Flowing to $\mathcal{N}=3$ Chern–Simons-matter theory

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arXiv:1910.06866
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Iberian Strings, January 16th 2020

Motivation: a supersymmetry-enhancing RG flow

- Gaiotto and Yin studied superconformal Chern–Simons-matter theories in [0704.3740]
- An RG flow was conceived between an $\mathcal{N} = 2$ theory with three chiral multiplets in the UV and the unique $\mathcal{N} = 3$ superconformal theory, with two chirals, in the IR
- Since both fixed points have a holographic dual in massive IIA [1504.08009], we wondered whether there is a dual to the GY RG flow

Motivation: the (weak coupling) RG flow by GY [0704.3740]

- Massive IIA ensures that we have a CS term on the worldvolume of D2-branes [0901.0969]

$$S_{CS} = \frac{k}{4\pi} \int \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right) + \cdots$$

- The SU(N) Yang–Mills term is irrelevant compared to the CS one; YM term can be integrated out and a superconformal theory may exist

- In the UV we have SU(3) global symmetry, and

$$\mathcal{W}_{UV} = \operatorname{tr} \Phi^1 [\Phi^2, \Phi^3]$$

with R-charge assignment $\Delta_{\Phi^i} = 2/3$ for each of the three chiral fields

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- The SU(N) Yang–Mills term is irrelevant compared to the CS one; YM term can be integrated out and a superconformal theory may exist

- In the IR we have SU(2) global symmetry, and

$$\mathcal{W}_{IR} = \frac{2\pi}{k}\,\mathrm{tr}\,[\Phi^1, \Phi^2]^2$$

  with R-charge assignment $\Delta_{\Phi^i} = 1/2$ for both chirals

Motivation: the (weak coupling) RG flow by GY [0704.3740]

$$\mathcal{W}_{UV\ \text{def}} = \text{tr}\,\Phi^1[\Phi^2, \Phi^3] + \frac{M}{2}\,\text{tr}(\Phi^3)^2$$

- To trigger the flow add a scale to the UV theory: a mass deformation for one of the chirals
- Naive dimension $M = 2/3$ from R-charge analysis. Instead impose that each term makes $\mathcal{W}$ marginal separately, then $\Delta_{\Phi^{\{1,2\}}} = 1/2$ and $\Delta_{\Phi^3} = 1$
- Integrate out Φ^3
- And coupling runs to $\frac{1}{M} = \frac{4\pi}{k}$ [0704.3740]

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$$\mathcal{W}_{\Phi^3 \text{ out}} = \frac{1}{2M} \operatorname{tr} [\Phi^1, \Phi^2]^2$$

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$$\mathcal{W}_{\Phi^3 \text{ out}} = \frac{2\pi}{k} \operatorname{tr} [\Phi^1, \Phi^2]^2 = \mathcal{W}_{IR}$$

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A strong coupling perspective [1910.06866]

- The free energy of SU(N)$_k$ CS coupled to N_f chirals, on S^3, can be obtained by localisation [1507.05817]

$$F = \frac{3\sqrt{3}\,\pi}{20 \cdot 2^{1/3}} \left[1 + \sum_{a=1}^{N_f} (1-\Delta_a)\left[1 - 2\,(1-\Delta_a)^2\right]\right]^{2/3} k^{1/3}\, N^{5/3}$$

- SU(N$_f$) and F-maximization imply

$$\begin{aligned} &\text{UV:} \qquad N_f = 3 \qquad \Delta_1 = \Delta_2 = \Delta_3 = 2/3 \\ &\text{IR:} \qquad N_f = 2 \qquad \Delta_1 = \Delta_2 = 1/2 \end{aligned}$$

- With $F_{\mathcal{N}=2} > F_{\mathcal{N}=3}$, thus an RG flow may exist [1103.1181]

ISO(7) gauged-sugra and mIIA supergravity [1508.04432, 1509.02526]

- Massive IIA sugra has solutions dual to CS-matter theories with simple gauge groups (e.g. $\mathrm{SU(N)}_k$)

- Brane configurations are quite complicated (70 scalars and metric): better go to consistent truncations as organisational and constructive principle

- mIIA can be consistently truncated on S^6 to a maximal supergravity in 4d with gauge group ISO(7)

ISO(7) gauged-sugra and mIIA supergravity [1508.04432, 1509.02526]

- The gauging is of dyonic type, with the magnetic coupling m related to the Romans mass and the dual CS level

$$F_0 = m = \frac{k}{2\pi\ell_s}$$

- There are several AdS_4 solutions dual to CS theories with adjoint matter and simple non-Abelian gauge groups

