Momentum/Complexity Duality

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Based on arXiv:1912.05996 with
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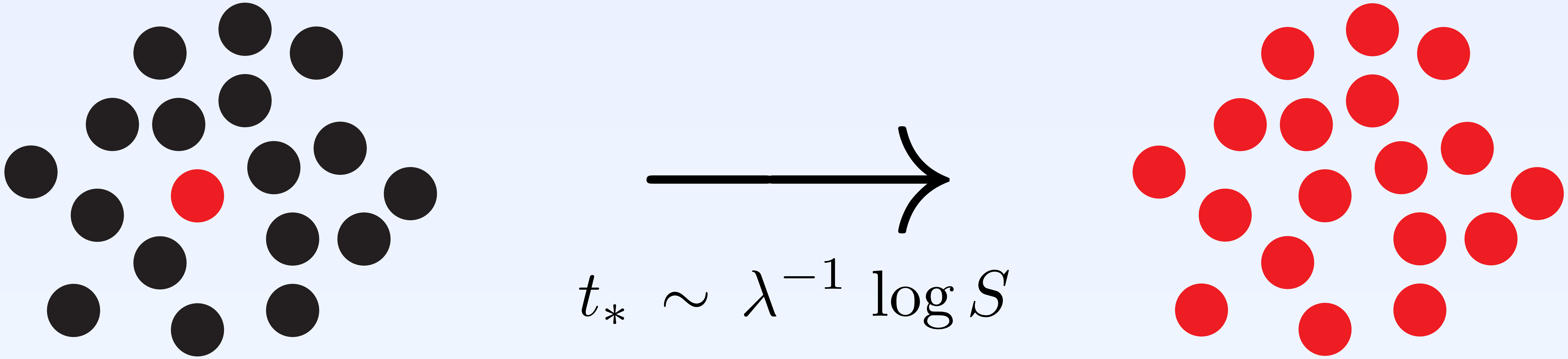
1. Operator growth

An indicator of chaos in a many body quantum systems is that the average size of an operator grows exponentially fast in time (with Lyapunov exponent λ)



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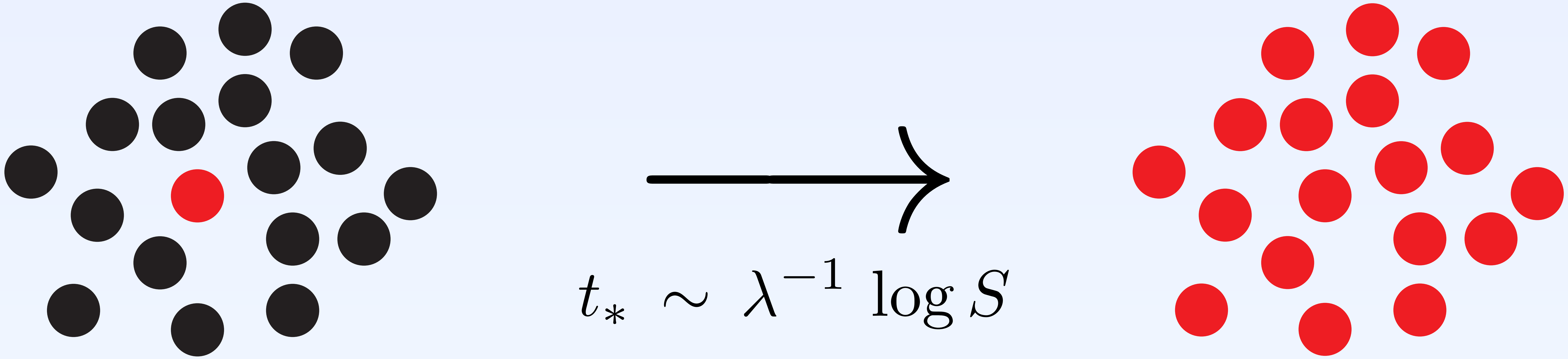

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For k-local Hamiltonians, fast scrambling (recently SYK)

