Discrete Symmetries in Dimer Diagrams

Eduardo García-Valdecasas, **Alessandro Mininno**, Ángel M. Uranga

Instituto de Física Teórica IFT-UAM/CSIC,
C/ Nicolás Cabrera 13-15, Campus de Cantoblanco, 28049 Madrid, Spain

arXiv: 1907.06938 (hep-th) (2019)

Instituto de Física Teórica UAM-CSIC

Universidad Autónoma de Madrid

Iberian Strings 2020

Santiago de Compostela

15th January 2020

"la Caixa" Foundation

A brief history of discrete symmetries

Discrete symmetries are very important in many theories of physics beyond the Standard Model for phenomenological reasons.

They are invoked, for instance,

- to solve the flavor puzzle: to explain the origin of fermions masses and their mixing;
- to prevent the proton to decay in MSSM, using $R-$parity or baryon triality.[1]

However their role remain obscure, and we should still investigate them.

[1] L. E. Ibanez, G. G. Ross, *Nucl. Phys.* **B368**, 3–37 (1992).

The Swampland program

"The Swampland program aims to distinguish effective theories which can be completed into quantum gravity in the ultraviolet from those which cannot."[2]

A consistent theory of quantum gravity cannot admit global symmetries[3], e.g. black hole evaporation.[4]

An exact symmetry should be **gauged**.

[2] E. Palti, arXiv: 1903.06239 (hep-th) (2019).

[3] T. Banks, N. Seiberg, arXiv: 1011.5120 (hep-th) (2011).

[4] N. Arkani-Hamed *et al.*, arXiv: hep-th/0601001 (hep-th) (2007).

Discrete symmetries in string theory

There has been a lot of studies on the realization of discrete symmetries in string theory:

- Abelian gauge symmetries have been explored in MSSM-like models or D-brane models.[5]
- Non-abelian discrete symmetries have been studied in
  1. 4d string compactifications[6]
  2. AdS/CFT correspondence[7]
- Attempts to refine the Swampland conjectures considering effects coming from discrete symmetries.[8]

[5] P. G. Camara *et al.*, arXiv: 1106.0060 (hep-th) (2011); M. Berasaluce-Gonzalez *et al.*, arXiv: 1106.4169 (hep-th) (2011); L. E. Ibanez *et al.*, arXiv: 1205.5364 (hep-th) (2012); M. Berasaluce-Gonzalez *et al.*, arXiv: 1211.5317 (hep-th) (2013); M. Berasaluce-Gonzalez *et al.*, arXiv: 1305.6788 (hep-th) (2013).

[6] M. Berasaluce-Gonzalez *et al.*, arXiv: 1206.2383 (hep-th) (2012).

[7] S. Gukov *et al.*, arXiv: hep-th/9811048 (hep-th) (1998); B. A. Burrington *et al.*, arXiv: hep-th/0602094 (hep-th) (2006); B. A. Burrington *et al.*, arXiv: hep-th/0701028 (hep-th) (2008).

[8] N. Craig *et al.*, *JHEP* **05**, 140 (2019).

Contents

Abelian gauge symmetries in 4d

A residual $\mathbf{Z}_N$ symmetry is present once a U(1) charged particle is higgsed.

The phase ϕ of the Higgs field will have the following kinetic term

$$|\partial_\mu \phi - N A_\mu|^2 \ ,$$

which is invariant under the simultaneous transformation

$$A_\mu \longrightarrow A_\mu + \partial_\mu \Lambda \quad \text{and} \quad \phi \longrightarrow \phi + N\Lambda \ .$$

The dualization of such system results in the usual BF theory[9]

$$\frac{1}{2} H_3 \wedge \star H_3 + N B_2 \wedge F_2 + \frac{1}{2} F_2 \wedge \star F_2 \ ,$$

with

$$H_3 = dB_2 = \star d\phi \quad \text{and} \quad F_2 = dA_1 \ .$$

[9] T. Banks, N. Seiberg, arXiv: 1011.5120 (hep-th) (2011).

