α' corrections of Reissner-Nördstrom black holes

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KU LEUVEN

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with Samuele Chimento, Roman Linares, Tomas Ortín and Pedro F. Ramírez

Iberian Strings

Santiago de Compostela, January 17th 2020

1. Introduction
2. Embedding the RN BH in string theory
3. α' corrections
4. Thermodynamics
5. Conclusions

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Introduction

At low energies String Theory **predicts General Relativity**...plus something more

- **Superstring effective actions**

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- The leading term $S_{10}^{(0,0)}$ corresponds to Einstein gravity coupled to certain matter: **10-dimensional supergravity theories**
- The rest of terms contain **higher-derivative interactions**. Deviations with respect to GR.

Introduction

The study of α' corrections is especially relevant for black holes

- Supersymmetric + special extremal BHs $\rightarrow$ precision counting of entropy
  Sen; Dabholkar, Kallosh, Maloney; Castro, Murthy; Prester, Terzic; Kutasov, Larsen, Leigh; PAC, Chimento, Messen, Ortín, Ramírez, Ruipérez ...
- Decay of charged black holes (WGC)
  Cheung, Liu, Remmen; Charles; Loges, Noumi, Shiu; PAC, Ortín, Ramírez ...
- Non-extremal BHs $\rightarrow$ thermodynamics ($T \neq 0$)
  Kats, Motl, Padi; Loges, Noumi, Shiu

Known α'-corrected solutions in Heterotic String Theory

Extremal BHs → Pedro's talk

- Susy black holes ✓ PAC, Chimento, Messen, Ortín, Ramírez, Ruipérez
- (some) non-susy black holes ✓ PAC, Ortín, Ramírez

Uncharged BHs (mostly in EdGB and dCS theories)

- Schwarzschild black hole ✓Moura, Schiappa; Yunes, Stein ...
- Kerr black hole ✓Campbell, Duncan, Kaloper; Mignemi, Stewart; Yunes, Pretorius; Pani, Macedo, Crispino, Cardoso; PAC, Ruipérez ... → See Alejandro's talk!

Charged but non-extremal

- GHS black hole ✓ Kats, Motl, Padi
- Reissner-Nordstrom black hole → Today's talk

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Embedding the RN BH in string theory

Einstein-Maxwell-Dilaton theory: truncation of zeroth-order-in-α' HST

$$S = \frac{1}{16\pi G_N^{(4)}} \int d^4x\sqrt{|g|}\left[R + 2(\partial\phi)^2 + \frac{e^{-2(\phi-\phi_\infty)}}{4}F^2\right]$$

Charged BHs typically have a non-trivial dilaton profile Gibbons; Gibbons, Maeda; Garfinkle, Horowitz, Strominger

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Exception: symmetric dyonic solution

RN solution with equal electric and magnetic charges

$$\begin{aligned} ds^2 &= \left(1 - \frac{2M}{r} + \frac{p^2/2}{r^2}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{2M}{r} + \frac{p^2/2}{r^2}\right)} - r^2[d\theta^2 + \sin^2\theta d\phi^2] \\ \phi &= \phi_\infty \\ F &= \frac{p}{r^2}(dt\wedge dr + r^2\sin\theta d\theta\wedge d\phi) \quad \Rightarrow \quad F^2 = 0 \end{aligned}$$

Embedding the RN BH in string theory

10d effective action of HST at zeroth order in α'

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x\sqrt{|\hat{g}|}\, e^{-2\hat{\phi}} \left[\hat{R} - 4(\partial\hat{\phi})^2 + \tfrac{1}{2\cdot 3!}\hat{H}^2\right], \quad \text{where } \hat{H} = d\hat{B}$$

Embedding in 10 dimensions

$$\begin{aligned} d\hat{s}^2 &= a^2 dt^2 - \frac{dr^2}{a^2} - r^2[d\theta^2 + \sin^2\theta d\phi^2] - dz^2 - d\vec{y}_{(5)}^2 \\ \hat{\phi} &= \hat{\phi}_\infty \\ \hat{H} &= \frac{p}{r^2}(dt\wedge dr + r^2 \sin\theta d\theta \wedge d\phi)\wedge dz \end{aligned}$$

