Holographic complex CFTs

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Introduction

- Renormalization Group (RG): organize physics according to scales.
- Fixed points (FPs) of the RG $\rightarrow$ scale (conformal) invariance. Control second order phase transitions: infinite correlation length.
- Conformal field theories (CFTs) defined at these fixed points.
- Conformallity lost through fixed point annihilation (FPA) [Kaplan, Lee, Son, Stephanov '09].

- FPA gives rise to exponential hierarchies: weakly first-order phase transitions or walking.

- Appears in many physical systems:
  - Conformal window of QCD and $3d$ QED.
  - Superconducting transitions in Abelian-Higgs models.
  - Ferromagnetic transitions in Potts models.
  - O(N) models ...

Fixed-point annihilation

- RG flow captured by β-functions for couplings g

$$\beta(g) = \frac{\partial g}{\partial \log \mu}$$

- Fixed points are solutions to $\beta(g) = 0$.

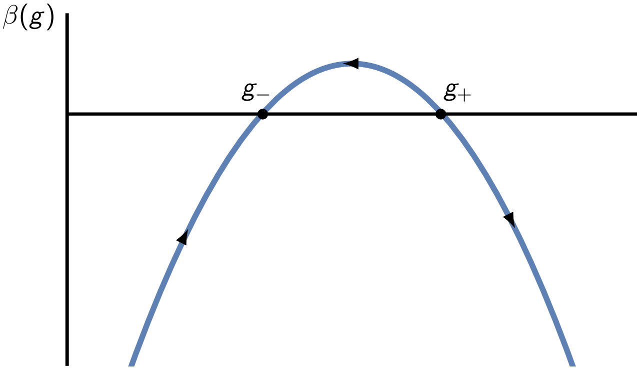
- FPA when β of the form

$$\beta(g) = (\alpha - \alpha_*) - (g - g_*)^2 + \mathcal{O}(g - g_*)^3$$

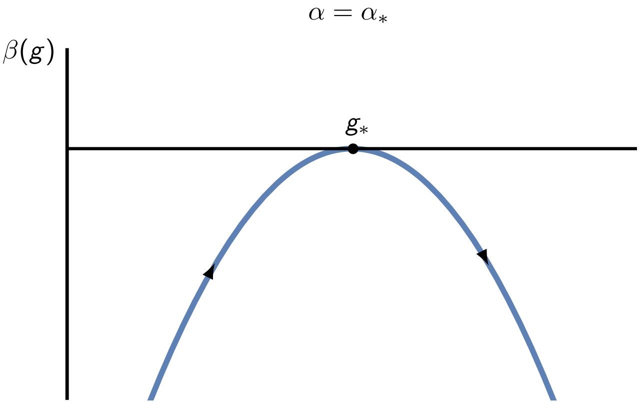
with α an external, tunable parameter

$$\frac{N_f}{N_c},\ N_H,\ Q,\ d\ \ldots$$

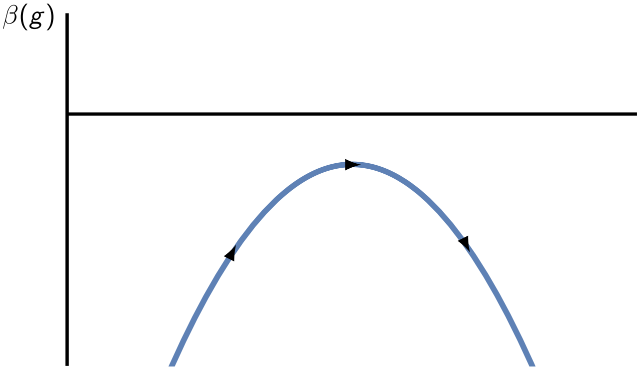
$\alpha > \alpha_*$



$$g_\pm = g_* \pm \sqrt{\alpha - \alpha_*}.$$


$\alpha = \alpha_*$
$\beta(g)$
g_*

$\alpha < \alpha_*$



$$\log \frac{\mu_{UV}}{\mu_{IR}} = \frac{\pi}{\sqrt{\alpha_* - \alpha}} \qquad \text{when} \qquad |g_{UV,IR} - g_*| \gg \sqrt{\alpha_* - \alpha}$$

Miransky scaling or Walking.

Complex CFTs

- Zeroes do not disappear but move to $\mathbb{C}$: complex fixed points.

$$g_\pm = g_* \pm i\sqrt{\alpha_* - \alpha}$$

  Walking flow passes in between.

- Define complex CFTs at these complex conjugate FPs [Gorbenko, Rychkov, Zan '18]. Exotic, but some properties known
  - Non-unitary.
  - Come in pairs.
  - Complex conjugate pair has spectrum of complex conjugate Δ.

