Stringy extremal black holes

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Outline

- Introduction.
- Supersymmetric four-charge black hole.
    - Stringy-corrected solution.
    - Shift in charges and mass.
    - Wald entropy.
- Charge-to-mass ratio of extremal non-supersymmetric black holes.
    - Reissner-Nordström and possible corrections in Einstein-Maxwell.
    - Mild version of the WGC.
    - Corrections to non-supersymmetric heterotic charged BH's.
    - Un-proving the mild WGC.
- Conclusions.

Heterotic theory as a series expansion

Frequently, effective field theories are defined through a series expansion.

The *zeroth-order theory* is intended to approximate, **under certain conditions**, the dynamics of the fields encoding the relevant d.o.f.

Heterotic theory, bosonic fields $g_{\mu\nu}$, ϕ, $B_{\mu\nu}$,

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left[R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 + \dots \right] .$$

$$H = dB \,, \qquad\qquad dH = 0 \,.$$

Additional terms involve an expansion in g_s and α'.

Heterotic theory as a series expansion

If we are interested in performing **precision studies** or, alternatively, if we are **away from good conditions**, the effect of additional terms in the expansion can be very relevant and should be understood.

Specially important can be to determine if the effective theory (maybe including some corrections) can still be used when not working in optimal conditions.

First-order heterotic stringy corrections Bergshoeff, de Roo

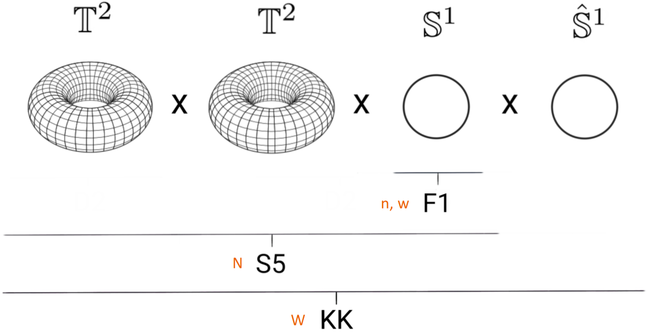
$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x \sqrt{|g|}\, e^{-2\phi} \left[R - 4(\partial\phi)^2 + \frac{1}{12} H^2 - \frac{\alpha'}{8} R_{(-)\mu\nu}{}^{a}{}_{b} R_{(-)}{}^{\mu\nu\, b}{}_{a} + \ldots \right]$$

$$H = dB + \frac{\alpha'}{4}\Omega^{\mathrm{L}}_{(-)}\,, \qquad dH = \frac{\alpha'}{4} R_{(-)}{}^{a}{}_{b} \wedge R_{(-)}{}^{b}{}_{a}\,.$$

Supersymmetric four-charge black hole

Supersymmetric four-charge black hole

Heterotic theory on $\mathbb{T}^6$,



Supersymmetric four-charge black hole

A **supersymmetric** black hole configuration has

$$
\begin{aligned}
ds^2 &= \frac{2}{\mathcal{Z}_-} du \left(dv - \tfrac{1}{2} \mathcal{Z}_+ du \right) - \mathcal{Z}_0 d\sigma^2_{(4)} - dy^i dy^i \,, \\
H &= d{\mathcal{Z}_-}^{-1} \wedge du \wedge dv + \star_{(4)} d\mathcal{Z}_0 \,, \\
e^{-2\phi} &= g_s^{-2} \frac{\mathcal{Z}_-}{\mathcal{Z}_0} \,, \\
d\sigma^2_{(4)} &= \mathcal{V}^{-1} (dz + \chi)^2 + \mathcal{V} d\vec{x}^2_{(3)} \,,
\end{aligned}
$$

with $d\sigma^2_{(4)}$ a hyperkähler (Gibbons-Hawking) space, for 4d BH we have

$$
\mathcal{V} = 1 + \frac{R_z \mathbf{W}}{2r} \,, \qquad d\mathcal{V} = \star_{(3)} d\chi \,.
$$

