Backreacted D3-D5 intersections

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AdS/CFT: consider it "a tool to address QFTs at strong coupling"

• 1st version of the correspondence (Maldacena '97):

$$\mathcal{N} = 4 \text{ SYM}, SU(N) \Leftrightarrow \text{type IIB strings on } AdS_5 \times S^5 \tag{1}$$

• Extended to different amounts of supersymmetry (SUSY), compactifications, dimensionalities... ⇒ "gauge/gravity" duality

• Only adjoint matter? Fundamentals? ⇒ Add N_f flavor D-branes (Karch '01)

- N_f small → Flavors added as probe branes. (non-dynamical, infinitely massive quarks)
- N_f large → Backreaction of flavor branes important! → $S_{SUGRA} + S_{branes}$ (more difficult problem)

• Branes = sources to supergravity (SUGRA) eoms → violation of Bianchi id. for fluxes $dF \neq 0$

Localized sources → $dF \sim \delta(x)$ Challenging!

⇒ Use smeared sources: "a continuous distribution of branes" → avoids $\delta(x)$ (easier problem...)

D3-D5 setup:

- Dual field theory: (2+1)-d fundamental matter coupled with (3+1)-d gauge theory (DeWolfe '02)
- well studied in the quenched approximation
- rich phase diagram at $T \neq 0$, charge density and magnetic field
- Holographic model of graphene
- Quantum Hall effect

Backreacted D3-D5 intersection?

$\rightarrow$ We construct backreacted geometries with massless and massive flavors, and at $T \neq 0$.

MASSLESS BACKGROUND: based on [arXiv:1607.04998] (E. Conde, H. Lin, J. M. P., A. V. Ramallo, D. Zoakos)

Consider the brane setup:

	0	1	2	3	4	5	6	7	8	9
D3 (N_c)	x	x	x	x	-	-	-	-	-	-
D5 (N_f)	x	x	x	-	x	x	x	-	-	-

Veneziano limit: $N_c \to \infty$, $N_f \to \infty$, $\frac{N_c}{N_f} = \text{finite} \Rightarrow S = S_{SUGRA} + S_{branes}$. $S_{branes} = S_{DBI} + S_{WZ}$

- smeared D5 sources along the transverse space
- internal space: Sasaki-Einstein $ds^2_{SE} \sim ds^2_{KE} + (d\tau + A)^2$ (Kähler-Einstein+fiber)
- D5 wrapping a 3d submanifold in the Calabi-Yau cone
- D5 branes $\Rightarrow$ violation of Bianchi id. for F_3 flux. $dF_3 \sim \Xi$, (smearing form)
- Hint: flavor deformation $\rightarrow$ squashing between KE base and fiber (as with D3-D7, Benini '07)

Ansatz of metric and fluxes:

$$ds^2 = h(r)^{-\frac{1}{2}}[-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + e^{2m(r)}(dx^3)^2] + h(r)^{\frac{1}{2}}[dr^2 + e^{2g(r)}ds^2_{KE} + e^{2f(r)}(d\tau + A)^2] \quad (2)$$

$$F_5 = K(r)(1+\star)d^4x \wedge dr, \quad \phi = \phi(r) \quad (3)$$

Smeared sources (Ξ smearing form):

$$\sum^{N_f} \int_{\mathcal{M}_6} \hat{C}_6 \Rightarrow \int_{\mathcal{M}_{10}} \Xi \wedge C_6 \quad (4)$$

$$dF_3 = 2\kappa_{10}^2 T_5 \Xi \quad (5)$$

$$F_3 = Q_f \, dx^3 \wedge Im(\hat{\Omega}_2) \quad (6)$$

where $\hat{\Omega}_2$ is 2-form on the internal space:

$$\hat{\Omega}_2 = e^{3i\tau}\Omega_2, \text{ with } \Omega_2 = (e^1 + ie^2) \wedge (e^3 + ie^4) \quad (7)$$

such that the vielbeins leave the Kähler form in canonical form:

$$J_{KE} = e^1 \wedge e^2 + e^3 \wedge e^4 \text{ with } ds^2_{KE} = \sum_i (e^i)^2 \quad (8)$$

