Iberian Strings

Santiago de Compostella, 16 January 2019

Composite and holographic axions

Elias Kiritsis

CCTP/ITCP University of Crete APC, Paris

Bibliography

with

F. Bigazzi, A. Cotrone, M. Jarvinen http://arxiv.org/abs/arXiv:1906.12132

Y. Hamada, F. Nitti, L. Witkowski J.Phys. A52 (2019) no.45, 454003 http://arxiv.org/abs/arXiv:1905.03663

P. Anastasopoulos, P. Betzios, M. Bianchi (Rome) D. Consoli (Vienna) JHEP 1910 (2019) 113 http://arxiv.org/abs/arXiv:1811.05940

D. Arean, I. Iatrakis, M. Jarvinen Phys.Rev. D96 (2017) no.2, 026001 http://arxiv.org/abs/arXiv:1609.08922

with F. Nitti unpublished

and based on earlier ideas in

E. Kiritsis EPJ Web Conf. 71 (2014) 00068; e-Print: arXiv:1408.3541 [hep-ph]

Introduction

- The SM of particle physics is an effective QFT.

- Its low-dimension operators O_i couple to "couplings" $g_i(x)$

$$S_{SM} = \int d^4x \sum_i g_i(x)\ O_i(x)$$

- The couplings $g_i(x)$ (also known as sources) may be dynamical:

♠ The coupling to the energy-momentum tensor $T_{\mu\nu}$ is the space-time metric and is dynamical (gravity).

♠ We also believe that the QCD θ-angle is dynamical (QCD axion).

♠ The Yukawa couplings are believed to be dynamical scalars that in string theory are known as (quasi) moduli.

♠ The "coupling" multiplying the identity operator, is the potential of all scalars not belonging to the SM. It is also known as the Cosmological Constant.

• All such potential couplings are known currently as the "portals" of the SM.

• They allow the SM to "interact" with other sectors of the total theory.

• The most sensitive portals correspond to gauge invariant, low-dimension operators of the SM, and they are two:

♠ The Higgs mass term $HH^\dagger$ whose existence is responsible for the Hierarchy problem of the SM.

♠ The field strength of the hypercharge gauge field, $F^Y_{\mu\nu}$ that can couple with the field strength of hidden U(1) gauge boson $F^D_{\mu\nu}$ via an operator of dimension four: $F^D_{\mu\nu}F^{\mu\nu}_Y$.

• If the gauge boson D_μ is light enough, such a coupling is experimentally excluded by about $>$ 10 orders of magnitude.

The string theory picture

• Such a state of affaires is generic in string theory.

• The geometrical picture of the SM depends to a certain degree on the (weakly-coupled) string theory it is embedded into.

• The most clear picture comes from type II orientifolds.

• In that, the SM is realized on a stack of D-branes.

• Other stacks that are at distances longer than the string scale correspond to standard "hidden sectors".

• The type II closed string sector is the "gravitational sector": apart from the graviton it supplies graviphotons (dark photons), (RR) axions and other scalar (quasi) moduli.

• The SM parameters (couplings and vevs) are functions of scalars and vector fluxes and are determined dynamically.

The QFT picture

• In QFT the “hidden sector” particles may arise in a weakly coupled QFT.

• In that case, scalars, axions and vectors may be considered as weakly coupled elementary particles.

• The “hidden” QFT may also be strongly coupled.

• In that case the “hidden sector” particles are tightly-bound composites made of other elementary particles.

• A controllable example of this, is a holographic theory.

• The gauge interactions provide (singlet) bound-states of the generalized (super)-gluons, that are in one to one correspondence with the bulk (string theory) singlet fields.

Attempts at quantizing gravity

• Many attempts in the past tried to capitalize on a winning strategy: resolving non-renormalizable interactions.

• A good example for us will be the low-energy theory of the strong interactions: It is the IR-free (but non-renormalizable) theory of pions, that reminds quite well the problems with quantizing gravity.

• In that theory, it was eventually understood, that one can quantize the low energy degrees of freedom (pions) in the chiral Lagrangian, but this description has a cutoff, $\Lambda \sim GeV$.

• Instead, the high-energy degrees of freedom (quarks+gluons) are different and the QFT associated to them is UV complete (and effectively strongly-coupled in the IR)

• Taking this as clue, it would suggest that the non-renormalizability of the graviton appears because of its compositeness: the graviton is a low-energy bound-state.

• This idea is VERY old: Many attempts were made in the past to construct gravity theories where the graviton is a composite field, made out of more elementary fields, of all types: scalars, fermions, vectors etc.

• All such attempts failed to go beyond the classical and provide a dynamical explanation of why the bound state appears "feature-less" at low energies.

• Holographic theories are modern example of composite gravitons.

• Their interactions are higher-dimensional and non-renormalizable in the usual sense.

• The underlying (string theory) has an effective cutoff at the string scale and that renders it finite.

• It is not UV complete as perturbative string theory cannot answer questions at or above the Planck scale.

• The dual (QFT) description in terms of gluons is however UV complete and well defined at all scales.

Enter holography

• Holography gives a concrete idea on how to realize the graviton and other (gauge-singlet) fields as composites of more elementary fields (like gluons etc).

• The large N limit suppresses large gravitational loop fluctuations.

• Strong coupling suppresses stringy physics.

• If we want observable gravity to be emergent in this sense, we must couple the SM to a holographic theory.

• We are led to consider a (holographic) large N QFT$_N$ coupled in the UV by bifundamental massive "messenger" fields to the SM.

• Much below the messenger mass scale, they can be integrated out to generate direct couplings between the holographic operators and the SM operators.

• In the geometrical picture of the holographic theory this is described by a SM brane inserted and coupled to a (holographic) gravitational bulk theory.

Betzios+Kiritsis+Niarchos

• A priori, all QFT$_N$ operators can couple to all SM operators. However, at low energies only "light" composites are relevant.

♠ One of the special QFT$_N$ operators is $T_{\mu\nu}$ that appears as an external metric for the SM. This generates gravity.

♠ Another class of "protected" operators are instanton densities and η's. They will appear as axions in the SM

♠ A third class corresponds to global conserved currents. They generate "dark" photons or graviphotons.

