

Quantum Technology for Cyclotron Radiation Detection

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Cyclotron Radiation Detection

The microwave engineering requirements for the cyclotron radiation detection for neutrino mass measurement are focussed in following areas:

- Ultra high energy resolution (optimally $\sim 0.01\text{eV}$)
- Fast response and short dead-time to improve statistics
- High linearity or calibration capability
- The accuracy of measurement of the cyclotron resonant frequency ($\sim 1\text{ppm}$) at 1T magnetic field

Quantum Technology for Cyclotron Radiation Detection

- The overall aim is to develop practical amplifiers operating at and beyond the ***standard quantum limit*** of sensitivity at higher microwave frequency than has currently been developed.
- Cyclotron radiation emission detection electronics will be developed, comprising a low-noise front end at a centre frequency of around 28 GHz and a bandwidth of ~ 2 GHz with 1T field.
- Demand for such amplifiers goes beyond the search for neutrino mass and includes rapidly developing areas of fundamental and applied physics and engineering, e.g. QIP, quantum computing, quantum metrology, radio-astronomy, communications, etc.
- ***This objective requires amplifiers working close to a region of high magnetic field (~ 1 T)***
- ***Critical measurement requirement is the accuracy of measurement of the cyclotron resonant frequency (~ 1 ppm).***

Quantum Technology for Cyclotron Radiation Detection

The overall aim is to develop practical amplifiers operating at and beyond the ***standard quantum limit*** of sensitivity at higher microwave frequency than has currently been developed.

Quantum limited amplifiers based on SQUID and/or Josephson parametric amplifier detector for microwave radiation will address this challenge.

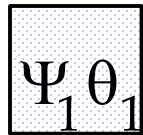
Two microwave amplifiers at quantum limit have been selected:

- NPL Microwave SLUG amplifier (based on SQUIDs)
- Cambridge Josephson Travelling Wave Parametric Amplifier (JTWPA)

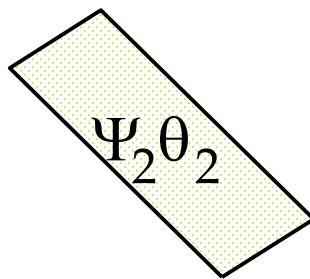
Superconductivity: d.c. Josephson effect

Consider two pieces of superconductor which have three possible relationships:

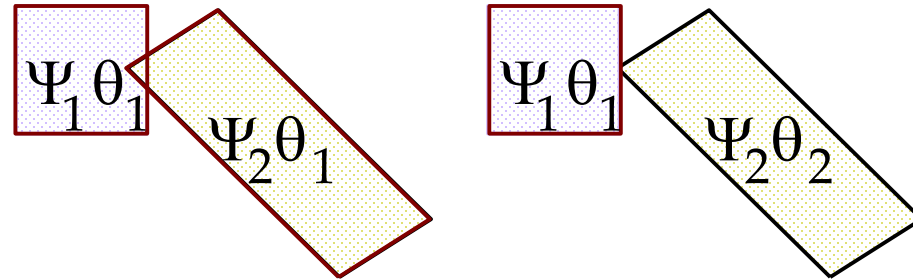
separated



in good contact



weakly linked

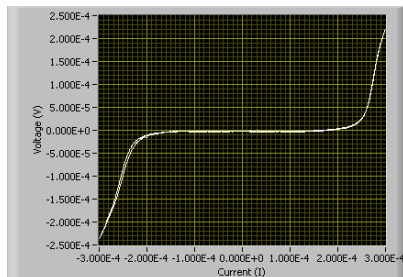


In the weakly coupled situation a small **supercurrent** can flow through the link with a corresponding phase shift:

$$i = i_c \sin(\theta_2 - \theta_1) = i_c \sin(\varphi)$$



Josephson junction



experimental I-V curve

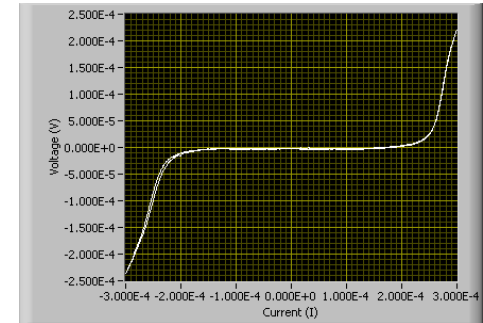
(here i_c is the critical current of the weak link)

The Josephson junction can be seen as a parametric inductor, variable as a function of φ which can be adjusted by the current through the junction.

Superconductivity: a.c. Josephson effect

What happens when a *d.c. external current* $i > i_c$ is driven through the junction?

- A *direct voltage* V appears across the junction (as if it were a resistance)

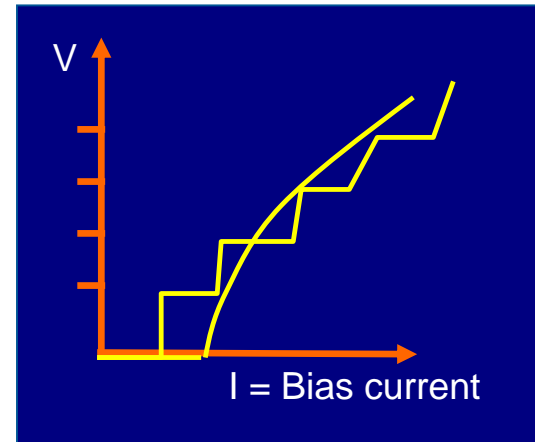


- But also an *oscillating supercurrent* flows with a frequency f given by:

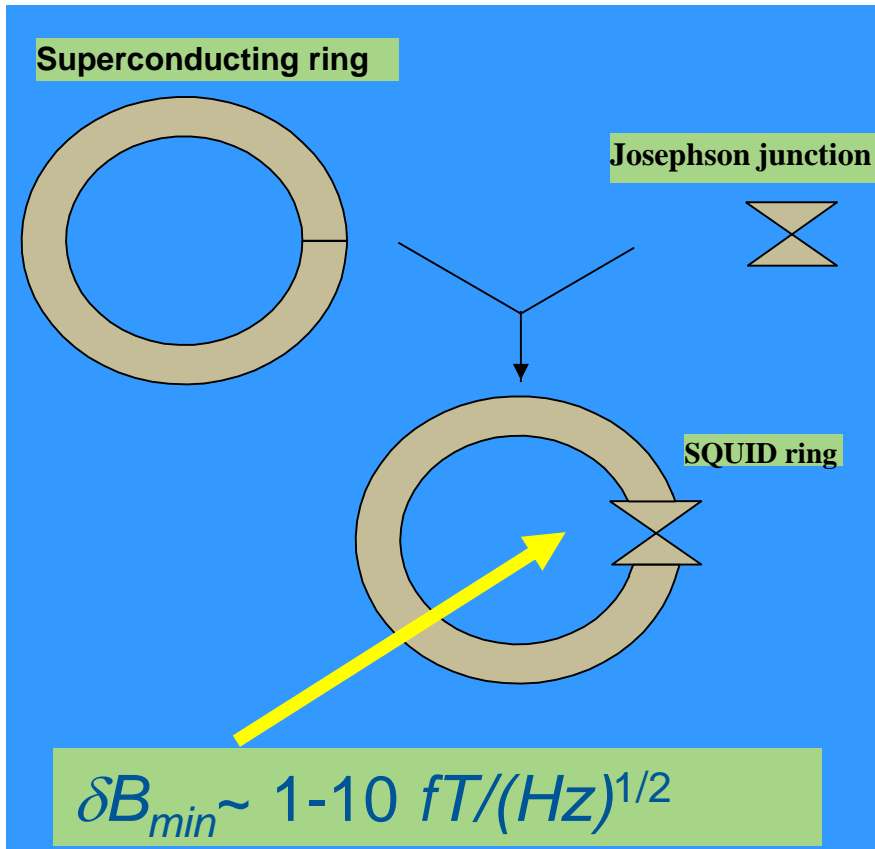
$$f = \frac{2e}{h} V$$

This is the *a.c Josephson effect*

- basis for the Josephson voltage standard
- voltage tunable THz oscillator and detector



SQUID: Superconducting Quantum Interference Device



SQUIDs have been called ‘the most sensitive measuring devices for any physical parameter’

A SQUID combines:-

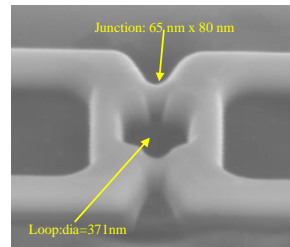
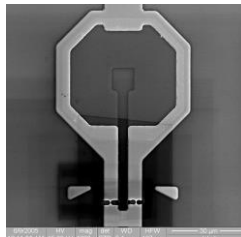
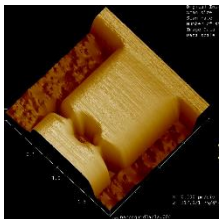
- ❑ *Flux Quantisation*
- ❑ *Josephson junction*

The maximum response frequency of a SQUID is set by the *Superconducting Energy Gap Δ*

$$2\Delta \sim 4k_B T_C$$
$$f \sim 2\Delta/h \sim 1-10 \text{ THz}$$

allowing very small changes in magnetic field to be measured

$$\delta B \sim 1-10 \text{ fT} / \text{Hz}^{1/2}$$



Quantum measurement with SQUIDs

SQUID: Superconducting QUantum Interference Device

SQUIDs have been called ‘the most sensitive measuring devices for any physical parameter’

This may be too good to be true....(e.g. alkali-metal magnetometer...)

But SQUIDs are certainly the most sensitive measuring devices for a wide range of physical parameters:

Magnetic Flux

Flux density

Current

Voltage

Magnetisation

Magnetic Resonance

Temperature

Energy

Quantum Limits & QIP

Displacement

SQUID evolution:

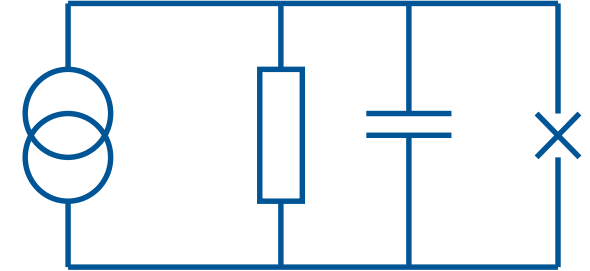
- 1) **SQUIDs for Quantum computing (Google, IBM)**
- 2) **SQUIDs at the nanoscale and high frequency**

Energy Sensitivity of SQUIDs

In classical regime thermal noise in a SQUID limits energy sensitivity, due to **voltage and current noise** in the shunt resistances of the Josephson junctions.