- SUSY solution $\Rightarrow$ BPS equations $\{\delta\lambda = 0, \, \delta\psi_\mu = 0\}$,

$\Rightarrow$ system of first order equations for the functions $\{f(r), g(r), m(r), \phi(r), h(r)\}$

- System further reduced to a single master equation for a master function $G = e^{2g}$, completely specifying the geometry

Solutions: 1. Unflavored solution (warm-up):

Define $\zeta = e^{g}$, $k(\zeta) = 1 - \frac{b^6}{\zeta^6}$

$$ds^2_{unflav} = h^{-\frac{1}{2}} dx^2_{1,3} + h^{\frac{1}{2}} \left[\frac{d\zeta^2}{k(\zeta)} + \zeta^2 ds^2_{KE} + \zeta^2 k(\zeta)(d\tau + A)^2 \right] \tag{9}$$

$\Rightarrow$ N_c smeared D3-branes on the blown-up cycle of the Calabi-Yau.

• Dual QFT: deformation of the SCFT due to the insertion of a VEV of a dimension 6 operator (Benvenuti '05)

Asymptotically $AdS_5 \times \mathcal{M}_5$ for large ζ

Solutions: 2. Massless flavored metric:

$$ds^2_{10} = ds^2_5 + d\hat{s}^2_5 \tag{10}$$

with

$$ds^2_5 = \frac{r^2}{R^2} \left[dx^2_{1,2} + \left(\frac{4Q_f}{3} \right)^{\frac{4}{3}} \frac{(dx^3)^2}{r^{\frac{4}{3}}} \right] + R^2 \frac{dr^2}{r^2} \tag{11}$$

and

$$d\hat{s}^2_5 = \bar{R}^2 \left[ds^2_{KE} + \frac{9}{8}(d\tau + A)^2 \right] \tag{12}$$

$\Rightarrow$ Solution invariant under anisotropic Lifshitz-like scale transformations:

$$r \to r/\lambda, \quad x^{0,1,2} \to \lambda x^{0,1,2}, \quad x^3 \to \lambda^{\frac{1}{z}} x^3 \tag{13}$$

with $z = 3$

Solutions: 3. Massive solution (first attempt):

Flavor branes do not reach origin $r = 0$. There is a cavity $r < r_q$ without sources
$\Rightarrow$ ansatz for F_3:

$$Q_f \to Q_f\, p(r) \tag{14}$$

with:

$$p(r < r_q) = 0, \quad p(r \to \infty) = 1 \tag{15}$$

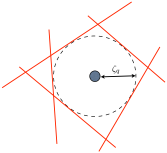
(deep UV region quarks effectively massless)

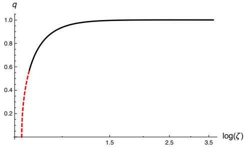
$$F_3 = Q_f\, p(r) dx^3 \wedge Im(\hat{\Omega}_2) \tag{16}$$

$p(r)$ for a given mass distribution of flavor branes? unknown
Geometry obtained after solving:

$$\begin{cases} \dot{\phi} = Q_f\, p e^{\frac{3}{2}\phi} \\ \ddot{G} - 6G^2 = \frac{\dot{\phi}}{2}\dot{G} \end{cases}$$

Shooting technique: unflavored IR, scaling UV, determines b. Take $p(\rho) = \Theta(\rho - \rho_q)$. Plot $q \equiv e^{f-g}$:





BLACK HOLE: based on [arXiv:1710.00548] (J. M. P., A. V. Ramallo, D. Zoakos)

Non zero QFT dual? $\Rightarrow$ Holographic prescription: black hole geometry
Horizon radius r_h related to temperature:

$$T = \frac{1}{2\pi}\left[\frac{1}{\sqrt{g_{rr}}}\frac{d}{dr}\left(\sqrt{-g_{x^0x^0}}\right)\right]\Big|_{r=r_h} \tag{17}$$

- Same internal space. Particularize to squashed S^5
- Solution describes a multilayer system (each layer $\sim x^3$)

• Main result: "system with intralayer dynamics corresponding to an effective stack of D2-branes"