Preview

- Generic Composite axions.

- Composite axions emerging from a holographic theory.

- Holographic axions and the dynamics of instanton densities.

Preview II

• Axion-like particles (ALPs) are omnipresent in physics beyond the SM

• They were introduced to solve the strong CP problem.

• Because of the protection rendered by PQ symmetry , they can serve as dark matter, dark energy, and drive inflation.

• They can be around without "serving" anything in particular.

• They are ubiquitous in string theory and typical vacua have hundreds of axions.

• Here we will study emergent/composite axions dual to instanton densities that DO NOT have an exact PQ symmetry.

Composite axions

• Consider a large-N gauge theory coupled to the SM via a coupling of the form

$$S_{12} = \lambda \int d^4x \; Tr[F \wedge F] \; Tr[G \wedge G] = \lambda \int d^4x \; O_1(x) O_2(x)$$

and the generating functional of the coupled theory

$$Z(J_1, J_2) = \langle 0| e^{iS_{12} + i\int d^4x \; (J_1(x)O_1(x) + J_2(x)O_2(x))} |0\rangle \quad , \quad e^{iW(J_1,J_2)} \equiv \frac{Z(J_1, J_2)}{Z(0,0)}$$

By performing a Hubbard-Stratonovich transformation we can write

$$e^{iS_{12}} = N_0 \int \mathcal{D}\zeta_1 \mathcal{D}\zeta_2 \; e^{\int d^4x \; \left(-\frac{i}{\lambda}\zeta_1(x)\zeta_2(x) - i\zeta_1 O_1 - i\zeta_2 O_2\right)}$$

that allows us to express the complete generating functional as follows

$$Z(J_1, J_2) = \langle 0| e^{iS_{12} + i\int d^4x \; (J_1(x)O_1(x) + J_2(x)O_2(x))} |0\rangle =$$

$$= N_0 \int \mathcal{D}\zeta_1 \mathcal{D}\zeta_2 \; e^{\int d^4x \; \left(-\frac{i}{\lambda}\zeta_1(x)\zeta_2(x)\right)} Z_1(J_1 - \zeta_1) Z_2(J_2 - \zeta_2)$$

where Z_1, Z_2 are the Schwinger functionals of the respective uncoupled theories

$$Z_1(J_1) = \langle 0| e^{i\int d^4x \; J_1(x)O_1(x)} |0\rangle_1 \quad , \quad Z_2(J_2) = \langle 0| e^{i\int d^4x \; J_2(x)O_2(x)} |0\rangle_2 \quad ,$$

We henceforth work at the quadratic order in which

$$Z_1(J_1) = e^{\frac{i}{2}\int d^4x d^4x' \; J_1(x)J_1(x')G_{11}(x-x')} = e^{\frac{i}{2}\int \frac{d^4p}{(2\pi)^4} J_1(p)J_1(-p)G_{11}(p)}$$

$$G_{11}(x-x') = \langle O_1(x)O_1(x')\rangle_1$$

is the translationally invariant, unperturbed two-point correlation function of O_1 in theory T_1.

• Performing the integral over ζ_1, ζ_2 explicitly, we obtain the quadratic order generating functional (expressed in momentum space) as

$$W(J_1, J_2) = \frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\left[\begin{pmatrix} J_1(p), & J_2(p)\end{pmatrix}\begin{pmatrix} \frac{1}{G_{11}(p)} & -\lambda \\ -\lambda & \frac{1}{G_{22}(p)}\end{pmatrix}^{-1}\begin{pmatrix} J_1(-p) \\ J_2(-p)\end{pmatrix}\right]$$

More explicitly, one can rewrite the matrix as

$$\begin{pmatrix} \frac{1}{G_{11}(p)} & -\lambda \\ -\lambda & \frac{1}{G_{22}(p)}\end{pmatrix}^{-1} = \frac{1}{1-\lambda^2 G_{11}(p)G_{22}(p)}\begin{pmatrix} G_{11}(p) & \lambda G_{11}(p)G_{22}(p) \\ \lambda G_{11}(p)G_{22}(p) & G_{22}(p)\end{pmatrix}$$

• We notice that the interaction between the two theories modifies the non-interacting correlators and create cross-correlations between the two sectors.

• The new correlator for O_2 in momentum space is

$$i\langle O_2(p)O_2(-p)\rangle = \frac{G_{22}(p)}{1-\lambda^2 G_{11}(p)G_{22}(p)} = G_{22}(p) + \lambda^2 \frac{G_{11}(p)G_{22}^2(p)}{1-\lambda^2 G_{11}(p)G_{22}(p)}$$

• We now consider the IR expansion of the correlators.

• In a theory with a single scale, which is also its mass gap m (like YM), the IR expansion in $p \ll m$ reads

$$i\langle O_2(p)O_2(-p)\rangle_2 = b_0 + b_2 p^2 + b_4\ p^4 \log\frac{p^2}{m^2} + \cdots \quad , \quad b_n \sim m^{4-n}$$

• for $p \gg m$ the UV expansion of the (renormalized) correlator is

$$i\langle O_2(p)O_2(-p)\rangle_2 = p^4 \left[\log\frac{p^2}{m^2}\left(a_0 + a_2\frac{m^2}{p^2} + \mathcal{O}\left(\frac{m^4}{p^4}\right)\right) + \right.$$

$$\left. + c_0 + c_2\frac{m^2}{p^2} + \mathcal{O}\left(\frac{m^4}{p^4}\right)\right]$$

• This expansion reflects the fact that at short distances $p \gg m$, the correlator in configuration space asymptotes to the CFT value, proportional to $|x|^{-8}$.

• If the theory has a UV scale Λ, but also other smaller IR scales like $m \ll \Lambda$ then for generic scalar operators, the larger scale dominates the coefficients in the expansion

$$b_n \sim \Lambda^{4-n}\left[1 + \mathcal{O}\left(\frac{m^2}{\Lambda^2}\right)\right]$$

• There is however a scalar operator in the gauge theory that is special and for which this scaling is not valid. This is the (CP-odd) instanton density.