The limiting flux noise spectral density S_ϕ may be written:

$$\epsilon_n = \frac{\langle S_\phi^2 \rangle}{2L} = 16k_B T (LC)^{1/2}$$



where ϵ_n is **minimum detectable energy change**, **C** and **L** are the capacitance and inductance of the SQUID and **T** is temperature.

So ϵ_n can be improved by **reducing C or L** (as well as the more obvious **T**). **That means nanobridges as JJ and NanoSQUIDs !!**

The Josephson junctions are also characterised by a plasma frequency ω_p where

$$\omega_p = \frac{1}{\sqrt{\Phi_0 C / i_c}}$$

At low temperatures if

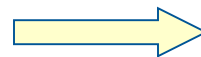
$$\hbar\omega_p \geq k_B T$$

this classical treatment breaks down and the back reaction noise arising from circulating noise currents in the SQUID loop must be taken into account. In this case it is found that

$$\mathcal{E}_n \geq \frac{\hbar}{2}$$

(agrees with the ***Uncertainty Principle!***)

As SQUID dimensions reduce towards nano-scale the energy sensitivity improves to Quantum limit



SQUID are evolving:
SQUIDs at the nanoscale

Approach to nanoscale-SQUIDs

NPL has pioneered the development of nanoscale SQUIDs (NanoSQUIDs)

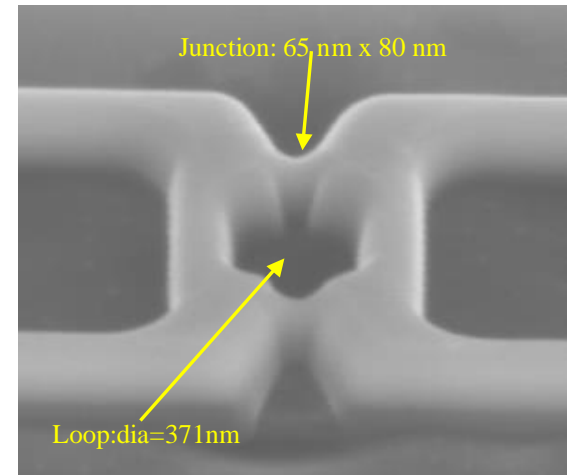
Our SQUID fabrication (based on nanobridge junctions) has high current density j_c and low capacitance C .

For *our high frequency requirements* these properties are also needed.

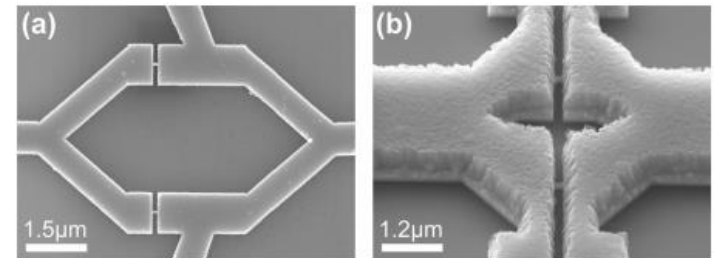
We have two fabrication methods:

- ❑ FIB (focussed ion beam) milling
- ❑ e-beam direct write lithography

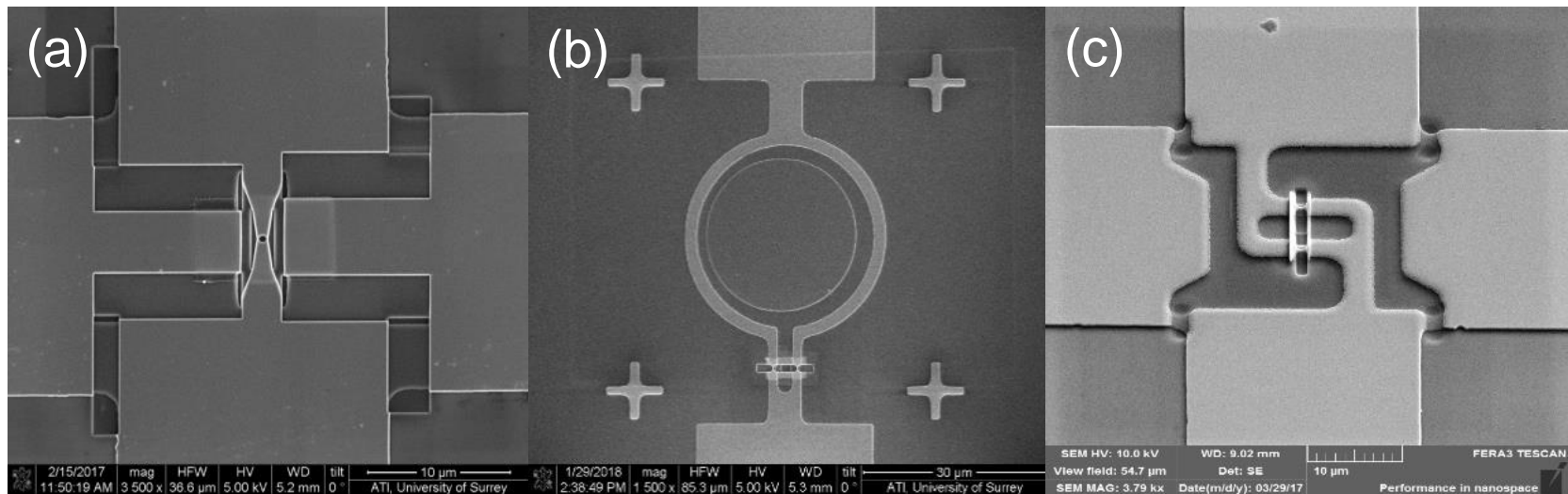
Nanometre sized FIB fabrication



E-beam: nanoSQUIDS



FIB Fabrication SQUIDs and structures

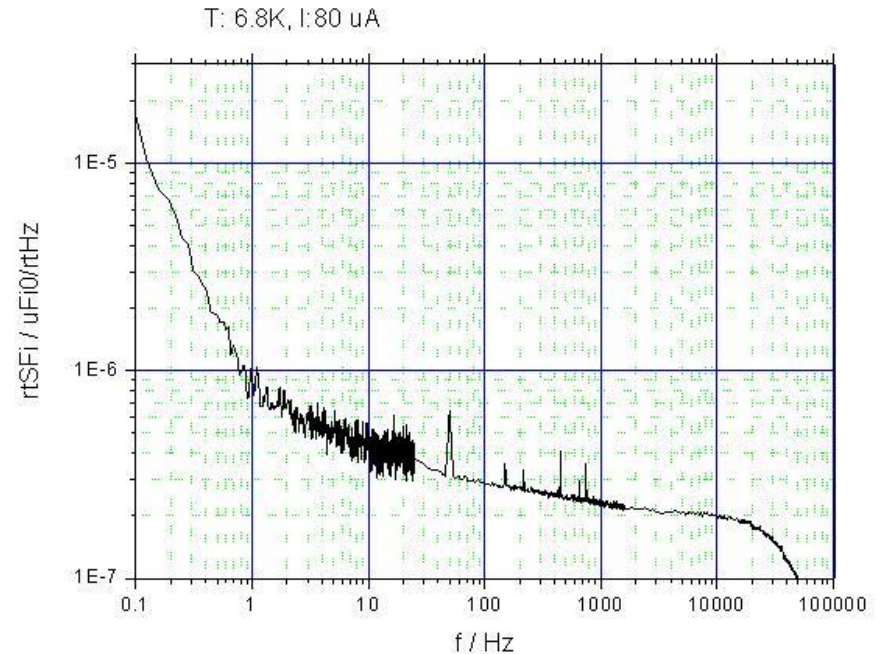


SEM images of three SQUID-based devices fabricated from single layer Nb films (200nm thick)

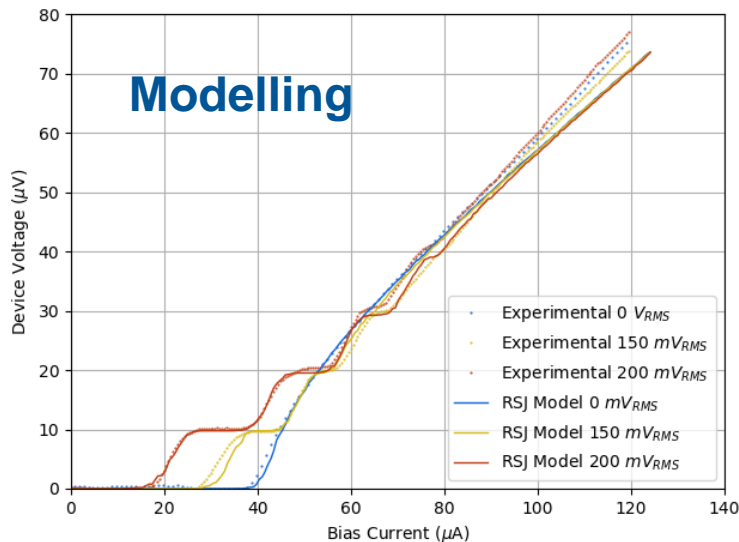
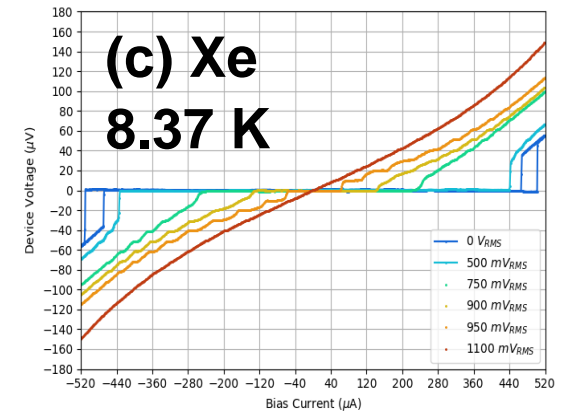
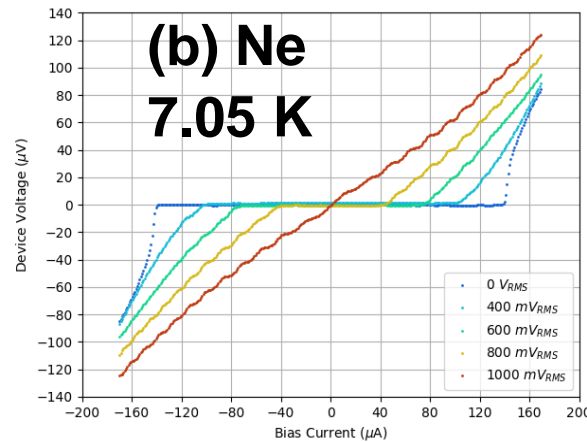
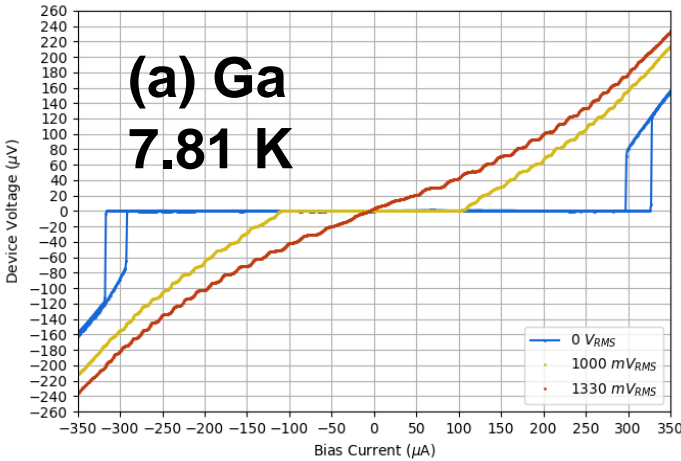
- a) nanoSQUID for single spin detection fabricated with Ga FIB
- b) ISTED SQUID based single photon based detector fabricated using Ne FIB
- c) slotted nanoSQUID design for NEMS detection fabricated with Xe FIB.

