Anomalies in F-theory

Pierre Corvilain

Instituto de Física Teórica UAM-CSIC

Based on

[1710.07626] with T. Grimm and D. Regalado and [200x.xxxx] P.C. to appear

Iberian Strings 2020 - Santiago de Compostela

F-Theory

Book-keeping device to describe Type IIB backgrounds with non-constant ϕ.

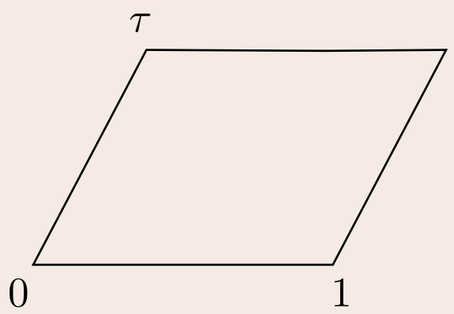
Recall $g_s = e^{\langle\phi\rangle}$. Necessary for global models.

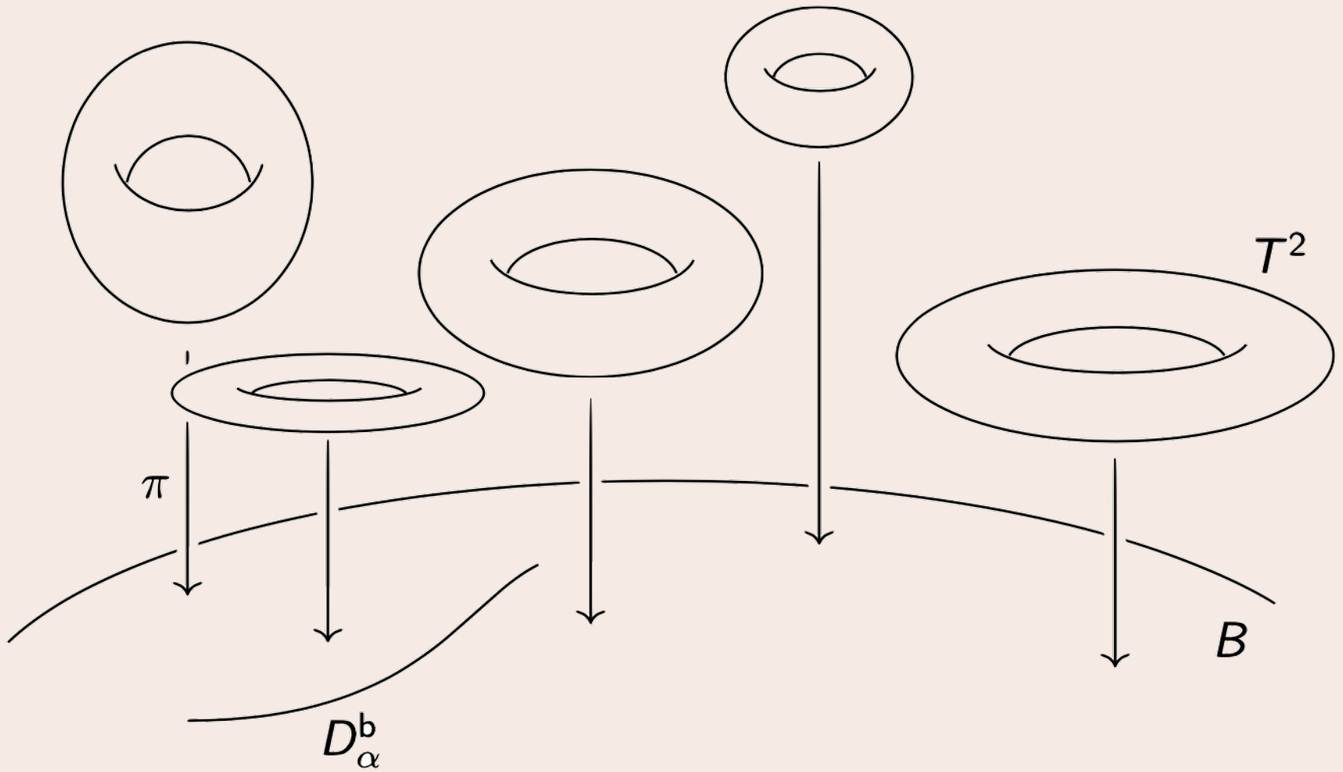
Vafa '96

axio-dilaton $\tau = C_0 + ie^{-\phi}$

identified with

Structure constant of a torus





With the extra torus F-Theory is 12D

- Compactification on a Calabi-Yau **threefold** yields **6D** EFT
- Compactification on a Calabi-Yau **fourfold** yields **4D** EFT

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Low energy physics ⟷ geometry of the internal space, e.g.

Non-abelin gauge group	⟷	Singularities of the CY
U(1) gauge factors	⟷	Extra sections
Spectrum	⟷	Hodge numbers of the CY
⋮		

Morisson, Vafa '96,…

Anomalies

For a $\mathcal{N} = (1,0)$ 6D theory with abelian gauge group

Pure gravitational anomaly $$n_H - n_V = 273 - 29\, n_T$$
$$a \cdot a = 9 - n_T$$

Mixed $$a \cdot b_{ij} = -\frac{1}{6} \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}$$

Pure gauge $$b_{ijkl} := b_{ij} \cdot b_{kl} + b_{ik} \cdot b_{jl} + b_{il} \cdot b_{jk} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}$$

a and b_{ij} are the Green-Schwarz parameters.

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a and b_{ij} are the Green-Schwarz parameters.

They have been matched to **geometrical quantities** Park '11

$$a^\alpha = K^\alpha$$ Coeff of the anti-canonical class

$$b_{ij}^\alpha = -\pi(D_i \cap D_j)^\alpha$$ Divisors

EFTs from F-Theory

The anomaly equations thus correspond to **geometrical equations**

Need to be satisfied for the effective theory to be anomaly free

Park ‘11

Grassi, Morrison ‘11

No proof in general

Bies, Mayerhofer, Weigand ‘17

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Physics proof of these geometrical identities

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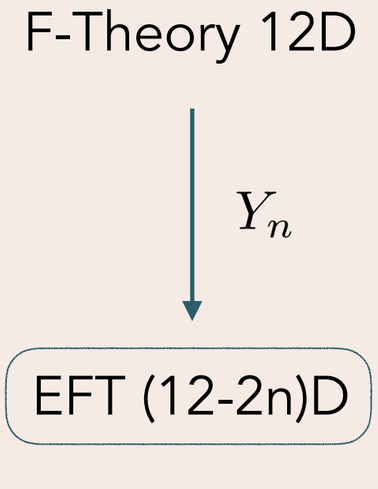
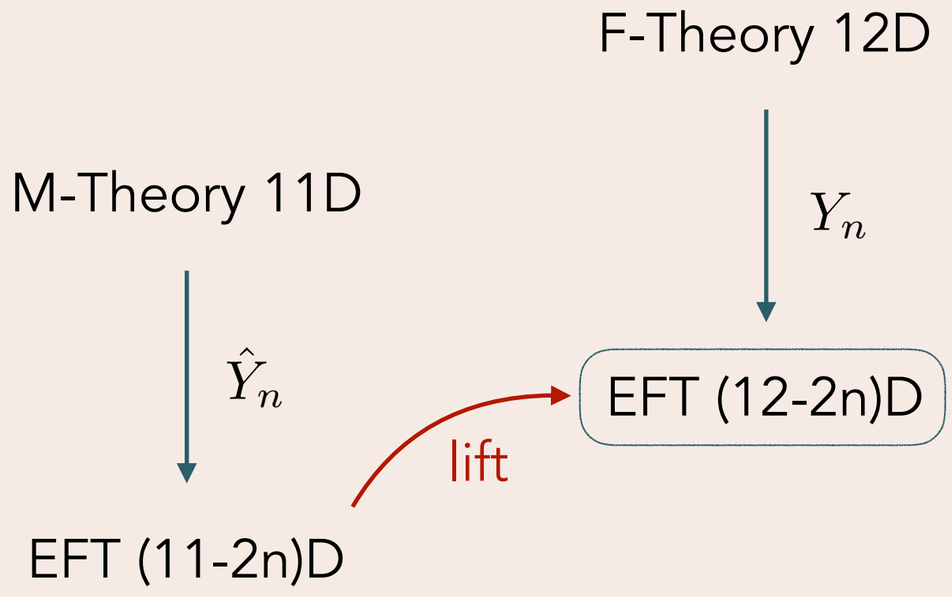
How to obtain EFTs from F-theory ?