Geometry:

$$ds^2_{10} = ds^2_5 + d\hat{s}^2_5 \tag{18}$$

$$ds^2_5 = \frac{r^2}{R^2}\left[-b(r)(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + \left(\frac{4Q_f}{3}\right)^{\frac{4}{3}}\frac{(dx^3)^2}{r^{\frac{4}{3}}}\right] + R^2\frac{dr^2}{b(r)r^2} \tag{19}$$

$$d\hat{s}^2_5 = \bar{R}^2\left[ds^2_{KE} + \frac{9}{8}(d\tau + A)^2\right] \tag{20}$$

with

$$b(r) = 1 - \left(\frac{r_h}{r}\right)^{\frac{10}{3}} \tag{21}$$

$\Rightarrow$ Study observables (WL, EE, thermodynamics of probes):

Quark-antiquark potential (Maldacena, Rey and Yee) & Entanglement entropy (Ryu and Takayanagi):

$$V_{q\bar{q}}(\text{intralayer}) \sim \frac{N_c^{\frac{2}{3}}}{N_f^{\frac{1}{3}}}\frac{1}{d_{\parallel}^{\frac{4}{3}}}, \quad V_{q\bar{q}}(\text{interlayer}) \sim \frac{N_c^2}{N_f^2}\frac{1}{d_{\perp}^4} \tag{22}$$

$$S_{\parallel} \sim \frac{N_f^{\frac{2}{3}} N_c^{\frac{5}{3}}}{l_{\parallel}^{\frac{4}{3}}}, \quad S_{\perp} \sim \frac{N_c^4}{N_f^4}\frac{1}{d_{\perp}^6} \tag{23}$$

Same dependence on N_c and $l_{\parallel}$ for $S_{\parallel}$ as for a D2-brane. We will find this equivalence several times

THERMODYNAMICS:

$$T = \frac{5r_h}{6\pi R^2}, \quad s = \frac{2\pi}{\kappa_{10}^2} = \alpha_s N_f^{\frac{2}{3}} N_c^{\frac{5}{3}} T^{\frac{7}{3}} \tag{24}$$

$$E_{ADM} = \epsilon V_3 = \beta_s Q_f^{\frac{2}{3}} Q_c^{\frac{5}{3}} T^{\frac{10}{3}} \tag{25}$$

$$f = \epsilon - Ts = -\frac{3}{7}\beta_s Q_f^{\frac{2}{3}} Q_c^{\frac{5}{3}} T^{\frac{10}{3}} \tag{26}$$

What if the number of D5-branes change? $\Rightarrow$ Introduce chemical potential Φ:

$$g = f - \Phi Q_f = -\frac{1}{7}\beta_s Q_f^{\frac{2}{3}} Q_c^{\frac{5}{3}} T^{\frac{10}{3}} \tag{27}$$

As argued (Mateos, Trancanelli '11) $f = -p_{xy}$, $g = -p_z$. In our case:

$$p_{xy} = \frac{3}{7}\epsilon, \quad p_z = \frac{1}{7}\epsilon \quad \Rightarrow \quad v_{xy}^2 = \frac{3}{7}, \quad v_z^2 = \frac{1}{7} \quad \text{same as D2} - \text{brane} \tag{28}$$

STRESS-ENERGY TENSOR:
ϵ and p_{xy}, p_z can be obtained through the holographic stress-energy tensor $\langle T^{\mu}_{\nu} \rangle$

- Brown-York tensor at the boundary (Balasubramanian, Kraus '99)
- and/or holographic renormalization (reduce the action to 5d (or 4d))

• Brown-York tensor:

$$\tau_{ij} = \frac{1}{\kappa_{10}^2}(K_{ij} - K\gamma_{ij}), \quad \Rightarrow \quad \langle T^{\mu}_{\nu} \rangle = V_{SE}\sqrt{-\gamma_{Mink}}\,\tau^{\mu}_{\nu}|_{reg\,, r_\Lambda \to \infty} \tag{29}$$

Divergent quantity regularized through (subtract the $T = 0$ SUSY value):

$$\langle T^{\mu}_{\nu} \rangle = V_{SE} \lim_{r_\Lambda \to \infty} \left[\sqrt{-\gamma_{Mink}}\,\tau^{\mu}_{\nu} - b^{\frac{1}{2}} \lim_{r_h \to 0} (\sqrt{-\gamma_{Mink}}\,\tau^{\mu}_{\nu}) \right] \Big|_{r=r_\Lambda} \tag{30}$$