- Measured flux noise spectral density, expressed in units of the flux quantum Φ_0 , as a function of frequency from 0.1Hz to 100kHz.
- Note there is a region at low frequency where the noise spectrum has a $1/f$ form.
- Above 1 Hz there is a much weaker frequency dependence.
- Even at 1Hz the spectral density is as low as $0.8 \mu\Phi_0/\text{Hz}^{1/2}$
- *In the white noise region around 1kHz the flux noise is as low as $0.2 \mu\Phi_0/\text{Hz}^{1/2}$.*
- The frequency roll-off at higher frequencies represents the flux-locked loop bandwidth of the measurements.

Flux noise spectral density at $T=6.8\text{K}$, bias current $80 \mu\text{A}$.



This is lowest reported NanoSQUIDs noise above 4K.

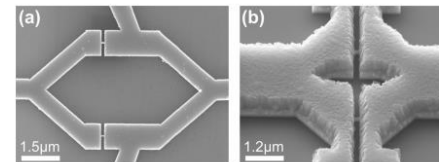
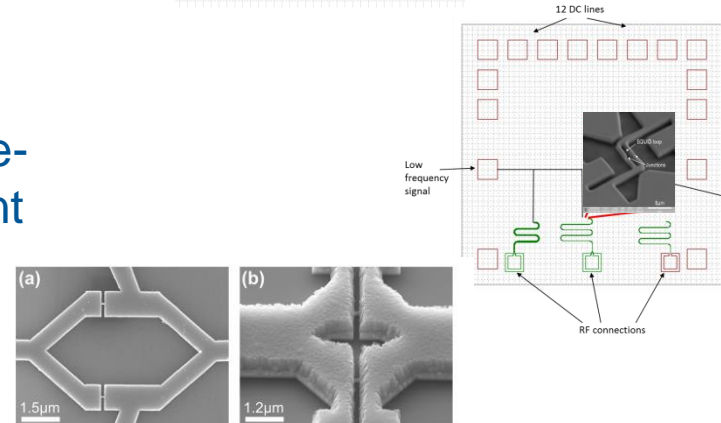
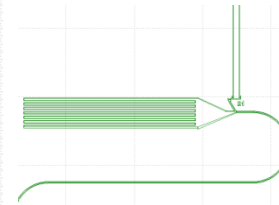
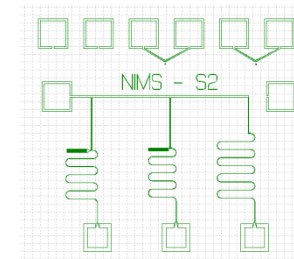
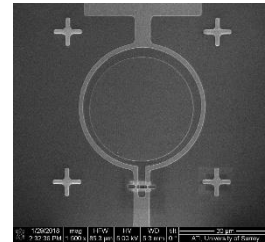
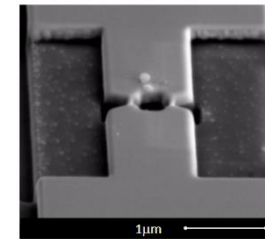
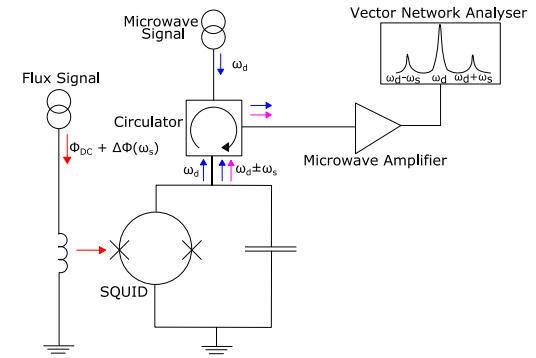


- As well as low-noise nanobridges we also require that they respond at high frequencies.
- We have measured the microwave response of a variety of FIB junctions by irradiating them and measuring the induced Shapiro steps.
- Even at operating temperatures of > 4K we see induced steps showing that there are internal Josephson currents up to at least 100GHz.

$$I_{dc} + I_{rf} \sin(2\pi ft) = I_c \sin(\delta) + \frac{\Phi_0}{2\pi R} \frac{d\delta}{dt}$$

Microwave SQUID technology

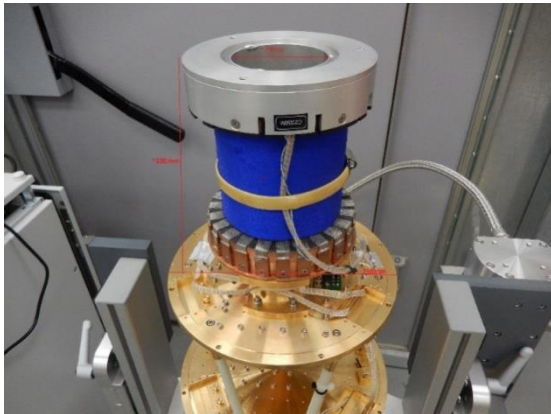
- NPL's main contribution to the project would be development of beyond-state-of-art microwave SQUIDs amplifiers.
- NPL's current SQUID technologies using nanobridge junctions have shown some of the best low noise flux sensitivity.
- Even better sensitivities are expected at microwave frequencies using microwave resonators with cryogenic microwave amplifiers and circulators.
- The nanobridge SQUIDs operate at around 4K (Nb-based) or millikelvin temperatures (Au-Ti based)
- The devices for mK have been fabricated using e-beam nanobridges (in collaboration with UCL – joint Ph.D student using UCL fab facilities)



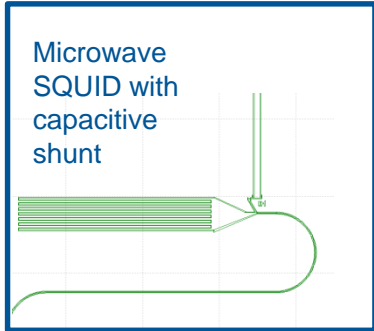
Existing Microwave SQUID System

We are developing this microwave SQUID system at NPL for single spin detection for Quantum Technologies applications.

5T magnet, 100mm bore, HTS leads

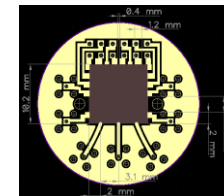
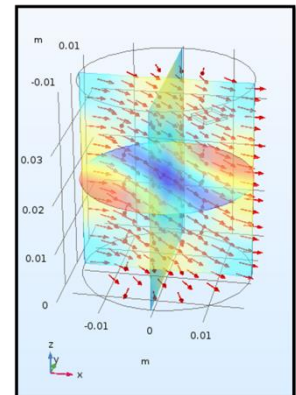
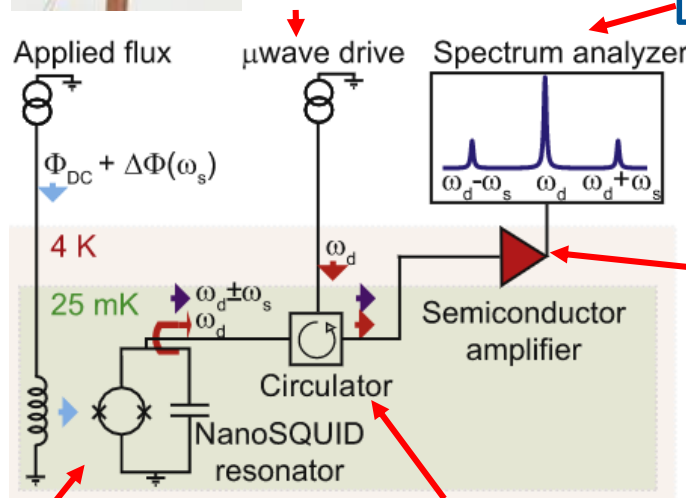


Low-vibration Entropy pulse-tube cooler base temp ~2.5 K



High Q OFHC Cu cavity TM010 6 GHz

Microwave synthesised source to 40GHz,

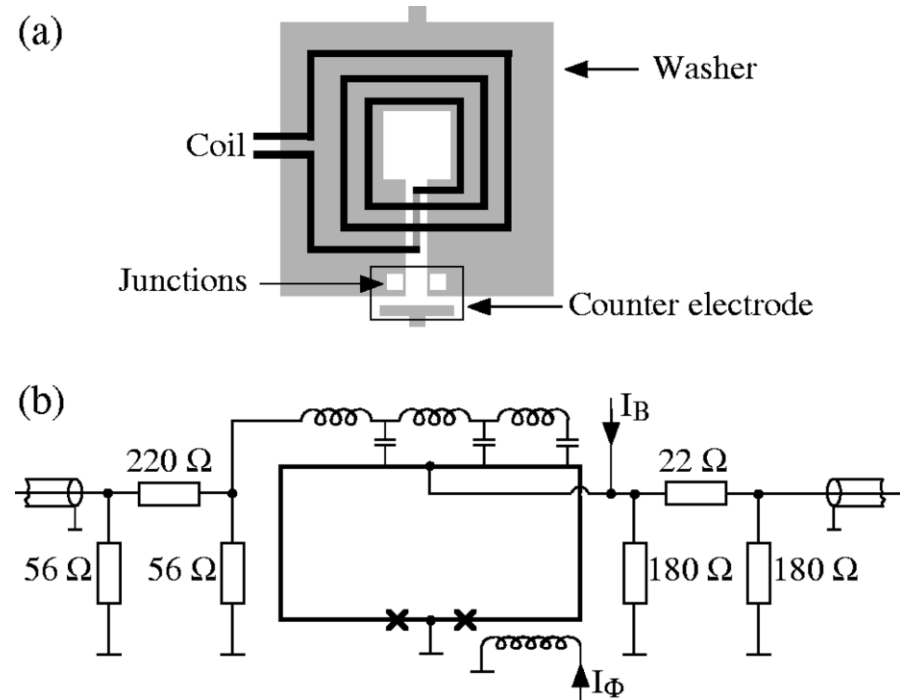


Early SQUID microwave amplifier design:

The first idea was set out by John Clarke's group: '*Radio-frequency amplifier based on a niobium dc SQUIDs with microstrip input coupling*' (APL v. 72 p2885-8 1998).

