T-duality equivalences beyond string theory

Alejandro Vilar López

Iberian Strings 2020, Santiago de Compostela

[Edelstein, Sfetsos, Sierra, AVL, 1803.04517]
[Edelstein, Sfetsos, Sierra, AVL, 1903.05554]

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Low energy String Theory

Effective action

The background on which a string propagates must satisfy the EoM derived from the following effective action, to leading order in α’:

$$\mathcal{I}_0 = \int \mathrm{d}^{D+1}x\sqrt{-G}e^{-2\Phi}\left[R - 2\Lambda + 4\left(\nabla_M\nabla^M\Phi - \nabla_M\Phi\nabla^M\Phi\right) - \frac{1}{12}H_{MNR}H^{MNR}\right] ,$$

where $H_{MNR} = 3\partial_{[M}B_{NR]}$.

This is only the **universal massless NS-NS** sector. We include also a cosmological constant.

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Low energy String Theory

Leading order T-duality

Whenever we have an **isometry** along a spacelike direction, the previous action can be shown to be invariant under the following Buscher rules:

$$\widetilde{G}_{\psi\psi} = \frac{1}{G_{\psi\psi}} \, , \quad \widetilde{G}_{\psi\mu} = -\frac{B_{\psi\mu}}{G_{\psi\psi}} \, , \quad \widetilde{G}_{\mu\nu} = G_{\mu\nu} - \frac{G_{\psi\mu}G_{\psi\nu} - B_{\psi\mu}B_{\psi\nu}}{G_{\psi\psi}} \, ,$$

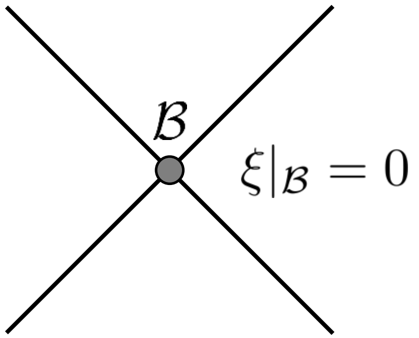
$$\widetilde{B}_{\psi\mu} = -\frac{G_{\psi\mu}}{G_{\psi\psi}} \, , \quad \widetilde{B}_{\mu\nu} = B_{\mu\nu} - \frac{G_{\psi\mu}B_{\psi\nu} - B_{\psi\mu}G_{\psi\nu}}{G_{\psi\psi}} \, ,$$

$$e^{-2\widetilde{\Phi}} = e^{-2\Phi} G_{\psi\psi} \, .$$

Adapted coordinates: $\{x^{\mu}, \psi\}$

T-duality invariances

Background with a bifurcate Killing horizon generated by the Killing field ξ.



It can be proved that:

- The dual background has a bifurcate Killing horizon generated by ξ.
- The Hawking temperature is the same for both backgrounds.
- The entropy of the bifurcate Killing horizon is invariant.

[Horowitz, Welch, 9308077]

Higher-order corrections

Action

A generalized four-derivative action with two free parameters can be written from the Double Field Theory (DFT) formalism, ensuring symmetry under T-duality:

$$\mathcal{I} = \int \mathrm{d}^{D+1}x\sqrt{-G}e^{-2\Phi}\left[R - 2\Lambda + 4\left(\nabla_M\nabla^M\Phi - \nabla_M\Phi\nabla^M\Phi\right) - \frac{1}{12}H'_{MNR}H'^{MNR} + \frac{a}{8}R^{(-)}_{MNA}{}^{B}R^{(-)MN}{}_{B}{}^{A} + \frac{b}{8}R^{(+)}_{MNA}{}^{B}R^{(+)MN}{}_{B}{}^{A}\right] .$$

[Marqués, Núñez, 1507.00652]

Some definitions:

$[a,\, b = \mathcal{O}(M_*^{-2})]$

$$H' = H - \frac{3}{2}\left(a\Theta^{(-)} - b\Theta^{(+)}\right) , \quad \Theta^{(\pm)} = \frac{1}{6}\mathrm{tr}\left[\Omega^{(\pm)} \wedge \mathrm{d}\Omega^{(\pm)} + \Omega^{(\pm)} \wedge \Omega^{(\pm)} \wedge \Omega^{(\pm)}\right]$$

$$\Omega^{(\pm)}_{MA}{}^{B} = \Omega_{MA}{}^{B} \pm \frac{H_{MA}{}^{B}}{2} , \quad R^{(\pm)}{}_{A}{}^{B} = \mathrm{d}\Omega^{(\pm)}{}_{A}{}^{B} + \Omega^{(\pm)}{}_{A}{}^{C} \wedge \Omega^{(\pm)}{}_{C}{}^{B} .$$