[Roberts, Stanford, Streicher '18]
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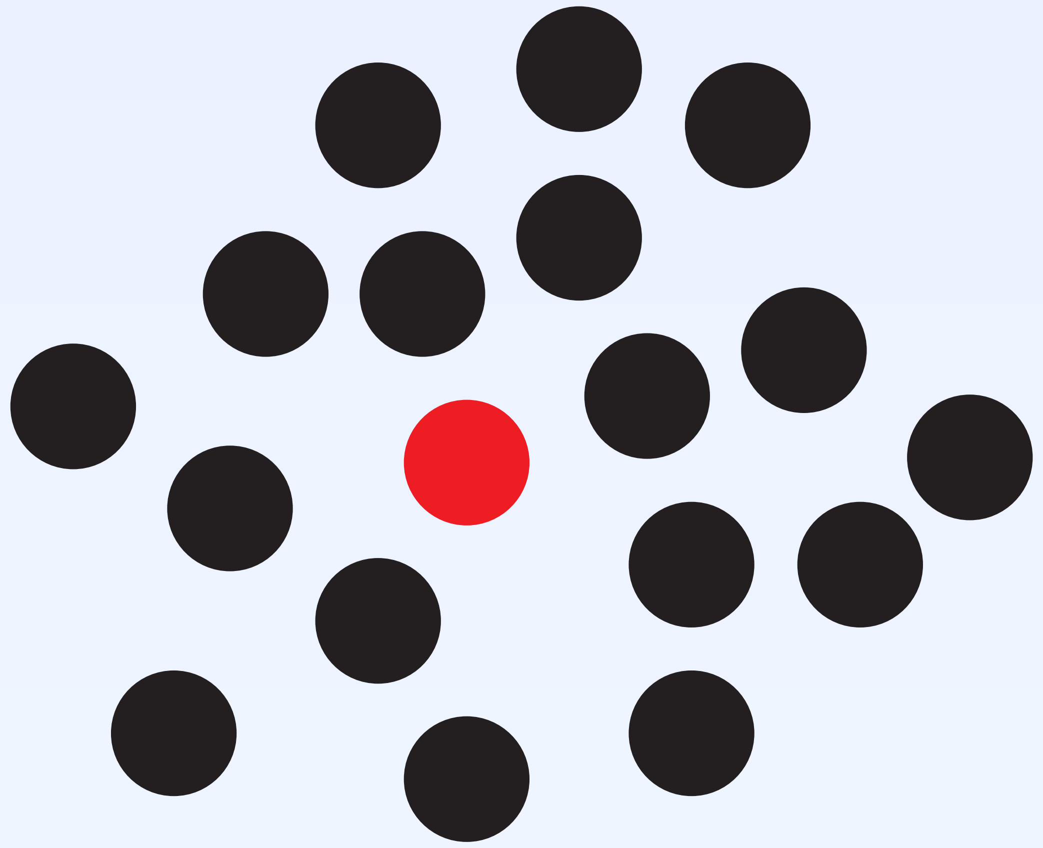

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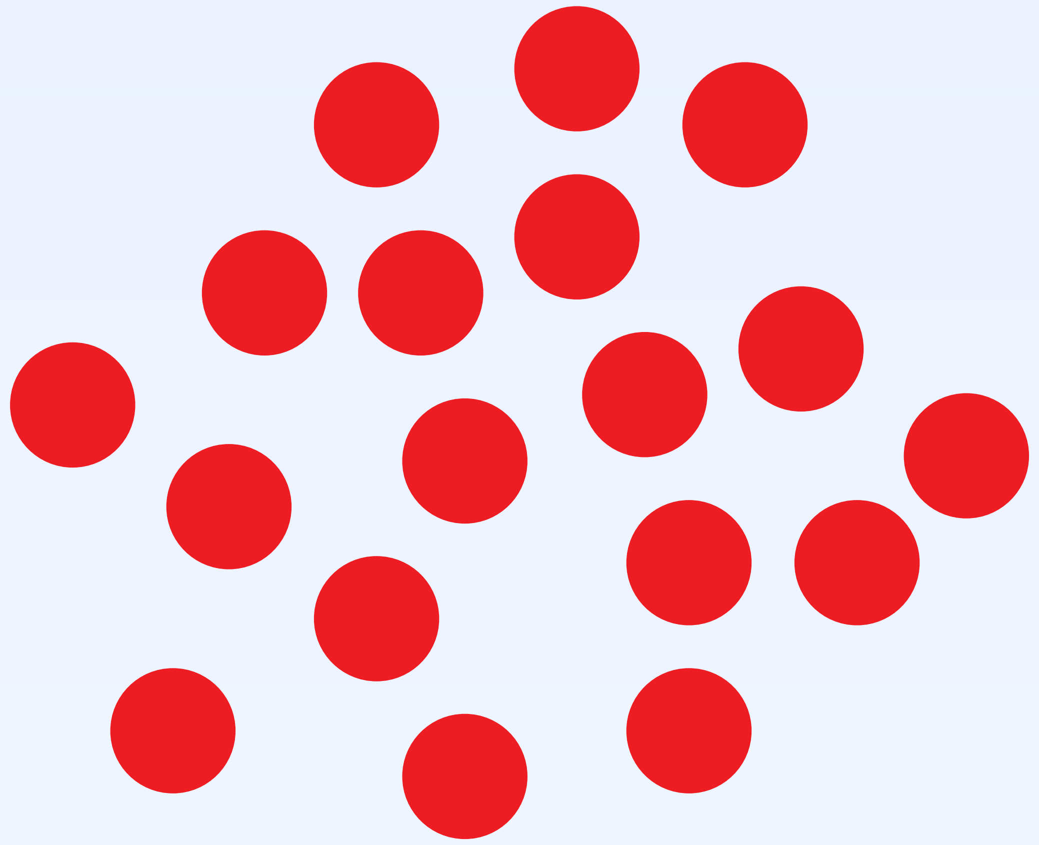

