Twisted $\mathcal{N} = 1$ SCFTs and their AdS$_3$ duals

Iberian Strings

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based on: [Christopher Couzens, HL, Kilian Mayer, arXiv: 1912.07605]

AdS_5/CFT_4 examples

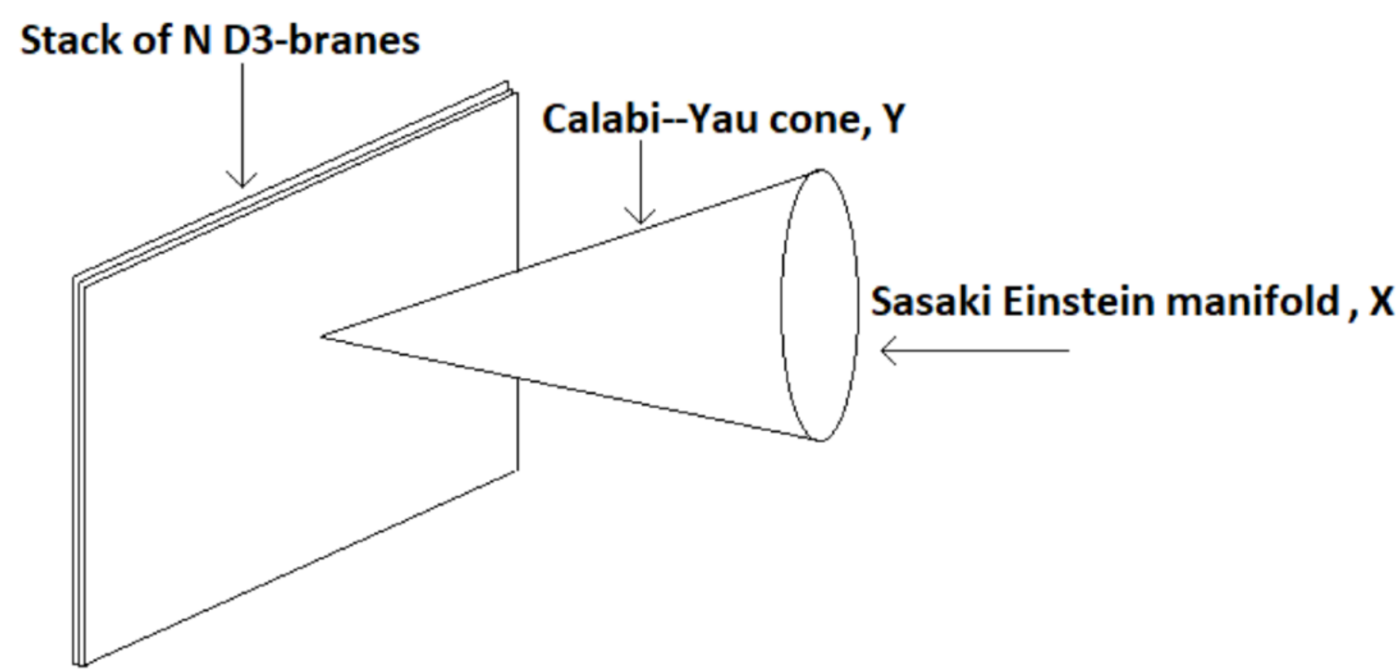
- $\mathrm{AdS}_5 \times \mathrm{S}^5 - \mathcal{N} = 4$ SYM
- $\mathrm{AdS}_5 \times Y_5 - \mathcal{N} = 1$ quiver gauge theories

Dualities can be understood using D3-branes probing

Calabi–Yau cone singularities

Explicit metrics for:

- $Y_5 = Y^{p,q}$ [Gauntlett, Martelli, Sparks, Waldram '04]
- $Y_5 = L^{a,b,c}$ [Cvetic, Lu, Page, Pope '05]



Constructing AdS_3/CFT_2 examples

Compactify 4d theories on Riemann surface

- Topological twist to preserve supersymmetry
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Expectation for dual geometry:

- $\mathrm{AdS}_3 \times Y_7$
- Y_7 fibration of Sasaki–Einstein manifold Y_5 over Riemann surface