• It is well known from studies in QFT, and holography, that the correlators of the instanton density are UV insensitive.

• The reason is that the θ angle in QCD is not UV-renormalized, as shown rigorously on the lattice.

• This is also true in holography, whereby the bulk axion field dual to the instanton density does not have a potential and the procedure of holographic renormalization allows to derive its correlation functions.

• However, in holographic QCD there is a non-trivial (and non-perturbative) β-function for θ driven by the vev of the instanton density on the (non-trivial) YM vacuum (encoded in the topological susceptibility).

• Notwithstanding this, all θ-dependent contributions to the vacuum energy are cutoff independent.

• Therefore, even though there is a non-trivial UV structure in the gauge theory, the two-point function is insensitive to the the messenger mass.

• *This is an important feature that distinguishes the instanton density operator from all other scalar operators.*

Integrating-in a pseudoscalar

• We would like now to interpret the presence of the interaction S_{12} from the point of view of theory T_2.

• We imagine that we probe theory T_2 and we can perform experiments involving only the operator O_2 of T_2.

• We can represent the effects of T_1 and its interaction to T_2 as coming from an "emergent" dynamical field coupled linearly to O_2.

• We consider a new scalar field χ coupled to the operator O_2 as follows:

$$S_{eff} = \int d^4x \left[\frac{1}{2}\chi \ K \ \chi + \chi \ O_2\right] + S_2 =$$

$$= \int \frac{d^4p}{(2\pi)^4}\chi(p)K(p)\chi(-p) + \chi(p) \ O_2(-p) + S_2$$

• K is a kinetic operator that we want to determine using consistency with the previous results.

• In order to determine the correct form of K, we now compute the O_2 correlator by integrating out the scalar field.

• We find

$$K(p) = -\frac{1}{\lambda^2 G_{11}} \sim \frac{M^8}{G_{11}}$$

• The IR structure of this emergent axion kinetic term is

$$iK(p) = \frac{M^8}{a_0 + a_2 p^2 + a_4 p^4 + \cdots} \simeq \frac{M^8}{a_0}\left[1 - \frac{a_2}{a_0}p^2 + \frac{a_2^2 - a_0 a_4}{a_0^2}p^4 + \cdots\right]$$

If we parametrize

$$iK(p) = f_a^2(p^2 + {m_a}^2) + \mathcal{O}(p^4)$$

we obtain

$$m_a^2 \sim \frac{a_0}{a_2} \quad , \quad f_a^2 \sim \frac{a_2}{a_0^2} M^8$$

• For a generic scalar operator O_1, as argued before, we have $a_n \sim M^{4-n}$ and we obtain

$$m_a^2 \sim M^2 \quad , \quad f_a^2 \sim M^2$$

• Such a scalar is irrelevant at low energy despite the weakness of the interaction.

• On the other hand if the operators $O_{1,2}$ are the instanton densities, then their two-point function is not UV sensitive

$$a_n = \bar{a}_n \, m_1^{4-n} \quad , \quad b_n = \bar{b}_n \, m_2^{4-n}$$

• $m_{1,2}$ are the IR mass scales of the hidden and visible theories.

• If the hidden theory were YM then m_1 is Λ_{YM}. In this special case we obtain instead

$$m_a^2 \sim m_1^2 \quad , \quad f_a^2 = m_1^2 \left(\frac{M}{m_1}\right)^8$$

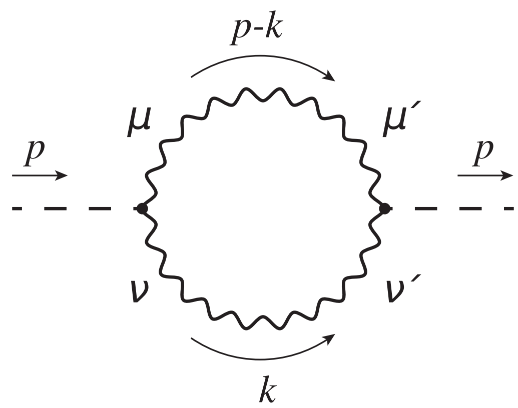
• If $m_1 \ll M$ then this is an emergent weakly-coupled axion-like field that couples to the SM instanton densities.

• We conclude that an emergent axion has a mass determined by the hidden theory scale m_1, and a coupling f_a depending on the messenger scale M.

The SM contributions

• If the axion is elementary then its mass so far would have been zero, because it is protected by the Peccei-Quinn symmetry.

• However SM corrections can contribute to the axion mass.



• In perturbation theory, there is no mass generation (protected by the approximate PQ symmetry of the instanton density.

• However, as is well known the QCD dynamics will generate a mass beyond perturbation theory.

$$-i\langle\chi\chi\rangle(p) = \frac{-i}{K(p) + G_{22}(p)} = \frac{1}{\frac{M^8}{iG_{11}} + iG_{22}}$$

- We obtain the QCD corrections

$$f_r^2 = \frac{M^8}{m_2^6} + \ m_2^2 + \cdots \simeq f_a^2 + \Lambda_{QCD}^2$$

$$m_r^2 = m_1^2 + \frac{m_1^6 m_2^4}{M^8} + \cdots \simeq m_1^2 + \frac{\Lambda_{QCD}^4}{f_a^2} + \cdots$$

- When $m_{1,2} \ll M$ they are subleading.

The holographic emergent axion

- We now assume that T_1 is a large N holographic theory.

- The general action can be written as

$$S = S_1 + S_{12} + S_2 \quad , \quad S_{12} = \lambda \int d^4x \; O_1 O_2$$

$$\langle e^{iS_{12}} \rangle_{T1} = \int_{\lim_{z\to 0} a(x,z) = O_2(x)} \mathcal{D}a \; e^{iS_{\text{bulk}}[a]}$$

- We insert a functional δ-function

$$\langle e^{iS_{12}} \rangle = \int_{\lim_{z\to 0} a(x,z) = \phi(x)} \mathcal{D}a(x,z)\mathcal{D}\phi(x)\mathcal{D}k(x) \; e^{iS_{\text{bulk}}[a] + i\int k(x)(\phi(x) - O_2(x))}$$

- We now integrate $\phi(x)$ first in the path integral to obtain the Legendre transform of the Schwinger functional of the bulk axion which becomes the bulk effective action.