F/M-Theory duality



Vafa ‘96

F/M-Theory duality



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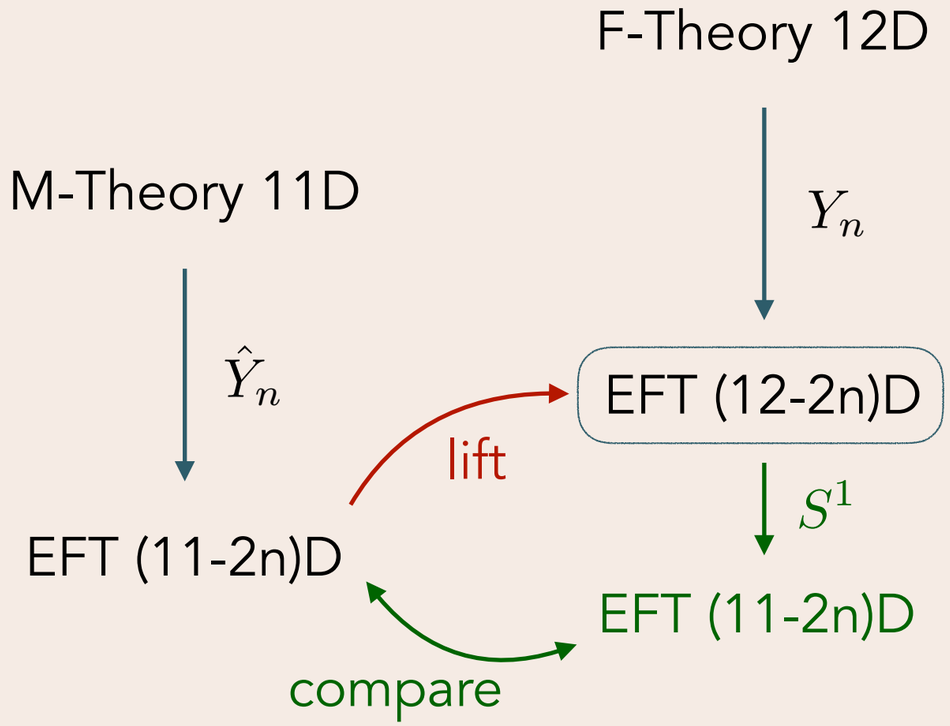

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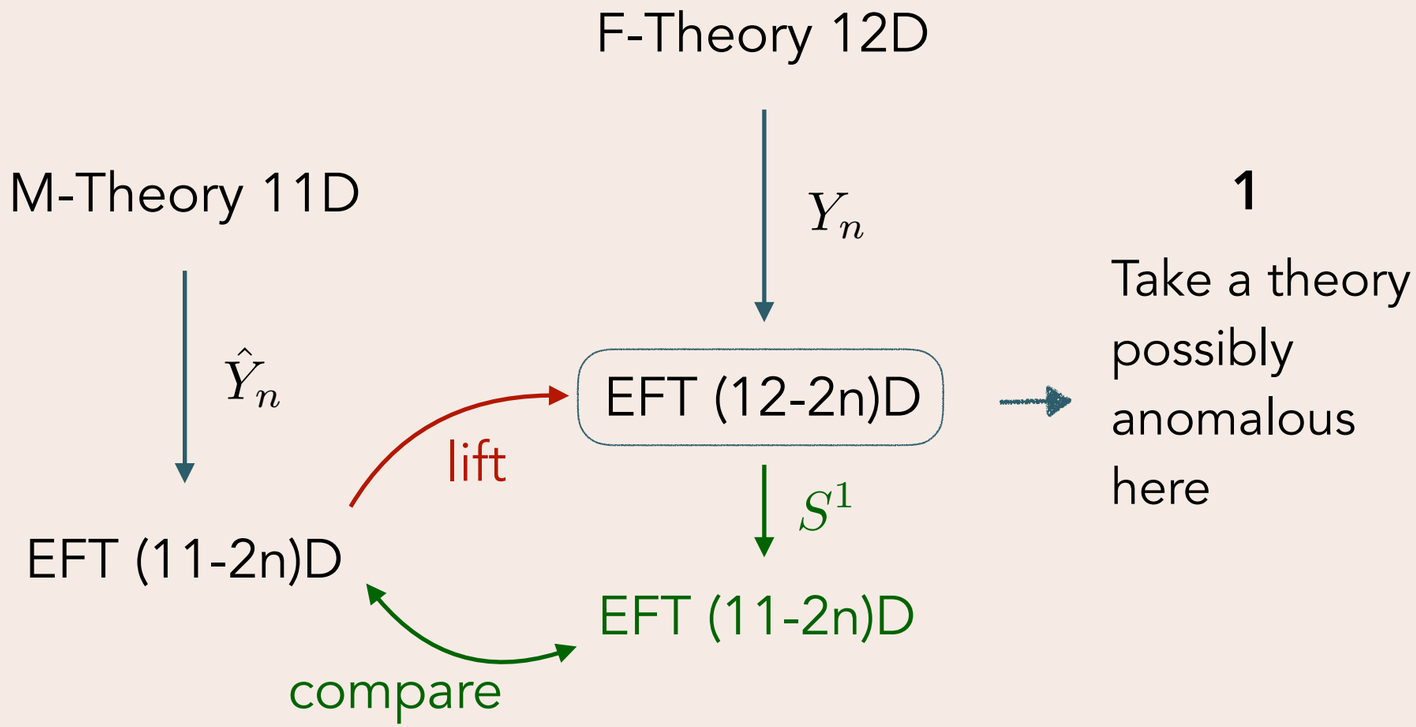

