \mathrm{Tr}\,\ln S^{-1}_{\mathrm{MF}}[m] - \frac{V}{T}\mathcal{U}(\Phi,\bar{\Phi};T)\right\}$$

The grand potential has the form: $\boxed{\Omega_{\mathrm{PNJL}} = \mathcal{U}(\Phi,\bar{\Phi};T) + G_s\langle\bar{q}q\rangle^2 + \Omega_q}$

$$\Omega_q = -2N_cN_f\int\frac{d^3p}{(2\pi)^3}E_{\mathrm{p}} - 2N_fT\int\frac{d^3p}{(2\pi)^3}\mathrm{Tr_c}\left[\ln(1+L^\dagger e^{-\beta(E_p-\mu)}) + \ln(1+Le^{-\beta(E_p+\mu)})\right]$$

with

$$N_\Phi^+(E_p) = \left[1+3\left(\Phi+\bar{\Phi}e^{-\beta E_p^+}\right)e^{-\beta E_p^+} + e^{-3\beta E_p^+}\right]$$

$$N_{\bar{\Phi}}^-(E_p) = \left[1+3\left(\bar{\Phi}+\Phi e^{-\beta E_p^-}\right)e^{-\beta E_p^-} + e^{-3\beta E_p^-}\right]$$

$$\boxed{\frac{\partial\Omega_{\mathrm{MF}}}{\partial<\bar{q}q>} = 0,\quad \frac{\partial\Omega_{\mathrm{MF}}}{\partial\Phi} = 0,\quad \frac{\partial\Omega_{\mathrm{MF}}}{\partial\bar{\Phi}} = 0}$$

GAP EQUATION

The quark propagator now includes the gauge field from covariant derivative in Lagrangian:

$$iS(p) = i\frac{1}{\hat{p} + \gamma_0(-iA_4) + M}$$

Gap equation:

$$m = m_0 - 2G_s \int \frac{dp}{(2\pi)^4} \mathrm{Tr_D Tr_c Tr_f} \left\{ \frac{1}{\hat{p} + \gamma_0(-iA_4) - m} \right\}$$

Making Matsubara summation:

$$m = m_0 + 8N_f G_s m \; iT \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \frac{1}{(\hat{p} + \gamma_0(-iA_4 + \mu))^2 - m^2}$$

$$\boxed{i\mathrm{Tr_c} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E_\mathrm{p}} (1 - f_\phi[E^+] - f_\phi[E^-])}$$

$$\mathrm{Tr_c}\left\{f[E_\mathrm{p} - (-iA_{4cc} + \mu)]\right\} = \sum_{c=1}^{N_c} \frac{1}{e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{4cc}} + 1} =$$

$$\begin{aligned}
= \; & [(e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{422}} + 1)(e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{433}} + 1) \\
+ \; & (e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{411}} + 1)(e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{433}} + 1) \\
+ \; & (e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{411}} + 1)(e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{422}} + 1)] \times \\
\times \; & [(e^{\beta(E_\mathrm{p}-\mu)} e^{i\beta A_{411}} + 1)(e^{(E_\mathrm{p}-\mu)} e^{i\beta A_{422}} + 1)(e^{(E_\mathrm{p}-\mu)} e^{i\beta A_{433}} + 1)]^{-1} = \\
= \; & N_c \frac{\bar{\Phi} e^{-\beta(E_\mathrm{p}-\mu)} + 2\Phi e^{-2\beta(E_\mathrm{p}-\mu)} + e^{-3\beta(E_\mathrm{p}-\mu)}}{1 + 3(\bar{\Phi} + \Phi e^{-\beta(E_\mathrm{p}-\mu)}) e^{-\beta(E_\mathrm{p}-\mu)} + e^{-3\beta(E_\mathrm{p}-\mu)}} = N_c f_\Phi^+(E_\mathrm{p} - \mu).
\end{aligned}$$

PHASE TRANSITION: NJL VS. PNJL