Abelian gauge symmetries in 4d

It is possible to identify the objects charged under such discrete symmetries:[10]

- $\mathbf{Z}_N$−charged particles

$$\mathcal{O}_p \sim e^{2\pi i n \int_C A_1}$$

- Instanton operators which annihilate N charged particles

$$e^{-2\pi i \phi}\mathcal{O}_p$$

- $\mathbf{Z}_N$−charged strings

$$\mathcal{O}_s \sim e^{2\pi i k \int_\Sigma B_2}$$

- String junction which annihilate N charged strings

$$e^{-2\pi i \int_L \tilde{A}_1}\mathcal{O}_s$$

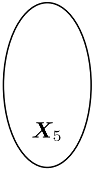
A particle $\mathcal{O}_p$ with charge n will pick up a Aharonov-Bohm phase[11] $e^{2\pi i \frac{nk}{N}}$ when moved around a charge k string $\mathcal{O}_s$.

[10] M. Berasaluce-Gonzalez *et al.*, arXiv: 1206.2383 (hep-th) (2012).

[11] Y. Aharonov, D. Bohm, *Phys. Rev.* **115**, [,95(1959)], 485–491 (1959).

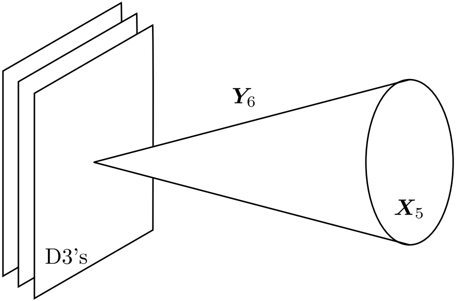
Introducing dimers

Take IIB string theory on $AdS_5 \times \boldsymbol{X}_5$ with N units of RR $5-$form flux over $\boldsymbol{X}_5$.



Introducing dimers

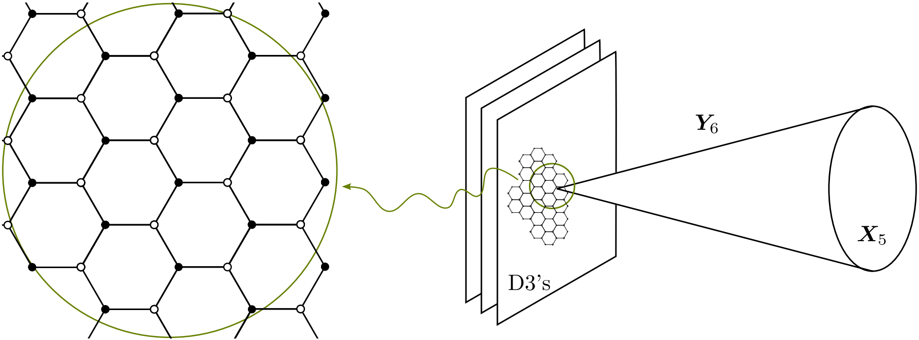
The theory is dual to the CFT living on N $D3-$branes probing a CY singularity.[12]



[12] J. M. Maldacena, arXiv: hep-th/9711200 (hep-th) (1999).

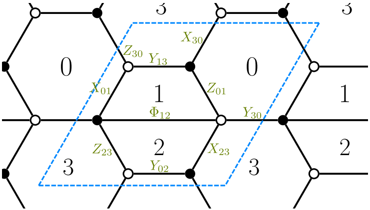
Introducing dimers

If the CY is toric, it is possible to represent the gauge theory using dimer diagrams.[12]



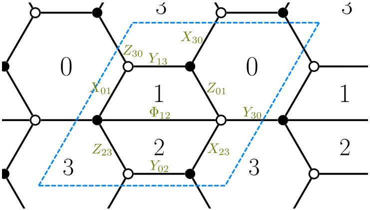
[12]K. D. Kennaway, arXiv: 0706.1660 (hep-th) (2007); S. Franco *et al.*, arXiv: hep-th/0504110 (hep-th) (2006); A. Hanany, K. D. Kennaway, arXiv: hep-th/0503149 (hep-th) (2005).