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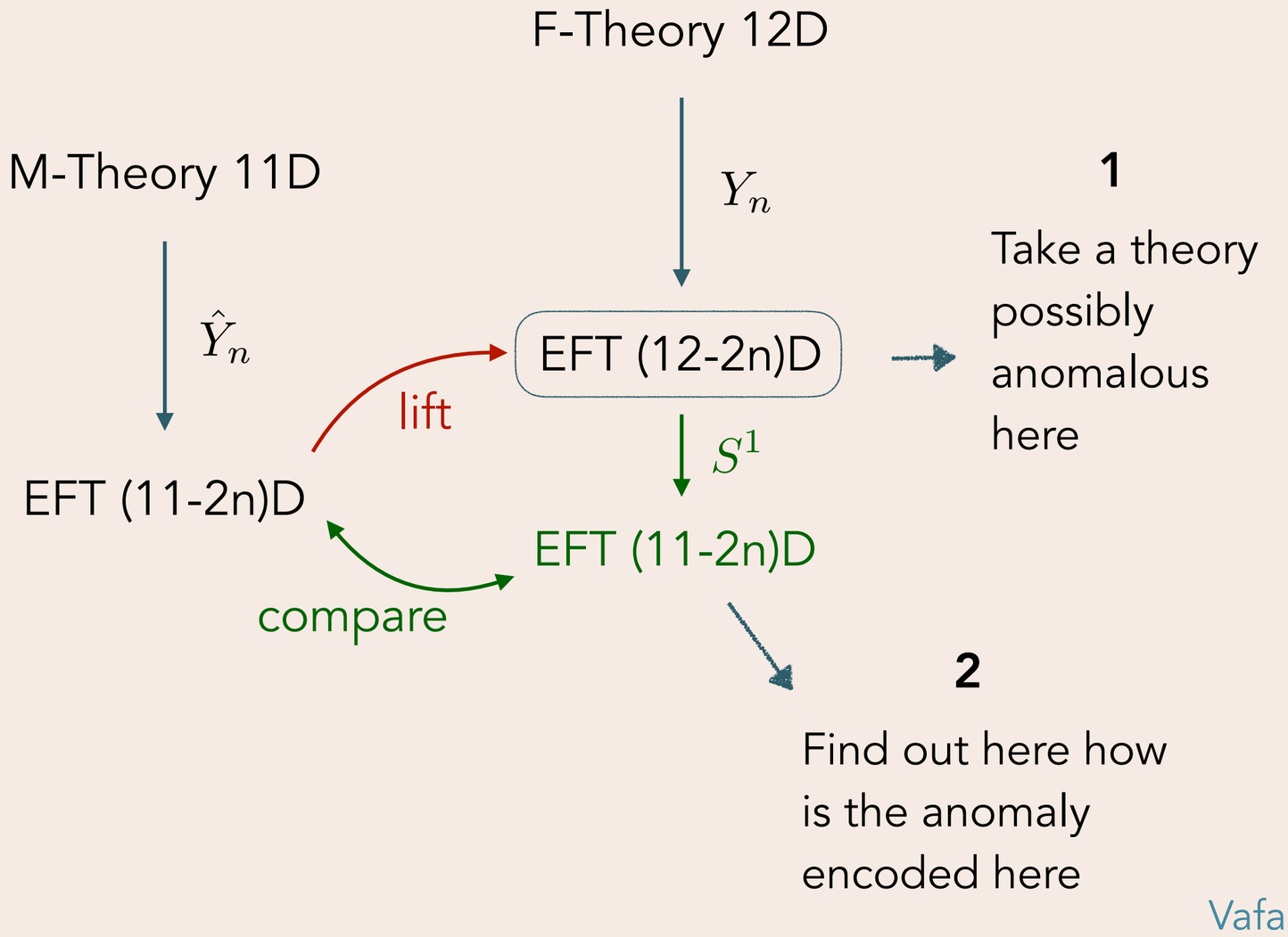

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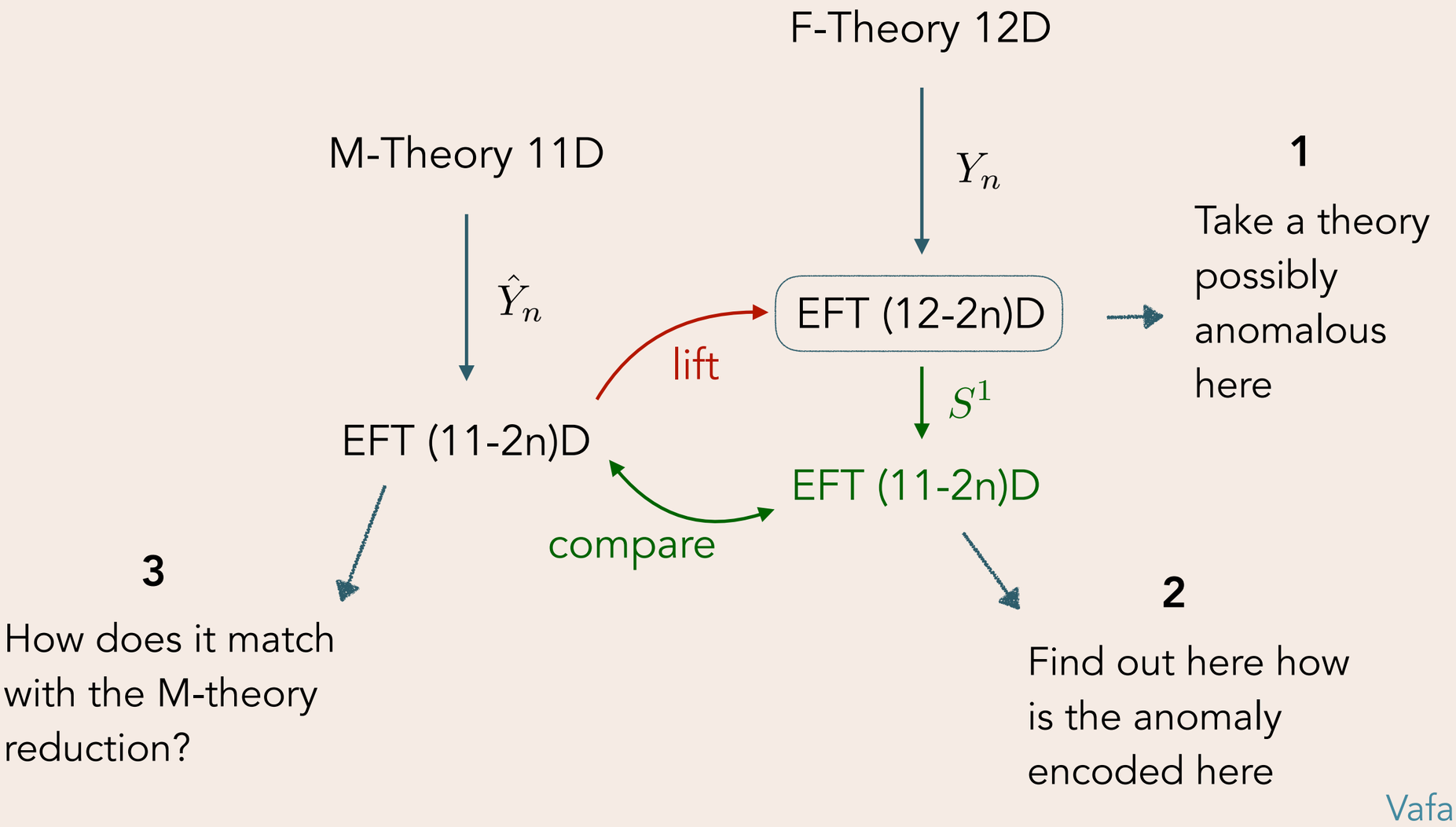