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Successfully studied (e.g. match central charges) for theories dual to $\mathrm{AdS}_5 \times Y^5$ where:

- $Y_5 = \mathrm{S}^5$ e.g. [Bershadsky, Johansen, Sadov, Vafa, '95; Maldacena, Nuñez '00; Benini, Bobev '12 '13]
- $Y_5 = Y^{p,q}$ [Benini, Bobev, Crichigno '15]

Our work

Compactify theory dual to $\mathrm{AdS}_5 \times L^{a,b,c}$ on Riemann surface

Test duality by computing central charges and R-charges of the $\mathcal{N} = (0,2)$ SCFTs:

- Field theory side: use c-extremization [Benini, Bobev '12 '13]
- Geometry side: assuming a solution, topological data is sufficient using geometric dual of c-extremization [Couzens, Gauntlett, Martelli, Sparks '18; Gauntlett, Martelli, Sparks '18]

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Construction of explicit solutions dual to $\mathcal{N} = (0,2)$ SCFTs and matching of central charges to c-extremization results

Field theory

- Bifundamental matter and vector multiplets in the $L^{a,b,c}$ quiver and their charges ($ck + bl = 1,\ \ d = a + b - c,\ \ 0 < a < d < c < b$) [Benvenuti, Kruczenski '05; Franco, Hanany, Martelli, Sparks, Vegh, Wecht '05; Butti, Forcella, Zaffaroni '05]

Field	multiplicity	$U(1)_{F_1}$	$U(1)_{F_2}$	$U(1)_B$	$U(1)_R$
Z	$N^2 a$	0	k	b	-1
U_2	$N^2 c$	-1	$-k-l$	$-d$	0
Y	$N^2 b$	1	0	a	-1
U_1	$N^2 d$	0	l	$-c$	0
V_1	$N^2(b-c)$	-1	$-l$	$c-a$	0
V_2	$N^2(c-a)$	0	$k+l$	$b-c$	0
λ	$(N^2-1)(a+b)$	0	0	0	1

- Background flux: $T_{\text{backgr}} = \frac{\kappa}{2} T_R + f_1 T_{F_1} + f_2 T_{F_2} + B T_B$

- 2d theory depends on 7 parameters

Example

- Torus with purely baryonic flux, i.e. $f_1 = f_2 = 0$:

$$c_R = 12N^2B\frac{abc(a+b-c)}{(a-c)(b-c)}$$

$$R[Z] = R[Y] = \frac{ab}{(a-c)(b-c)} \qquad R[U_1] = R[U_2] = -\frac{cd}{(a-c)(b-c)}$$

- Recall: $0 < a < d < c < b$, so not all R-charges positive

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Cured by taking 'a' negative

Dual solutions

- Most general supersymmetric AdS_3 solution of type IIB with only five-form flux has the form: [Kim '05]

$$
\begin{aligned}
\mathrm{d}s^2 &= L^2 e^{-\frac{w}{2}}\left(\mathrm{d}s^2_{\mathrm{AdS}_3} + \frac{1}{4}(\mathrm{d}z+P)^2 + e^w \mathrm{d}s^2(\mathscr{M}_6)\right) \\
F_5 &= (1+\star)\mathrm{dvol}_{\mathrm{AdS}_3}\wedge\left(-2J + \frac{1}{2}\mathrm{d}\left[e^{-w}(\mathrm{d}z+P)\right]\right) \\
e^w &= \frac{1}{8}R \\
\mathrm{d}P &= \rho
\end{aligned}
$$

- Master equation: $\Box R = \frac{1}{2}R^2 - R_{ij}R^{ij}$

- Flux quantization: $\frac{1}{(2\pi\ell_s)^4 g_s}\int_{\Sigma_A} F_5 = N_A \in \mathbb{Z}$