Higher-order corrections

Action

$$\mathcal{I} = \int \mathrm{d}^{D+1}x\sqrt{-G}e^{-2\Phi}\left[R - 2\Lambda + 4\left(\nabla_M\nabla^M\Phi - \nabla_M\Phi\nabla^M\Phi\right) - \frac{1}{12}H'_{MNR}H'^{MNR} + \frac{a}{8}R^{(-)}_{MNA}{}^{B}R^{(-)MN}{}_{B}{}^{A} + \frac{b}{8}R^{(+)}_{MNA}{}^{B}R^{(+)MN}{}_{B}{}^{A}\right] .$$

String theory values for the parameters were previously known:

Bosonic ______________ $a = b = -\alpha'$,

Heterotic ______________ $a = -\alpha'$, $b = 0$,

Type II ______________ $a = b = 0$.

[Metsaev, Tseytlin, Nucl. Phys. B **293** (1987) 385]

[Bergshoeff, de Roo, Phys. Lett. B **218** (1989) 210]

[Bergshoeff, de Roo, Nucl.Phys. B **328** (1989) 439]

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Higher-order corrections

Corrected T-duality rules

DFT allows to write (D+1)-dimensional actions as manifestly invariant under T-duality by going to a 2(D+1)-dimensional space. We can read from there the corrected T-duality transformation rules:

Kalb-Ramond 2-form

$$\widehat{B}_{\mu\nu} = \widetilde{B}_{\mu\nu} + \sum_{k=\pm} \frac{a_k}{4} \frac{2}{G_{\psi\psi}} \left(\Omega^{(k)2}_{\psi[\mu|} - \frac{\Omega^{(k)2}_{\psi\psi}}{G_{\psi\psi}} G_{\psi[\mu|} \right) B_{\psi|\nu]} \ ,$$

$$\widehat{B}_{\psi\mu} = \widetilde{B}_{\psi\mu} + \sum_{k=\pm} \frac{a_k}{4} \frac{1}{G_{\psi\psi}} \left(\Omega^{(k)2}_{\psi\mu} - \frac{\Omega^{(k)2}_{\psi\psi}}{G_{\psi\psi}} G_{\psi\mu} \right) \ .$$

Auxiliary results

$$\Omega^{(k)2}_{MN} = \Omega^{(k)}_{MA}{}^{B} \Omega^{(k)}_{NB}{}^{A} \ ,$$

$$a_- = a \ , \quad a_+ = b \ .$$

Higher-order corrections

Corrected T-duality rules

Metric

$$\widehat{G}_{\mu\nu} = \widetilde{G}_{\mu\nu} + \sum_{k=\pm} \frac{a_k}{4} \left(\widetilde{\Omega}^{(k)2}_{\mu\nu} - \Omega^{(k)2}_{\mu\nu} + \frac{2\Omega^{(k)2}_{\psi(\mu} G_{\nu)\psi}}{G_{\psi\psi}} - \frac{\Omega^{(k)2}_{\psi\psi}}{G^2_{\psi\psi}} (G_{\psi\mu} G_{\psi\nu} - B_{\psi\mu} B_{\psi\nu}) \right) ,$$

$$\widehat{G}_{\psi\psi} = \widetilde{G}_{\psi\psi} + \sum_{k=\pm} \frac{a_k}{4} \left(\widetilde{\Omega}^{(k)2}_{\psi\psi} + \frac{\Omega^{(k)2}_{\psi\psi}}{G^2_{\psi\psi}} \right) .$$

$$\widehat{G}_{\psi\mu} = \widetilde{G}_{\psi\mu} + \sum_{k=\pm} \frac{a_k}{4} \left(\widetilde{\Omega}^{(k)2}_{\psi\mu} - \frac{\Omega^{(k)2}_{\psi\psi}}{G^2_{\psi\psi}} B_{\psi\mu} \right) .$$

Auxiliary results

$$\Omega^{(k)2}_{MN} = \Omega^{(k)}_{MA}{}^{B} \Omega^{(k)}_{NB}{}^{A} ,$$

$$a_- = a , \quad a_+ = b .$$

Dilaton

$$e^{-2\widehat{\Phi}} \sqrt{-\widehat{G}} = e^{-2\Phi} \sqrt{-G} .$$

Entropy

Two-parameter theory

Using a generalization of the Wald procedure, the full expression for the entropy in this theory is shown to be:

$$S = S_0 - 2\pi \int_{\mathcal{B}} \mathrm{d}^{D-1}x \sqrt{G_{\mathcal{B}}}\, e^{-2\Phi} \Big[\gamma_+ \Big(R^{MNRS} - \frac{3}{4} H^{TMN} H_T{}^{RS}\Big) n_{MN} n_{RS} - \gamma_- H^{TMN} \Omega_T{}^{RS} n_{MN} n_{RS}\Big] \,,$$

where:

$$S_0 = 4\pi \int_{\mathcal{B}} \mathrm{d}^{D-1}x \sqrt{G_{\mathcal{B}}}\, e^{-2\Phi} \,.$$