Their system was relatively narrow band and low frequency.

- The SQUID input coil was used as a microstrip resonator.
- The input signal is no longer coupled to the two ends of the input coil, but rather between one end of the coil and the SQUID loop, which acts as a ground-plane for the coil.
- The resonator is thus formed by the inductance of the input coil and its ground-plane and the capacitance between them.
- The resonator can be half wavelength or quarter wavelength, depending on the termination of the stripline.
- At 4.2K and 600MHz the noise temperature was 3.0K, gain +18dB

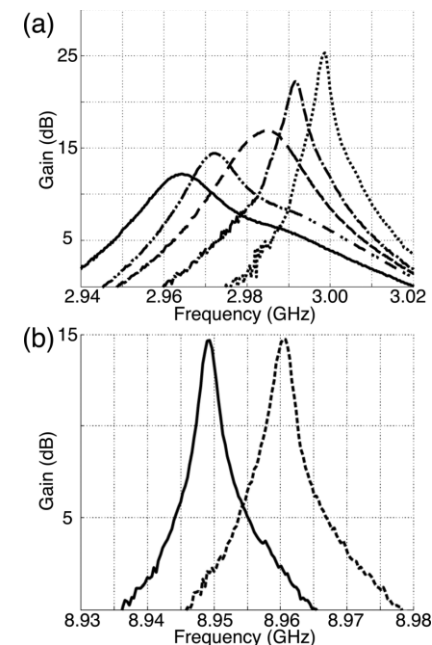
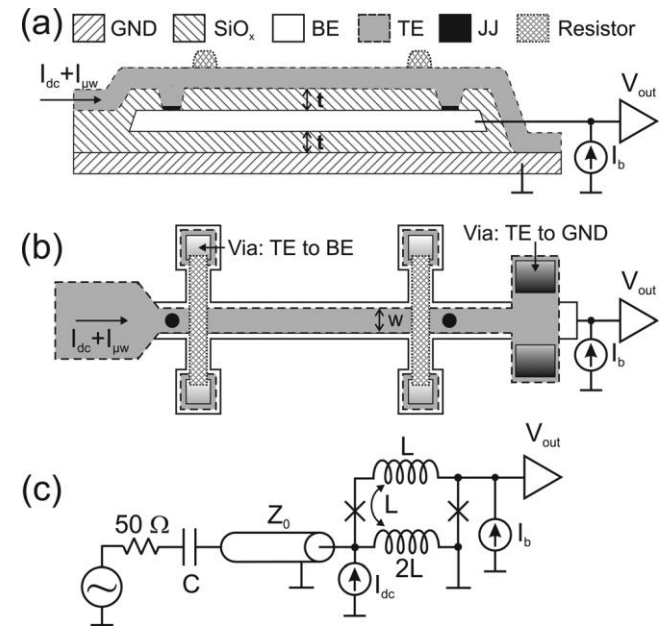


SLUG Microwave Amplifier

D. Hover, Y.-F. Chen, G. J. Ribeill, S. Zhu, S. Sendelbach and R. McDermott, 'SLUG microwave Amplifier' APL. V. 100, 063503 (2012)

- For our application we need higher frequency operation (>20 GHz)
- The SLUG is a rather new approach. **It uses a SQUID geometry where the r.f. current is applied to the SQUID loop (long and thin). The loop also acts as a resonator.**
- **High gain depends on high $dV/d\Phi$ so it benefits from high critical currents.**
- To achieve tuneable broadband operation (25 to 28 GHz) will require additional r.f. SQUID with low critical current to provide high kinetic inductance, coupled to the resonator.

SLUG amplifier (A very contrived acronym, invented by John Clarke in 1965 – Superconducting Low inductance Undulating Galvanometer....)



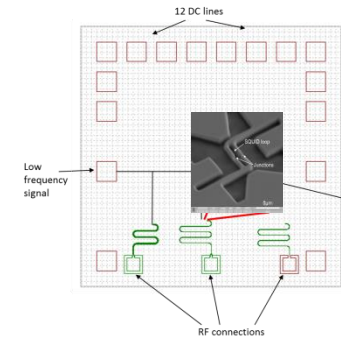
SLUG: 15 dB at 9GHz at 30mK (APL, 2012)

Capacitively coupled CPW resonator



Flux tuned rf
SQUID

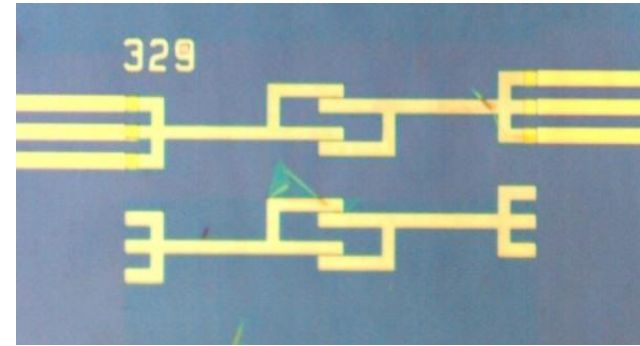
SLUG broadband
microwave
amplifier



- The entire design requires only single layer thin film of superconductor on a low-loss single crystal substrate.
- We have shown that nanobridge Josephson junctions operate with high frequency Josephson supercurrents (up to $> 100\text{GHz}$)
- These junctions are themselves intrinsically high critical current, low noise & low capacitance.
- Nanobridge junctions may be made using Nb, NbN, Al or even graphene.

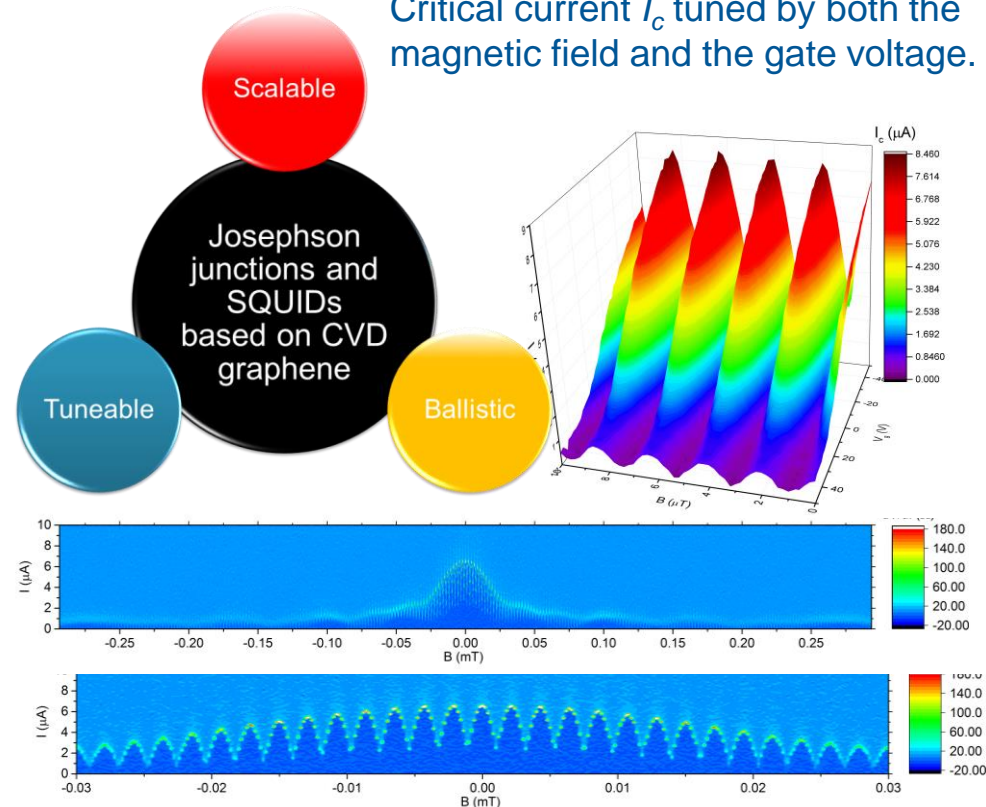
Tunable SQUIDs based on Graphene Barriers

- Is there any way we could use graphene barrier SQUIDs in a parametric amplifier design?
- That would give us an advantage.....?
- Tuning I_c with electric field and flux seems to allow different parametric modes of operation.
- But the E-field gating would have to be carefully designed to give large bandwidth.
- We don't yet know about the noise level in these devices.



Operating temperature range below 1K

Critical current I_c tuned by both the magnetic field and the gate voltage.