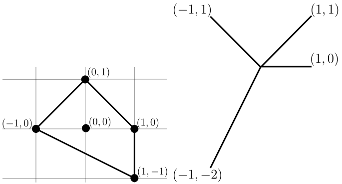
Dimer diagrams



Dimer diagrams	$\longleftrightarrow$	**Gauge theory**
Face F_a	$\longleftrightarrow$	gauge factor $\mathrm{U}(N_a)$
Edge E_i between F_a and F_b	$\longleftrightarrow$	bifundamental chiral $(\square_a, \bar{\square}_b)$
Black vertex V_α	$\longleftrightarrow$	superpotential term $+\mathrm{tr}\left[\Phi_{E_1}\dots\Phi_{E_n}\right]$
White vertex V'_α	$\longleftrightarrow$	superpotential term $-\mathrm{tr}\left[\Phi_{E_1}\dots\Phi_{E_n}\right]$

Dimer diagrams





- encode gauge theories on $D3-$branes at toric CY $3-$fold singularities.
- are bipartite tilings of $\mathbf{T}^2$.
- can describe theories for general ranks N_a, satisfying

$$\sum_a N_a I_{ab} = 0 \, , \forall b \, .$$

- describe **regular** $D3-$branes for $N_a = N$ for all a.
- contains the information about toric diagrams and $(p, q)-$web diagrams of the CY.

Mesonic and baryonic symmetries

A toric CY 3−fold enjoys a $\mathrm{U}(1)^3$ symmetry:

1. R−symmetry $\Longrightarrow$ a−maximization.[13]
2. 2 **mesonic** symmetries $\Longrightarrow$ perfect matchings.

A general dimer will also have $\mathrm{U}(1)_a$ symmetries associated to the faces F_a, called **baryonic** symmetries, in general anomalous.

The non-anomalous combinations of the generator Q_a of the $\mathrm{U}(1)_a$ define baryonic charges

$$Q_{\boldsymbol{B}} = \sum_a n_a Q_a \text{ with } n_a \text{ such that } \sum_a n_a I_{ab} = 0\,.$$

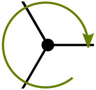
[13] M. Bertolini *et al.*, arXiv: hep-th/0411249 (hep-th) (2004); S. Benvenuti *et al.*, arXiv: hep-th/0411264 (hep-th) (2005).

Continuous symmetries from a Dimer diagram

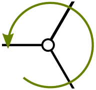
1. Assign a charge to an edge $E_i \Longrightarrow$ 1-form γ such that E_i has charge $\gamma(E_i)$.

2. Invariance of the superpotential $\Longrightarrow$

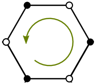
$$\partial V_\alpha \longrightarrow \sum_{E_i \in V_\alpha} \gamma(E_i) = 0$$



$$\partial V'_\alpha \longrightarrow \sum_{E_i \in V'_\alpha} \gamma(E_i) = 0$$



3. Anomaly cancellation conditions $\Longrightarrow \tilde{\partial} F_a \longrightarrow \sum_{E_i \in F_a} \gamma(E_i) = 0$



Geometric identities

For any dimer diagram,

$$F + V = E \ ,$$

F = Faces, V = Vertices and E = Edges.

There are linear relations among the equations:

Geometric identities:

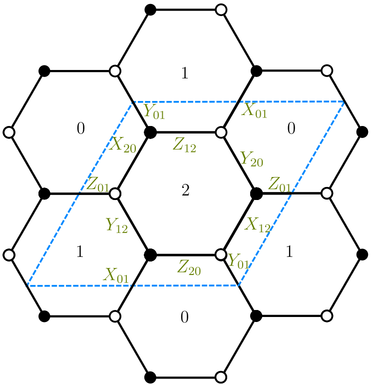
$$\sum_\alpha \partial V_\alpha - \sum_\alpha \partial V'_\alpha = 0$$

$$\sum_a \tilde{\partial} F_a - \sum_\alpha \partial V_\alpha - \sum_\alpha \partial V'_\alpha = 0$$