Ansatz for Kähler metric

$$\mathrm{d}s^2(\mathscr{M}_6) = H(\eta,\xi)\mathrm{d}s^2(\Sigma_g) + \frac{\eta-\xi}{F(\xi)}\mathrm{d}\xi^2 + \frac{F(\xi)}{\eta-\xi}\left(\mathrm{d}\phi + \eta\mathrm{d}\psi + \partial_\xi(H)A_g\right)$$
$$+\frac{\eta-\xi}{G(\eta)}\mathrm{d}\eta^2 + \frac{G(\eta)}{\eta-\xi}\left(\mathrm{d}\phi + \xi\mathrm{d}\psi + \partial_\eta(H)A_g\right)$$

- $\xi \in [\xi_-,\xi_+], \quad \eta \in [\eta_-,\eta_+], \quad \eta_- > \xi_+$
- $F, G, H > 0$

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Steps to be taken:

1. Solve master equation
2. Study global regularity
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$F'_\pm \equiv F'(\xi_\pm) \quad G'_\pm \equiv G'(\eta_\pm)$

$$a + b - c - d = 0, \qquad \frac{a}{F'_-} + \frac{b}{F'_+} + \frac{c}{G'_-} + \frac{d}{G'_+} = 0, \qquad \frac{a\xi_-}{F'_-} + \frac{b\xi_+}{F'_+} + \frac{c\eta_-}{G'_-} + \frac{d\eta_+}{G'_+} = 0$$

Flux quantization

- Integrals computable before solving master equation and requiring regularity
- $N_A = \frac{1}{(2\pi\ell_s)^4 g_s} \int_{\Sigma_A} F_5 \in \mathbb{Z}$
- Results are expressed in roots $\xi_\pm, \eta_\pm$, and the functions F, G, H and their first derivatives evaluated at the roots

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Same holds for central charge and R-charges

$$c_R = \frac{1}{4}\frac{3L^8}{(2\pi)^6 g_s^2 \ell_s^8}\int_{Y_7}(\mathrm{d}z + P)\wedge\rho\wedge J^2 \qquad R[S_a] = \frac{L^4}{(2\pi)^3 g_s \ell_s^4}\int_{S_a} e^{-w}\mathrm{dvol}(S_a)$$

$$R_{\eta_+} = -\frac{1}{F'_+ G'_-}\frac{L^4}{2\pi a g_s \ell_s^4}(\xi_+ - \xi_-)\left[(\eta_+ - \eta_-)F'_+ + (\eta_+ - \xi_+)G'_- + (\xi_+ - \eta_-)G'_+\right]$$

Solving master equation

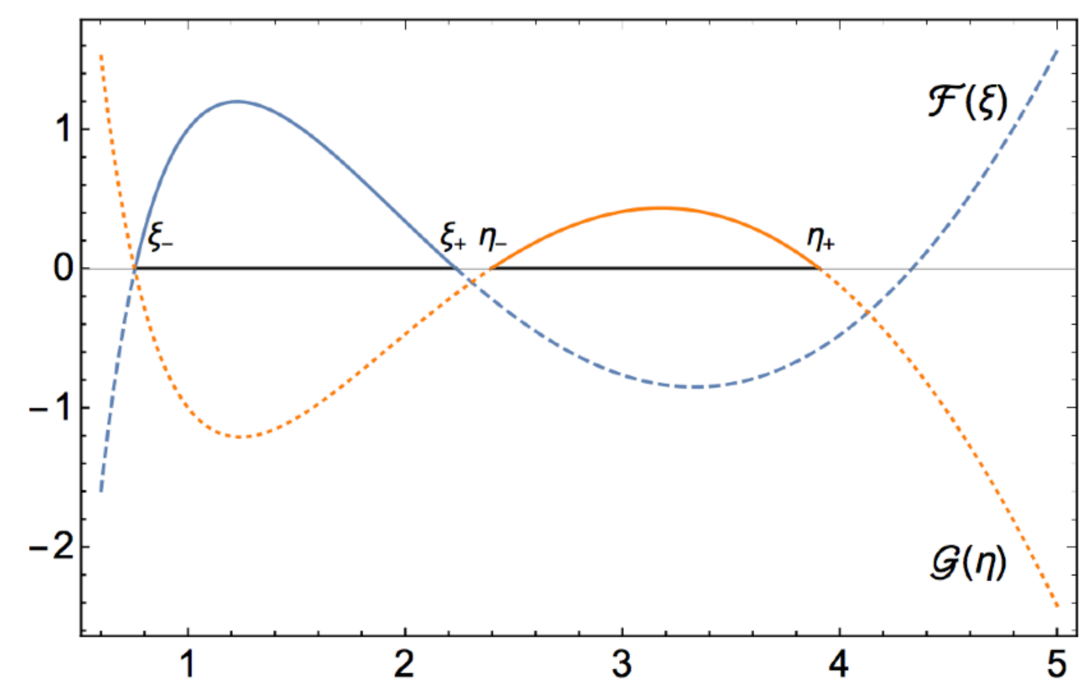
- Use symmetries of metric to fix $H = \eta\xi$
- Master equation $\Box R = \frac{1}{2}R^2 - R_{ij}R^{ij}$ is solved using polynomial ansätze for F, G :