• This corresponds in holography to switching boundary conditions at the AdS boundary from Dirichlet to Neumann, and where $k(x)$ is the expectation value of the operator O_1.

$$\langle e^{iS_{12}}\rangle = \int_{\lim_{z\to 0}\partial_z a(x,z)=z^3 k(x)} \mathcal{D}a(x,z)\mathcal{D}k(x)\ e^{iS_N[a]-i\int k(x)O_2(x)}$$

• We may imagine the SM action as coupled at the radial scale $z_0 \sim 1/M$ to the bulk action.

• We may then rewrite the full bulk+brane action of the emergent axion as

$$S_{total} = S_{bulk} + S_{brane}$$

$$S_{bulk} = M_P^3 \int d^5x\sqrt{g}\left[Z(\partial a)^2 + \mathcal{O}((\partial a)^4)\right]$$

$$S_{brane} = \delta(z-z_0)\int d^4x\sqrt{\gamma}\left[\lambda\hat{a}(x)O_2(x) + M^2(\partial\hat{a})^2 - \Lambda^4\hat{a}^2 + \cdots\right]$$

where $\hat{a}(x) \equiv a(z_0,x)$ is the induced axion on the brane.

• The equation that determines the bulk-to-bulk propagator of the axion is

$$M_P^3\left[\partial_z^2+\left(\frac{Z'}{Z}+4A'\right)\partial_z-e^{-2A}p^2\right]G(p,z)-\delta(z-z_0)(M^2p^2+\Lambda^4)G(p,z)=\delta(z-z_0$$

where $p^2=p^ip^i$ is the (Euclidean) momentum squared.

• The general solution is

$$G(p,z;z_0)=\frac{G_0(p,z;z_0)}{1+(M^2p^2+\Lambda^4)G_0(p,z_0;z_0)}$$

• The propagator on the brane is obtained by setting $z=z_0$ and becomes

$$G(p,z_0;z_0)=\frac{G_0(p,z_0;z_0)}{1+(M^2p^2+\Lambda^4)G_0(p,z_0;z_0)}$$

- For generic holographic RG flows and generic scalars

$$G_0(p,z_0;z_0)=\frac{1}{2(M_P\ \ell)^3}\begin{cases}\frac{\ell^3}{p}, & p\gg R_0\\ \frac{d_0}{m^4}-d_2\frac{p^2}{m^6}-d_4\frac{p^4}{m^8}+\cdots\ , & p\ll R_0.\end{cases}$$

- We may now extract the effective axion parameters in the far IR

$$f_{eff}^2=M^2+N^2\ m^2\quad ,\quad m_{eff}^2=\frac{\Lambda^4+N^2\ m^4}{f_{eff}^2}\ .$$

- When $Nm>>M$ this reduces to the non-holographic result.

- The axion interaction is 4d at short and long distances but can be 5d in intermediate distances.

Phenomenology

- Axions are of various types:

♠ *Dark Matter axions (with QCD constraints)*.

$$10^{-25}\text{ eV} < m_a^{DM} < 10^{-18}\text{ eV}$$

$$10^{10}\text{ GeV} < f_a^{DM} < 10^{16}\text{ GeV}$$

♠ *Dark Energy axions (with QCD constraints)*.

$$10^{-33}\text{ eV} < m_a^{DE} < 10^{-30}\text{ eV}$$

$$10^{10}\text{ GeV} < f_a^{DE} < 10^{15}\text{ GeV}$$

♠ *Axions as Inflatons*. These are highly model dependent and there are no strict bounds.

♠ *Heavy Axions*. Masses are larger than 1 eV. The allowed axion masses and lifetimes are

$$m_a > 10 \text{ MeV} \quad \text{and} \quad \tau_{a\gamma} < 10^{-2} \text{ s}$$

or

$$m_a < 10 \text{ eV} \quad \text{or} \quad \tau_{a\gamma} > 10^{24} \text{ s}$$

♠ *QCD axion*.

$$10^{-12} \text{ eV} < m_a^{QCD} < 10^{-3} \text{ eV}$$

$$10^{9} \text{ GeV} < f_a^{QCD} < 10^{15} \text{ GeV}$$

• Generic composite axions can play the role of inflatons or heavy axions. They can be more general but their consequences need reexamination, as in some regimes their kinetic terms are non-local.

• Holographic composite axions can play also the role of dark matter, dark energy or QCD axions.

Holographic axions

• Axions abound in string theory: all forms including the IIB axion are sources of such examples upon compactification.

• Because of the associated gauge symmetry, they have no potential in perturbation theory.

• This implies the existence of a perturbative global symmetry.

• This symmetry is always broken by string instantons to a discrete symmetry.

• The breaking may or may not generate a potential for the axion.

• Such a potential is exponentially suppressed in the coupling constant.

• In holography, it is $\sim e^{-N}$.

• In the prototypical AdS/CFT paradigm the type-IIB RR axion is dual to the YM Instanton density

$$a \quad \leftrightarrow \quad \frac{i}{64\pi^2} Tr[F \wedge F] \qquad\qquad (Euclidean)$$

• The topological properties of this operator and its irrelevance in perturbation theory are related to the perturbative PQ symmetry of axions in type IIB string theory.

• According to the AdS/CFT dictionary the near AdS boundary expansion of the axion is

$$a(r) = a_{UV} + r^d \, Q + \cdots$$

where a_{UV} is the source and Q is related to the vev of the instanton density.

• The source is related in a many to one ways to the field theory source, the θ-angle as

$$a_{UV} = c\, \frac{\theta_{UV} + 2\pi k}{N_c}$$

where $\theta_{UV} \in [0, 2\pi)$, $k \in \mathbf{Z}$ and c a dimensionless number of $\mathcal{O}(N_c^0)$.

General properties

• There is a general line of arguments about the θ dependance of the vacuum energy or partition function of a gauge theory at large N_c.