For k-local Hamiltonians, fast scrambling (recently SYK) [Roberts, Stanford, Streicher '18] [Qi, Streicher '19]

Observation: The growth of the radial momentum of a test particle falling into a BH follows the same exponential growth

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{size} \sim P}$$

[Susskind '18 '19]

Scrambling occurs when the particle reaches the stretched horizon. After?

Before scrambling, complexity $\sim$ size. After scrambling, size saturates while complexity continues to grow linearly. The notion of K-complexity incorporates both [Altman et al. '18] [Barbón, Ravinovici, Shir, Sinha '19]

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For our purposes, we will define the complexity of the operator $\mathcal{O}$ in terms of the difference in state complexity. Given $|\Psi_0\rangle$ and $|\Psi\rangle = \mathcal{O}\,|\Psi_0\rangle$

$$\mathcal{C}_{\mathcal{O}}(t) = \mathcal{C}\left[|\Psi\rangle_t\right] - \mathcal{C}\left[|\Psi_0\rangle_t\right]$$

together with some prescription (VC) for the state complexity. This guarantees a linear growth at late times.

2. Holographic setup

Two copies of a CFT on $\mathbf{S}^{d-1}$ and $|\mathrm{TFD}\rangle$ at temperature β^{-1} as reference state

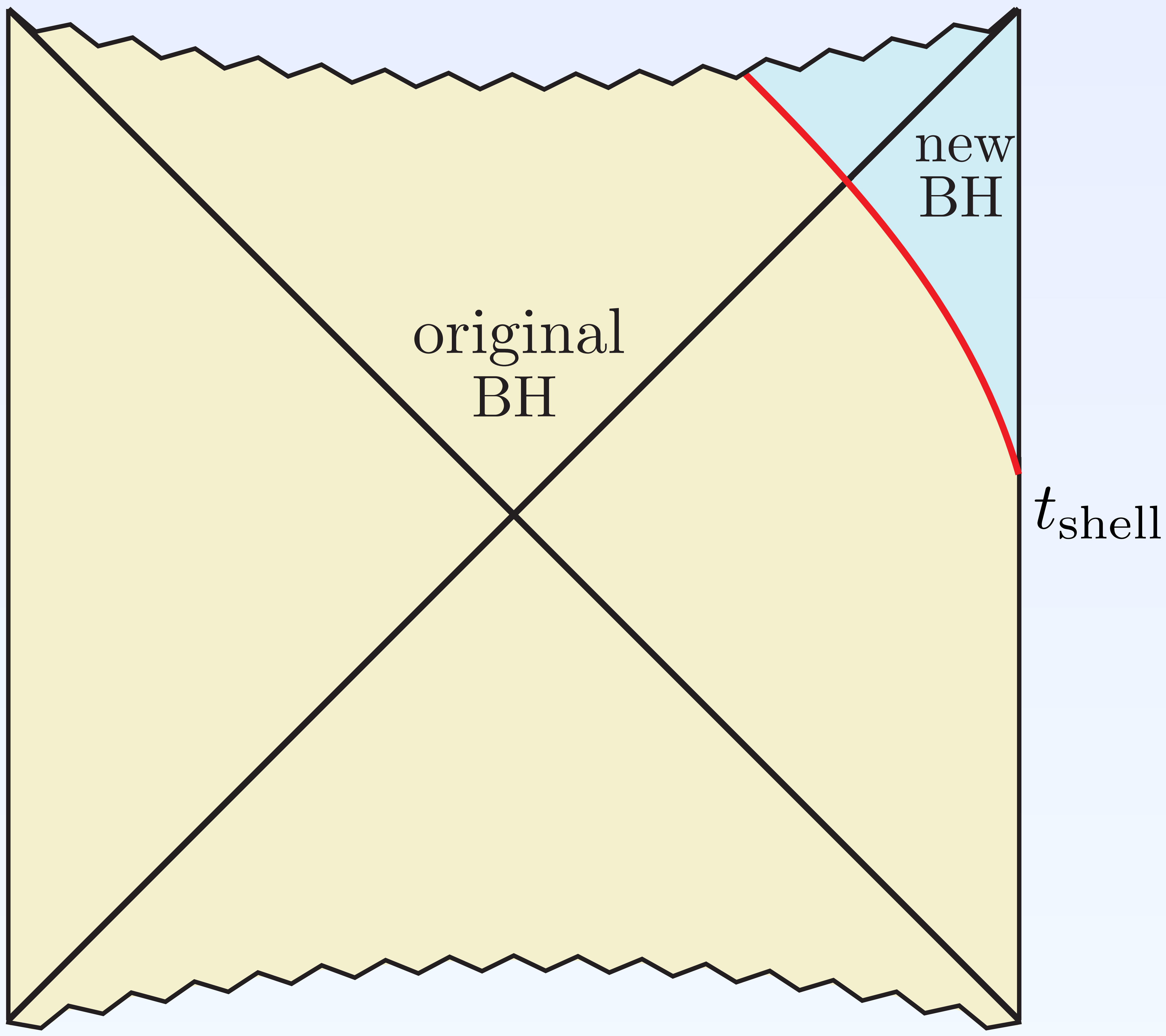
Shell operator in CFT$_{\mathrm{R}}$

$$\mathcal{O}_{\text{shell}} \sim \prod_{\mathbf{S}^{d-1}} \mathcal{O}(\Omega, t_{\text{shell}})$$

and the corresponding state

$$|\Psi\rangle \; = \; \mathcal{O}_{\text{shell}} \; |\mathrm{TFD}\rangle$$

Time-evolution will be given by $\quad H \; = \; H_{\mathrm{L}} \, + \, H_{\mathrm{R}}$