$b^{\frac{1}{2}}$ is introduced to match the geometries at the cutoff. We obtain:

$$\langle T^{x^0}_{x^0} \rangle = -\epsilon, \quad \langle T^{x^1}_{x^1} \rangle = \langle T^{x^2}_{x^2} \rangle = \frac{3}{7}\epsilon, \quad \langle T^{x^3}_{x^3} \rangle = \frac{1}{7}\epsilon \tag{31}$$

Thus:

$$\langle T^{\mu}_{\nu} \rangle = \mathrm{diag}(-\epsilon, p_{xy}, p_{xy}, p_z) \tag{32}$$

Which coincides with the results of the static analysis

- Reduced action in 4d (system of lower-dimensional gravity) $ds^2_{10} \to ds^2_4$:

$$ds^2_{10} = e^{\frac{10}{3}\gamma - \beta} g_{mn} dz^m dz^n + e^{\frac{10}{3}\gamma + 2\beta} (dx^3)^2 + e^{-2(\gamma+\lambda)} ds^2_{\mathbb{CP}^2} + e^{2(4\lambda-\gamma)} (d\tau + A)^2 \quad (33)$$

$S_{4d} = S_{4d}$(gravity+kinetic terms for scalars + scalar potential V_{4d})

$$S_{eff(4D)} = \frac{V_5 V_{x^3}}{2\kappa^2_{10}} \int d^4x \sqrt{-g_4} \left[R_4 - \frac{40}{3}(\partial\gamma)^2 - 20(\partial\lambda)^2 - \frac{3}{2}(\partial\beta)^2 - \frac{1}{2}(\partial\phi)^2 - V \right] \quad (34)$$

$$V = 4e^{\frac{16}{3}\gamma + 12\lambda - \beta} - 24e^{\frac{16}{3}\gamma + 2\lambda - \beta} + Q_f^2 e^{4\gamma+4\lambda-3\beta+\phi} + \frac{Q_c^2}{2} e^{\frac{40}{3}\gamma - \beta} + 6Q_f e^{\frac{14}{3}\gamma - 2\lambda - 2\beta + \frac{\phi}{2}} \quad (35)$$

We can construct a superpotential W_{4d} such that $V_{4d} = V_{4d}(W_{4d}, \partial_\Psi W_{4d})$

$$V = \frac{1}{2}\left[\frac{3}{80} (\partial_\gamma W_{4d})^2 + \frac{1}{40} (\partial_\lambda W_{4d})^2 + \frac{1}{3} (\partial_\beta W_{4d})^2 + (\partial_\phi W_{4d})^2 \right] - \frac{3}{8} W^2_{4d} \quad (36)$$

$$W_{4d} = -6\, e^{\frac{8}{3}\gamma - 4\lambda - \frac{\beta}{2}} - 4\, e^{\frac{8}{3}\gamma + 6\lambda - \frac{\beta}{2}} + Q_c\, e^{\frac{20}{3}\gamma - \frac{\beta}{2}} + 2\, Q_f\, e^{2\gamma + 2\lambda + \frac{\phi}{2} - \frac{3\beta}{2}} \quad (37)$$

Stress-Energy → on-shell act.+bdy. terms: Gibbons-Hawking+superpotential (Battrachenko, Liu... '05)

$$S_{boundary} = \frac{V_5\, V_{x^3}}{2\, \kappa^2_{10}} \int_{r\to\infty} d^3x \sqrt{\gamma} \left(2\, K + W_{4d} \right) , \quad (38)$$

$$\frac{S_{renormalized}}{V_3\, V_{x^3}} = \frac{S_{eff\, , on-shell} + S_{boundary\, , on-shell}}{V_3\, V_{x^3}} = \frac{3}{7} \beta_s\, Q_f^{\frac{2}{3}}\, Q_c^{\frac{5}{3}}\, T^{\frac{10}{3}} = -f \quad (39)$$

$$\langle T^\mu{}_\nu \rangle = \mathrm{diag}(-\epsilon\, ,\, p_{xy}\, ,\, p_{xy}\,) \quad (40)$$