Witten

• The proper variable that is kept fixed in the large N_c limit is $\zeta = \frac{\theta}{N_c}$.

$$F(\theta) = N_c^2\ f(\zeta) \quad , \quad f(\zeta) = f(-\zeta) \quad , \quad F(\theta) = F(\theta + 2\pi)$$

and

$$F(\theta) = N_c^2\ f(0) + \chi\ \theta^2 + \mathcal{O}\left(\frac{\theta^4}{N_c^2}\right)$$

at tree level in string theory.

• $\chi > 0$ is the famous topological susceptibility of the gauge theory.

• The periodicity of is restored in the large N_c theory by the existence of an infinite number of saddle points (known as oblique vacua)

$$F_k(\theta) = N_c^2\ f\left(\frac{\theta + 2\pi k}{N_c}\right) = N_c^2\ f(0) + \chi\ (\theta + 2\pi k)^2 + \mathcal{O}\left(\frac{(\theta + 2\pi k)^4}{N_c^2}\right)$$

• The true ground state has free energy that is discontinuous

$$E(\theta) - E(0) = Min_{k \in Z} \;\; \chi \; (\theta + 2\pi k)^2$$

• The minimum of $E(\theta)$ is always at $\theta = 0$ because the integrant is real and positive in that case.

• There is a non-analyticity for $\theta = \pi$ that signals the existence of two vacuum states and a phase transition.

Witten

• In string theory the correct N_c dependence is there because the axion is a RR state

$$S = \int d^{10}x \; e^{-2\phi} \left[R - \frac{1}{2} e^{2\phi} (\partial a)^2 + \cdots \right]$$

• Remembering that $\lambda \sim N_c e^{\phi}$ is fixed, the scaling follows.

• The fact that $\chi \neq 0$ is related in QCD to the resolution of the $U(1)_A$ problem.

General axion flows

- We will parametrize the bulk axion action as follows

$$S = M_p^{d-1} \int d^{d+1}x\sqrt{-g}\left[R - \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi - \frac{1}{2}Y(\Phi)g^{ab}\partial_a a\partial_b a - V(\Phi)\right] + S_{GHY},$$

  - There are always couplings of the axion a to scalars, Φ.

- Standard RG flows have Φ varying along the flow but $a = \theta =$ constant.

- The $a = \theta =$ constant solution is always a regular solution.

- This is the solution relevant for $N = 4$ sYM and corresponds to

$$\langle Tr[F \wedge F]\rangle = 0$$

- If $\dot{a}(u) \neq 0$ then $\langle Tr[F \wedge F]\rangle \neq 0$.

For a Lorentz invariant ansatz

$$ds^2 = du^2 + e^{2A(u)}dx^\mu dx_\mu \quad , \quad \Phi(u) \quad , \quad a(u)$$

the axion equation is

$$\dot{a} = \frac{Q}{Y e^{dA}} \, ,$$

- Q is proportional to the vev $\langle Tr[F \wedge F] \rangle$

- If the flow ends at a finite endpoint Φ_*, the flow is regular if Y diverges sufficiently fast as $\Phi \to \Phi_*$.

- However, Y coming from string theory effective actions never has poles in the middle of scalar space.

- The conclusion is that in regular flows with a standard holographic CFT in the IR limit, the axion flow is always trivial.

- In this parametrization, possible "regular" axion flows can appear only if $\Phi \to \pm\infty$.

- The function $Y(\Phi)$ must properly diverge so that the flow is regular (à la Gubser).

- The black D_4 Witten solution describing a non-susy YM on $R^4 \times S^1$ is a good guiding principle

• In this 5d YM theory the analogue f the instanton term is

$$S_{inst} = \int d^4x \int d\tau \; A \wedge Tr[F \wedge F]$$

where A is the RR one-form of type II string theory.

• Upon dimensional reduction to 4d

$$S_{CP-odd,D_4} \sim \int d\tau \; A_\tau \int d^5x \; Tr[F \wedge F] \quad , \quad \theta_{YM} \sim \int d\tau \; A_\tau = 2\pi R_\tau A_\tau$$

• Therefore A_τ plays here the role of the axion, and it is protected by the gauge symmetry of the RR one-form.

• The radius of the cigar geometry becomes the scalar Φ in our 5d bulk description. The tip of the cigar corresponds to $\Phi \to \infty$.

• The correct boundary (regularity) condition for A_τ at the tip of the cigar is $A_\tau \to 0$.

• With this boundary condition Witten has found the correct solution for the running of the θ angle.

• The solution looks singular in 5 bulk dimensions as $\Phi \to \infty$. But in 6d the solution is regular, and the singularity is an artifact of the KK reduction.

• The lesson for the general case we discuss is that in the case of non-trivial flows that correct IR regularity condition is

$$a(u_{IR}) = 0 \quad , \quad \Phi(u_{IR}) = \pm\infty$$

• This implies the axion solution

$$a(u) = Q \int_{u_{IR}}^{u} \frac{du}{Y e^{dA}} \quad , \quad a_{UV} = Q \int_{u_{IR}}^{u_{UV}} \frac{du}{Y e^{dA}}.$$

• This is as it should: given a_{UV}, the IR condition fixes the vev Q as a function of a_{UV}.

The first order formalism

- We may rewrite the gravitational-dilaton-axion equations as

$$\dot{A} = -\frac{W(\Phi)}{2(d-1)} \quad , \quad \dot{\Phi} = S(\Phi) \quad , \quad \dot{a} = \text{sign}(Q)\frac{\sqrt{T}}{Y}\,.$$

where

$$T(\Phi) \equiv \frac{Q^2}{e^{2dA}}\,.$$

and

$$S^2 - W'S + \frac{T}{Y} = 0 \quad , \quad \frac{T'}{T} = \frac{d}{d-1}\frac{W}{S}$$

$$\frac{d}{4(d-1)}W^2 - \frac{S^2}{2} - \frac{T}{2Y} + V = 0\,.$$

- S and T can be solved algebraically and W satisfies a second order non-linear differential equation.

- The two integration constants are interpreted as the vevs of Φ and a.

The general IR boundary condition

• In the IR we parametrize

$$V \sim e^{b\Phi} \quad , \quad b \leq \sqrt{\frac{2d}{d-1}} \quad , \quad Y \sim e^{\gamma\Phi}$$

• We find two types of IR solutions.