The function $a(r)$ is given by $a^2 = 1 - \frac{2M}{r} + \frac{p^2/2}{r^2}$

The extremal limit is **no supersymmetric** Khuri, Ortín

α' corrections

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We work in $d = 10$

Heterotic Superstring effective action at first order in α' Bergshoeff, de Roo

$$S = \frac{g_s^2}{16\pi G_N^{(10)}} \int d^{10}x\sqrt{|g|}\, e^{-2\phi} \left\{ R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!} H^2 - \frac{\alpha'}{8} R_{(-)\mu\nu}{}^{a}{}_{b} R_{(-)}{}^{\mu\nu\, b}{}_{a} \right\}$$

$$H = dB + \frac{\alpha'}{4}\left[d\Omega_{(-)}{}^{a}{}_{b} \wedge \Omega_{(-)}{}^{b}{}_{a} - \frac{2}{3}\Omega_{(-)}{}^{a}{}_{b} \wedge \Omega_{(-)}{}^{b}{}_{c} \wedge \Omega_{(-)}{}^{c}{}_{a} \right]$$

$$R_{(-)}{}^{a}{}_{b} = d\Omega_{(-)}{}^{a}{}_{b} - \Omega_{(-)}{}^{a}{}_{c} \wedge \Omega_{(-)}{}^{c}{}_{b}$$

$\Omega_{(-)}{}^{a}{}_{b} = \omega^{a}{}_{b} - \frac{1}{2} H_{\mu}{}^{a}{}_{b}\, dx^{\mu}$ is the torsionful spin connection

How to solve the higher-derivative EOMs

Step 1: write down sufficiently general ansatz

$$
\begin{aligned}
d\hat{s}^2 &= A^2 dt^2 - B^2 dr^2 - r^2[d\theta^2 + \sin^2\theta d\phi^2] - C^2[dz + Fdt]^2 - d\vec{y}_5^2 \\
\hat{\phi} &= \hat{\phi}(r) \\
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Step 2: assume perturbative corrections over 0th-order solution

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A &\sim a + \alpha'\delta_A\,, \quad B \sim a^{-1} + \alpha'\delta_B\,, \quad C \sim 1 + \alpha'\delta_C\,, \quad F \sim \alpha'\delta_F\,, \\
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\end{aligned}
$$

Step 3: Expand EOMs at $\mathcal{O}(\alpha')$ and solve imposing boundary conditions

- Regularity
- Asymptotic flatness
- Fixed charges and mass

α' CORRECTIONS

4-dimensional metric $\rightarrow$ $ds^2 = Ce^{-2(\hat{\phi}-\hat{\phi}_\infty)}\left[A^2dt^2 - B^2dr^2 - r^2d\Omega^2_{(2)}\right]$

We introduce $\rho = rC^{1/2}e^{-(\hat{\phi}-\hat{\phi}_\infty)}$

$$ds^2 = N(\rho)^2 f(\rho)dt^2 - \frac{d\rho^2}{f(\rho)} - \rho^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

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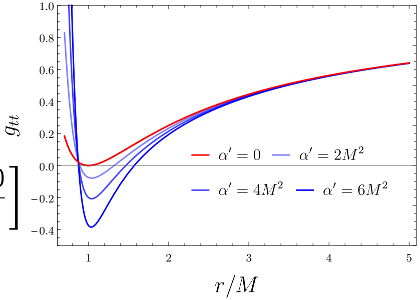
$$\begin{aligned} N(\rho)^2 &= 1 + \alpha' \frac{p^2/8}{\rho^4} \\ f(\rho) &= 1 - \frac{2M}{\rho} + \frac{p^2/2}{\rho^2} \\ &- \alpha' \frac{p^2/4}{\rho^4}\left[1 - \frac{3M/2}{\rho} + \frac{11p^2/40}{\rho^2}\right] \end{aligned}$$

Solution with $M = p/\sqrt{2}$



Event horizon $\rightarrow$ placed at $f(\rho_h) = 0$

Far from extremality: $M^2 - p^2/2 >> \alpha'$

$$\rho_{\rm h} = r_+ + \frac{\alpha'}{M}\left\{-\frac{13}{20}(M/p)^2 + \frac{8}{5}(M/p)^4 - \frac{1}{\sqrt{1-\frac{1}{2}(p/M)^2}}\left[\frac{9}{80} - \frac{21}{20}(M/p)^2 + \frac{8}{5}(M/p)^4\right]\right\}$$