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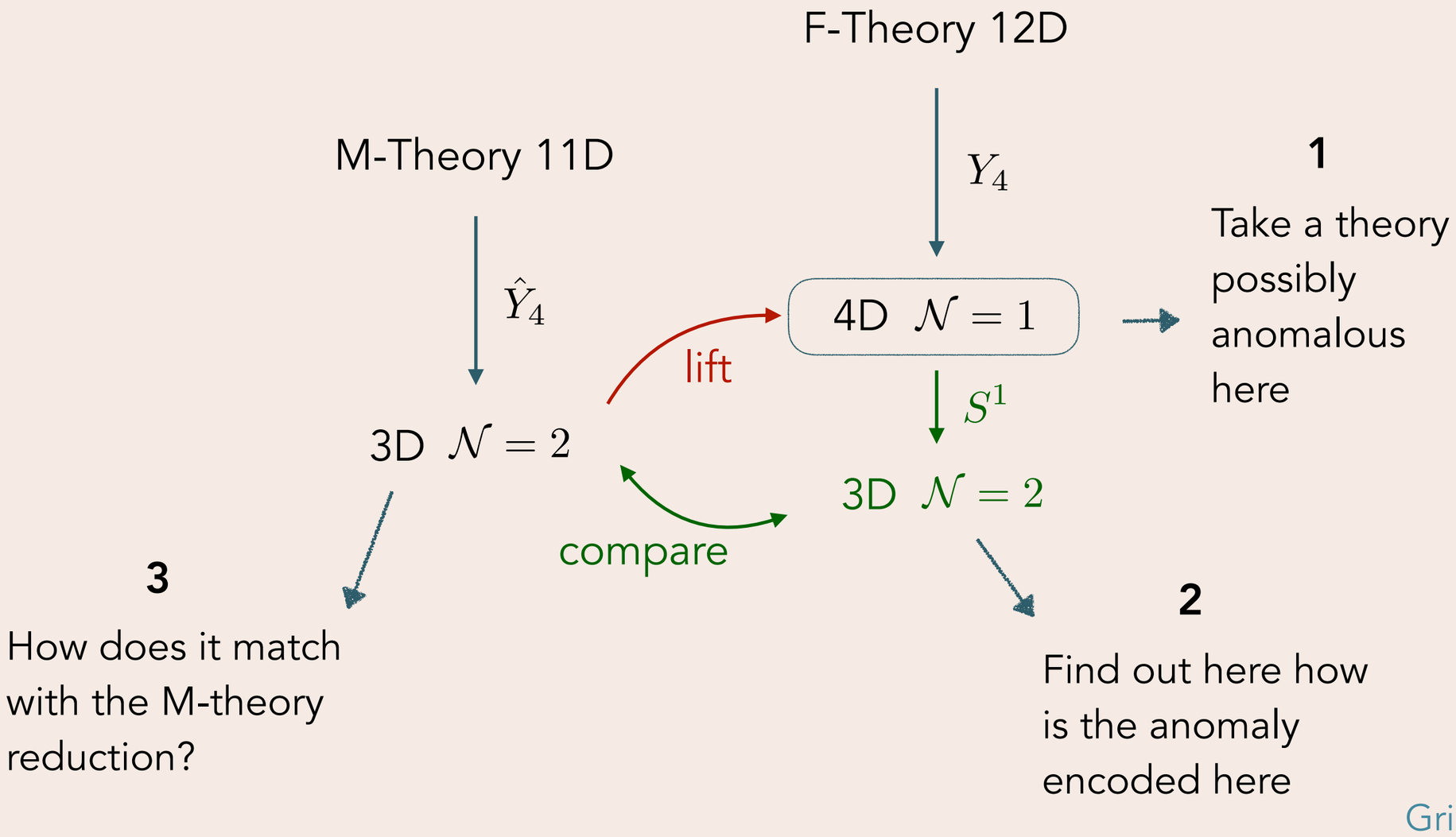
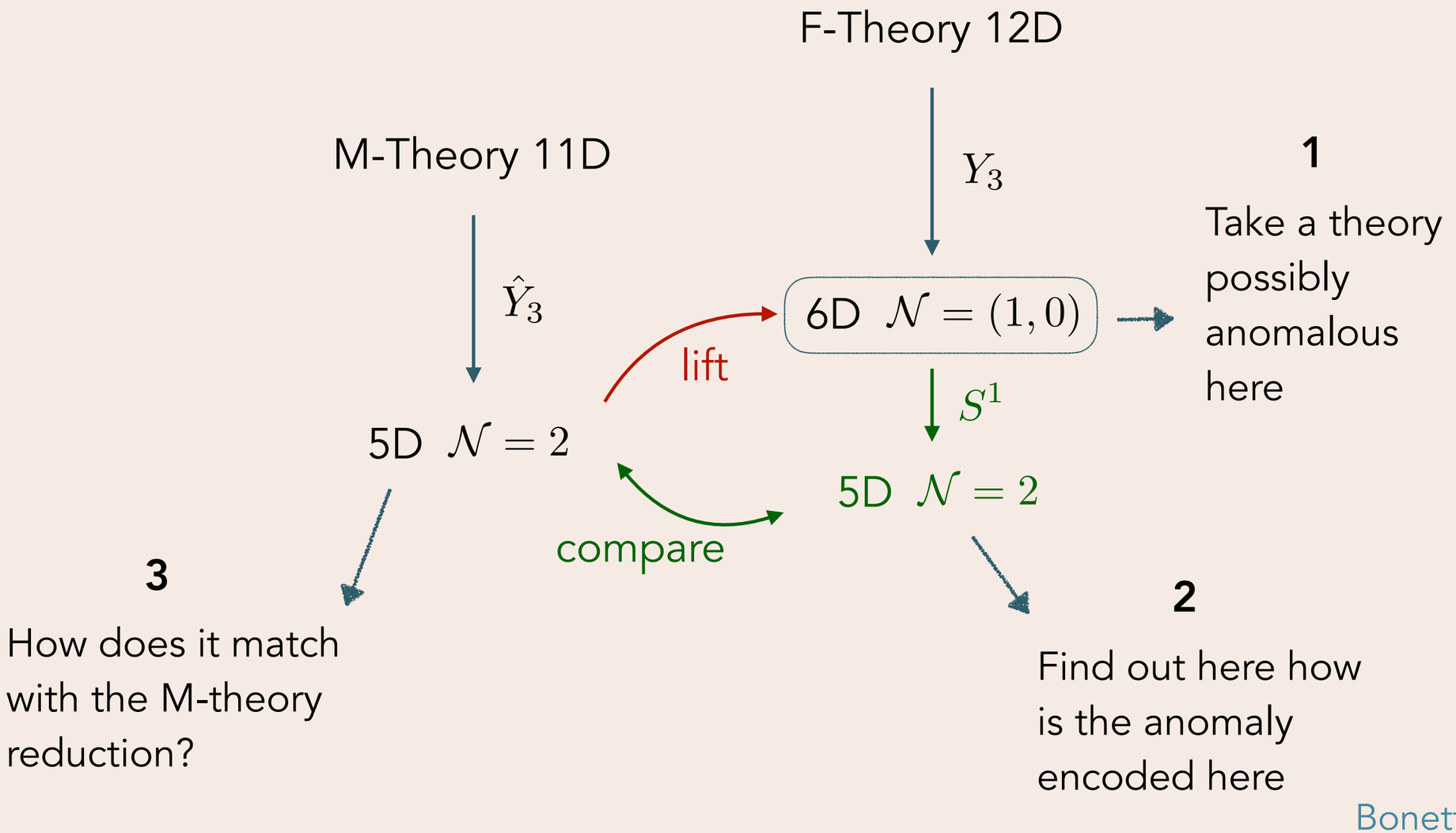


Grimm ‘10

F/M-Theory duality



Bonetti, Grimm '11

Anomalies on a circle

We start a 4D or 6D theory with a local anomaly, we reduce on the circle

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4D

$\downarrow s^1$

3D

6D

$\downarrow s^1$

5D

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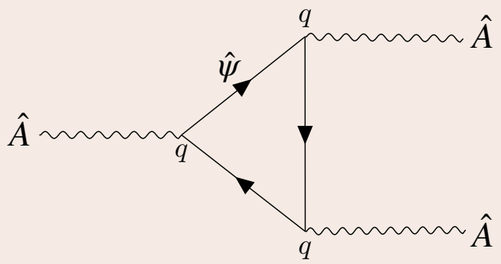
We start a 4D or 6D theory with a local anomaly, we reduce on the circle

For simplicity, we consider a single chiral fermion, charge q. 1PI action varies as