♠ The first back-reacts to leading order to the Φ flow and fixes completely the integration constant Q in terms of other parameters.

• Such a solution is holographically unacceptable.

♠ The second solution is subleading to the Φ and Q is a free parameter as it should.

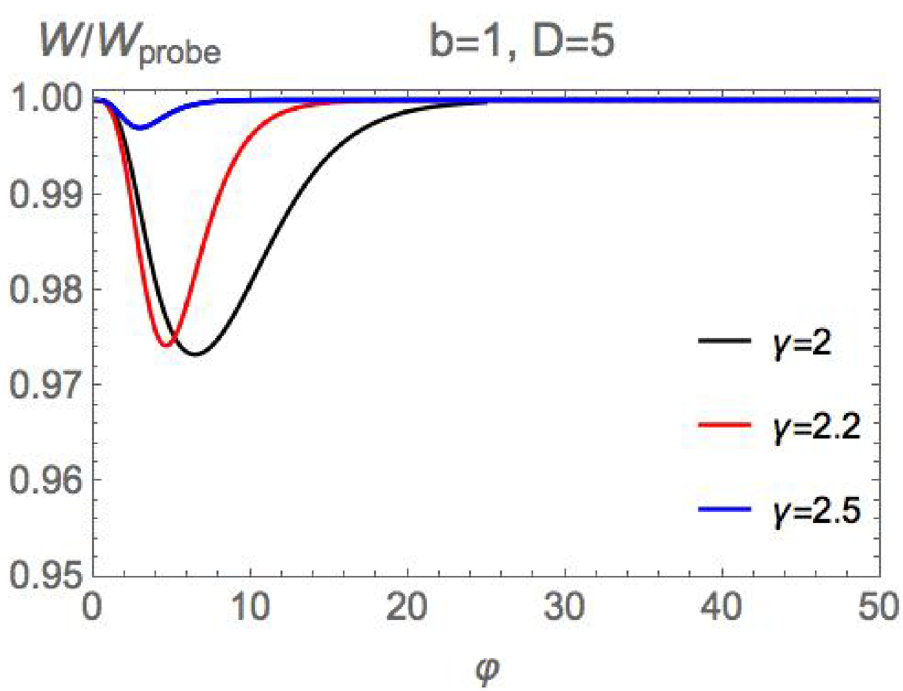
• Therefore the IR regular solution is unique.

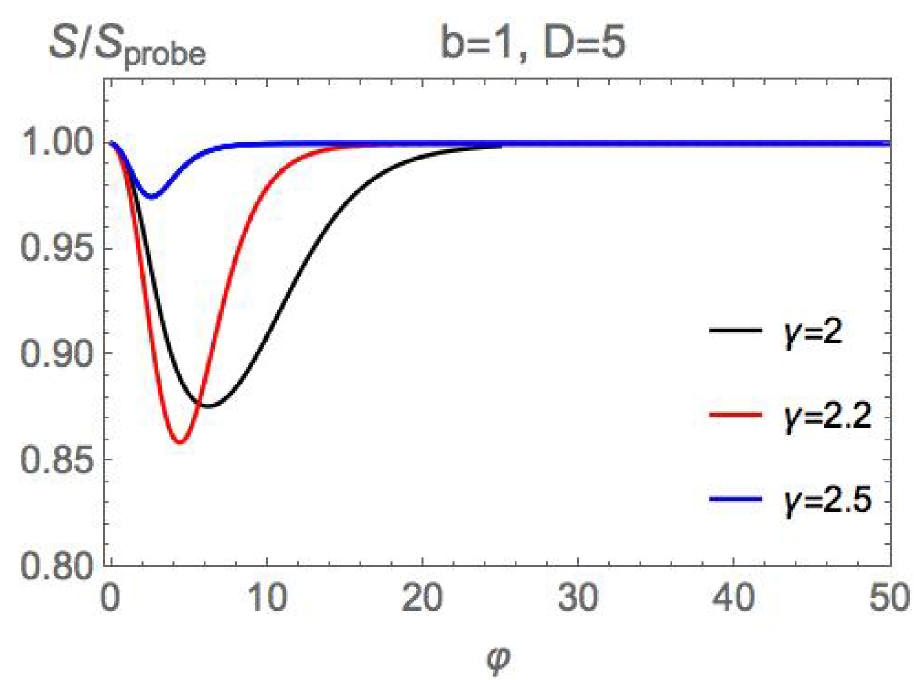
• The analogue of the Gubser bound for γ is