Supersymmetric four-charge black hole

The solution (including corrections) is given by

$$\mathcal{Z}_- = 1 + \frac{q_-}{r}\,,$$
$$\mathcal{Z}_0 = 1 + \frac{q_0}{r} - \alpha' \left[\frac{(r+q_v)(r+2q_0)+q_0^2}{4q_v(r+q_v)(r+q_0)^2} + \frac{(r+q_v)(r+2q_v)+q_v^2}{4q_v(r+q_v)^3} \right]\,,$$
$$\mathcal{Z}_+ = 1 + \frac{q_0}{r} + \frac{q_+\alpha'}{2q_vq_0}\frac{r^2+r(q_0+q_-+q_v)+q_vq_0+q_vq_-+q_0q_-}{(r+q_v)(r+q_0)(r+q_-)}\,,$$

with

$$q_+ = \frac{\alpha'^2 g_s^2 \mathbf{n}}{2R_zR_u^2}, \qquad q_- = \frac{\alpha' g_s^2 \mathbf{w}}{2R_z}, \qquad q_0 = \frac{\alpha' \mathbf{N}}{2R_z}.$$

One can check

$$\lim_{r\to\infty} \mathcal{Z}_0 = 1 + \left(q_0 - \frac{\alpha'}{2q_v}\right)\frac{1}{r}\,, \qquad \lim_{r\to\infty} \mathcal{Z}_+ = 1 + \left(q_0 + \frac{q_+\alpha'}{2q_vq_0}\right)\frac{1}{r}\,,$$

Supersymmetric four-charge black hole. Properties.

The metric is a 4d black hole, with $AdS_2 \times S^2$ **near-horizon unmodified**

$$ds^2 = \frac{r^2}{L^2}(nwNW)^{-1/2}dt^2 - L^2(nwNW)^{1/2}(\frac{dr^2}{r^2} + d\Omega^2_{(2)})\,,$$

If any (n, w, N, W) vanishes, the horizon becomes singular.

The computation of the charges yields

Shift in the charges

$$Q_N = N - \frac{2}{W}, \qquad Q_W = W, \qquad Q_w = w, \qquad Q_n = n\left(1 + \frac{2}{NW}\right)\,.$$

The ADM mass is just

$$M = \frac{R_u}{\ell_s^2}Q_n + \frac{1}{R_u}Q_w + \frac{R_u}{g_s^2\ell_s^2}Q_N + \frac{R_z^2 R_u}{g_s^2\ell_s^4}Q_W\,,$$

Supersymmetric four-charge black hole. Exact entropy.

After α' corrections, the complete solution changes, but the near-horizon remains the same.

The entropy is now given by Wald's formula,

$$S = -2\pi \int_{\Sigma} d^8x \sqrt{|h|} \frac{\delta \mathcal{L}}{\delta R_{abcd}} \epsilon_{ab} \epsilon_{cd} \,,$$

which yields

$$S_{\mathrm{BH}} = 4\pi \sqrt{nwNW} \left(1 + \frac{2}{NW}\right) = 4\pi \sqrt{Q_n Q_w \left(Q_N Q_W + 4\right)}, ,$$

in **exact agreement** with Cardy formula of the dual CFT. Kutasov, Larsen, Leigh

Charge-to-mass ratio of extremal non-supersymmetric black holes

Reissner-Nordström

Einstein-Maxwell theory,

$$S = \frac{1}{16\pi} \int d^4x \sqrt{|g|} \left(R - F_{\mu\nu} F^{\mu\nu} \right) .$$

The equations of motion admit the static solution

$$\begin{aligned} ds^2 &= \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 , \\ A &= \frac{Q}{r} dt . \end{aligned}$$

There is a regular horizon when $\mathbf{M} \geq \mathbf{Q}$.

Extremal Reissner-Nordström + higher derivatives

Taking Einstein-Maxwell theory as the zeroth-order term of some effective theory, there are many higher-derivative terms that can be added to the action: $h_1 R^2$, $h_2 R_{\mu\nu} R^{\mu\nu}$, $h_3 (F_{\mu\nu} F^{\mu\nu})^2$, $h_4 R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$, ...

The original extremal bound at $Q/M|_{\rm ext} = 1$ might now be shifted,

$$\frac{Q}{M}|_{\rm ext} = 1 + f(M, h_i)\,.$$

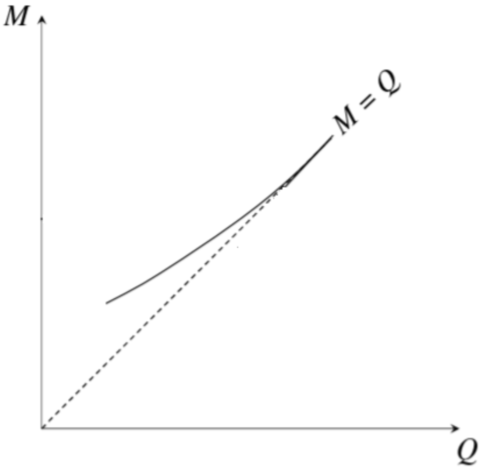
Properties of the shift depend on the UV completion taken.