$t_{\text{shell}} \gg \beta$

Thin shell of massive dust

$$T_{\mu\nu} = \sigma\, u_\mu\, u_\nu\, \delta(\ell)$$

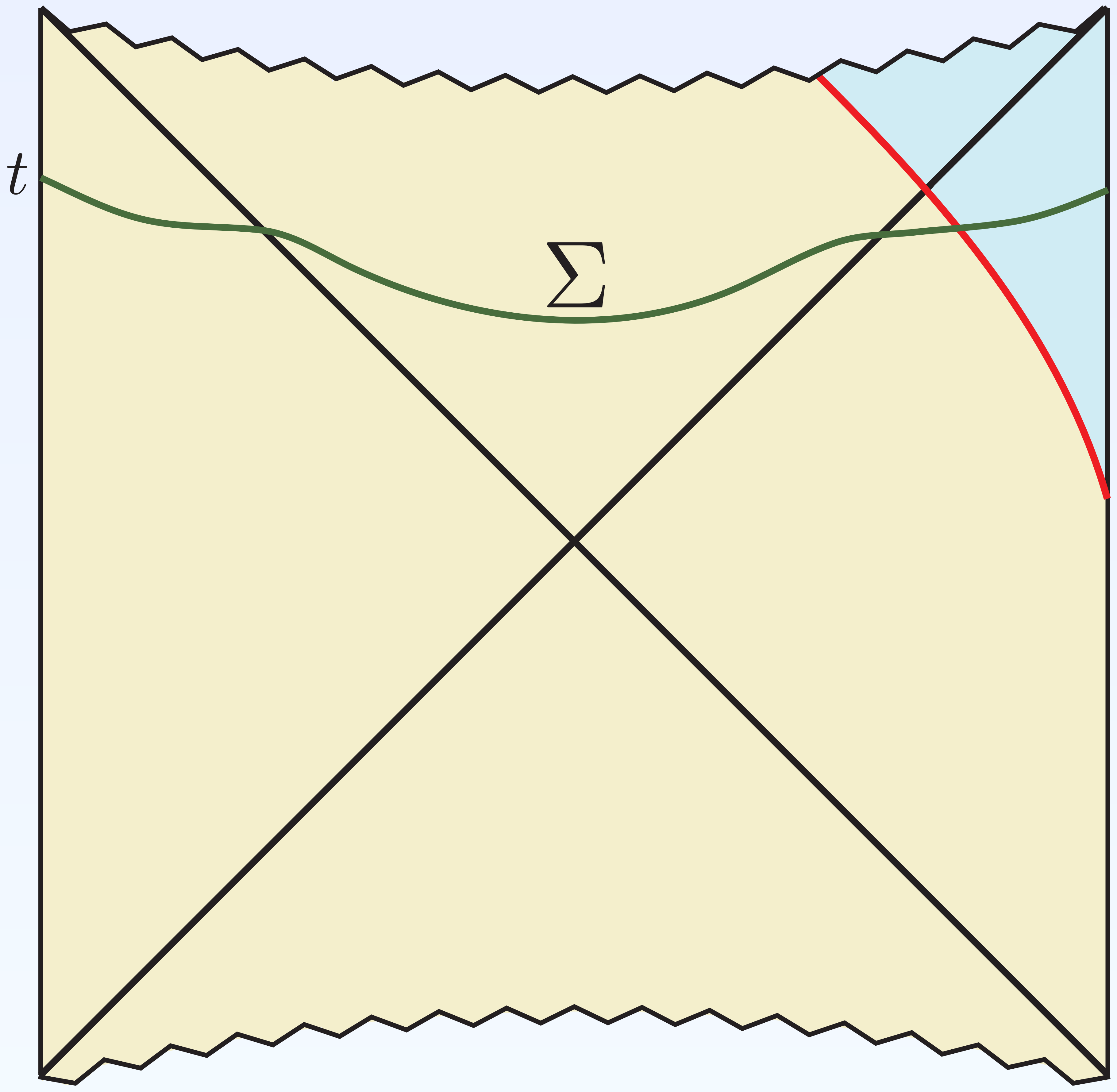