$$F(\xi) = \frac{-A(\xi + C)^2 + \kappa\xi^2 \pm \xi^3}{\xi}, \qquad G(\eta) = \frac{-B(\eta + C)^2 - \kappa\eta^2 \mp \eta^3}{\eta}$$

- Needs to obey: $\frac{a}{F'_-} + \frac{b}{F'_+} + \frac{c}{G'_-} + \frac{d}{G'_+} = 0, \qquad \frac{a\xi_-}{F'_-} + \frac{b\xi_+}{F'_+} + \frac{c\eta_-}{G'_-} + \frac{d\eta_+}{G'_+} = 0$

- 5-parameter solution

$$a = -1, \quad b = 19, \quad c = 16$$
$$\kappa = 0, \qquad C = -1$$



Matching central charges and R-charges

- Central charges and R-charges can be matched to the field theory
- However, only formal matching for torus, sphere and a subset of higher genus Riemann surface fibrations
- Torus, sphere and a subset of the $g > 1$ Riemann surface fibrations characterized by integers $a < 0 < d < c < b$
- Field theory characterized by integers $0 < a < d < c < b$
- There are $g > 1$ Riemann surface fibrations that have $0 < a < d < c < b$
- This difference has been noted earlier for specific $Y^{p,q}$, $L^{a,b,c}$ solutions
  [Couzens, Martelli, Schäfer-Nameki '17; Couzens, Gauntlett, Martelli, Sparks '18; Gauntlett, Martelli, Sparks '18]

Conclusion

- We computed the central charges and R-charges of the $L^{a,b,c}$ quiver gauge theories compactified on a Riemann surface
- We constructed explicit solutions dual to $\mathcal{N} = (0,2)$ 2d SCFTS
- We showed that the central charges and R-charges of the solutions match the field theoretic expressions
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Thank you for your attention

Global regularity

Method: see e.g. [Cvetic, Lü, Page, Pope '05]

- Determine Killing vector V (linear combination of $\partial_z, \partial_\phi, \partial_\psi$) that degenerates at $\xi_\pm, \eta_\pm$ and has its surface gravity equal 1

$$\kappa^2_{\text{grav}} = \frac{\partial_\mu |V|^2 \partial^\mu |V|^2}{4|V|^2} = 1$$

- V corresponds to local coordinate which should have period 2π

- 4 Killing vectors: $k_{\eta_\pm} = \partial_z - \frac{2}{G'_\pm}(\partial_\psi - \eta_\pm \partial_\phi)$ $\quad l_{\xi_\pm} = \partial_z + \frac{2}{F'_\pm}(\partial_\psi - \xi \pm \partial_\phi)$

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$F'_\pm \equiv F'(\xi_\pm)$

- Linear relation Killing vectors $ck_{\eta_-} + dk_{\eta_+} - al_{\xi_-} - bl_{\xi_+} = 0$ imposes conditions:

$$a + b - c - d = 0, \qquad \frac{a}{F'_-} + \frac{b}{F'_+} + \frac{c}{G'_-} + \frac{d}{G'_+} = 0, \qquad \frac{a\xi_-}{F'_-} + \frac{b\xi_+}{F'_+} + \frac{c\eta_-}{G'_-} + \frac{d\eta_+}{G'_+} = 0$$