- Operator associated to g almost marginal

$$\Delta_\pm = d + \beta'(g_\pm) \simeq d \mp 2i\sqrt{\alpha_* - \alpha}$$

  in terms of which

$$\log \frac{\mu_{UV}}{\mu_{IR}} = \frac{2\pi}{|\mathrm{Im}\,\Delta|}$$

Holographic realization

- (Complex)FPs often perturbative. Holography provides non-perturbative definition with expected characteristics implemented.
- Previous realization of FPA in terms of BF bound violation (real FP unstable) [Kaplan, Lee, Son, Stephanov '09]. Propose alternative, better suited for FPA and cFPs.
- In its most basic incarnation, holographic duality between AdS_{d+1} spaces and d-dimensional CFTs. AdS solution to

$$(R - \Lambda) \qquad \text{with} \qquad \Lambda = -\frac{d(d-1)}{L^2}$$

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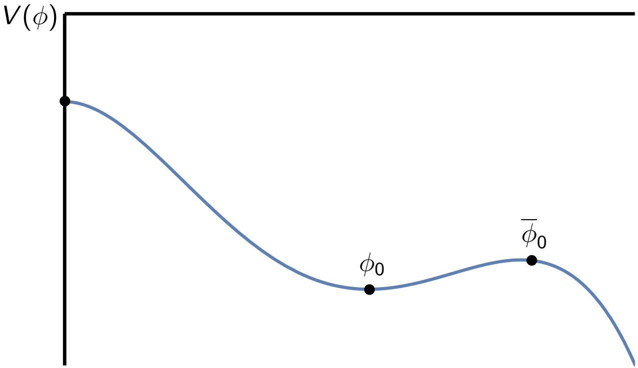
$$\left[R - \tfrac{1}{2}(\partial\phi)^2 - V(\phi)\right] \qquad \text{with} \qquad V'(\phi_c) = 0$$

$$\downarrow$$

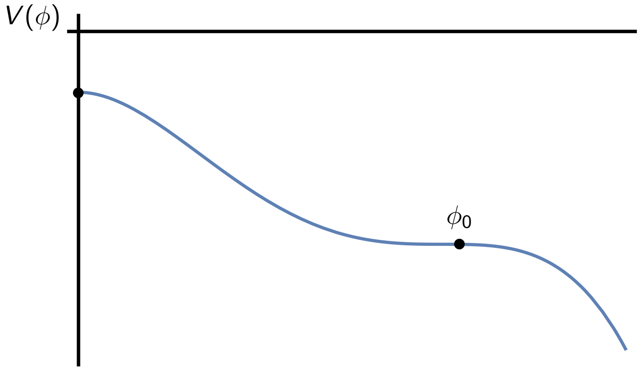
$$(R - \Lambda) \qquad \text{with} \qquad \Lambda = V(\phi_c) < 0$$

$$V'(\phi) \sim \phi(\phi - \phi_0)(\phi - \overline{\phi}_0)$$



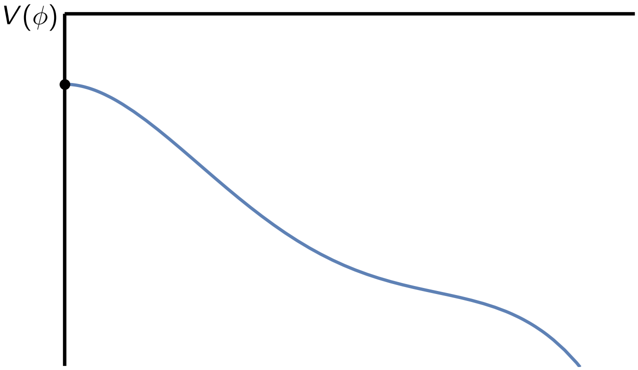
$$\phi_0 < \overline{\phi}_0 \in \mathbb{R}.$$