Mild version of the WGC: The Q/M ratio of extremal black holes should increase as the mass of the BH decreases. Kats et al.

They argued that this is occurs in string theory, although not completely satisfactorily:

- Example without dilaton and **without ST embedding**.
- Example with dilaton, GHS, **not a BH** in the extremal limit.

Extremal charge-to-mass?



Connection with the WGC

The question has recently attracted interest. Several studies related to the extremal RN, among which:

- Imposing unitary and causality for the corrected theory, agreement with the mild WGC. Cheung, Shiu, ...
- Claims that it has been proved that, as a **consecuence of entropy increase**, $Q/M|_{\mathrm{ext}} > 1$. Remmen, Goon, ...

$$\Delta M_{\text{ext}} = -T_0(M, \vec{Q})\Delta S(M, \vec{Q})\big|_{M \approx M_{\text{ext}}^{(0)}} .$$

But this is **strange**, as we know that susy BH have the mass given as a linear combination of the charges and this is not (expected to be) modified by the corrections...

We can use our tools to try to solve this puzzle.

Connection with the WGC

Hence, at least one of the following options is true:

1. Supersymmetric BH's do not preserve the (linear) relation between M and Q.
2. The (universal) entropy-extremality relation between ΔS and ΔM is not always true.
3. The entropy-extremality relation does not actually imply the mild WGC.

Stringy (non-susy) extremal RN-like black hole

We have studied 3 families of solutions of the heterotic theory and computed the higher-derivative corrections. (non-extremal → Pablo's talk)

- Start with a non-supersymmetric embedding of (dyonic) Reissner-Nordström in string theory.
- Work directly with the full (10d) theory, solving the e.o.m. perturbatively.

Embedding,

$$\begin{aligned} d\hat{s} =& e^{2(\phi-\phi_\infty)} ds^2 - c^2 (dz + V/c_\infty)^2 - dy^i dy^i \,, \\ \hat{H} =& F \wedge (c_\infty dz + V) + H \,, \\ e^{-2\hat{\phi}} =& \frac{1}{c} e^{-2\phi} \,, \end{aligned}$$

Compactification on $\mathbb{S}^1_z \times \mathbb{T}^5$, where we truncate all the fields that have indices on $\mathbb{T}^5$, while the KK reduction on $\mathbb{S}^1_z$ is **general**.

Stringy (non-susy) extremal RN-like black hole

At zeroth-order, the 4d effective action for these fields is

$$S = \frac{1}{16\pi G_N^{(4)}} \int d^4x \sqrt{|g|} \left\{ R + 2(\partial\phi)^2 + \frac{(\partial c)^2}{c^2} + \frac{e^{-4(\phi-\phi_\infty)}}{2\cdot 3!} H^2 \right. \\ \left. + \frac{e^{-2(\phi-\phi_\infty)}}{4} \left(G^2 + \frac{c_\infty^2}{c^2} F^2 \right) \right\},$$

One could truncate V, H and c, obtaining Einstein-Maxwell-Dilaton. However, this is inconsistent once α' corrections are taken into account.

Higher-derivative corrections to the Einstein-Maxwell-Dilaton effective model in the context of string theory **may require** the activation of **additional fields!**

Stringy (non-susy) extremal RN-like black hole

Departing solution:

$$
\begin{aligned}
ds^2 &= \left(1+\frac{Q}{r}\right)^{-2} dt^2 - \left(1+\frac{Q}{r}\right)^{2}\left(dr^2 + r^2 d\Omega^2_{(2)}\right) ,\\
A &= \frac{2q_A}{(r+Q)}dt - 2p_A \cos\theta d\varphi \,,\\
V &= \frac{2q_V}{(r+Q)}dt - 2p_V \cos\theta d\varphi \,,\\
\phi &= \phi_\infty \,, \quad c = c_\infty \,, \quad H = 0 \,.
\end{aligned}
$$

with

$$
|q_A| = |p_A| \,, \quad |q_V| = |p_V| \,, \qquad Q = \sqrt{q_A^2 + p_A^2 + q_V^2 + p_V^2} \,,
$$

$$
q_A p_V + p_A q_V = 0 \,.
$$

Two (*simple*) inequivalent possibilities, $q_A = -q_V = p_A = p_V = Q/2$ and $q_A = q_V = p_A = -p_V = Q/2$.