NPL Microwave SLUG Amplifier at Quantum limit

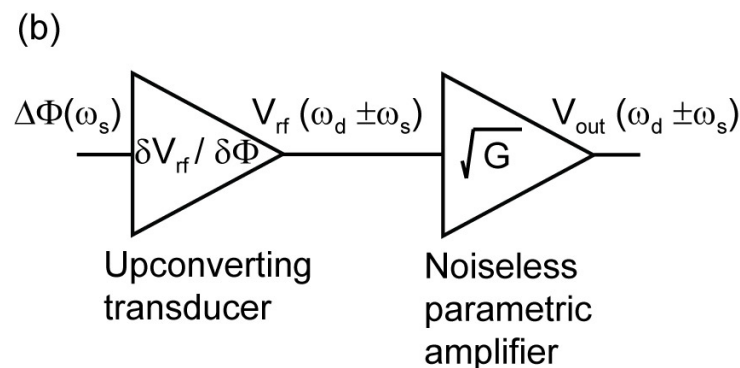
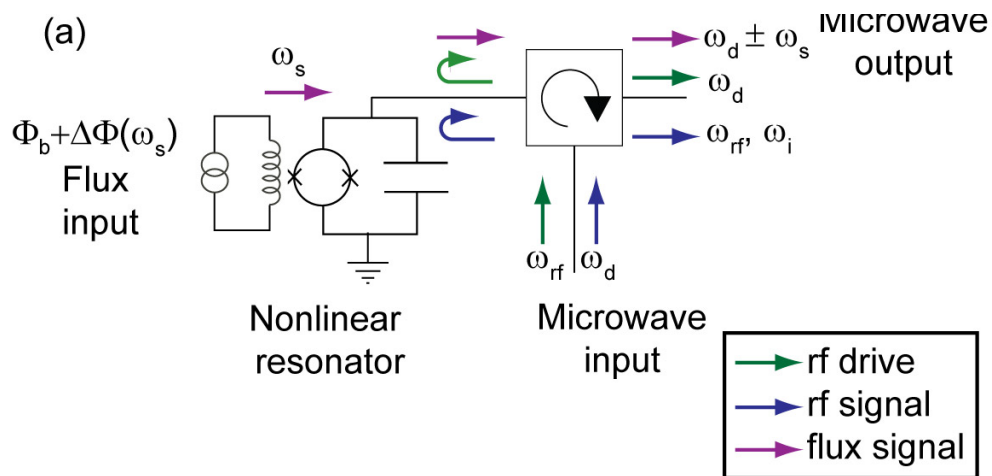
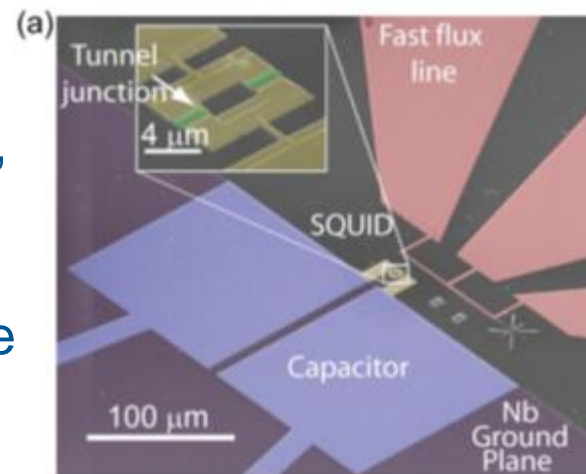
- High frequency SQUID amplifier based on the SLUG idea.
- This would use the nanobridge Josephson junctions which have been developed at NPL and shown to be both low noise (for low frequency nanoSQUIDs) but also demonstrate Josephson supercurrent performance to above 100GHz.
- Initial tests of the SLUG geometry would be carried out in a pulse tube cooler operating down to 2.5K.
- Further improvements will then be tested in a $^3\text{He} - ^4\text{He}$ dilution fridge closed cycle refrigerator, cooling to below 50mK.

Cambridge JTWPA at Quantum limit

- In parallel Cambridge will design fabricate and test a Josephson Travelling Wave Parametric Amplifier (JTWPA) with microwave pump electronics.
- Josephson junction technology will use tri-layer tunnel junction design, with in-house fabrication.
- This would operate in the Cambridge adiabatic demagnetisation refrigerator (ADR) operating down to 50mK.

Quantum limited SQUID parametric amplifier

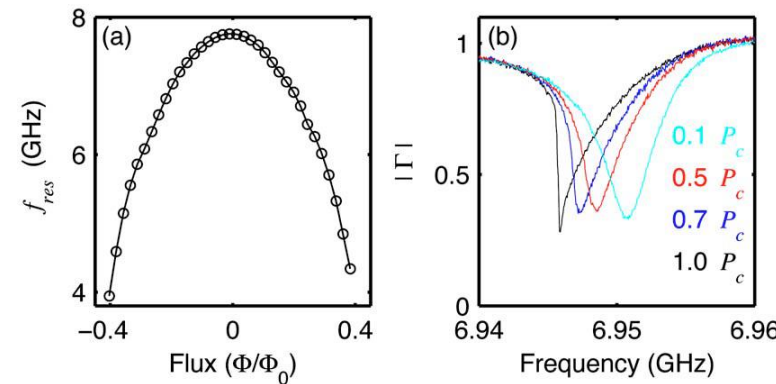
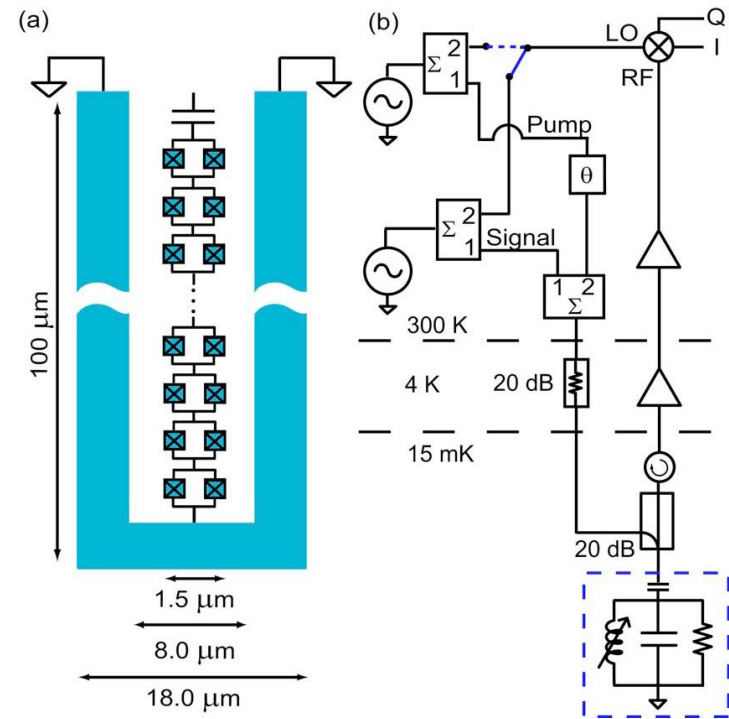
- This is operable over a tuning range from 4 to 7GHz
- SQUID was fabricated with Al-AlOx-Al junctions, $I_c = 2.1 \mu\text{A}$.
- Operating temperature 30mK
- The bandwidth can be as high as 20MHz but the lowest noise figure is found for much narrower bandwidths when it is quantum limited.



M. Hatridge, R. Vijay, D. H. Slichter, John Clarke and I. Siddiqi Dispersive magnetometry with a quantum limited SQUID parametric amplifier Phys. Rev. B 83, 134501 (2011)

SQUID Array Parametric Amplifier

- The first Clarke group SQUID amplifier was not tuneable (apparently) and the frequency was too low for our purposes.
- A better approach which solves both problems was reported by a NIST group, using a series SQUID array in the centre conductor of a CPW.
- D.c. SQUIDs are the usual Al/AIOx /Al double angle evaporated junctions
- This shows flux-based tuning over the range 4 – 7.8 GHz and gains of 28dB.
- Operating at 15mK the estimated critical current of a single SQUID is 1.5 μ A
- The bandwidth with a power gain of >15dB only seems to be ~ a few hundred kHz.

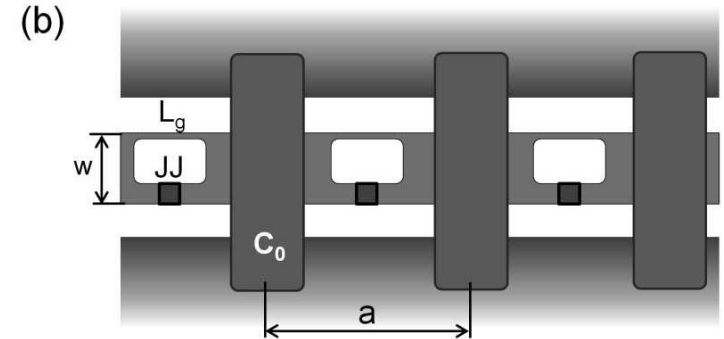
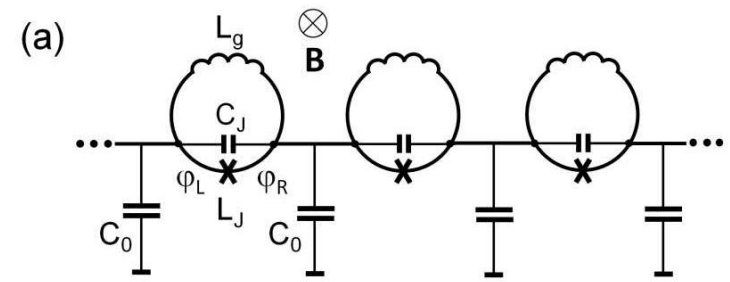


M. A. Castellanos-Beltrana and K. W. Lehnert, 'Widely tunable parametric amplifier based on a SQUID array resonator', Appl. Phys. Lett. Vol. 91 083509 2007

r.f. SQUID Travelling Wave Parametric Amplifier

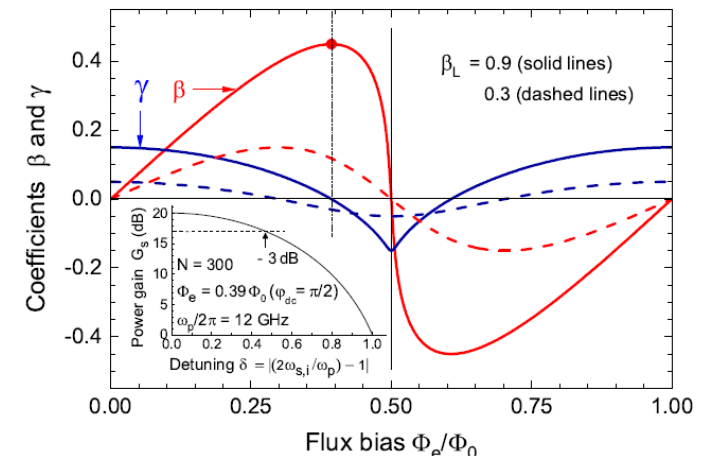
- A third approach to Josephson junction based Josephson Parametric Amplifiers uses an array of r.f. SQUIDs (that s single junction SQUIDs).
- This circuit possesses both quadratic and tertiary non-linearity terms in the expansion of the r.f. current flowing through the array.
- This means that 3-wave mixing is permitted, not just 4-wave.
- The advantage is that pump and signal frequencies are well separated.
- Phase modulation and gain are separately controlled by the two separate non-linearities.
- With 400 r.f. SQUIDs in the array the gain predicted is greater than 20dB at 10GHz with up to 1GHz bandwidth.

