All higher-curvature gravities as Generalized quasi-topological gravities

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Introduction

- The gravitational effective action expected to have infinite number higher-derivative terms (including the E-H term).
- From EFT perspective, higher-curvature gravities (HCG) thought as effective description of underlying UV-complete theory.
- Examples of HCG:
  - Lovelock gravities [Lovelock, 1971]:

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \sum_{n=2}^{[(d-1)/2]} \lambda_n \, \ell^{2n-2} \, \chi_{2n}\right) \; ,$$

where $\quad \chi_{2n} = \frac{(2n)!}{2^n}\delta^{[\mu_1}_{\nu_1} \ldots \delta^{\mu_{2n}]}_{\nu_{2n}} R_{\mu_1\mu_2}{}^{\nu_1\nu_2} \ldots R_{\mu_{2n-1}\mu_{2n}}{}^{\nu_{2n-1}\nu_{2n}} \, .$

  - $f(R)$ theories [Sotiriou, Faraoni, 2010]:

$$\mathcal{L} = \frac{1}{16\pi G} f(R) \, .$$

Introduction

- Examples of HCG:
  - Generalized quasi-topological gravities[1] (GQ) [Bueno, Cano, Hennigar, Kubizňak, Mann,...]. Non-trivial examples in $D = 4$. Simplest one: Einsteinian Cubic Gravity [Bueno, Cano, 2016]:

$$\mathcal{L} = \frac{1}{16\pi G}\Big[R + \beta\ell^4 (12 R_{a\ b}^{\ c\ d} R_{c\ d}^{\ e\ f} R_{e\ f}^{\ a\ b} + R_{ab}^{cd} R_{cd}^{ef} R_{ef}^{ab}$$
$$-12 R_{abcd} R^{ac} R^{bd} + 8 R_a^b R_b^c R_c^a)\Big] .$$

- These theories may seem fine-tuned from EFT perspective. But, on considering field redefinitions... Can we map any HCG into a particular class of HCG? $\Longrightarrow$ Strong evidence supports that any HCG may be mapped into a GQ!

[1] Lovelock gravities are a subclass of GQs.

Generalized quasi-topological gravities (GQs)

- Consider a certain theory of gravity:

$$S = \int \mathrm{d}^D x \sqrt{|g|} \mathcal{L}(g^{ab}, R_{abcd}, \nabla_e R_{abcd}, \dots).$$

- Assume a general static and spherically symmetric (SSS) ansatz

$$\mathrm{d}s^2 = -N(r)^2 f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\Omega^2_{(D-2)}.$$

- Define $L_{N,f}(r,(f(r), N(r), f'(r), N'(r), ...)$ as the effective Lagrangian after evaluating the action on SSS ansatz and $L_f \equiv L_{1,f}$.

Generalized quasi-topological gravities (GQs)

Definition 1

We say that $\mathcal{L}(g^{ab}, R_{abcd}, \nabla_a R_{bcde}, \dots)$ *belongs to the GQ family if the Euler-Lagrange equation of* L_f *vanishes identically, that is, if*

$$\frac{\partial L_f}{\partial f} - \frac{d}{dr}\frac{\partial L_f}{\partial f'} + \frac{d^2}{dr^2}\frac{\partial L_f}{\partial f''} - \cdots = 0\,, \quad \forall\ f(r)\,.$$

This definition implies GQs admit generalizations of Schwarzchild BH
$\rightarrow$ There exist SSS solutions with $N =$ const., characterized uniquely by $f(r)$.

Features of GQ theories

Most important features of GQ theories: [Bueno, Cano, Hennigar, Mann, Kubizňak,...]

- Around max. sym. background, lin. EoM of GQs are order 2.
- Continuous and well-defined Einstein gravity limit.
- Metric function determined from ODE of order $\leq 2m+2$ (m is maximum number of cov. derivatives at any term).
- If ODE for $f(r)$ is order 2, unique BH solution characterized by ADM mass.
- Thermodynamic properties of BHs computed analytically[2].

Some examples of GQ: Lovelock gravities, ECG.

[2] This property is only known to be true for GQs without cov. derivatives of the Riemann.

Field redefinitions in HCG

- Consider the action

$$S = \frac{1}{16\pi G} \int \mathrm{d}^D x \sqrt{|g|} \left[R + \sum_{n=2}^{\infty} \ell^{2(n-1)} \mathcal{L}^{(n)} \right] ,$$

where ℓ a length scale and $\mathcal{L}^{(n)}$ the most general Lagrangian with $2n$ derivatives of metric.

- Perform the field redefinition

$$g_{ab} = \tilde{g}_{ab} + \ell^{2k} \tilde{Q}_{ab}^{(k)}(\tilde{g}_{ab}, \partial_c \tilde{g}_{ab}, \ldots) .$$

Field redefinitions in HCG

- The action for $\tilde{g}$ reads

$$\tilde{S}[\tilde{g}_{ab}] = \int \frac{\mathrm{d}^D x \sqrt{|\tilde{g}|}}{16\pi G} \left[\tilde{R} + \sum_{n=2}^{k} \ell^{2(n-1)} \tilde{\mathcal{L}}^{(n)} \right. \\ \left. + \ell^{2k} \left(\tilde{\mathcal{L}}^{(k+1)} - \tilde{R}^{ab} \hat{Q}_{ab}^{(k)} \right) + \sum_{n=k+2}^{\infty} \ell^{2(n-1)} \tilde{\mathcal{L}}'^{(n)} \right] ,$$

where $\hat{Q}_{ab}^{(k)} = \tilde{Q}_{ab}^{(k)} - \frac{1}{2} \tilde{g}_{ab} \tilde{Q}^{(k)}$, $\quad \tilde{Q}^{(k)} = \tilde{g}^{ab} \tilde{Q}_{ab}^{(k)}$.