- An example is given in [1504.08009], where the holographic dual of $\mathcal{N}=2$ $\mathrm{SU(N)}_k$ CS-matter theory with SU(3) global symmetry was proposed

$\mathcal{N} = 1$ sugra coupled to 4 chiral fields [1910.06866]

- Recall: ISO(7) gauged supergravity comes with 70 scalars, and that is a lot to chew

- Fortunately we can restrict to consistent truncations, in particular a $\mathcal{N} = 1$ sugra coupled to 4 chiral fields

$$K = -2 \sum_{i=1}^{3} \log[-i(z_i - \bar{z}_i)] - \log[-i(z_4 - \bar{z}_4)]$$
$$W = 2\,m + 2\,g\,\left[\,4\,z_1\,z_2\,z_3 + \left(z_1^2 + z_2^2 + z_3^2\right)\,z_4\,\right]$$

- This model captures supersymmetric AdS_4 solutions of 4d ISO(7) supergravity, and they all solve the BPS conditions

AdS_4 solutions

$\mathcal{N} = 1$ & G_2

[1209.3003]

$\mathcal{N} = 2$ & SU(3)×U(1)

[1504.08009]

$\mathcal{N} = 3$ & SO(4)

[1410.0711]

$\mathcal{N} = 1$ & SU(3)

[1508.04432]

$\mathcal{N} = 1$ & U(1)

[1910.06866]

$\mathcal{N} = 1$ & U(1)

[1910.06866]