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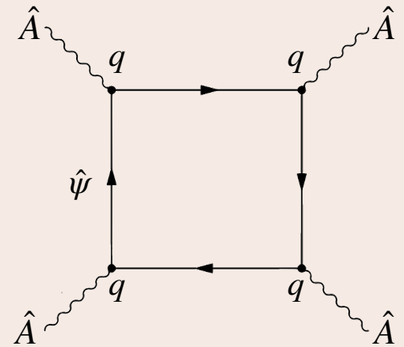
$$q^4 \int \hat{\lambda}\, \hat{F} \wedge \hat{F} \wedge \hat{F}$$



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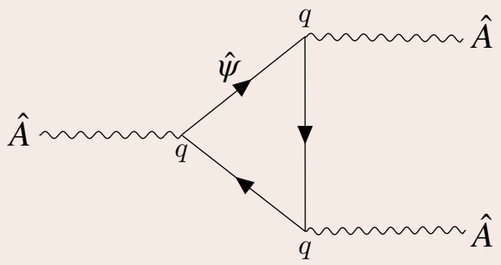

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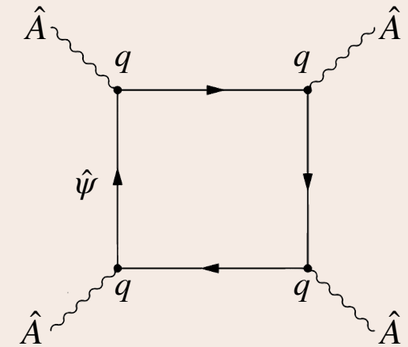
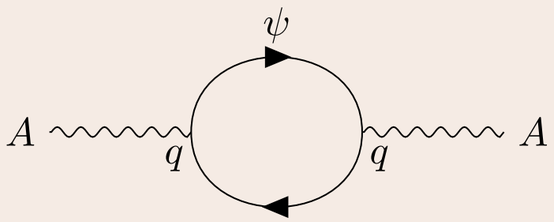
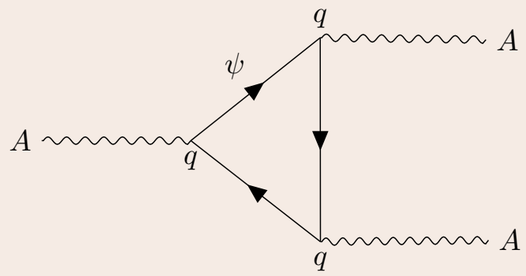


In one dimension lower, anomaly related to the Chern-Simons terms $\hat{A} = A + \zeta\ \mathrm{d}y$



$$\Theta\, A \wedge F$$

$$k\, A \wedge F \wedge F$$



Generated at one-loop by chiral fermions running in the loop

One-loop CS terms

Integrating out a single fermion of mass m

$$\Theta = \frac{1}{2} q^2 \, \mathrm{sign}(m)$$

$$k = \frac{1}{2} \, q^3 \, \mathrm{sign}(m)$$

One-loop CS terms

Integrating out a single fermion of mass m

$$\Theta = \frac{1}{2} q^2 \, \mathrm{sign}(m) \qquad\qquad k = \frac{1}{2} \, q^3 \, \mathrm{sign}(m)$$

Whole tower of KK-modes with mass $\frac{n}{r} + q \, \langle \zeta \rangle$ $\qquad \mu = q \, \langle \zeta \rangle \qquad\qquad \hat{A} = A + \zeta \, \mathrm{d}y$

$$\Theta = \frac{1}{2} \sum_{n \in \mathbb{Z}} q^2 \, \mathrm{sign} \left(\frac{n}{r} + \mu \right) \qquad\qquad k = \frac{1}{2} \sum_{n \in \mathbb{Z}} q^3 \, \mathrm{sign} \left(\frac{n}{r} + \mu \right)$$

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Sum diverge, regularization with zeta-function

$$\Theta^{\mathrm{reg}} = q^2 \left(\frac{1}{2} + \lfloor \mu \rfloor - \mu \right) \qquad\qquad k^{\mathrm{reg}} = q^3 \left(\frac{1}{2} + \lfloor \mu \rfloor - \mu \right)$$

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Do not reproduce the anomaly. Regularization should preserve the **UV symmetries**, in particular **higher dimensional Lorenz invariance** (UV ↔ short distances)

PC, Grimm, Regalado ‘17

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$$\Theta^{\mathbf{reg}} = q^2 \left(\frac{1}{2} + \lfloor \mu \rfloor - \frac{2}{3} \, q \, \zeta \right) \qquad\qquad k^{\mathbf{reg}} = q^3 \left(\frac{1}{2} + \lfloor \mu \rfloor - \frac{3}{4} \, q \, \zeta \right)$$

PC, Grimm, Regalado ‘17 — Reproduce the anomaly correctly! — PC to appear

Field-dependent CS terms

Include several abelian gauge fields, the KK-photon and all the fermions.

Assume $\lfloor \mu \rfloor = 0$. We find

$$\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}} \left(\frac{1}{2} - \frac{2}{3}\, q_k^{\mathsf{f}}\, \zeta^k\right) A^i \wedge F^j$$

$$+ 2 \sum_{\mathsf{f}} q_i^{\mathsf{f}} \left(-\frac{1}{12} + \frac{1}{6}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, \zeta^j \zeta^k\right) A^i \wedge F^0$$

PC, Grimm, Regalado '17

$$\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}} \left(\frac{1}{2} - \frac{3}{4}\, q_l^{\mathsf{f}}\, \zeta^l\right) A^i \wedge F^j \wedge F^k$$

$$+ 3 \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}} \left(-\frac{1}{12} + \frac{1}{4}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}\, \zeta^k \zeta^l\right) A^i \wedge F^j \wedge F^0$$

$$- \frac{1}{4} \sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}\, \zeta^j \zeta^k \zeta^l\, A^i \wedge F^0 \wedge F^0$$

$$+ \frac{1}{120} \left(n_H - n_V - n_T - 3\right) A^0 \wedge F^0 \wedge F^0$$

PC to appear

The anomaly is captured by the **field-dependent CS terms** (non gauge invariant)

They appears **if and only if** the original theory is **anomalous**

Green-Schwarz mechanism

Coefficients crucial when anomalies are canceled by a **Green-Schwarz-Sagnotti mechanism**

Actually for **6D abelian theories**, they **have to** (anomaly is $\propto q^4$)

Green-Schwarz '84
Sagnotti '92

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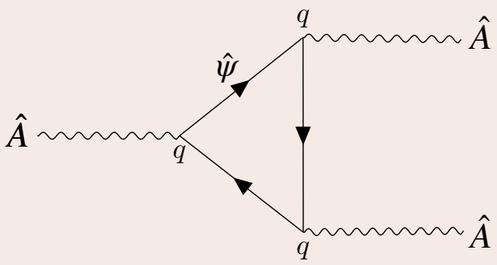
1PI variation



$$\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}} \int \hat{\lambda}^i\, \hat{F}^j \wedge \hat{F}^k$$

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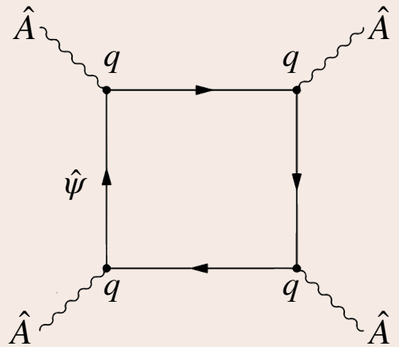
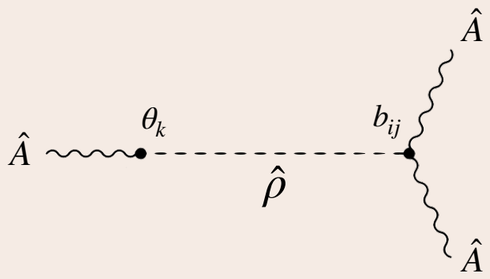