A.B.Zorin, *Josephson traveling-wave parametric amplifier with three-wave mixing* Phys. Rev. Applied **6**, 034006 (2016)

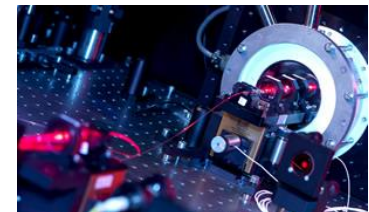
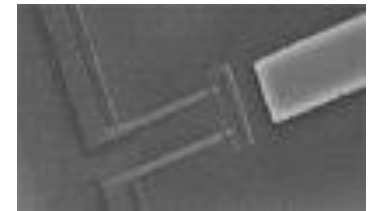
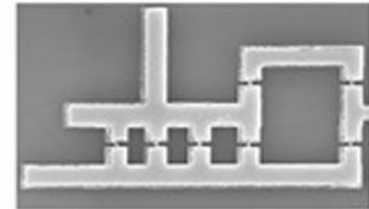


(a) Transmission line with array of r.f. SQUIDs. The tunnel JJ is connected in parallel with Josephson inductance L_J and capacitance C_J . (b) Proposed layout of transmission line with parallel plate capacitor cross-overs.

$$I/I_c = (\beta_L^{-1} + \cos \varphi_{dc})\varphi - \tilde{\beta}\varphi^2 - \tilde{\gamma}\varphi^3 - \dots,$$

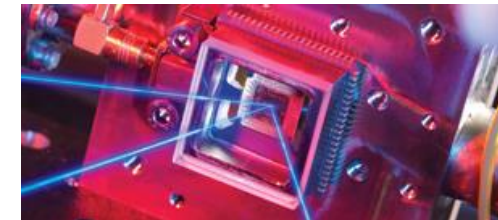
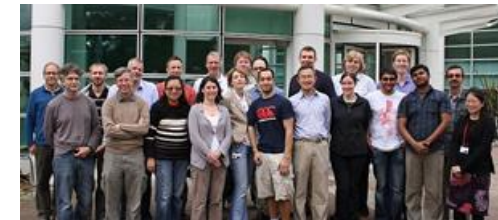


- Nanobridge Microwave SQUID Detectors
 - close to quantum-limited detection at 4.2K and microwave readout using cold preamplifiers.
- Cryogenic Signal Processing
 - NPL is developing superconducting devices and associated measurement systems for signal processing at cryogenic temperatures.
- Solid-State Quantum Information Processing
 - Applying precision frequency metrology techniques and analysis to our solid-state system to measure noise due to TLFs within a variety of systems.
- MICROPHOTON
 - This is a European project funded by the EURAMET EMRP programme. The goal of this project is to develop devices and methods that can be used to control single photon microwave radiation on a chip.



NPL Cryogenic and Magnet Infrastructure

- Low noise pulse tube cooler to 2.5K with 5T magnet.
- Bluefors dry dilution refrigerator to 10mK with 14T magnet, second Bluefors fridge to 10mK with 9T magnet.
- Heavily shielded rooms with filtered supplies and feed throughs.
- New Advanced Quantum Measurement Laboratory (AQML) opening 2020, where most of the work outlined above will be housed.
- Quantum Metrology Institute (QMI) with > 100 scientists
- Precise measurement infrastructure provided by 800 scientists (mainly physics-based) with unrivalled range of capability.



Relevant Recent NPL Publications

- T Godfrey, JC Gallop, DC Cox, EJ Romans, J Chen, L Hao, 'Investigation of Dayem Bridge NanoSQUIDs Made by Xe Focused Ion Beam', IEEE Transactions on Applied Superconductivity 28 1-5 (2018)
- X Wang, T Godfrey, T Li, D Cox, J Gallop, L Wang, Q Zhong, J Li, Y Zhong, 'Niobium Nano-SQUIDs for Inductive Superconducting Transition Edge Detectors', Conference on Precision Electromagnetic Measurements (CPEM 2018), 1-2 (2018)
- T Li, J Gallop, L Hao, E Romans, 'Ballistic Josephson junctions based on CVD graphene', Superconductor Science and Technology 31 (4), 045004 (2018)
- B Li, T Godfrey, D Cox, T Li, J Gallop, S Galer, A Nisbet, E Romans, L Hao, 'Investigation of properties of nanobridge Josephson junctions and superconducting tracks fabricated by FIB', Journal of Physics: Conference Series 964 (1), 012004 (2018)
- C. D. Shelly, P. See, J. Ireland, E. J. Romans and J. M. Williams, 'Weak link nanobridges as single flux quantum elements', Superconductor Science & Technology, 30, 095013 (2017)
- C. Yan, S. Kumar, M. Pepper, P. See, I. Farrer, D. Ritchie, J. Griffiths, and G. Jones, 'Interference Effects in a Tunable Quantum Point Contact Integrated with an Electronic Cavity' Phys. Rev. Appl, 8, 024009 (2017)
- R. E. George, J. Senior, O. P. Saira, J. P. Pekola, S. E. de Graaf, T. Lindstrom, and Y. A. Pashkin, 'Multiplexing Superconducting Qubit Circuit for Single Microwave Photon Generation' J. Low Temp. Phys., 189, 60-75 (2017)
- T. Patel, B. Li, T. Y. Li, R. Wang, J. C. Gallop, D. C. Cox, J. Chen, E. J. Romans, and L. Hao, 'Toward the Use of NanoSQUIDs to Measure the Displacement of an NEMS Resonator' IEEE Trans. Appl. Supercond., 27, 1602005 (2017)
- T. Lindstrom, R. Lake, Y. A. Pashkin, and A. Manninen, 'Controlling Single Microwave Photons: A New Frontier in Microwave Engineering', Microwave Journal, 60, 118-130 (2017)
- L. Hao, and C. Granata, 'Recent trends and perspectives of nanoSQUIDs: introduction to 'Focus on nanoSQUIDs and their applications', Superconductor Science & Technology, 30, 050301 (2017)
- N. Johnson, J. D. Fletcher, D. A. Humphreys, P. See, J. P. Griffiths, G. A. C. Jones, I. Farrer, D. A. Ritchie, M. Pepper, T. J. B. M. Janssen, and M. Kataoka, 'Ultrafast voltage sampling using single-electron wavepackets', Appl. Phys. Lett., 110, 102105 (2017)
- M. Kataoka, J. D. Fletcher, and N. Johnson, 'Time-resolved single-electron wave-packet detection', Physica Status Solidi B-basic Solid State Physics, 254, 1600547 (2017)
- S. E. de Graaf, A. A. Adamyant, T. Lindstrom, D. Erts, S. E. Kubatkin, A. Y. Tzalenchuk, and A. V. Danilov, 'Direct Identification of Dilute Surface Spins on Al₂O₃: Origin of Flux Noise in Quantum Circuits', Phys. Rev. Lett., 118, 057703 (2017)
- A. Blois, S. Rozhko, L. Hao, J. C. Gallop, and E. J. Romans, 'Heat propagation models for superconducting nanobridges at millikelvin temperatures', Superconductor Science & Technology, 30, 014003 (2017)
- L Lolli, T Li, C Portesi, E Taralli, N Acharya, K Chen, M Rajteri, D Cox, 'Micro-SQUIDs based on MgB₂ nano-bridges for NEMS readout', Superconductor Science and Technology 29 (10), 104008 (2016)
- J Gallop, L Hao, 'Nanoscale Superconducting Quantum Interference Devices Add Another Dimension', ACS nano 10 (9), 8128-8132 (2016)
- X Wang, T Li, D Cox, J Gallop, J Li, Y Zhong, W Cao, Q Zhong, Z Li, 'Investigation of niobium nanoSQUIDs based on nanobridge junctions', Precision Electromagnetic Measurements (CPEM 2016), 1-2 (2016)
- J Gallop, D Cox, L Hao, 'Nanobridge SQUIDs as calorimetric inductive particle detectors', Superconductor Science and Technology 28 (8), 084002 (2015)

Relevant Capability at Cambridge

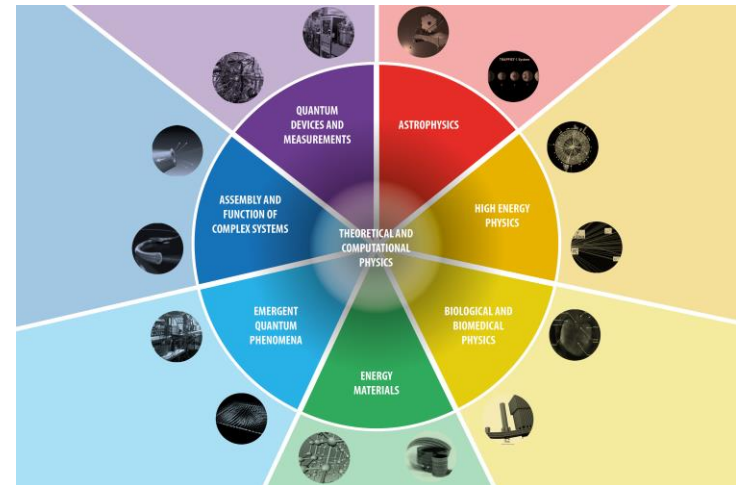
- State-of –art clean room and other fabrication facilities
- Cryogenic measurement facilities to mK

Key Publications:

D. Tihon, S. Withington, C.N. Thomas and C. Craeye, Identification of the absorption processes in periodic plasmonic structures using energy absorption interferometry, *J. Opt. Soc. Am. A*, 36, 12 (2019).

S. Zhao, S. Withington, D.J. Goldie, and C.N. Thomas, *Electromagnetic models for multilayer superconducting transmission lines*, *Supercond. Sci. Tech.* **31**, 085012 (2018)

S. Zhou, D. J. Goldie, C.N. Thomas, and S. Withington, *Calculation and measurement of critical temperature in thin superconducting multilayers*, *Supercond. Sci. Tech.* **31**, 105004 (2018)



- <https://www.phy.cam.ac.uk/directory/withingtons>



Email: stafford@mrao.cam.ac.uk

Thank you!



Department for
Business, Energy
& Industrial Strategy

FUNDED BY BEIS

The National Physical Laboratory is operated by NPL Management Ltd, a wholly-owned company of the Department for Business, Energy and Industrial Strategy (BEIS).

- **Nanobridge readout of single photon detectors:** Superconducting nanobridge devices can be used to process the signal from superconducting nanowire single photon detector arrays. We are developing this technology as part of an Innovate UK funded feasibility study, with the University of Glasgow and the London Centre for Nanotechnology.
- **Quantum Voltage Synthesizer:** Arrays of Josephson junctions can be used to generate voltages at practical laboratory levels. NPL has developed a quantum voltage synthesizer for generation of waveforms directly in terms of the Josephson effect.
- **High Bandwidth Quantum Voltage Digitiser:** Delta-sigma modulation techniques can be used in combination with a Josephson junction to provide a quantum-accurate digitizer. NPL is developing this technology in collaboration with European partners.
- **Superconducting Nanobridges:** Josephson junctions can be created using nanoscale weak links fabricated out of thin superconducting films. A first application we are studying is a Josephson comparator.