- Reduced action in 5d $ds^2_{10} \to ds^2_5$:

$$ds^2_{10} \;=\; e^{\frac{10}{3}\,\gamma}\, g_{pq}\, dz^p\, dz^q \;+\; e^{-2(\gamma+\lambda)}\, ds^2_{\mathbb{CP}^2} \;+\; e^{2(4\lambda-\gamma)}\,(d\tau + A)^2 \tag{41}$$

$S_{5d} \;=\; S_{5d}$(gravity+kinetic terms for scalars+scalar potential+effective brane action in 4d)

$$S_{eff} \;=\; \frac{V_5}{2\,\kappa^2_{10}} \int d^5 z\,\sqrt{-g_5}\,\Big[R_5 \;-\; \frac{40}{3}\,(\partial\gamma)^2 \;-\; 20\,(\partial\lambda)^2 \;-\; \frac{1}{2}\,(\partial\phi)^2 \;-\; \frac{1}{2}\,e^{4\gamma+4\lambda+\phi}\,(\partial\mathcal{V})^2 \;-\; U\Big] \;- \tag{42}$$

$$-\;\frac{V_5}{2\,\kappa^2_{10}} \int d^5 z\,\sqrt{-\hat{g}_4}\,\big[\,6\,Q_f\,e^{\frac{14}{3}\,\gamma-2\lambda+\frac{\phi}{2}}\,\big]\;, \tag{43}$$

$$U = 4\,e^{\frac{16}{3}\,\gamma+12\lambda} \;-\; 24\,e^{\frac{16}{3}\,\gamma+2\lambda} \;+\; \frac{Q_c^2}{2}\,e^{\frac{40}{3}\,\gamma} \tag{44}$$

We can construct a superpotential W_{5d} such that $V_{5d} \;=\; V_{5d}(W_{5d}\,,\,\partial_\Psi\,W_{5d})$

$$U \;=\; \frac{1}{2}\,\Big[\frac{3}{80}\,\big(\partial_\gamma\,W_{5d}\big)^2 + \frac{1}{40}\,\big(\partial_\gamma\,W_{5d}\big)^2 + \big(\partial_\phi\,W_{5d}\big)^2\Big] - \frac{1}{3}\,W^2_{5d} \quad\Rightarrow W_{5d} \;=\; -6\,e^{\frac{8\gamma}{3}-4\lambda} \;-\; 4\,e^{\frac{8\gamma}{3}+6\lambda} + Q_c\,e^{\frac{20}{3}\gamma} \tag{45}$$

We also need a "flavor superpotential" $W_{\rm flavor}$

$$W_{flavor} \;=\; 2\,Q_f\,e^{2\lambda+2\gamma+\frac{\phi}{2}} \tag{46}$$

$\Rightarrow$

$$S_{boundary} \;=\; \frac{V_5}{2\,\kappa^2_{10}} \int_{r\to\infty} d^4 x\,\sqrt{\gamma}\,\Big(2\,K \;+\; W_{5d}\Big) \;+\; \frac{V_5}{2\,\kappa^2_{10}} \int_{r\to\infty} d^4 x\,\sqrt{\hat{\gamma}}\,W_{flavor} \tag{47}$$

$$\frac{S_{renormalized}}{V_3\,V_{x^3}} \;=\; \frac{S_{eff\,,\,on-shell} \;+\; S_{boundary\,,\,on-shell}}{V_3\,V_{x^3}} \;=\; \frac{3}{7}\,\beta_s\,Q_f^{\frac{2}{3}}\,Q_c^{\frac{5}{3}}\,T^{\frac{10}{3}} \tag{48}$$

$$\langle T^{\mu}{}_{\nu}\rangle = {\rm diag}\big(-\epsilon,\,p_{xy}\,,\,p_{xy}\,,\,p_z\big) \tag{49}$$

HYDRODYNAMICS (only modes in the (x^1, x^2) plane):

Compute the transport coefficients (Kovtun, Starinets '05):
$\Rightarrow$ Fluctuate 4d reduced metric and scalar fields:

$$\left\{ \begin{array}{l} g_{mn} \rightarrow g_{mn} + h_{mn} \\ \Psi \rightarrow \Psi + \delta\Psi \end{array} \right\}$$