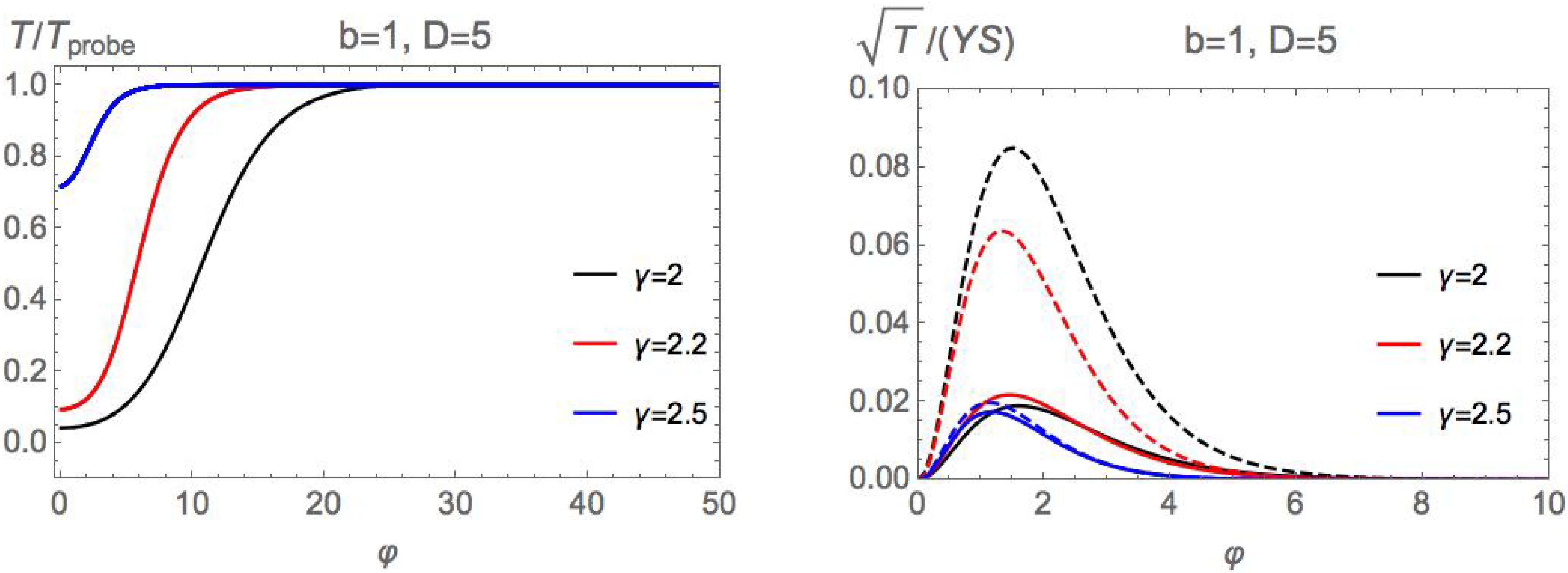
$$\gamma \geq \frac{2d}{(d-1)b} - b,$$

• Therefore, the correct axion solution is subleading, both in the UV and the IR.

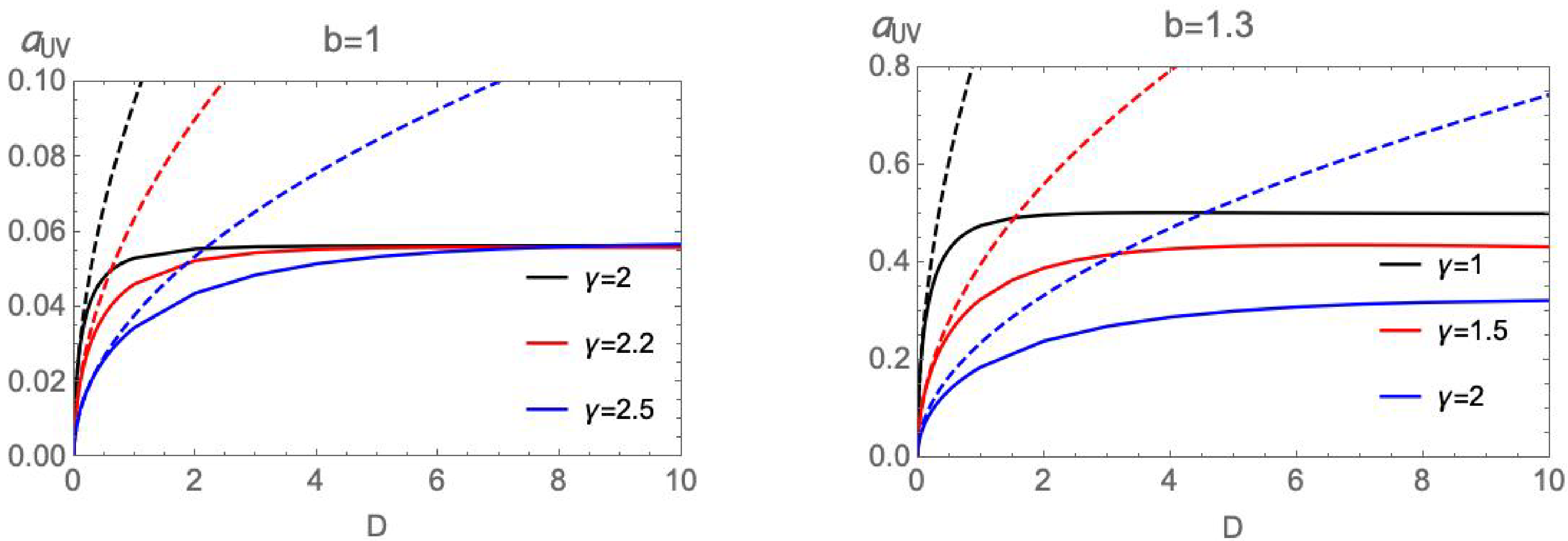
• For small Q (and therefore small θ) it can be found perturbatively around the Φ flow.







T/T_probe
b=1, D=5
1.0
0.8
0.6
0.4
0.2
0.0
0
10
20
30
40
50
φ
γ=2
γ=2.2
γ=2.5
√T /(YS)
b=1, D=5
0.10
0.08
0.06
0.04
0.02
0.00
0
2
4
6
8
10
φ
γ=2
γ=2.2
γ=2.5



Plot of a_{UV} as a function of the constant D that is related to the vev of the dual instanton density, for $b = 1$, $\gamma = 2, 2.2, 2.5$ (**left**) and $b = 1.3$, $\gamma = 1, 1.5, 2$ (**right**). The dashed and solid lines correspond to the probe and backreacted solutions, respectively. It is apparent from the backreacted results that as $D \to \infty$, a_{UV} saturates, and the range of possible a_{UV} values is compact.

• The compactness of the range of a_{UV} can be shown analytically:

$$|a_{UV}| \leq \int_0^\infty \frac{d\Phi}{\sqrt{Y}} \quad \to \quad finite\,,$$

• Since

$$a_{UV} = \frac{\theta_{UV} + 2\pi k}{N_c} \quad , \quad k \in Z$$

we deduce that the number of distinct (saddle-point) solutions with the same θ_{UV} is equal to the number of possible values the integer k can take.

• This number is

$$Z \ni n = \left\lfloor \frac{N_c \; a_{UV}^{max}}{2\pi} \right\rfloor$$

• For large N_c this is a large number, that should be compared to a similar number emerging from the chiral Lagrangian.