AdS_4 solutions

$\mathcal{N} = 1$ & G_2

$V \approx -19.987$

$\mathcal{N} = 2$ & $SU(3)\times U(1)$

$V \approx -20.785$

$\mathcal{N} = 3$ & $SO(4)$

$V \approx -23.277$

$\mathcal{N} = 1$ & $SU(3)$

$V \approx -23.796$

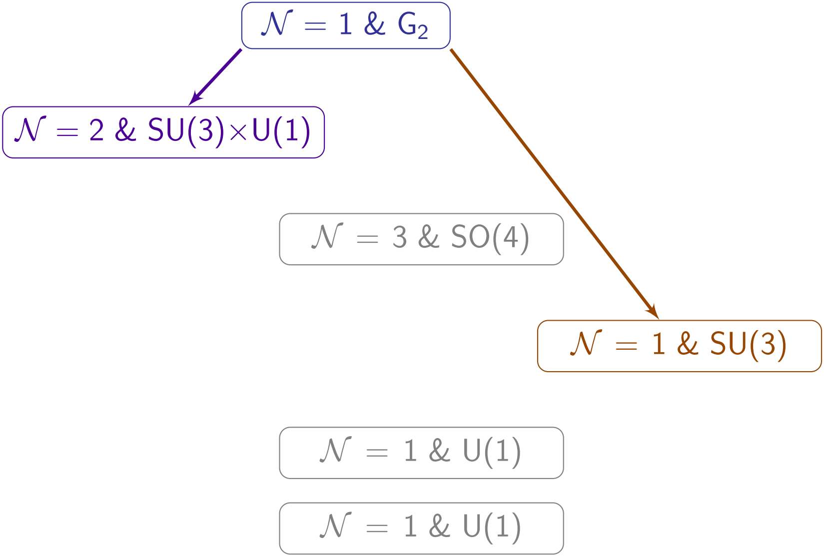
$\mathcal{N} = 1$ & $U(1)$

$V \approx -25.697$

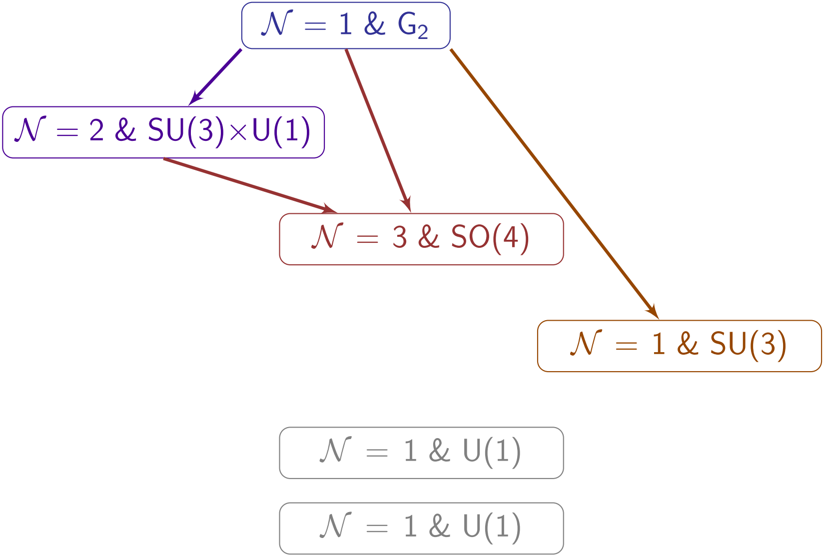
$\mathcal{N} = 1$ & $U(1)$

$V \approx -35.610$

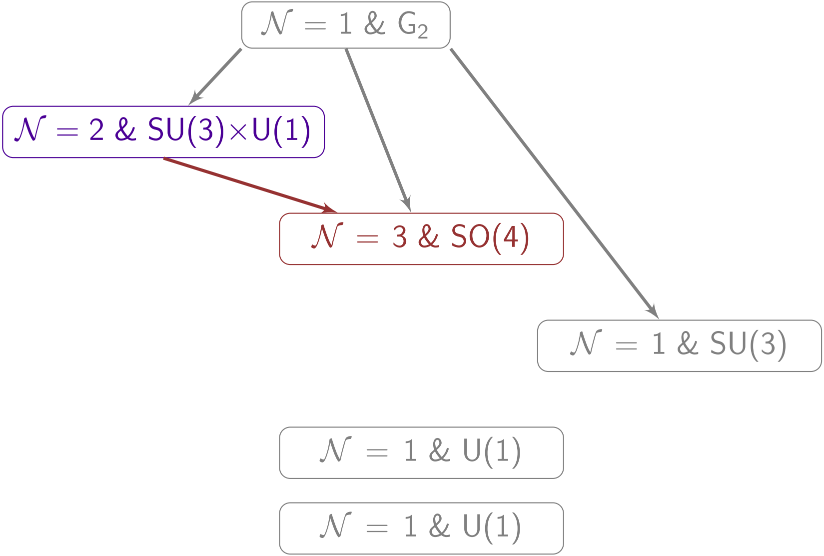
Domain walls between AdS_4 solutions [1605.09254]



Domain walls between AdS_4 solutions [1605.09254,1910.06866]



Flowing to $\mathcal{N}$=3 AdS$_4$ solution in the IR [1910.06866]



Flowing to $\mathcal{N}=3$ AdS solution in the IR

- The construction of the Domain Wall is performed numerically from the IR solution with SO(4) symmetry

$$z_1(\rho) = -\bar{z}_2(\rho) = z_1^*$$

$$z_3(\rho) = z_3^*$$

$$z_4(\rho) = z_4^*$$

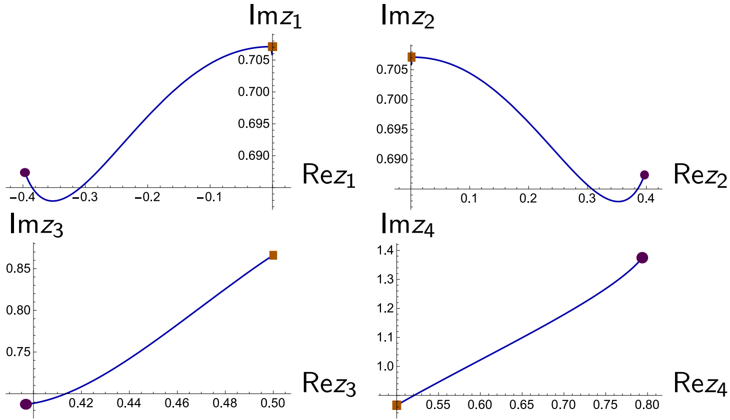
Flowing to $\mathcal{N}=3$ AdS solution in the IR