Green-Schwarz terms

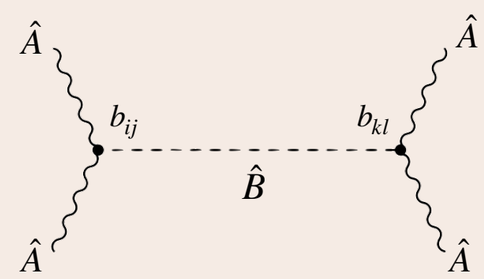


$$-b_{ij}\, \hat{\rho}\, \hat{F}^i \wedge \hat{F}^j$$

$$\delta\hat{\rho} = 2\,\theta_i \hat{\lambda}^i$$

$$-b_{ij}\, \hat{B} \wedge \hat{F}^i \wedge \hat{F}^j$$

$$\delta\hat{B} = 2\, b_{ij} \hat{\lambda}^i \hat{F}^j$$



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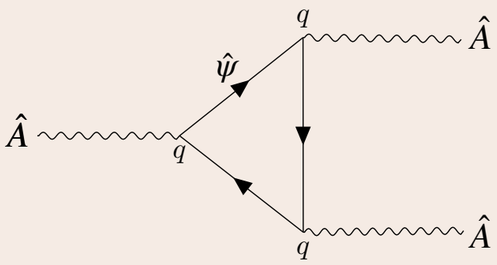

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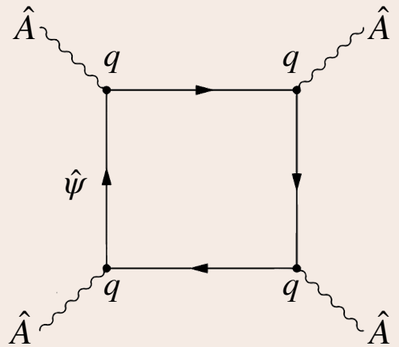

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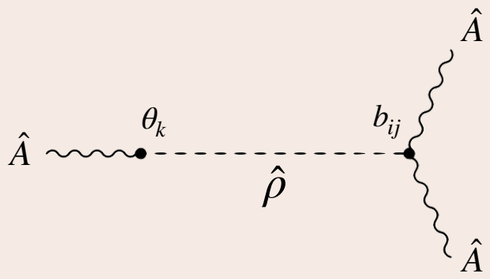

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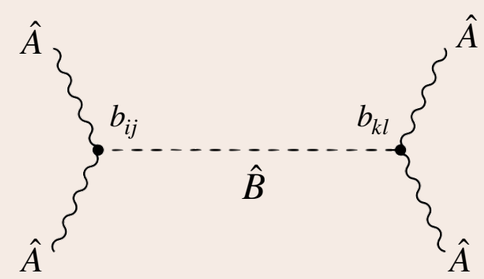

Anomaly cancellation equations

$$\theta_i b_{jk} = \sum_{\mathsf{f}}\, q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}$$

$$b_{ijkl} = \sum_{\mathsf{f}}\, q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}$$

GS terms on a circle

$$-b_{ij} \bullet \hat{\rho} \wedge \hat{F}^i \wedge \hat{F}^j$$

$$-b_{ij} \bullet \hat{B} \wedge \hat{F}^i \wedge \hat{F}^j$$

Reduce on S^1 and dualize

$$\rho \to A$$

$$B \to A$$

PC, Grimm, Regalado ‘17

PC to appear

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Resulting CS terms

$$\frac{1}{3} f \bullet \left(b_{ij} A^i \wedge F^j - A \wedge F^0 \right) + \frac{1}{3} \theta_i \bullet A \wedge F^i$$

$$- \frac{1}{24}\, \theta_i \cdot b_{jk} \left(\zeta^i A^j \wedge F^k - \frac{1}{2}\, \zeta^i \zeta^j A^k \wedge F^0 \right)$$

$$- \frac{1}{4} A^0 \wedge F \wedge \bullet F + \frac{1}{4}\, b_{ij} \bullet A \wedge F^i \wedge F^j \,,$$

$$- \frac{1}{16}\, b_{ijkl} \left(\zeta^i A^j \wedge F^k \wedge F^l - \zeta^i \zeta^j A^k \wedge F^l \wedge F^0 \right.$$

$$\left. + \frac{1}{3}\, \zeta^i \zeta^j \zeta^k A^l \wedge F^0 \wedge F^0 \right)$$

Field-dependent Chen-Simons terms. Because GS terms are not gauge invariant.

PC, Grimm, Regalado ‘17

PC to appear

GS terms on a circle

$$-b_{ij} \bullet \hat{\rho} \wedge \hat{F}^i \wedge \hat{F}^j \qquad\qquad -b_{ij} \bullet \hat{B} \wedge \hat{F}^i \wedge \hat{F}^j$$

Reduce on S^1 and dualize

$$\rho \to A \qquad\qquad B \to A$$

Resulting CS terms

$$\frac{1}{3} f \bullet \left(b_{ij} A^i \wedge F^j - A \wedge F^0\right) + \frac{1}{3} \theta_i \bullet A \wedge F^i$$
$$- \frac{1}{24}\, \theta_i \cdot b_{jk} \left(\zeta^i A^j \wedge F^k - \frac{1}{2}\, \zeta^i \zeta^j A^k \wedge F^0\right)$$

$$- \frac{1}{4} A^0 \wedge F \wedge \bullet F + \frac{1}{4}\, b_{ij} \bullet A \wedge F^i \wedge F^j \,,$$
$$- \frac{1}{16}\, b_{ijkl} \left(\zeta^i A^j \wedge F^k \wedge F^l - \zeta^i \zeta^j A^k \wedge F^l \wedge F^0 + \frac{1}{3}\, \zeta^i \zeta^j \zeta^k A^l \wedge F^0 \wedge F^0\right)$$

Field-dependent Chen-Simons terms. Because GS terms are not gauge invariant.

Precisely combine with the one-loop field-dep terms

$$- \frac{1}{24} \left(\theta_i b_{jk} - \textstyle\sum_{\mathsf{f}}\, q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\right) \times \left(\zeta^i A^j F^k - \frac{1}{2}\, \zeta^i \zeta^j A^k F^0\right)$$

$$- \frac{1}{16} \left(b_{ijkl} - \textstyle\sum_{\mathsf{f}}\, q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}\right) \times \left(\zeta^i A^j F^k F^l - \zeta^i \zeta^j A^k F^l F^0 + \frac{1}{3}\, \zeta^i \zeta^j \zeta^k A^l F^0 F^0\right)$$

PC, Grimm, Regalado ’17

Cancel iff the anomaly cancels

PC to appear

5D canonical form

5D $\mathcal{N}=2$ vector multiplets

$$S = -\frac{1}{2}\, G_{IJ}\left[\,\mathrm{d}M^I \wedge \star\, \mathrm{d}M^J + \bar{F}^I \wedge \star\, \bar{F}^J\right] - \frac{1}{12}\, k_{IJK}\bar{A}^I \wedge \bar{F}^J \wedge \bar{F}^K\ ,$$

G_{IJ} and k_{IJK} in terms of a cubic potential $\qquad \mathfrak{N} \equiv \frac{1}{3!}\, k_{IJK} M^I M^J M^K \stackrel{!}{=} 1,$

as $\qquad G_{IJ} = -\frac{1}{2}\partial_I \partial_J \log \mathfrak{N}\Big|_{\mathfrak{N}=1}\ , \qquad k_{IJK} = \partial_I \partial_J \partial_K \mathfrak{N}$