The on-shell effective action

• The expression of the regularized on-shell action is the same as in the case of Φ only flows

$$S_{\text{on-shell}} = M_p^{d-1} V_d \left[e^{dA} W \right]_{\text{UV}} .$$

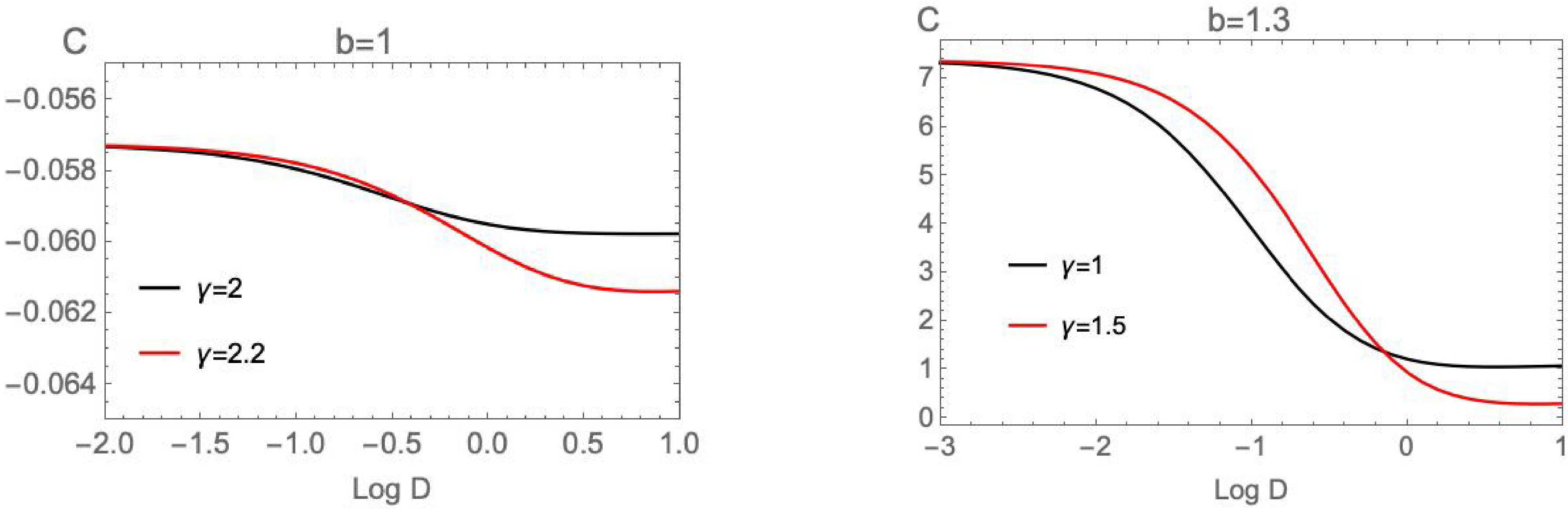
but $W(\Phi)$ and $e^A(\Phi)$ are affected by the axion flow.

• After renormalizing the result and then taking the cutoff to infinity we obtain

$$S^{\text{ren}}_{\text{on-shell}} = (M_p \ell)^{d-1}\, V_d\, |\Phi_-|^{\frac{d}{\Delta_-}} \Big(C(q) - C_{ct} \Big).$$

• For relevant Φ (like in YM) $|\Phi_-|^{\frac{d}{\Delta_-}} \to \Lambda^4_{YM}$ and

$$S_k \sim \Lambda^4_{\text{YM}} \tilde{C} \left(\frac{\theta_{UV} + 2\pi k}{N_c} \right)$$



Plot of C as a function of $\log D$ for $b = 1$ and $\gamma = 2, 2.2$ (**left**) and $b = 1.3$ and $\gamma = 1, 1.5$ (**right**).

The instanton density two-point function

• The Euclidean two-point function of the instanton density in any positive theory is negative definite.

$$C(x) \equiv \langle O(x)O(0)\rangle \quad , \quad O = Tr[F \wedge F]$$

• Despite this, it is known that the topological susceptibility in YM is positive.

$$\chi \sim \int d^4x \; C(x) \;\; > \;\; 0$$

• The lattice says so, and when $m_q = 0$ it is required from $m^2_{\eta'} > 0$.

• These statements are compatible because the correlator contains non-trivial contact terms.

• Moreover, the Fourier-space correlator is ill-defined and requires renormalization.

• All of the above, plus the independence of the correlator and contact terms on the UV cutoff can be obtained from the holographic description advanced before.

Conclusions and Outlook

• Axions may arise from strongly-coupled gauge theories as composite instanton densities.

♠ Their properties in theories with a semiclassical limit (large N_c) can be studied, and the properties of the composite axions determined.

♠ Such axions can couple to the SM once the hidden QFT that generates is coupled to the SM in the UV.

♠ Both composite and holographic axions can play several phenomenological roles: QCD axion, dark matter and dark energy axions, inflatons, or just heavy ALPs.

♠ The non-perturbative dynamics of instanton densities (holographic axions) can be studied in enough generality using a theory of a few scalars and pseudoscalars without potential.

♠ The non-trivial axions solutions exist only when solutions for the scalars run to the boundary of their space

♠ There is a unique regularity condition that gives sensible holographic results.

♠ The space of axion sources is always finite.

♠ The on-shell action has the form expected from general gauge theory principles.

• All of this can be used to obtain a holographic theory that at the same time self-tunes the cosmological constant and provides small Higgs masses solving in this way the hierarchy problem.

Charmousis+Kiritsis+Nitti, Hamada+Kiritsis+Nitti+Witkowski

.

THANK YOU

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 3 minutes
- The String Theory Picture 4 minutes
- The QFT Picture 5 minutes
- Attempts at quantizing gravity 8 minutes
- Enter holography 10 minutes
- Preview 11 minutes
- Preview II 12 minutes
- Composite axions 19 minutes
- Integrating-in a pseudoscalar 25 minutes
- The SM contributions 28 minutes
- The holographic axion 35 minutes
- Phenomenology 37 minutes

- Holographic axions 40 minutes
- General properties 43 minutes
- General axion flows 50 minutes
- The first order formalism 51 minutes
- The general IR boundary condition 56 minutes
- The on-shell effective action 58 minutes
- Conclusions and outlook 60 minutes

• Regular Curved RG flows 61 minutes