- The deviation from the AdS_4 solution is given by 2 parameters

$$
\begin{aligned}
z_1(\rho) = -\bar{z}_2(\rho) = z_1^* &+ \left(\frac{1+(2+\sqrt{3})i}{2\cdot 2^{1/3}}\right)\zeta_1\, e^{\frac{\sqrt{3}\,\rho}{L_{IR}}} \\
&+ \left(\frac{1-(2-\sqrt{3})i}{2\cdot 2^{1/3}}\right)\zeta_2\, e^{\frac{(\sqrt{3}-1)\rho}{L_{IR}}} + \cdots \\
z_3(\rho) = z_3^* &+ \left(-\frac{1-(2+\sqrt{3})i}{2\cdot 2^{1/3}}\right)\zeta_1\, e^{\frac{\sqrt{3}\,\rho}{L_{IR}}} \\
&+ \left(\frac{3-\sqrt{3}}{6\cdot 2^{1/3}} + \frac{9-5\sqrt{3}}{6\cdot 2^{1/3}}i\right)\zeta_2\, e^{\frac{(\sqrt{3}-1)\rho}{L_{IR}}} + \cdots \\
z_4(\rho) = z_4^* &+ \left(-\frac{(\sqrt{3}+1)-(\sqrt{3}-1)i}{2\cdot 2^{1/3}}\right)\zeta_1\, e^{\frac{\sqrt{3}\,\rho}{L_{IR}}} \\
&+ \left(\frac{2^{2/3}}{3}(3-2\sqrt{3}) - \frac{2^{2/3}}{\sqrt{3}}i\right)\zeta_2\, e^{\frac{(\sqrt{3}-1)\rho}{L_{IR}}} + \cdots
\end{aligned}
$$

Flowing to $\mathcal{N}=3$ AdS solution in the IR

- The domain wall dual to GY-RG flow is found by relating appropriately the two parameters ($\zeta_2^{\frac{1}{1-\sqrt{3}}} \simeq 15.54\, \zeta_1^{\frac{1}{-\sqrt{3}}}$)



Flowing from $\mathcal{N}=2$ AdS solution in the UV

To make some connection with the RG flow by Gaiotto and Yin, what is the UV description?

scalar/pseudoscalar	SU(3)×U(1)	M^2L^2	Δ	Osp(4|2) multiplet
$Z^a\bar{Z}_b - \frac{1}{3}\delta^a_b Z^c\bar{Z}_c$	$\mathbf{8}_0$	-2	1	massless vector
$Z^{(a}Z^{b)}$ $\bar{Z}_{(a}\bar{Z}_{b)}$	$\mathbf{6}_{-4/3}$ $\mathbf{6}_{4/3}$	$-\frac{20}{9}$	$\frac{4}{3}$	hypermultiplet
$Z^a\bar{Z}_a - 3Z^4\bar{Z}_4$	$\mathbf{1}_0$	$3-\sqrt{17}$	$\frac{1+\sqrt{17}}{2}$	long vector
$\mathrm{Re}(Z^4Z^4)$	$\mathbf{1}_0$	$3+\sqrt{17}$	$\frac{5+\sqrt{17}}{2}$	long vector
$\chi^a\bar{\chi}_b - \frac{1}{3}\delta^a_b\chi^c\bar{\chi}_c$	$\mathbf{8}_0$	-2	2	massless vector
$\chi^{(a}\chi^{b)}$ $\bar{\chi}_{(a}\bar{\chi}_{b)}$	$\mathbf{6}_{2/3}$ $\bar{\mathbf{6}}_{-2/3}$	$-\frac{14}{9}$	$\frac{7}{3}$	hypermultiplet
$\mathrm{Re}(\chi^4\chi^4)$	$\mathbf{1}_{-2}$	2	$\frac{3+\sqrt{17}}{2}$	long vector
$\mathrm{Im}(\chi^4\chi^4)$	$\mathbf{1}_2$	2	$\frac{3+\sqrt{17}}{2}$	long vector

Flowing from $\mathcal{N}=2$ AdS solution in the UV

- Deviations from AdS_4 governed by modes with $e^{-\frac{\tilde{\Delta}}{L_{UV}}\rho}$ dependence

$$\tilde{\Delta} = \frac{1-\sqrt{17}}{2},\ \frac{3-\sqrt{17}}{2},\ \frac{2}{3},\ 1(\times 2),\ \frac{4}{3},\ \frac{1+\sqrt{17}}{2},\ \frac{3+\sqrt{17}}{2}$$

- The constant of integration associated to $\tilde{\Delta} = 2/3$ corresponds to the only source term

- Recall that $\Delta_{1,2,3} = 2/3$, with usual assignment $[\theta^\alpha] = -1/2$ this gives $[\chi^a] = 7/6$, so a fermion bilinear has $\Delta = 7/3$, thus we have a mass deformation

Conclusions

- CS-matter theories with simple non-Abelian gauge groups can be studied holographically considering ISO(7)-gauged supergravity in 4d, or consistent truncations of it

- In particular, we have focused on a 4d supergravity coupled to 4 chirals that retains AdS solutions dual to conformal vacua and the domain walls connecting them

- Within that truncation we focused on the solution analogous to the GY RG flow: from an $\mathcal{N} = 2$ SU(3) CSm in the UV and to an $\mathcal{N} = 3$ SU(2) fixed point, via a mass deformation

Thank you