Zagoskin Squeezing

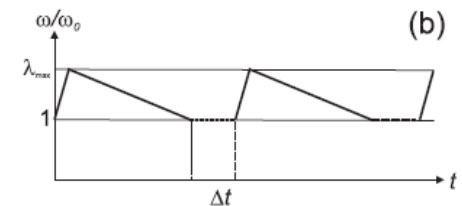
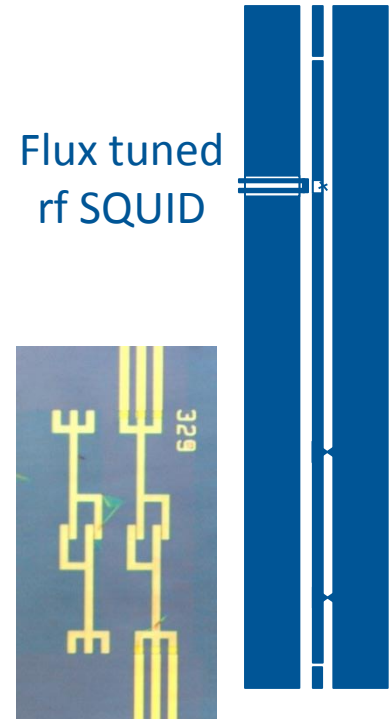
The inductively coupled r.f. SQUID in the CPW resonator gives wideband tuning of the CPW frequency and hence of the SLUG centre frequency.

Application of a careful tuning sequence to the same inductive r.f. SQUID (or if necessary a separate one within the CPW resonator) can allow **squeezing of the noise** in the SLUG amplifier.

A sequence of fast frequency shifts is applied to the CPW resonator, with controlled repetition rate. The fast shift must be on a timescale $\ll 1/\omega_0$ where ω_0 is the angular frequency of the CPW resonator.

This frequency is as high as 10 GHz so fast inductive switching requires careful circuit design but might be achieved with relaxation oscillator behaviour of a separate under-damped Josephson junction.

Other possibilities include electric field tuning of a graphene barrier SNS junction.



Limiting Energy Sensitivity

For any **linear detector** there is a minimum detectable energy sensitivity within unit bandwidth. If the detector senses a physical parameter X and the minimum detectable change in the parameter in a 1Hz bandwidth is dX then the limiting energy sensitivity $\delta\varepsilon$ is

$$\delta\varepsilon = \frac{\delta X^2}{Z}$$

where Z is the transfer function between the variable X and its conjugate variable Y :-

$$Z = \frac{\partial X}{\partial Y} \quad \text{Note that also} \quad \Delta X \cdot \Delta Y \geq \hbar/2$$

The conventional white noise limit for a detector at temperature T is

$$\delta\varepsilon \geq k_B T / df \quad \text{where } df \text{ is the detection bandwidth}$$

At sufficiently low temperature T such that

$$k_B T < h \cdot df$$

a **quantum limit applies** such that

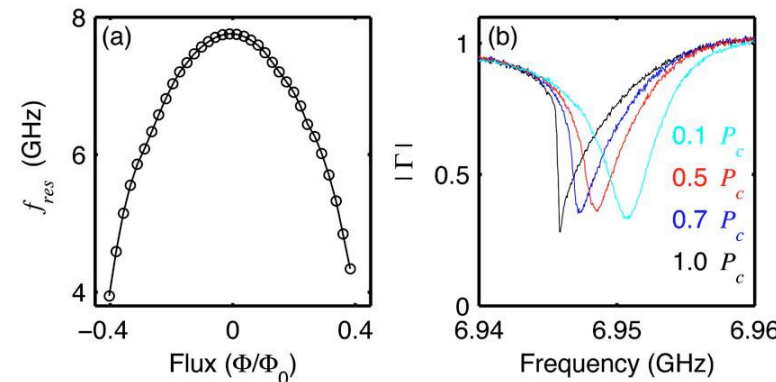
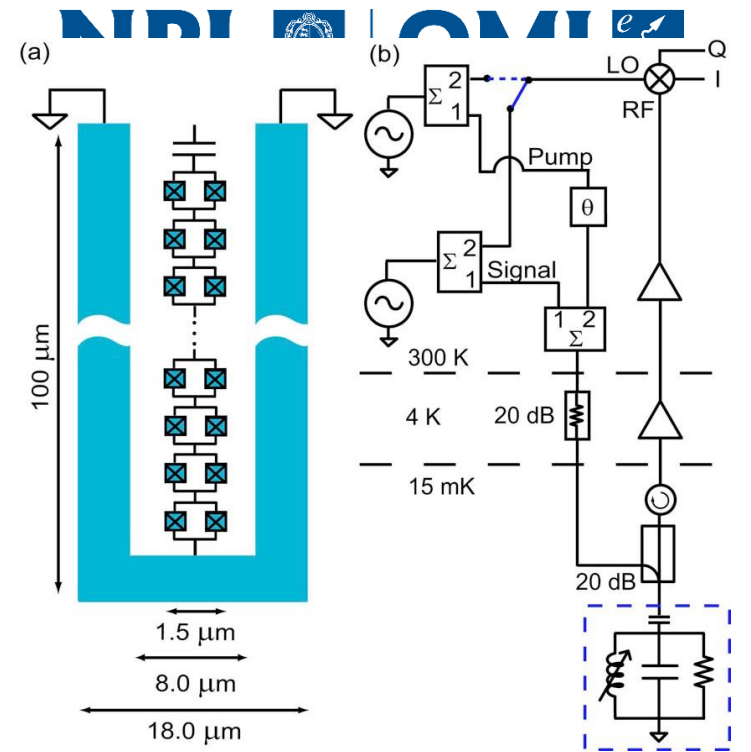
$$\delta\varepsilon \geq \hbar/2$$

SQUIDs probably reach this limit more closely than any other detector.

SQUID Array Parametric Amplifier

- The first Clarke group SQUID amplifier was not tuneable (apparently) and the frequency was too low for our purposes.
- A better approach which solves both problems was reported by a NIST group, using a series SQUID array in the centre conductor of a CPW.
- D.c. SQUIDs are the usual Al/AlOx /Al double angle evaporated junctions
- This shows flux-based tuning over the range 4 – 7.8 GHz and gains of 28dB.
- Operating at 15mK the estimated critical current of a single SQUID is $1.5\mu\text{A}$
- The bandwidth with a power gain of $>15\text{dB}$ only seems to be \sim a few hundred kHz.
- A similar device already in use in Yale Axion search.

M. A. Castellanos-Beltrana and K. W. Lehnert, 'Widely tunable parametric amplifier based on a SQUID array resonator', Appl. Phys. Lett. Vol. 91 083509 2007



Thank you!

Josephson junctions may also be characterised by a **plasma frequency** ω_p where

$$\omega_p = \frac{1}{\sqrt{\Phi_0 C / i_c}}$$

(this is the frequency at which the pairs oscillate back and forth within a J. junction)

At low temperatures if

$$\hbar\omega_p \geq k_B T$$

(i.e. plasma frequency photon has more energy than a typical thermal fluctuation)

this classical treatment breaks down and the **back reaction noise arising from circulating noise currents in the SQUID loop** must be taken into account. In this case it is found that

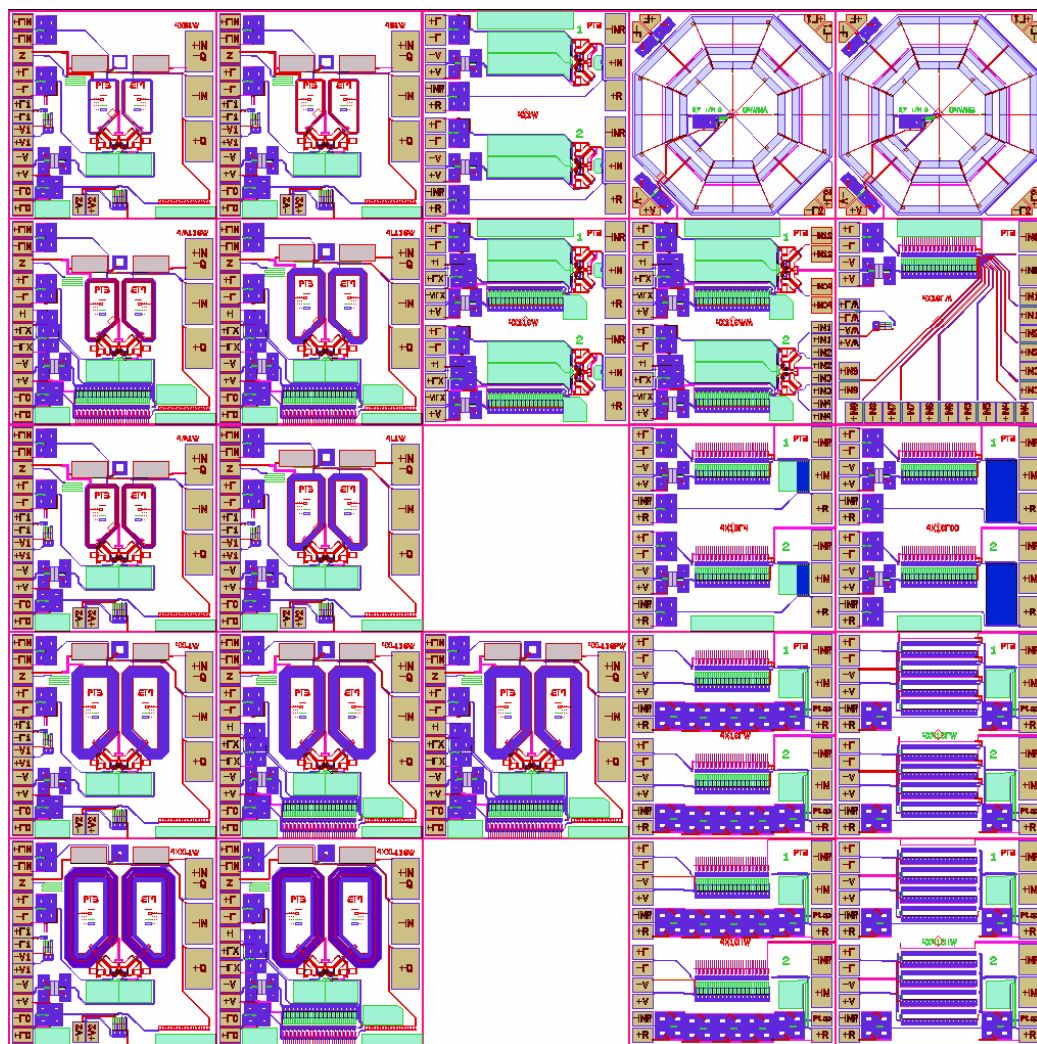
$$\delta\varepsilon \geq \frac{\hbar}{2}$$

(agrees with the Heisenberg Uncertainty Principle!)