Use junction conditions to solve EE

original BH $\longrightarrow M$

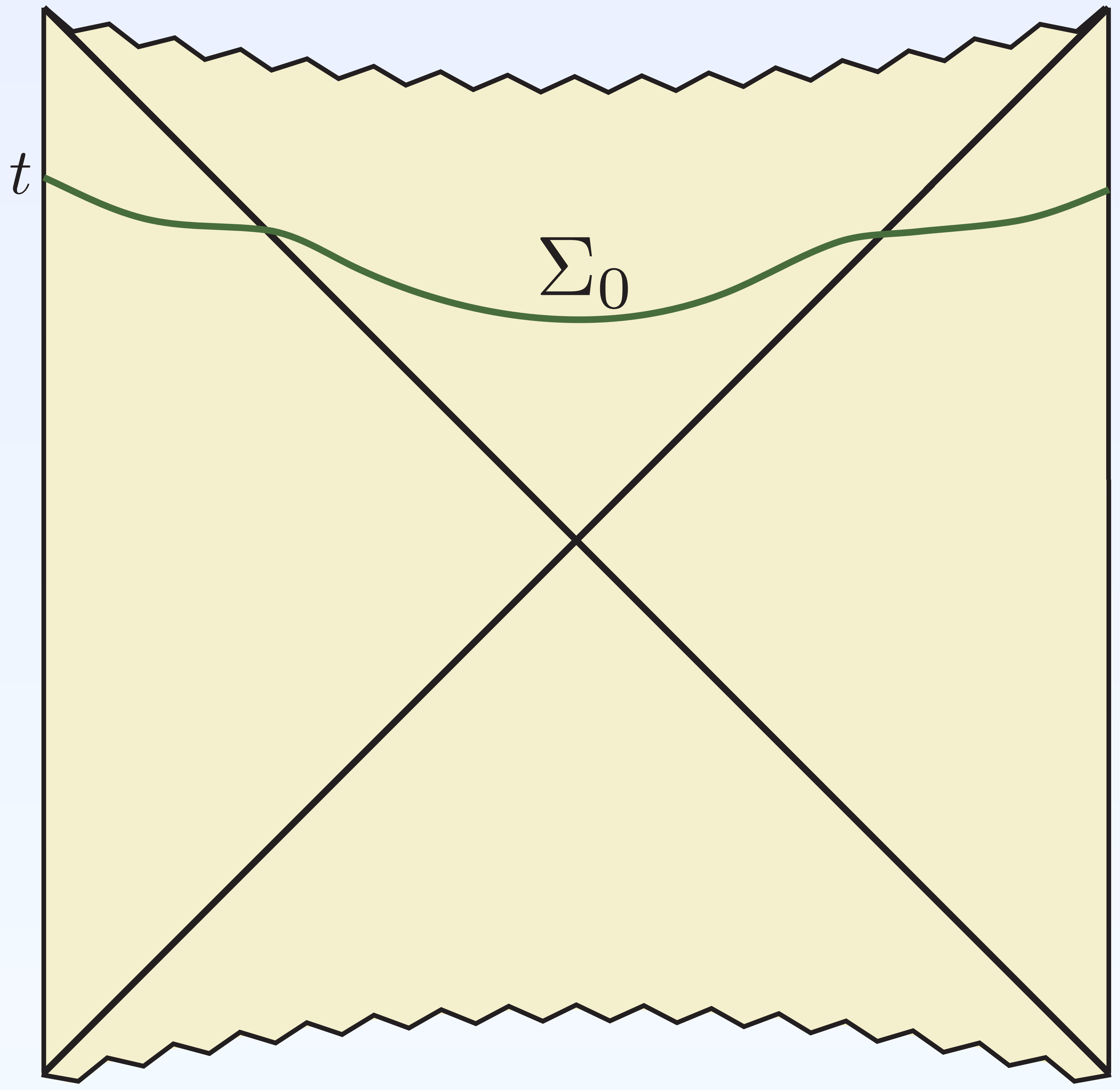
new BH $\longrightarrow M + m_{\text{shell}}$