$$\sum_a n_a \tilde{\partial} F_a = \text{ combination of vertices.}$$

A first Heisenberg group

Let us consider the gauge theory on $D3-$branes at the $\mathbf{C}^3/\mathbf{Z}_3$, $(k_1, k_2) = (1, 1)$, singularity.[14]



$$W_{i,i+1} = \begin{pmatrix} X_{i,i+1} \\ Y_{i,i+1} \\ Z_{i,i+1} \end{pmatrix} \text{ with } i = 0, 1, 2$$

$$A : \begin{cases} i & \Longrightarrow i+1 \\ SU(N_i) & \Longrightarrow SU(N_{i+1}) \\ W_{i,i+1} & \Longrightarrow W_{i+1,i+2} \end{cases}$$

$$B : (W_{01}, W_{12}, W_{20}) \Longrightarrow \left(\omega W_{01}, \omega^{-1} W_{12}, W_{20}\right)$$

$$C : (W_{01}, W_{12}, W_{20}) \Longrightarrow \left(\omega^{-2} W_{01}, \omega W_{12}, \omega W_{20}\right)$$

Heisenberg group: $AB = CBA$

[14] S. Gukov *et al.*, arXiv: hep-th/9811048 (hep-th) (1998).

Set up

Previous problems:[15]

- Symmetries were constructed as global discrete symmetries of the quiver theory.
- Heisenberg group was found solving the conditions of invariance of the superpotential and cancellation of discrete gauge anomalies.

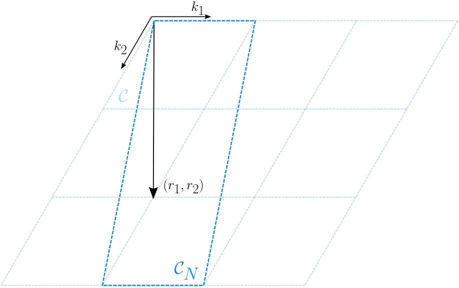
Our **work**:

Unravel the underlying structure of discrete symmetries in **general orbifolds** of **general toric singularities**.

[15] B. A. Burrington *et al.*, arXiv: hep-th/0602094 (hep-th) (2006); B. A. Burrington *et al.*, arXiv: hep-th/0701028 (hep-th) (2008); B. A. Burrington *et al.*, arXiv: hep-th/0604092 (hep-th) (2006); B. A. Burrington *et al.*, arXiv: hep-th/0603114 (hep-th) (2006).

General Orbifolds of General Toric Theories

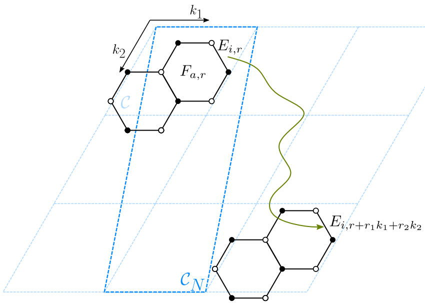
Consider a dimer with unit cell C, faces F_a, edges E_i and vertices V_α, V'_α and let's construct the general $\mathbf{Z}_N$ orbifolds of this theory.[16].



- A unit cell C_N is obtained taking N copies of the unit cell C.
- The infinite copies of C can be labeled by two indices (r_1, r_2).
- (k_1, k_2) determine the action of the orbifold on the mesons, coordinates of the toric geometry.

[16] M. Schmaltz, arXiv: hep-th/9805218 (hep-th) (1999); A. M. Uranga, arXiv: hep-th/9811004 (hep-th) (1999).

General structure



1. Any copy of a cell C is labeled by $r \in \mathbf{Z}$.
2. Jump $r \longrightarrow r + k_i$ in directions (k_1, k_2).
3. A motion by one unit cell corresponds to application of A^{k_i}, $i = 1, 2$.
4. B and C charges are defined for each edge $E_{i,r}$
   $Q_B(E_{i,r}) = b_{E_{i,r}}$ and $Q_C(E_{i,r}) = c_{E_{i,r}}$

C charges

Since the C charges are independent from r, i.e. $Q_C(E_{i,r}) = c_{E_i}$, they must be combinations of symmetries in the fundamental cell of the parent theory. In a given parent theory there are

- Two mesonic U(1)'s, Q_1 and Q_2.
- $N_{\boldsymbol{B}}$ baryonic U(1)'s, $Q_{\boldsymbol{B}_i}$.