Ceresole, Dall'Agata '00

5D canonical form

5D $\mathcal{N} = 2$ vector multiplets

$$S = -\frac{1}{2}\, G_{IJ}\left[\,\mathrm{d}M^I \wedge \star\, \mathrm{d}M^J + \bar{F}^I \wedge \star\, \bar{F}^J\right] - \frac{1}{12}\, k_{IJK}\, \bar{A}^I \wedge \bar{F}^J \wedge \bar{F}^K\,,$$

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Ceresole, Dall'Agata '00

Classically: because of the field dep CS terms coming from GS, potential not cubic

$$\mathfrak{N}^F_{\text{class}} = \frac{1}{2} M^0 M \cdot M - \frac{1}{2}\, b_{ij} \cdot M M^i M^j + \frac{1}{24}\, b_{ijkl}\, \frac{M^i M^j M^k M^l}{M^0}$$

Bonetti, Grimm '11

$$\begin{aligned} M^0 &= a\, r^{-4/3} \\ M^\alpha &= a\, \frac{1}{4}\, r^{2/3}\left(j^\alpha + 2 r^{-2} b^\alpha_{ij}\, \zeta^i \zeta^j\right) \\ M^i &= a\, r^{-4/3} \zeta^i \end{aligned}$$

$a = 32^{-1/3}$

At one loop

Including the **one-loop corrections**:

$$\mathfrak{N}^F_{\text{1-loop}} = \frac{1}{2}\,M^0 M\cdot M - \frac{1}{2}\,b_{ij}\cdot M M^i M^j + \frac{1}{24}\left(b_{ijkl} - \sum_{\mathsf{f}}\, q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, q_l^{\mathsf{f}}\right)\frac{M^i M^j M^k M^l}{M^0}$$
$$+\frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, q_k^{\mathsf{f}}\, M^i M^j M^k - \frac{1}{24}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\, q_j^{\mathsf{f}}\, M^0 M^i M^j + \frac{1}{720}(n_H - n_V - n_T - 3)(M^0)^3$$

At one loop

Including the **one-loop corrections**:

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Using the anomaly conditions $\quad b_{ijkl} = \sum_{\mathsf{f}}\, q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,q_l^{\mathsf{f}}$

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Including the **one-loop corrections**:

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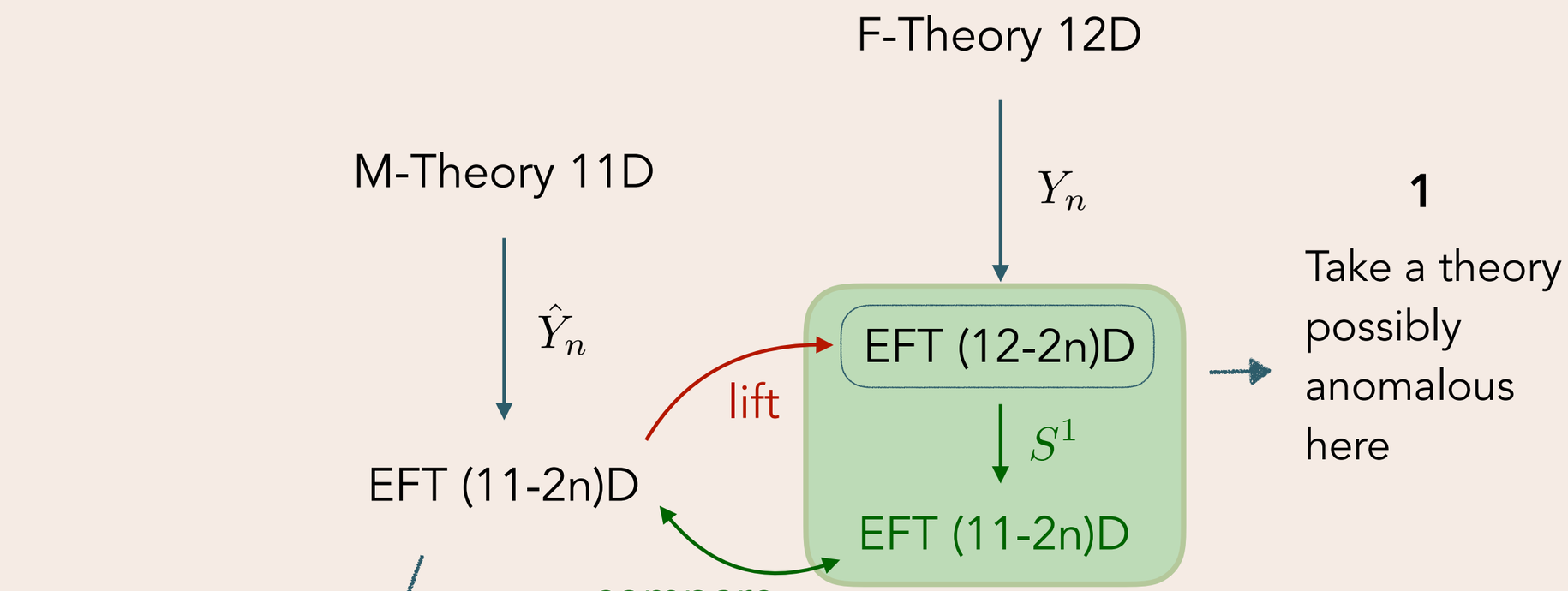
$$+\frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,M^i M^j M^k - \frac{1}{24}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,M^0 M^i M^j + \frac{1}{720}(n_H - n_V - n_T - 3)(M^0)^3$$

Using the anomaly conditions

$$b_{ijkl} = \sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,q_l^{\mathsf{f}}$$
$$n_H - n_V = 273 - 29\,n_T$$
$$a\cdot a = 9 - n_T$$
$$a\cdot b_{ij} = -\frac{1}{6}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}$$

$$\mathfrak{N}^F_{\text{1-loop}} = \frac{1}{2}\,M^0 M\cdot M - \frac{1}{2}\,b_{ij}\cdot M M^i M^j$$
$$+\frac{1}{12}\sum_{\mathsf{f}} q_i^{\mathsf{f}}\,q_j^{\mathsf{f}}\,q_k^{\mathsf{f}}\,M^i M^j M^k + \frac{1}{4}\,a\cdot b_{ij}\,M^0 M^i M^j + \frac{1}{24}\,a\cdot a\,(M^0)^3$$

F/M-Theory duality



1

Take a theory possibly anomalous here

2

Find out here how is the anomaly encoded here

Very interesting in itself!

3

How does it match with the M-theory reduction?