State Complexity = Volume

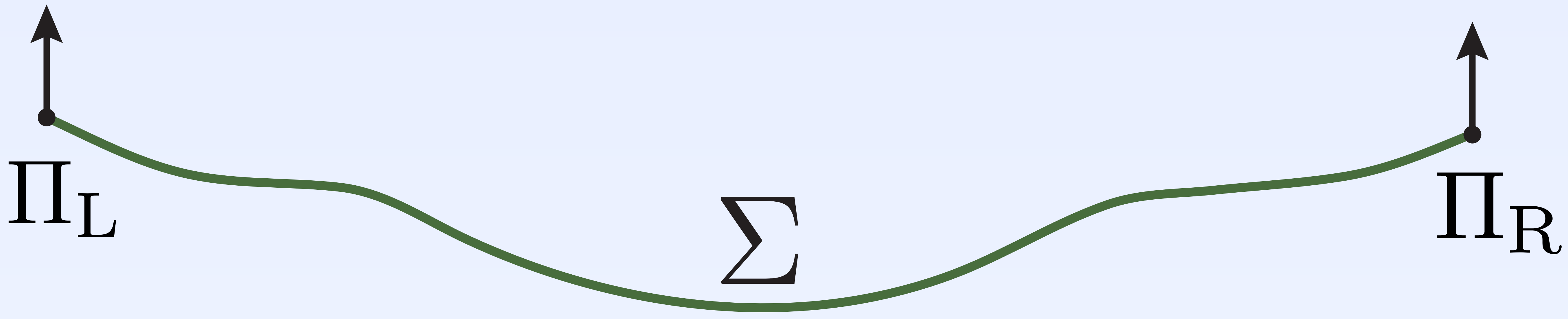
$$\mathcal{C}\left[|\Psi\rangle_t\right] \equiv \mathrm{Vol}(\Sigma)$$



$$\mathcal{C}\left[|\mathrm{TFD}\rangle_t\right] \equiv \mathrm{Vol}(\Sigma_0)$$



Extremal volume rates will follow a Hamilton-Jacobi equation



$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Vol}(\Sigma) = \Pi_L + \Pi_R$$