(very) near extremality: $M^2 - p^2/2 \sim \alpha'$

$$\rho_{\rm h\,nearext} = M\left\{1 + \sqrt{2}\sqrt{1 - \frac{p}{\sqrt{2}M} + \frac{1}{80}\frac{\alpha'}{M^2}} + \frac{3}{40}\frac{\alpha'}{M^2}\right\}.$$

Extremal limit:

$$\rho_{\rm h\,ext} = (p/\sqrt{2})\left[1 + \frac{\alpha'}{16}\frac{1}{(p/\sqrt{2})^2}\right] + \mathcal{O}(\alpha'^2)$$

$$M_{\rm ext} = (p/\sqrt{2})\left[1 - \frac{\alpha'}{80}\frac{1}{(p/\sqrt{2})^2}\right] + \mathcal{O}(\alpha'^2)$$

Additional fields
Besides $g_{\mu\nu}$ and Maxwell field, new fields become non-trivial

$$\text{Dilaton } \phi, \quad \text{KK scalar } C, \quad \text{axion } \chi \ (\leftrightarrow B_{\mu\nu}), \quad \text{KK vector } A_\mu$$

Charges are generated

$$C \sim 1 + \frac{Q_C}{\rho} \quad \Rightarrow \quad Q_C = \frac{\alpha' r_- \left(140 r_+^3 - 154 r_+^2 r_- + 35 r_+ r_-^2 - 9 r_-^3\right)}{140 r_+^3 \left(r_+^2 + 4 r_+ r_- + r_-^2\right)}$$

$$e^{\phi} \sim e^{\phi_\infty}\left[1 + \frac{Q_\phi}{\rho}\right] \quad \Rightarrow \quad Q_\phi = \alpha' \frac{16 r_-^4 - 77 r_-^3 r_+ - 49 r_-^2 r_+^2 + 70 r_+^3 (r_- + r_+)}{280 r_+^3 (r_-^2 + 4 r_- r_+ + r_+^2)}$$

$$F(A) \sim \frac{Q_A}{\rho^2} dt \wedge d\rho \quad \Rightarrow \quad Q_A = \frac{\alpha' r_- (r_+ - r_-)^2}{4\sqrt{2 r_+ r_-}\, r_+^3} \qquad r_\pm = M \pm \sqrt{M^2 - p^2/2}$$

No hair $\to$ everything determined by free parameters M, p

Thermodynamics

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Thermodynamics

The Hawking temperature is related to the surface gravity by the famous formula

$$T = \frac{\kappa}{2\pi}$$

Far from extremality, it reads

$$T = \frac{\sqrt{M^2 - \frac{p^2}{2}}}{2\pi\left(M + \sqrt{M^2 - \frac{p^2}{2}}\right)^2} + \frac{\alpha'\left(M + 3\sqrt{M^2 - \frac{p^2}{2}}\right)\left(M - \sqrt{M^2 - \frac{p^2}{2}}\right)^2}{160\pi\sqrt{M^2 - \frac{p^2}{2}}\left(M + \sqrt{M^2 - \frac{p^2}{2}}\right)^5}$$

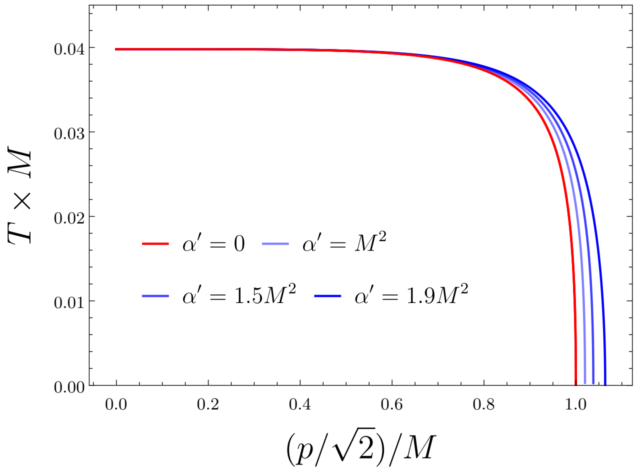
Formula diverges near extremality! correct value in that case:

$$T = \frac{1}{\pi p^2}\left[2^{1/4} p^{1/2}\sqrt{M - M_{\text{ext}}} + 4(M - M_{\text{ext}}) + \ldots\right]$$

where we recall that $M_{\text{ext}} = \frac{p}{\sqrt{2}} - \frac{\sqrt{2}\alpha'}{80p} + \mathcal{O}(\alpha'^2)$

$$T(M = p/\sqrt{2}) = \alpha'^{1/2}/(4\pi\sqrt{10}M^2)$$

Thermodynamics



Thermodynamics

Entropy→ Wald's formula

$$S = -2\pi \int_{\Sigma} d^2x \sqrt{|h|} \frac{\partial \mathcal{L}}{\partial R_{abcd}} \epsilon_{ab}\epsilon_{cd}$$