Highly coupled + redundant equations! $\Rightarrow$ gauge choice $h_{mr} = 0, \quad m = (t, x^1, x^2, r)$

$\rightarrow$ Two consistent truncations decouple

SHEAR CHANNEL:

$$(h_{tx^1}, h_{x^1x^2}) \sim e^{-i(\omega t - qx^2)}(H_{tx}(r), H_{xy}(r)) \tag{50}$$

Gauge invariant combination: $X = qH_{tx} + \omega H_{xy} \Rightarrow$ reduces the problem to a 2nd-order ODE

Solution found, with a dispersion relation:

$$\omega = -iD_\eta q^2(1 + \tau_s D_\eta q^2) \tag{51}$$

Shear viscosity $\frac{\eta}{s}$ and relaxation time τ_s:

$$\frac{\eta}{s} = TD_\eta = \frac{1}{4\pi}, \quad \tau_s = \frac{1}{4\pi T}[\gamma + \psi(\frac{8}{5})] \tag{52}$$

Coincides with the coefficents for a D2-brane!

SOUND CHANNEL:

$$(h_{tt}\,,\,h_{tx^2}\,,\,h_{x^1x^1}\,,\,h_{x^2x^2}) \tag{53}$$

A more complicated set of gauge invariant fluctuations simplify the problem: $(Z_\Phi\,,\,Z_\Gamma\,,\,Z_\Lambda\,,\,Z_B\,,\,Z_H)$

Another combination $Z_S \equiv 3Z_B + 2Z_\Phi$ decouples, with trivial solution

$\Rightarrow$ remaining equation for Z_H has solution with dispersion relation:

$$\omega = v_s q - i\Gamma q^2 + \mathcal{T} q^3 \tag{54}$$

With:

$$\Gamma = \frac{1}{2Ts}(\eta + \zeta), \quad \mathcal{T} = \frac{\Gamma}{v_s}\left[v_s^2\tau_{eff} - \frac{\Gamma}{2}\right] \tag{55}$$

where ζ is the bulk viscosity. Solution:

$$v_s = \sqrt{\frac{3}{7}}, \quad \Gamma = \frac{1}{7\pi T} \Rightarrow \frac{\zeta}{\eta} = \frac{1}{7}, \quad \mathcal{T} = \frac{\sqrt{3}}{28\sqrt{7}(\pi T)^2}(1 + \gamma + \psi(\frac{8}{5})) \Rightarrow \tau_{eff} = \frac{1}{4\pi T}[\frac{5}{3} + \gamma + \psi(\frac{8}{5})] \tag{56}$$

- v_s coincides with static analysis, for propagation in x^1x^2! (same as a D2-brane)
- $\frac{\zeta}{\eta}$ coincides same as D2-brane
- τ_{eff} same as D2-brane

MASSIVE BACKGROUND: based on [arXiv:1901.02020] (N. Jokela, J. M. P. , A. V. Ramallo, D. Zoakos)

The D3 and D5 branes can be separated in directions (7,8,9)

• (Non-) zero separation → (massive) massless flavors

Construct massive solutions. QFT duals with massive flavors ⇒ RG flow

Expectations (and results):

- large mass → flavors decouple → recover deformed $AdS_5 \times S^5$ (IR)
- massless flavors → anisotropic Lifshitz scaling solution (UV)
- Interpolating solution for finite non-vanishing quark mass
- IR geometry depends on a free parameter

ANSATZ:

$$ds^2_{10} = h^{-\frac{1}{2}} \left[-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + e^{-2\phi} (dx^3)^2 \right] + h^{\frac{1}{2}} \left[\zeta^2 e^{-2f} d\zeta^2 + \zeta^2 ds^2_{\mathbb{CP}^2} + e^{2f} (d\tau + A)^2 \right] \quad (57)$$

$$F_3 = Q_f \, p(\zeta) \, dx^3 \wedge \mathrm{Im}\, \hat{\Omega}_2 \quad (58)$$

$p(\zeta)$ is the profile:

→ Microscopic computation (κ-symmetric embeddings $\chi(r)$)
→ Macroscopic computation (DBI action with smearing and calibration forms)