Stringy (non-susy) extremal RN-like black hole: Case 1

Case 1 ($q_V < 0$):

$$\begin{aligned}
ds^2 &= \left(1+\frac{Q}{r}+\frac{\alpha' Q^2}{8(r+Q)^3 r}\right)^{-2} dt^2 - \left(1+\frac{Q}{r}+\frac{\alpha' Q^2}{8(r+Q)^3 r}\right)^{2}\left(dr^2 + r^2 d\Omega^2_{(2)}\right) \\
F &= \frac{Q}{(r+Q)^2}\left(1+\frac{\alpha' Q^2}{4(r+Q)^4}\right) dt\wedge dr + Q\left(1+\frac{\alpha' Q(Q+4r)}{2(r+Q)^4}\right)\sin\theta d\theta\wedge d\varphi\,, \\
V &= -\frac{Q}{(r+Q)} dt - Q\cos\theta d\varphi\,, \\
\hat{\phi} &= \hat{\phi}_\infty + \frac{\alpha' Q^2}{4(r+Q)^4}\,, \\
c &= c_\infty\left(1+\frac{\alpha' Q^2}{4(r+Q)^4}\right)\,, \quad H = 0\,.
\end{aligned}$$

Charge-to-mass ratio at extremality **unmodified**, **entropy shifted**

$$\frac{Q}{M} = 1 + \mathcal{O}(\alpha'^2)\,, \qquad S = \frac{\pi}{G_N^{(4)}}\left(Q^2 + \frac{\alpha'}{4}\right) + \ldots$$

Stringy (non-susy) extremal RN-like black hole: Case 2

Case 2 ($q_V > 0$):

$$ds^2 \;=\; A^2\left(1+\frac{Q}{r}\right)^{-2} dt^2 - B^2\left(1+\frac{Q}{r}\right)^2 \left(dr^2 + r^2 d\Omega^2_{(2)}\right)\,,$$

where

$$\begin{aligned} A =& 1 + \alpha' \frac{6Q^3 + 13Q^2 r + 8Qr^2 + 2r^3}{40Q(r+Q)^4}\,,\\ B =& 1 - \alpha' \frac{5Q^3 + 9Q^2 r + 7Qr^2 + 2r^3}{40Q(r+Q)^4}\,. \end{aligned}$$

Charge-to-mass ratio at extremality **modified**, with same **entropy shift**

$$\frac{Q}{M} = 1 + \frac{\alpha'}{20M^2} + \mathcal{O}(\alpha'^2)\,, \qquad S = \frac{\pi}{G_N^{(4)}}\left(Q^2 + \frac{\alpha'}{4}\right) + \ldots$$

Relation between shift in the entropy and Q/M?

The entropy-extremality relation Goon, Penco

$$\Delta M_{\text{ext}} = -T_0(M,\vec{Q})\Delta S(M,\vec{Q})\big|_{M\approx M_{\text{ext}}^{(0)}}\,,$$

does not imply $\Delta S(M,\vec{Q}) > 0 \to \Delta M_{\text{ext}} < 0$, as the relation trivializes to $0 = 0$ in some cases.

Proof: According to the prescription, the r.h.s. is evaluated at $M \approx M_{\text{ext}}^{(0)}$, which is defined as the mass slightly above extremality for which $\Delta T(M,\vec{Q})$ is subdominant with respect to $T_0(M,\vec{Q})$.

It follows that if for a solution $\Delta T(M,\vec{Q}) = 0$, by definition $\Delta M_{\text{ext}} = 0$. Then, the r.h.s. must be evaluated precisely at $M = M_{\text{ext}}^{(0)}$, where $T_0 = 0$ so this side vanishes as well regardless the sign of $\Delta S(M,\vec{Q})$.

Conclusion: The positivity of the corrections to the entropy does **not prove** the mild WGC.

Summary of results

Supersymmetric

- It is fundamental to distinguish between fundamental sources and charges,

$$Q_N = N - \frac{2}{W}, \qquad Q_W = W, \qquad Q_w = w, \qquad Q_n = n\left(1 + \frac{2}{NW}\right).$$

Non supersymmetric

- Stringy black holes we studied have $Q/M \geq 1$, with some configurations saturating the bound.
- The corrections seem to quite generally require the activation of new lower-dimensional fields, an important point which is (almost always) not considered.
- The positivity of the correction to the entropy does not imply $Q/M > 1$.

THANKS FOR YOUR ATTENTION