Definitions:

$$\gamma_\pm = \mp \frac{a \pm b}{4} \,, \quad \Omega_T{}^{RS} = \Omega_T{}^{AB} E_A{}^R E_B{}^S \,,$$

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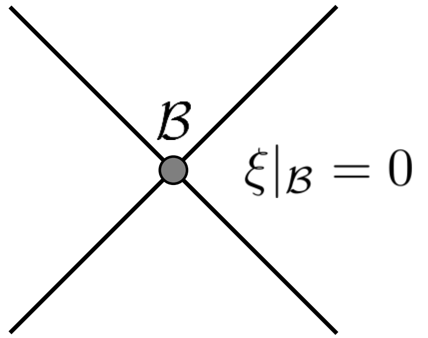
Corrected T-duality invariances

Invariance of temperature and entropy: general proof

Background with a bifurcate Killing horizon generated by the Killing field ξ.



It can be proved that:

- The dual background has a bifurcate Killing horizon generated by ξ.
- The Hawking temperature is the same for both backgrounds.
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Corrected T-duality invariances

An example: the BTZ

We can present an explicit example using the BTZ black hole and its dual:

BTZ solution

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{N^2} + r^2\left(\mathrm{d}\psi + N^\psi \mathrm{d}t\right)^2 \ ,$$

$$B_{t\psi} = \frac{r_+^2 - r^2}{\ell} \ , \qquad e^{-2\Phi} = 1 \ .$$

$$N^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2} \ ,$$

$$N^\psi = \frac{r_+ r_-}{\ell}\left(\frac{1}{r_+^2} - \frac{1}{r^2}\right) \ ,$$

$$J = \frac{2r_+ r_-}{\ell} \ , \qquad M = \frac{r_+^2 - r_-^2}{\ell^2} \ .$$

This is a solution with thermodynamic quantities given by:

$$\kappa = \frac{r_+^2 - r_-^2}{\ell^2 r_+} \ , \qquad S = 8\pi^2 r_+ + \frac{32\pi^2}{\ell^2}(\gamma_+ r_+ + \gamma_- r_-) \ .$$

Corrected T-duality invariances

An example: the BTZ

Dual solution (Black string)

$$[\Delta\widehat{\psi} = 2\pi]$$

$$d\hat{s}^2 = -N^2 dt^2 + \frac{dr^2}{N^2} + e^{-2\sigma}\left(d\widehat{\psi} + N^{\hat{\psi}} dt\right)^2 ,$$

$$\widehat{B}_{t\hat{\psi}} = \frac{e}{\ell}\left(1 - \frac{r_+^2}{r^2}\right) - \frac{2}{\ell r^2}\left[\gamma_+ \frac{J}{\ell}\left(1 - \frac{M\ell^2}{2r^2}\right) + \gamma_-\left(M - \frac{J^2}{2r^2}\right)\right] ,$$

$$e^{-2\widehat{\Phi}} = r^2(1 + \Delta_+) .$$

$$N^2 = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{\ell^2 r^2} ,$$

$$N^{\hat{\psi}} = \frac{r_+^2 - r^2}{\ell}(1 + e\Delta_-) ,$$

$$e = \frac{r_-}{r_+} , \quad e^{-\sigma} = \frac{1 - \Delta_+}{r^2} ,$$

$$\Delta_\pm = \frac{2}{r^2}\left(\gamma_\pm M + \gamma_\mp \frac{J}{\ell}\right) ,$$

$$J = \frac{2r_+ r_-}{\ell} , \quad M = \frac{r_+^2 - r_-^2}{\ell^2} .$$

Both temperature and entropy are preserved:

$$\widehat{\kappa} = \frac{r_+^2 - r_-^2}{\ell^2 r_+} = \kappa , \quad \widehat{S} = 8\pi^2 r_+ + \frac{32\pi^2}{\ell^2}(\gamma_+ r_+ + \gamma_- r_-) = S .$$

Summary and Conclusions

- We have shown that the entropy and temperature of a generic non-extremal black hole solution are invariant under corrected T-duality. This is so for all values of the parameters of the theory, not just the String Theory ones.

- This is surprising. We use T-duality as a symmetry principle to constrain possible effective actions, but for values of the parameters without a known sigma model one is not guaranteed to have a physical equivalence. Entropy, though, does not break the equivalence, as we have shown.

Summary and Conclusions

- Future possibilities would be to explore whether or not other thermodynamic quantities are also invariant for all values of the parameters.

- There is also the reasonable question about whether or not this equivalence remains to be true when we go to higher orders in the derivative expansion. It would be nice to check this, but building the actions rapidly becomes quite difficult.

Graciñas!