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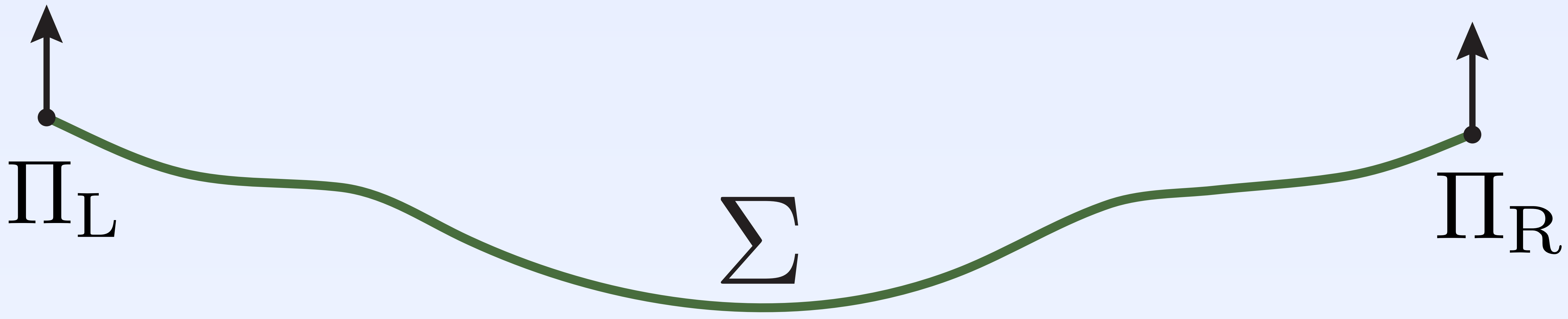
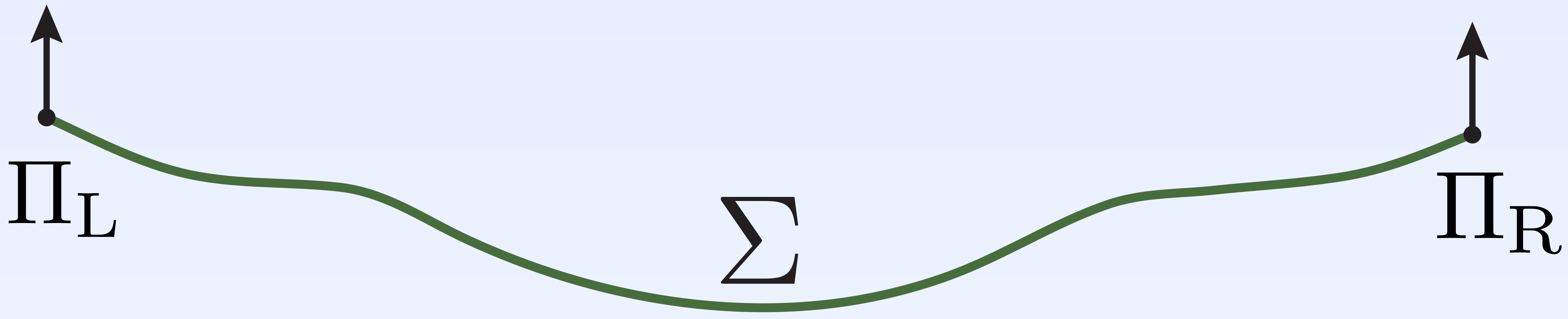


$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Vol}(\Sigma) = \Pi_L + \Pi_R$$

In our setup, we know that Π_L will be the same with/without the shell

Without shell, left and right contributions are the same by reflection symmetry

Extremal volume rates will follow a Hamilton-Jacobi equation



$$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{Vol}(\Sigma) = \Pi_{\rm L} + \Pi_{\rm R}$$

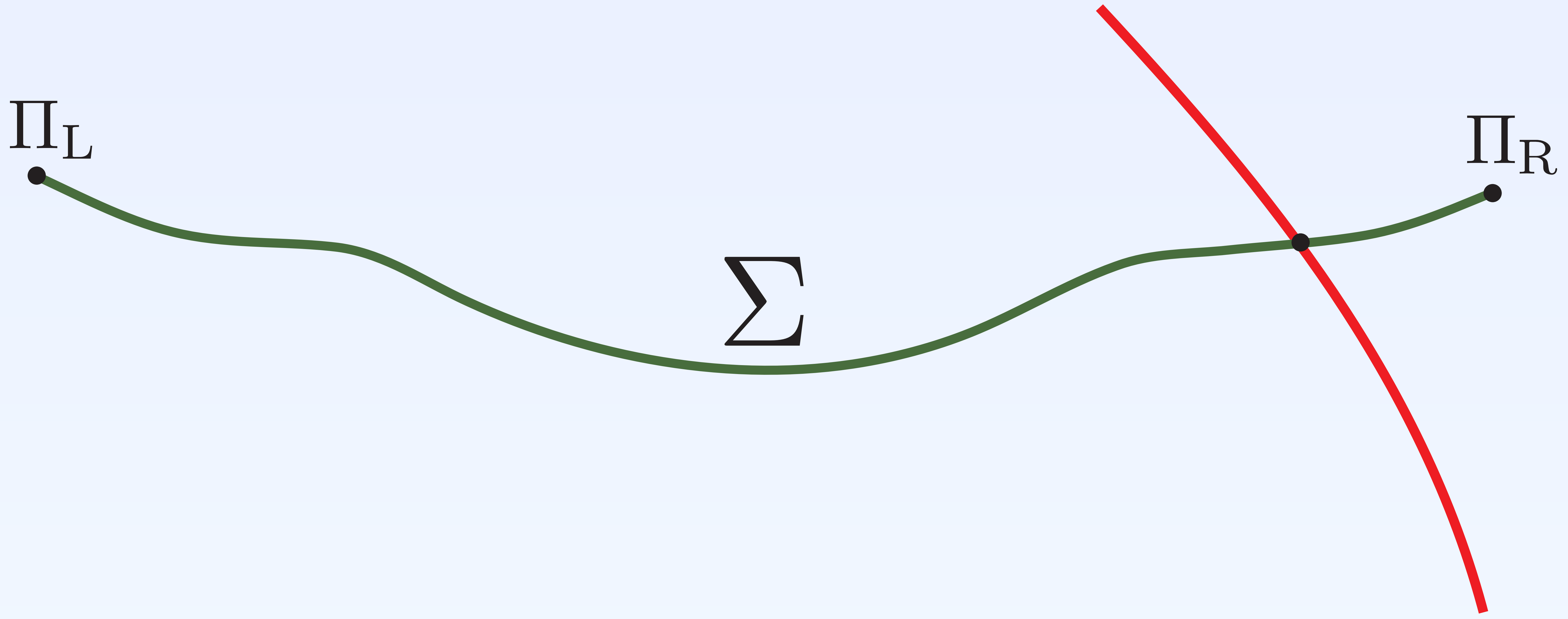
In our setup, we know that $\Pi_{\rm L}$ will be the same with/without the shell