$$V'(\phi) \sim \phi\,(\phi - \phi_0)^2$$

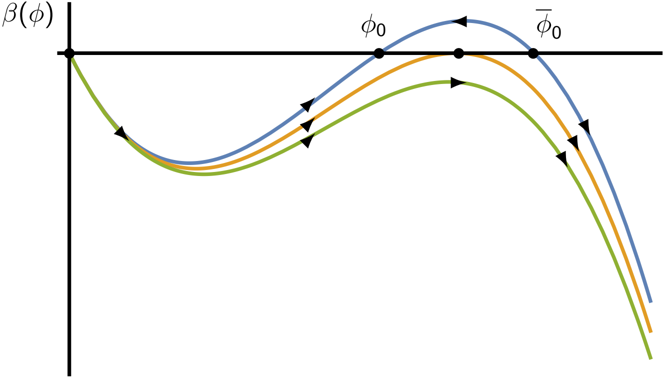


$$\phi_0 = \overline{\phi}_0 \in \mathbb{R}.$$

$$V'(\phi) \sim \phi(\phi - \phi_0)(\phi - \phi_0^*)$$



$$\phi_0 = \overline{\phi}_0^* \in \mathbb{C}.$$



$$\phi_0, \overline{\phi}_0 \quad \longleftrightarrow \quad \alpha$$

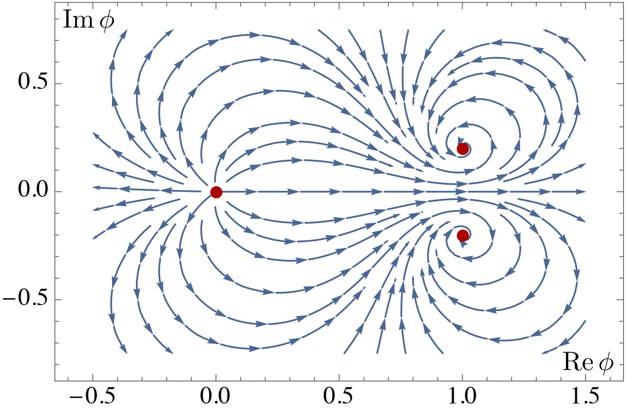
- For cFPs, extension of the coupling to $\mathbb{C} \leftrightarrow$ extension of ϕ to $\mathbb{C}$.

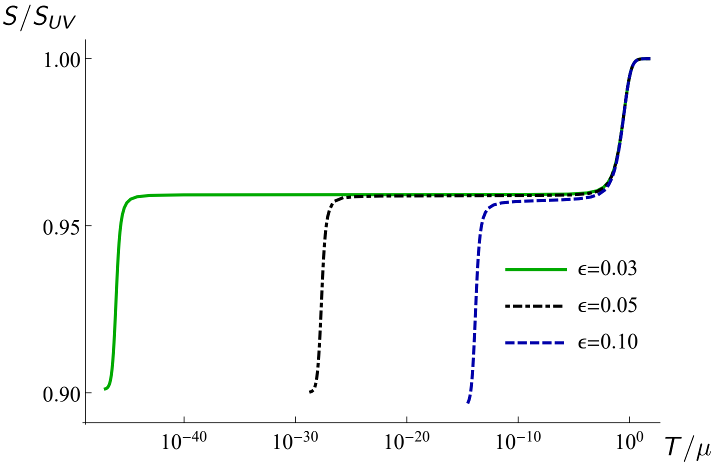
- Action holomorphic function of metric and scalar.

- Extrema of the potential at ϕ_0 and $\overline{\phi}_0 = \phi_0^*$, so complex AdS solutions with complex conjugate radius

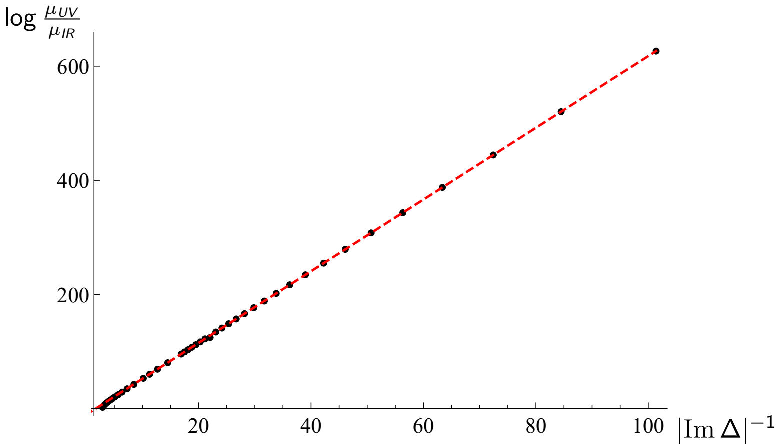
$$V(\phi_0) = -\frac{d(d-1)}{L^2} \in \mathbb{C}$$

- Straightforward extension of the AdS/CFT rules gives expected properties
  - Non-unitary.
  - Come in pairs.
  - Complex conjugate spectra, central charge etc.


Im ϕ
0.5
0.0
−0.5
Re ϕ
−0.5
0.0
0.5
1.0
1.5



$$\phi_0 = 1 + i\,\epsilon.$$



Miransky scaling $\longleftrightarrow$ $\log \frac{\mu_{UV}}{\mu_{IR}} = \frac{2\pi}{|\mathrm{Im}\Delta|}$

Summary and prospects

- Conformality lost through FPA in many physical systems. Weakly first-order transitions with Miransky (BKT, walking) scaling.
- Fixed points migrate into complex plane. Useful concept of complex CFTs.
- Strong-coupling holographic realization with expected properties.
- Embedding into string theory?