Nonperturbative Mellin Amplitudes: Existence, Properties, Applications

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Plan for the talk

- Motivation: Use bootstrap to study quantum gravity
- Results: Nonperturbative CFT sum rules
  - A scalar field (within a range of masses) minimally coupled to gravity in AdS cannot be UV completed
- Method: Nonperturbative Mellin amplitudes

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We would like to use the bootstrap equations to address these questions **nonperturbatively**

CFT Sum Rule

Consider a CFT four point function $\langle OOOO \rangle$, where the external operator has dimension Δ.

Suppose primary operators of twist τ and spin J are being exchanged. Then,

$$\text{Nonperturbative sum rule: } \sum_{\tau,J} \underbrace{C_{\tau,J}}_{OPE\ \ coefficient} \underbrace{\alpha^{d,\Delta}_{\tau,J}}_{functional} = 0$$

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Remarkable positivity property: $\alpha^{d,\Delta}_{\tau,J} \geq 0$ if $\tau \geq 2\Delta$

Checked the sum rule in free field theory and 3d Ising model

- Caveat: made technical assumptions that we believe are true, but that we cannot prove

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$$\sum_{0\leq\tau\leq 2\Delta,J} C_{\tau,J}\ \alpha^{d,\Delta}_{\tau,J} + \underbrace{\sum_{\tau\geq 2\Delta,J} C_{\tau,J}\ \alpha^{d,\Delta}_{\tau,J}}_{\geq 0} = 0$$

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We can apply this sum rule to an hypothetical CFT whose AdS dual is

$$S = \int d^{d+1}x\sqrt{g}\Big(\frac{1}{2}\nabla\phi\cdot\nabla\phi + \frac{1}{2}m^2\phi^2 + R\Big),$$

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We find

$$\sum_{0\leq\tau\leq 2\Delta,J} C_{\tau,J}\ \alpha_{\tau,J}^{d,\Delta} > 0.$$

for some range of masses.
So, if we trust the validity of our sum rule, then such a theory belongs to the Swampland.

Definition of a Mellin amplitude

$$\langle O(x_1)...O(x_4)\rangle = \frac{1}{x_{12}^{2\Delta}x_{34}^{2\Delta}} + \frac{1}{x_{13}^{2\Delta}x_{24}^{2\Delta}} + \frac{1}{x_{14}^{2\Delta}x_{23}^{2\Delta}}$$

$$+\frac{1}{x_{13}^{2\Delta}x_{24}^{2\Delta}}\int_{-i\infty}^{+i\infty}\frac{d\gamma_{12}d\gamma_{14}}{(2\pi i)^2}\Gamma(\gamma_{12})^2\Gamma(\gamma_{13})^2\Gamma(\gamma_{14})^2\underbrace{M(\gamma_{12},\gamma_{14})}_{Mellin\ \ Amplitude}u^{-\gamma_{12}}v^{-\gamma_{14}},$$

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What are Mellin amplitudes useful for?

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Do Mellin amplitudes exist nonperturbatively? Can we bootstrap them?

- Yes! Yes!

Nonperturbative Mellin Amplitudes. Existence.

A function of two complex variables $g(u,v)$ has a well defined Mellin transform if [Antipova; 08]

- it is **analytic** in a sectorial domain $\big(arg(u), arg(v)\big) \in \Theta \subset \mathbb{R}^2$
- it is **polynomially bounded**: $g(u,v) \leq \frac{C(c_u,c_v)}{u^{c_u} v^{c_v}}$

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We can prove analyticity of CFT correlators from convergence of OPE

[Simmons-Duffin, Maldacena, Zhiboedov; 15]

Polynomial boundedness we were not able to prove, but we believe that is true.

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- The construction of the functionals involve a lot of what is known about CFT's in $d>2$.

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What are the maximal functionals that we can construct?

Can we prove the assumptions that we made?

Can we use the functionals to bootstrap $\mathcal{N} = 4$ SYM?