- Remove all terms in the action involving contractions of Riccis!

Field Redefinitions in HCG

- Example: Most general 4-derivative action:

$$S^{(4)} = \frac{1}{16\pi G}\int d^D x \sqrt{|g|}\left[R + \ell^2(\alpha_1 R^2 + \alpha_2 R_{ab}R^{ab} + \alpha_3 \chi_4)\right] .$$

- By field redefinition

$$g_{ab} = \tilde{g}_{ab} + \alpha_2 \ell^2 \tilde{R}_{ab} - \ell^2 \frac{\tilde{R}}{D-2}\tilde{g}_{ab}(2\alpha_1 + \alpha_2) ,$$

the new action $\tilde{S}^{(4)}$ reads (staying to order 4 in derivatives)

$$\tilde{S}^{(4)} = \frac{1}{16\pi G}\int d^D x \sqrt{|\tilde{g}|}\left[R + \ell^2 \alpha_3' \tilde{\chi}_4\right] .$$

- We have mapped most general 4-derivative theory to a GQ! For 6-derivative terms, similar arguments yield same conclusion.

Field redefinitions in HCG

- Any term involving Riccis is meaningless[3] from EFT perspective.

Definition 2

A curvature invariant is said to be "reducible" if it is a total derivative when evaluated on any Ricci-flat metric. The rest of them are said to be "irreducible".

- For instance:
  1. $R_{abcd}R^{abc}{}_{e}R^{de}$ is reducible.
  2. $R_{ab}{}^{cd}R_{cd}{}^{ef}R_{ef}{}^{ab}$ is irreducible.
- Therefore, the most general HCG effective action is obtained by including all irreducible terms.

[3] The same holds after the inclusion of a cosmological constant.

All $\mathcal{L}(g^{ab}, R_{abcd})$ gravities as GQs

Definition 3

We say that a curvature invariant $\mathcal{L}$ is "completable to a Generalized quasi-topological density" (or just "completable" for short), if there exists a GQ density $\mathcal{Q}$ such that $\mathcal{L} - \mathcal{Q}$ is reducible.

- Hence $\mathcal{L}$ completable if adding reducible terms we get a GQ.
- Example: $R_{abcd}R^{abcd}$ completable to χ_4 by adding $R^2 - 4R_{ab}R^{ab}$.
- All HCG expressed as sums of GQs $\iff$ All irreducible densities are completable to a GQ.
- Start with $\mathcal{L}(g^{ab}, R_{abcd})$ terms.

All $\mathcal{L}(g^{ab}, R_{abcd})$ gravities as GQs

Combining our results with those of [Bueno, Cano, Hennigar, 2019]:

Theorem 4 (Bueno, Cano, Moreno, Murcia and Hennigar)

Any higher-derivative gravity Lagrangian involving an arbitrary sum of invariants constructed from the Riemann tensor and the metric can be mapped, order by order, to a sum of GQ terms through metric redefinitions.

Terms involving covariant derivatives of the Riemann tensor

- We would like to extend Theorem 4 to all HCG (possibly with cov. derivatives).
- We have checked:
  a) Also 8-derivatives terms are completable to GQs with no cov. derivatives.
  b) Any density

$$R^n \nabla R \nabla R$$

  is completable to a GQ which, evaluated on SSS ansatz, is equivalent to GQ with no cov. derivatives.

Terms involving covariant derivatives of the Riemann tensor

Theorem 5

Let $\mathcal{I}$ a certain higher-derivative term. If $\mathcal{I}$ contains at most two cov. derivatives of the Riemann, one and only one of the following holds:

1. $\mathcal{I} = R^n$ *and can be mapped to a GQ without cov. derivatives.*
2. $\mathcal{I} = R^n \nabla R \nabla R$ *and can be mapped to a sum GQs which, evaluated on a SSS metric, is equivalent to GQs without cov. derivatives.*

- Conjecture: Any HCG can be mapped, order by order, to a sum of GQs which, evaluated on a SSS metric, are equivalent to GQs without cov. derivatives.

Conclusions

- GQs are not mere fine-tuned HCG.
- Through field redefinitions, we have shown any higher-derivative term with at most two cov. derivatives of the Riemann can be mapped to GQs.
- We expect actually any HCG can be mapped to GQs.
- Physics of (at the very least, a large part of) HCG captured by their GQ counterparts, easier to characterize.

Thank you for your attention

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Proof of Theorem 4

- By [Bueno, Cano, Hennigar, 2019], always exists a GQ of $\mathcal{L}(g_{ab}, R_{abcd})$ at all orders.
- At order n, let $S^{(n)}$ be such GQ. Adding reducible terms, $S^{(n)} \to \mathcal{W}^{(n)}$, with $\mathcal{W}^{(n)} \sim (\mathrm{Weyl})^n$.
- By [Deser, Ryzhov, 2005], **any** contraction of Weyls satisfies

$$(\mathrm{Weyl})^n|_{SSS} = c\,F(r)^n\,.$$

- Then $\mathcal{W}^{(n)}|_{SSS} = c_{\mathrm{GQ}} F(r)^n$.
- If $\mathcal{R}^{(n)}$ is arbitrary irreducible term, converting it into Weyls $\mathcal{W}_{\mathcal{R}}$ we learn $\mathcal{W}_{\mathcal{R}}|_{SSS} = c_R F(r)^n$.
- Since $\mathcal{W}^{(n)}$ is completable to $S^{(n)}$, $\mathcal{W}_{\mathcal{R}}$ as well. Hence

$$\mathcal{R}^{(n)} = S^{(n)} + \mathrm{Riccis} + T^{(n)}\,.$$

with $T^{(n)}|_{SSS} = 0$ trivial GQs.