Experimentally the best SQUIDs have a minimum flux sensitivity of $\sim 1 \times 10^{-7} \Phi_0 / (\text{Hz})^{1/2}$, limited by room temperature amplifier noise.

But use of a cryogenic preamplifier following the SQUID it is quite possible to achieve quantum limited energy sensitivity.

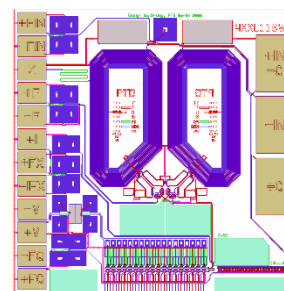
Collaboration with PTB:



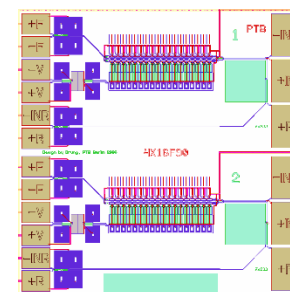
SSA: SQUID series array

Measure the nanoSQUID with very low noise SQUID array amplifier.

Other areas: nanomagnetic particle measurement, CNT Thermometry, NEMs etc.

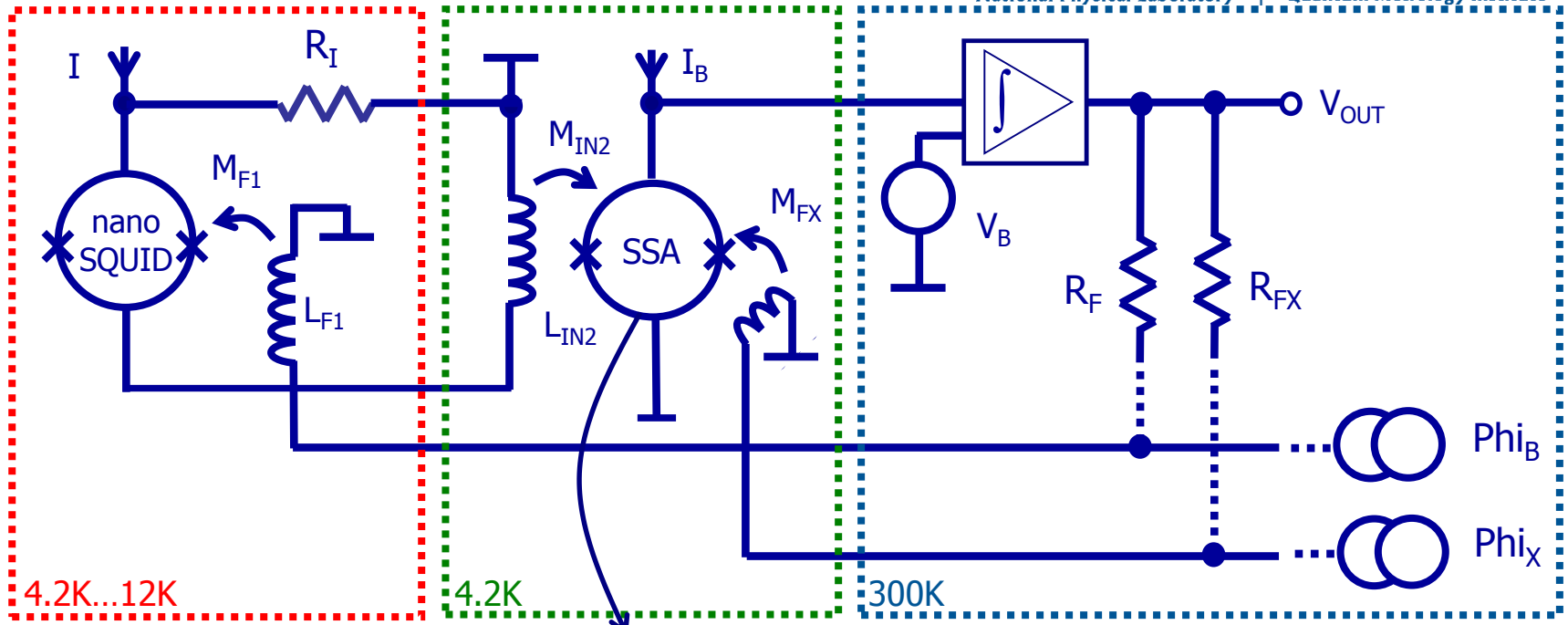


#23 = C4XL116N



#15 = C4X16F90

NanoSQUID characterization using 2-stage SQUID setup

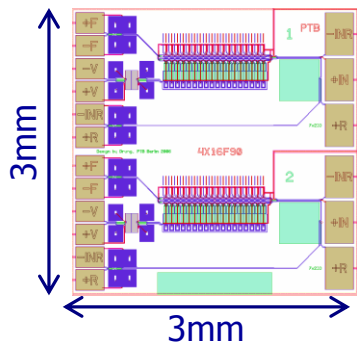


4.2K...12K

4.2K

300K

- 16-SQUID series array current sensor
- Input inductance < 3 nH
- Current noise $\sqrt{S_I} < 10 \text{ pA}/\sqrt{\text{Hz}}$ @ 4.2 K
- Magnetically unshielded operation



Schematic of 2-stage setup using a SQUID series array (SSA) to read out the nanoSQUID



Capacitively coupled CPW resonator



Flux tuned rf SQUID

SLUG broadband
microwave amplifier

Assume $w = 10\mu\text{m}$ and $s = 10\mu\text{m}$.
Then $l = 1.55 \mu\text{H/m}$
and $c = 108 \text{ pF/m}$
 $Z_0 = 120 \text{ ohms}$

Baseline activities:

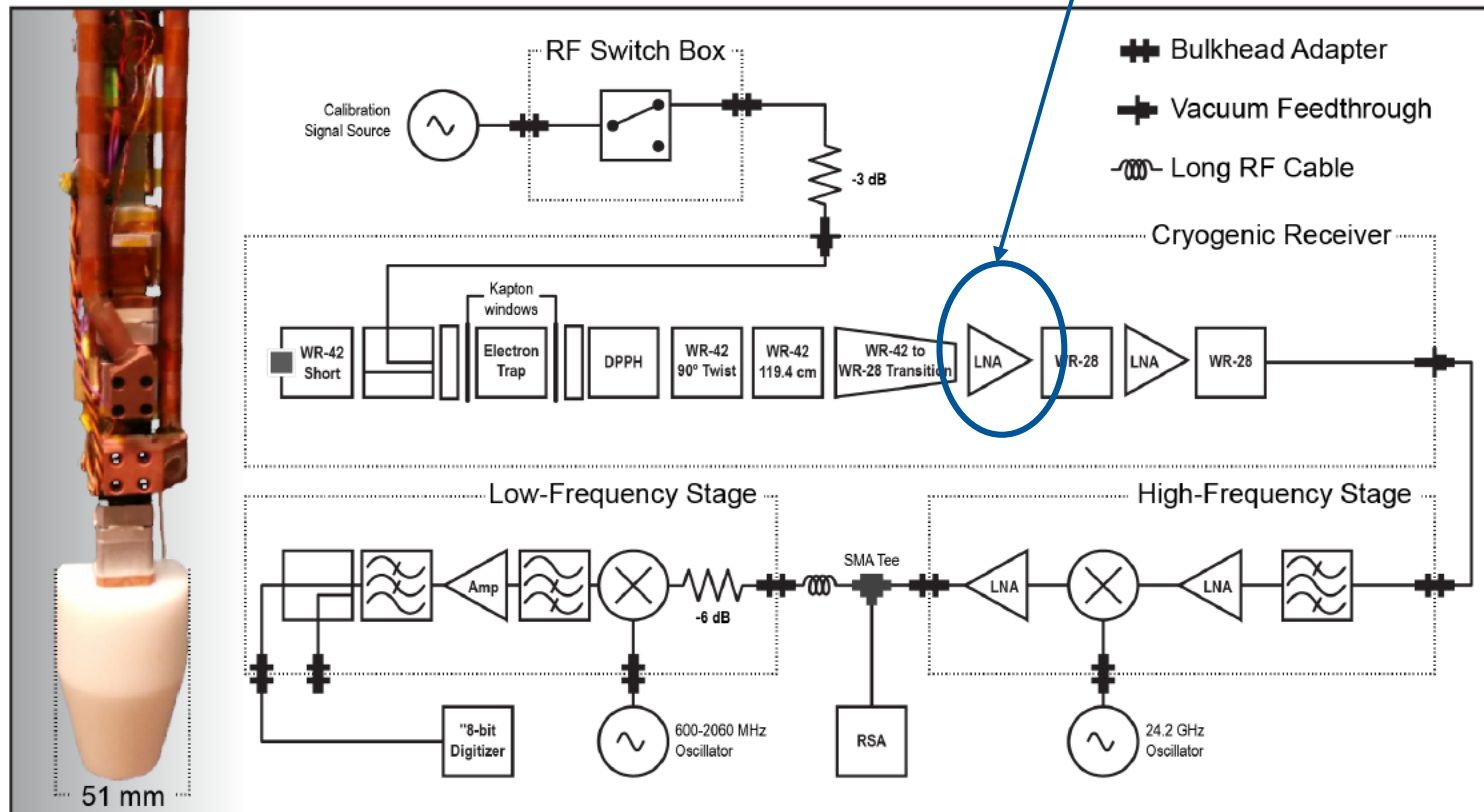
.4.1. Design, construct and test baseline receiver and measure noise temperature with operating temperature at 3K. (M18)

D.4.2 Evaluate gain and bandwidth of SLUG amplifier and JTWPA at 28 GHz and mK operation. (M24)

D.4.3 Development of a demonstrator to be delivered to UCL test facility based on the choice of one or other of SLUG or JTWPA. (M36)

Baseline activities with quantum limited amplifiers :

Quantum limited amplifiers



- Josephson or SQUID based amplifier with gain of $\sim 20\text{dB}$
- Tuneable over full 20-28 GHz range
- Operational from base of 10mK to as high a temperature as realistic.