- Requires knowing the compactified Lagrangian $\mathcal{L}$ → hard
- Work in 10d? → Ambiguities appear due to Chern-Simons terms

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Result for our field configuration

$$S = -\frac{1}{8} \int_{\Sigma} d^2x \sqrt{|h|} \epsilon_{ab}\epsilon_{cd} \left\{ g^{ab,\,cd} - \frac{\alpha'}{8} \left[-2R^{(0)\,abcd}_{(-)} + G^{(0)\,[a|c}G^{(0)\,|b]d} + G^{(0)\,ab}G^{(0)\,cd} \right] \right\}$$

where $G^{(0)}{}_{\mu\nu} = 2\partial_{[\mu}B^{(0)}{}_{\nu]}$, $\quad R^{(0)\,abcd}_{(-)}$ = 4d curvature of torsionful spin connection

Final result

$$S = \pi \rho_{\rm h}^2 \left\{ 1 + \alpha' \left[\frac{M}{\rho_{\rm h}^3} - \frac{3p^2}{8\rho_{\rm h}^4} \right] \right\}$$

Thermodynamics

Far from extremality: $M^2 - p^2/2 >> \alpha'$

$$S = \pi\left[\left(M + \sqrt{M^2 - \frac{p^2}{2}}\right)^2 + \frac{\alpha'\left(18M\sqrt{M^2 - \frac{p^2}{2}} + 21\left(M^2 - \frac{p^2}{2}\right) + M^2\right)}{40\sqrt{M^2 - \frac{p^2}{2}}\left(\sqrt{M^2 - \frac{p^2}{2}} + M\right)}\right]$$

We check the following equality: $\frac{\partial S}{\partial M} = \frac{1}{T}$ $\Rightarrow$ the 1st law holds!

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Near extremality: $M^2 - p^2/2 \sim \alpha'$

$$S/\pi = \frac{p^2}{2} + \frac{3\alpha'}{8} + 2^{3/4}p^{3/2}\sqrt{M-M_{\text{ext}}} + \sqrt{8}p(M-M_{\text{ext}}) + \dots$$

Two important values

$$S(M_{\text{ext}}) = \pi\left[\frac{p^2}{2} + \frac{3}{8}\alpha'\right] \qquad S(M^{(0)}_{\text{ext}}) = \pi\left[\frac{p^2}{2} + \frac{p\alpha'^{1/2}}{2\sqrt{5}} + \mathcal{O}(\alpha')\right]$$

Relation between ΔM_{ext} and ΔS Cheung, Liu, Remmen

We recall that the extremal mass is $M_{\text{ext}} = p/\sqrt{2} - \frac{\alpha'}{80}\frac{1}{p/\sqrt{2}}$

In the regime $\alpha' << M^2 - p^2/2 << p^2$ we observe the following Goon, Penco

$$\Delta M_{\text{ext}} \approx -T_0(M,p)\Delta S(M,p)$$

where T_0 is the uncorrected temperature

We also note a more precise relation

$$\Delta M_{\text{ext}} = -\frac{1}{2}T(M^{(0)}_{\text{ext}},p)\Delta S(M^{(0)}_{\text{ext}},p)$$

where $M^{(0)}_{\text{ext}} = p/\sqrt{2}$

But notice that ΔS_{ext} plays no role!

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- First example of non-extremal α'-corrected RN BH in ST
- Entropy computed satisfies 1st law
- Satisfies mild form of WGC ($Q \equiv p/\sqrt{2}$)

$$\left.\frac{Q}{M}\right|_{\text{ext}} = 1 + \frac{\alpha'}{80M^2} + \mathcal{O}(\alpha'^2)$$

- Additional fields activated. Important to determine EFT constraints beyond leading order in the derivative expansion.

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Thank you for your attention