The result is

$$Q_C = m_1 Q_1 + m_2 Q_2 + \sum_{i=1}^{N_{\boldsymbol{B}}} m_{\boldsymbol{B}_i} Q_{\boldsymbol{B}_i} \ .$$

Fixed by **geometric identities**.

B charges

$$Q_B(E_{i,r+r_1k_1+r_2k_2}) = Q_B(E_{i,r}) + (r_1k_1 + r_2k_2)Q_C(E_{i,r}) \,.$$

The invariance of the superpotential and the anomaly cancellation conditions can be imposed on the cell r, e.g.

$$Q_B(\tilde{\partial}F_{a,r+r_1k_1+r_2k_2}) = Q_B(\tilde{\partial}F_{a,r}) + (r_1k_1 + r_2k_2)Q_C(\tilde{\partial}F_{a,r}) \,.$$

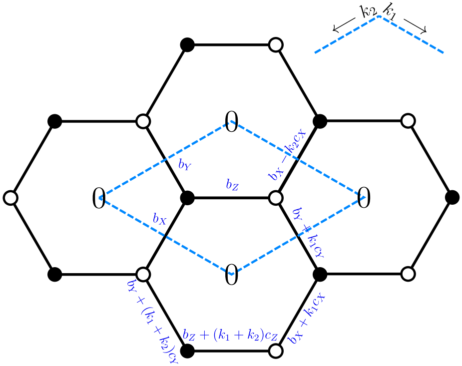
Fixed by solving the equations in the fundamental cell of the parent theory with **twisted** boundary conditions.

Conclusions

- We applied **dimer** techniques to uncover discrete global symmetries in the field theories on $D3-$branes at singularities given by **general orbifolds** of **general toric CY** $3-$**fold singularities**.
- The discrete symmetries are discrete Heisenberg group, with two $\mathbf{Z}_N$ generators A, B with commutation $AB = BAC$ and C a central element.
- The discrete symmetries for any orbifold of a given parent theory are defined from the parent theory.
- The general structure is as follows:
  - A is a **shift** in the dimer diagram;
  - B is determined by **equations** describing a $1-$form in the dimer graph in the unit cell of the parent theory with *twisted* boundary conditions.
  - C is a discrete subgroup of the non-anomalous U(1) symmetries (mesonic and baryonic), determined by **geometric identities** in the dimer of the parent theory.

Thank you!

$\mathbf{C}^3/\mathbf{Z}_N$



	Q_1	Q_2	Q_C
X	1	0	m_1
Y	0	1	m_2
Z	-1	-1	$-m_1 - m_2$

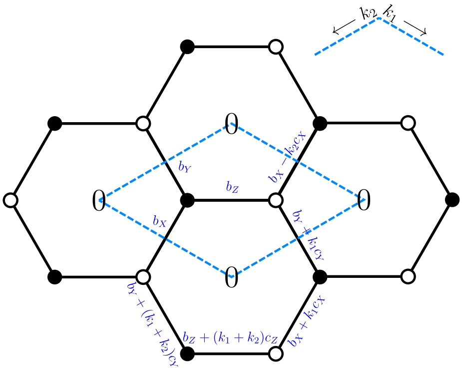
G. I.:
$$\partial V - \partial V' = 0\,,$$
$$\tilde{\partial} F - \partial V - \partial V' = 0\,.$$

$$\partial V \longrightarrow \quad b_X + b_Y + b_Z = 0\,,$$

$$\partial V' \longrightarrow \quad b_X - k_2 c_X + b_Y + k_1 c_Y + b_Z = 0\,,$$

$$\tilde{\partial} F \longrightarrow \quad 2b_X + k_1 c_X + 2b_Y + k_1 c_Y + (k_1 + k_2) c_Y + 2b_Z + (k_1 + k_2) c_Z = 0\,.$$