Comparison with M-theory

On the M-theory side, vectors come from expanding C_3 along harmonic 2-forms

$$C_3 = A^I \wedge \omega_I, \qquad I = 1, ..., h^{1,1}(\hat{Y}_3)$$

Morisson, Vafa '96
Ferrara, Khuri, Minasian '96
Ferrara, Minasian, Sagnotti '96

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On the M-theory side, vectors come from expanding C_3 along harmonic 2-forms

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such that the 11D CS term

$$-\frac{1}{12} C_3 \wedge G_4 \wedge G_4$$

yields 3D and 5D CS terms with coefficients

Morisson, Vafa '96
Ferrara, Khuri, Minasian '96
Ferrara, Minasian, Sagnotti '96

Haack, Louis '01

$$\Theta_{IJ} = \int_{\hat{Y}_4} \omega_I \wedge \omega_J \wedge \langle G_4 \rangle \qquad k_{IJK} = \int_{\hat{Y}_3} \omega_I \wedge \omega_J \wedge \omega_K$$

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Being intersection numbers, these are **not** field dependent, s.t. they can **only** be matched with the circle reduction of an **anomaly free** theory

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Being intersection numbers, these are **not** field dependent, s.t. they can **only** be matched with the circle reduction of an **anomaly free** theory

EFTs from F-theory anomaly free → Physics proof of the earlier identities

M-Theory reduction

The ω_I are Poincaré dual to divisors D_I

- The **vectors** A^α lifting to two-forms (tensors multiplets) come from **vertical divisors**

$$D_\alpha = \pi^*(D^{\rm b}_\alpha), \qquad \alpha = 1, ..., h^{1,1}(B_2)$$

- **KK-photon** A^0 comes from the **zero section** σ_0 shifted as

$$D_0 = \sigma_0 - \frac{1}{2}\left(\sigma_0 \cap \sigma_0 \cap D^\alpha\right) D_\alpha \,,$$

Park '11
Grimm, Savelli '11

- $U(1)$ **vectors** come from the **extra sections** σ_i's shifted as

$$D_i = \sigma_i - \sigma_0 - \Big[(\sigma_i - \sigma_0) \cap \sigma_i \cap D^\alpha\Big] D_\alpha$$

Park '11
Morisson Park '11

Intersection numbers $\quad \mathcal{K}_{IJK} = D_I \cap D_J \cap D_K, \qquad I = (0, \alpha, i)$

M-Theory reduction

Non-vanishing intersection numbers:

$$\mathcal{K}_{0\alpha\beta} = \eta_{\alpha\beta}\,, \quad \mathcal{K}_{000} = \frac{1}{4}\,K^\alpha K_\alpha\,, \qquad \mathcal{K}_{\alpha ij} = \pi(D_i \cap D_j)_\alpha\,, \quad \mathcal{K}_{0ij} = -\frac{1}{2}\,\mathcal{K}_{\alpha ij}K^\alpha\,, \quad \text{and} \quad \mathcal{K}_{ijk}$$

where $K_B^{-1} = K^\alpha D_\alpha^{\rm b}$

Cadavid, Ceresole, D'Auria, Ferrara '95 Bonetti, Grimm '11

M-Theory reduction

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where $K_B^{-1} = K^\alpha D_\alpha^{\rm b}$

They give the 5D CS terms, or equivalently, the cubic potential

$$\mathfrak{N}^M = \frac{1}{3!}\,\mathcal{K}_{IJK}L^I L^J L^K = 1$$

with $L^I = \dfrac{v^I}{\mathcal{V}^{1/3}}$ $\qquad J = v^I\omega_I$ $\qquad \mathcal{V} = \displaystyle\int_{Y_3} J\wedge J\wedge J = \mathcal{K}_{IJK}v^I v^J v^K$

Cadavid, Ceresole, D'Auria, Ferrara '95 Bonetti, Grimm '11

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$$\mathfrak{N}^M = \frac{1}{2}\,L^0\,L\circ L + \frac{1}{2}\,\mathcal{K}_{ij}\circ L\,L^i L^j + \frac{1}{6}\,\mathcal{K}_{ijk}\,L^i L^j L^k - \frac{1}{4}\,\mathcal{K}_{ij}\circ K\,L^0\,L^i L^j + \frac{1}{24}\,K\circ K\,(L^0)^3$$

Cadavid, Ceresole, D'Auria, Ferrara '95 Bonetti, Grimm '11

Match

Perfect match between $\mathfrak{N}^F_{1-\text{loop}}$ and $\mathfrak{N}^M$ with the identifications $M^I = L^I$ and

$$a^\alpha = K^\alpha, \qquad b^\alpha_{ij} = -\pi(D_i \cap D_j)^\alpha, \qquad \mathcal{K}_{ijk} = \frac{1}{2}\sum_{\mathsf{f}} q^{\mathsf{f}}_i q^{\mathsf{f}}_j q^{\mathsf{f}}_k$$

Park' 11
Grimm, Taylor '11

M-Theory reduction includes all the one-loop corrections

Witten '96
Intriligator, Morisson, Seiberg '97
Bonetti, Grimm '11

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Park' 11
Grimm, Taylor '11

M-Theory reduction includes all the one-loop corrections

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Coordinates modified at one loop

$$\begin{aligned} M^0 &= a\, r^{-4/3} \\ M^\alpha &= a\, \tfrac{1}{4}\, r^{2/3}\big(\sqrt{1-\Delta}\, j^\alpha + 2r^{-2} b^\alpha_{ij}\, \zeta^i\zeta^j\big) \\ M^i &= a\, r^{-4/3}\zeta^i \end{aligned}$$

$$\Delta = 8\, r^{-4}\Big(\tfrac{1}{6}\, a\bullet a + a\bullet b_{ij}\, \zeta^i\zeta^j + \tfrac{1}{2}\textstyle\sum_{\mathsf{f}} q^{\mathsf{f}}_i\, q^{\mathsf{f}}_j\, q^{\mathsf{f}}_k\, \zeta^i\zeta^j\zeta^k - \tfrac{1}{6}\, b_{ijkl}\, \zeta^i\zeta^j\zeta^k\zeta^l\Big)$$

PC to appear

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Perfect match between $\mathfrak{N}^F_{1-\text{loop}}$ and $\mathfrak{N}^M$ with the identifications $M^I = L^I$ and

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Corrections to the gauge coupling function can be computed

PC to appear

Summary

- **Anomalies on a circle**
  - Regularization preserving higher dimensional Lorentz invariance
  - Field-dependent Chern-Simons terms
  - Include Green-Schwarz mechanism (only option in 6D)
  - Found correct 5D cubic potential

- **Match with M-Theory**
  - CS coeff are intersection numbers → EFTs from F-theory are **anomaly free**
  - Perfect match of circle and M-Theory cubic potential in 5D
  - Read-off corrections to the coordinates → to the gauge coupling functions

Thank you!

Back-up slides

Canonical form

5D $\mathcal{N}=2$ vector multiplets

$$S = -\frac{1}{2}\, G_{IJ}\big[\,\mathrm{d}M^I \wedge \star\,\mathrm{d}M^J + \bar{F}^I \wedge \star\,\bar{F}^J\big] - \frac{1}{12}\, k_{IJK}\bar{A}^I \wedge \bar{F}^J \wedge \bar{F}^K\,,$$

G_{IJ} and k_{IJK} in terms of a cubic potential $\quad \mathfrak{N} \equiv \frac{1}{3!}\, k_{IJK} M^I M^J M^K \stackrel{!}{=} 1,$

as $\qquad G_{IJ} = \Big[-\frac{1}{2}\partial_I\partial_J \log \mathfrak{N}\Big]_{\mathfrak{N}=1}\,, \qquad k_{IJK} = \partial_I\partial_J\partial_K \mathfrak{N}\,.$

Because of the field rep CS terms, potential not cubic

$$\mathfrak{N}^F_{\rm class} = \underbrace{\frac{1}{2} M^0 M\cdot M - \frac{1}{2}\, b_{ij}\cdot M M^i M^j}_{\mathfrak{N}_{\rm p}} + \underbrace{\frac{1}{24}\, b_{ijkl}\, \frac{M^i M^j M^k M^l}{M^0}}_{\mathfrak{N}_{\rm np}}\,,$$

And CS obtained as

$$S_5^{\rm CS} = \int_{\mathcal{M}_5} -\frac{1}{12}(\mathfrak{N}_{\rm p})_{IJK}\bar{A}^I \wedge \bar{F}^J \wedge \bar{F}^K - \frac{1}{16}(\mathfrak{N}_{\rm np})_{iJK}\bar{A}^i \wedge \bar{F}^J \wedge \bar{F}^K$$