Comparison $S_{smeared} \sim N_f \, S_{brane}$ allows to determine $p(\zeta)$

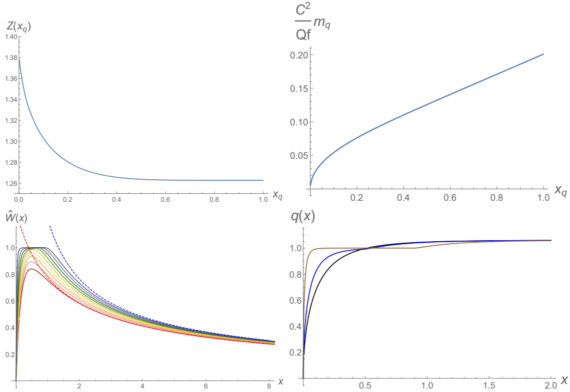
(h, f, g, ϕ) obtained through a master function $W(\zeta)$ satisfying a "master equation":

$$\frac{d}{d\zeta}\left(\zeta \frac{dW}{d\zeta}\right) + 6 \frac{dW}{d\zeta} = -\frac{6 \, Q_f \, p(\zeta)}{\zeta^2 \sqrt{W}} \quad (59)$$

Numerical integration: change $x \equiv \frac{\zeta - b}{\zeta_q}$, $x_q = 1 - \frac{b}{\zeta_q}$ ((old) new position of the cavity in (ζ_q) x_q). With $\hat{W} \equiv \frac{W}{C} \Rightarrow$

$$\frac{d}{dx}\left[(x+1-x_q)\frac{d\hat{W}}{dx}\right]+6\frac{d\hat{W}}{dx} = -\frac{6}{Z(x_q)}\frac{p(x)}{\sqrt{\hat{W}}}, \quad p(x) = \theta(x-x_q)\frac{\sqrt{x-x_q}\sqrt{x-x_q+2}}{2(x+1-x_q)^3}\left[1+2(x+1-x_q)^2\right] \quad (60)$$

$$\hat{W}_{\rm unflav}(x) = 1 - \frac{(1-x_q)^6}{(1+x-x_q)^6}, \quad 0 \le x \le x_q, \quad \hat{W}_{\rm massless\ flav}(x) \approx \frac{9}{8}\left[\frac{\sqrt{2}}{Z(x_q)}\right]^{\frac{2}{3}} x^{-\frac{2}{3}}, \ (x \to \infty) \quad (61)$$



WILSON LOOPS:

$$S = \frac{1}{2\pi} \int d\tau d\sigma \, e^{\frac{\phi}{2}} \sqrt{-\det g_2} \tag{62}$$

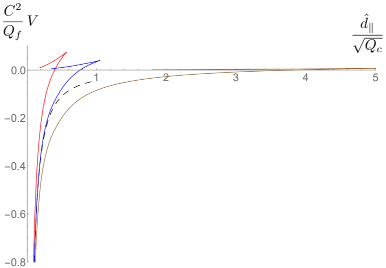
"Find the embedding of a string hanging from the UV that minimizes S"

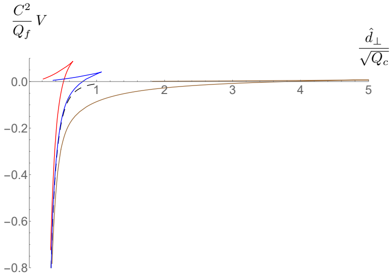
Wilson loops are divergent. Regularization: subtract two fundamental strings stretched from $x = 0$ to $x = x_{max}$

Two embeddings:

1. intralayer (extending along x^1) $(t, x(\hat{x}^1))$
2. interlayer (extending along x^3) $(t, x(\hat{x}^3))$

UV behaviour coincides with the results found in the previous section





- String breaks for large $\hat{d}$ (pair creation) $\Rightarrow$ Disconnected configuration dominates (external quarks screened)
- Critical d grows with x_q. (Breaking when string penetrates cavity, whose size is maximal for large x_q)

ENTANGLEMENT ENTROPY FOR SLABS:

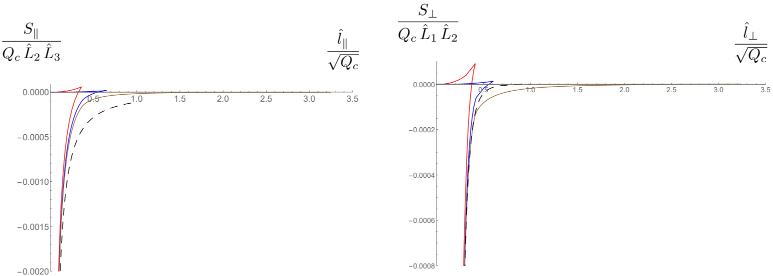
$$S_A = \frac{1}{4G_{10}} \int_\Sigma d^8\xi \sqrt{\det g_8} \tag{63}$$

"Find the embedding of a surface ending on the boundary of the slab at the UV that minimizes S"

Entanglement entropies are divergent. Regularization: subtract a flat embedding
Two embeddings:

1. intralayer ($A = \{-\frac{\hat{l}_\parallel}{2} \leq \hat{x}^1 \leq \frac{\hat{l}_\parallel}{2}, -\infty < \hat{x}^2, \hat{x}^3 < +\infty\}$)
2. interlayer ($A = \{-\infty < \hat{x}^1, \hat{x}^2 < +\infty, -\frac{\hat{l}_\perp}{2} \leq \hat{x}^3 \leq \frac{\hat{l}_\perp}{2}\}$)

UV behaviour coincides with the one in the previous section:



- $S > 0$ for large $\hat{d} \Rightarrow$ Disconnected surface dominates
- Critical d grows with x_q

THERMODYNAMICS OF A MASSLESS PROBE:

Embed a D5-probe as in the original array for the intersection

Switch on a worldvolume gauge field: $A = A_0 dx^0$, dual to a chemical potential $\mu = A_0|_{boundary}$:

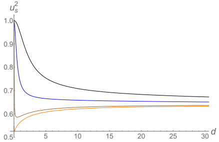
Massless embeddings $\chi = \chi_*$ in which the probe reaches IR. Solution if $\frac{e^{2g-\frac{\phi}{2}}A_0'}{\sqrt{1-e^{-\phi}A_0'^2}} = d$

$$\mu = d\int_0^\infty \frac{e^{\frac{\phi}{2}}}{\sqrt{d^2+e^{4g}}}dr \quad (64)$$

$T = 0$ thermodynamics:

$$\Omega = -S_{on-shell} \to \rho = -\frac{\partial\Omega}{\partial\mu} \to \epsilon = \Omega + \mu\rho \to u_s^2 = -\frac{\partial\Omega}{\partial\epsilon} \quad (65)$$

UV limit: $u_s^2(\text{massless flav} = \frac{2}{3})$
IR limit: $u_s^2(\text{unflav} = \frac{1}{2}) \Rightarrow$ agress with a conformal worldvolume theory in (2+1)-d
Does not match numerics (Subtlety. Limits do not commute $\hat{d} \to 0$ & $x_q \to 1$)



IR controlled by x_q. For large x_q, the long range (IR) behaviour becomes more isotropic and the theory retains the (3+1)-d character

Summary

- We computed the backreacted geometries of D3-D5 intersections along a (2+1)-d subspace in the Veneziano limit, with smeared D5 sources, creating a multilayer system
- We got solutions for massless and massive fundamentals and generalized the results for massless quarks at $T \neq 0$
- Massless solution with anistropic scale invariance
- Hydrodynamics suggests that our system behaves as an effective set of D2-branes
- Massive solutions show that by varying x_q we can 'change' the dimensionality of the theory in the IR ((3+1)-d for large x_q vs (2+1)-d behaviour for small x_q). UV given by the scaling solution

Future directions

- Solutions with branes not reaching UV (Work in progress with N. Jokela, C. Hoyos and A. V. Ramallo)
- Backreaction of chemical potential?
- D3-D5 in other compact spaces?
- Study of hydrodynamic modes propagating along x^3?
- Application to neutron stars? (In the spirit of C. Hoyos, D. Rodriguez Fernandez, N. Jokela, A. Vuorinen '16)

Thanks for your attention!