$\mathbf{C}^3/\mathbf{Z}_N$



- $C-$charges solutions:

$$c_X = m_1 = k_1 \ ,$$

$$c_Y = m_2 = k_2 \ ,$$

$$c_Z = -m_1 - m_2 = -k_1 - k_2 \ .$$

- The $B-$charges need to satisfy

$$b_X + b_Y + b_Z = 0$$

General Orbifolds of General Toric Theories

Consider a dimer with unit cell C, faces F_a, edges E_i and vertices V_α, V'_α and let's construct the general $\mathbf{Z}_N$ orbifolds of this theory.[17].

A generator θ of $\mathbf{Z}_N$ will act on

- Fields: $\Phi_{E_i} \longrightarrow \exp\left(2\pi i \frac{k_{E_i}}{N}\right) \Phi_{E_i}$.
- Gauge d.o.f. : $\lambda_a \longrightarrow \gamma_{\theta,a}\lambda_a\gamma_{\theta,a}^{-1}$ with λ_a generator of $\mathrm{U}(N_a)$ and

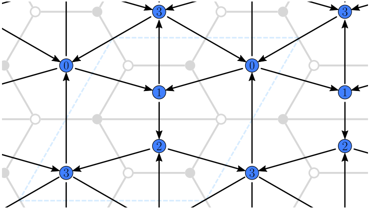
$$\gamma_{\theta,a} = \mathrm{diag}\left(\mathbf{1}_{n_{a,0}}, e^{2\pi i/N}\mathbf{1}_{n_{a,1}}, \dots, e^{2\pi i(N-1)/N}\mathbf{1}_{n_{a,N-1}}\right) .$$

Demanding the invariance under the orbifold action:

- $\bigotimes_a \mathrm{U}(N_a) \Longrightarrow \bigotimes_a \bigotimes_{r=0}^{N-1} \mathrm{U}(N_{a,r})$,
- $(\square_a, \bar{\square}_b) \Longrightarrow \left(\square_{a,r}, \bar{\square}_{b,r+k_{E_i}}\right)$.

[17] M. Schmaltz, arXiv: hep- th/9805218 (hep-th) (1999); A. M. Uranga, arXiv: hep- th/9811004 (hep-th) (1999).

Periodic quivers



Dimer diagrams	$\longleftrightarrow$	**Gauge theory**
Node F_a	$\longleftrightarrow$	gauge factor $\mathrm{U}(N_a)$
Arrow E_i connecting F_a and F_b	$\longleftrightarrow$	bifundamental chiral $(\square_a, \bar{\square}_b)$
Clockwise plaquette V_α	$\longleftrightarrow$	superpotential term $+\mathrm{tr}\left[\Phi_{E_1}\ldots\Phi_{E_n}\right]$
Counter-clockwise plaquette V'_α	$\longleftrightarrow$	superpotential term $-\mathrm{tr}\left[\Phi_{E_1}\ldots\Phi_{E_n}\right]$

Remarks on the gravity dual

- The discrete symmetries in $\mathbf{C}^3/\mathbf{Z}_3$ theory are associated to torsion classes in the 5d horizon $\mathbf{S}^5/\mathbf{Z}_3$ of the orbifold theory.[18]
- Objects charged under the generators of the discrete Heisenberg group correspond to branes wrapped on torsion cycles.

Take for instance $\mathbf{X}_5 = \mathbf{S}^5/\mathbf{Z}_N$:

1. $H_3(\mathbf{X}_5, \mathbf{Z}_N) = \mathbf{Z}_N \implies$ wrapped $D5-$branes and $NS5-$branes produce 5d codimension 2 objects.
2. Around these objects the theory experiences monodromies associated to the A and B generators.
3. The intersection of such branes of two torsion $3-$cycles over a torsion $1-$cycle in $H_1(\mathbf{X}_5, \mathbf{Z}_N) = \mathbf{Z}_N$, generates a $D3-$brane associated to the C generator.

[18] S. Gukov, arXiv: hep-th/9806180 (hep-th) (1998).