Without shell, left and right contributions are the same by reflection symmetry

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{C}_{\mathcal{O}} = (\Pi_{\rm L} + \Pi_{\rm R}) - 2\Pi_{\rm L} = \Pi_{\rm R} - \Pi_{\rm L}$$

3. Proof of the PC duality

Localization on the worldvolume of the shell



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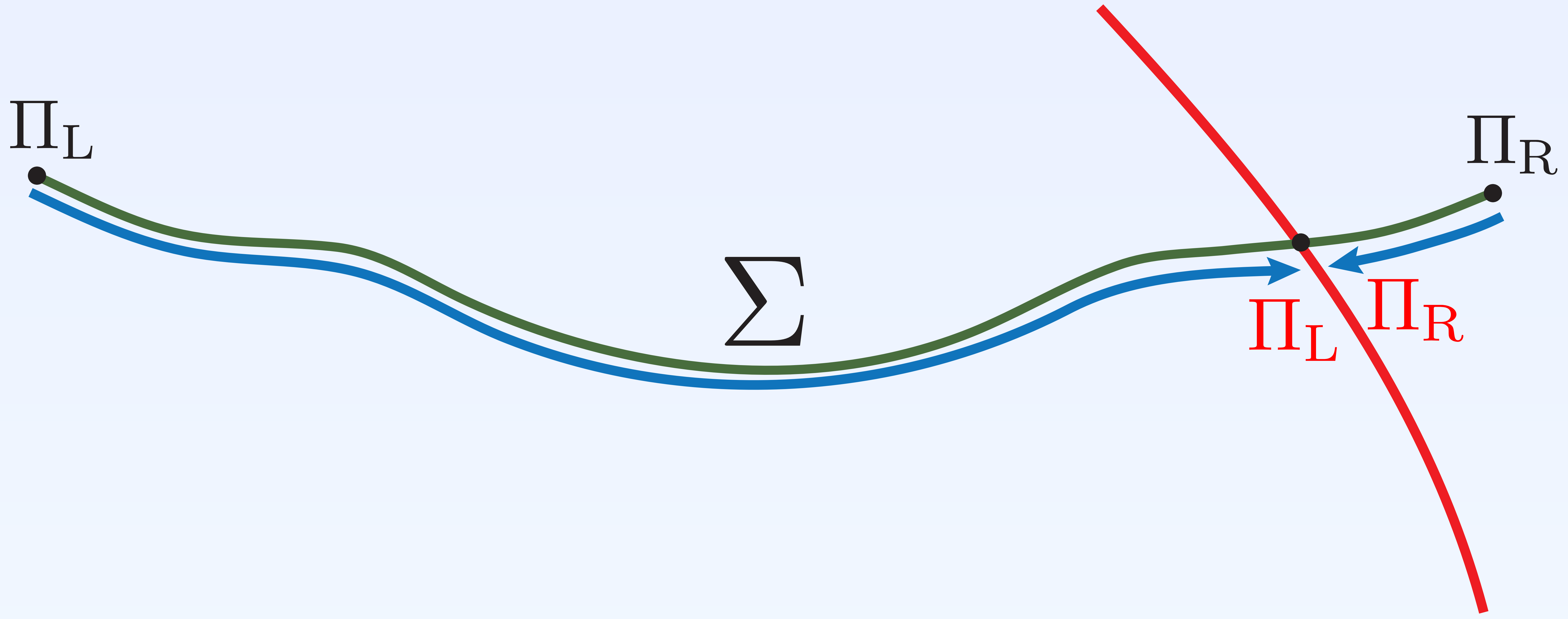
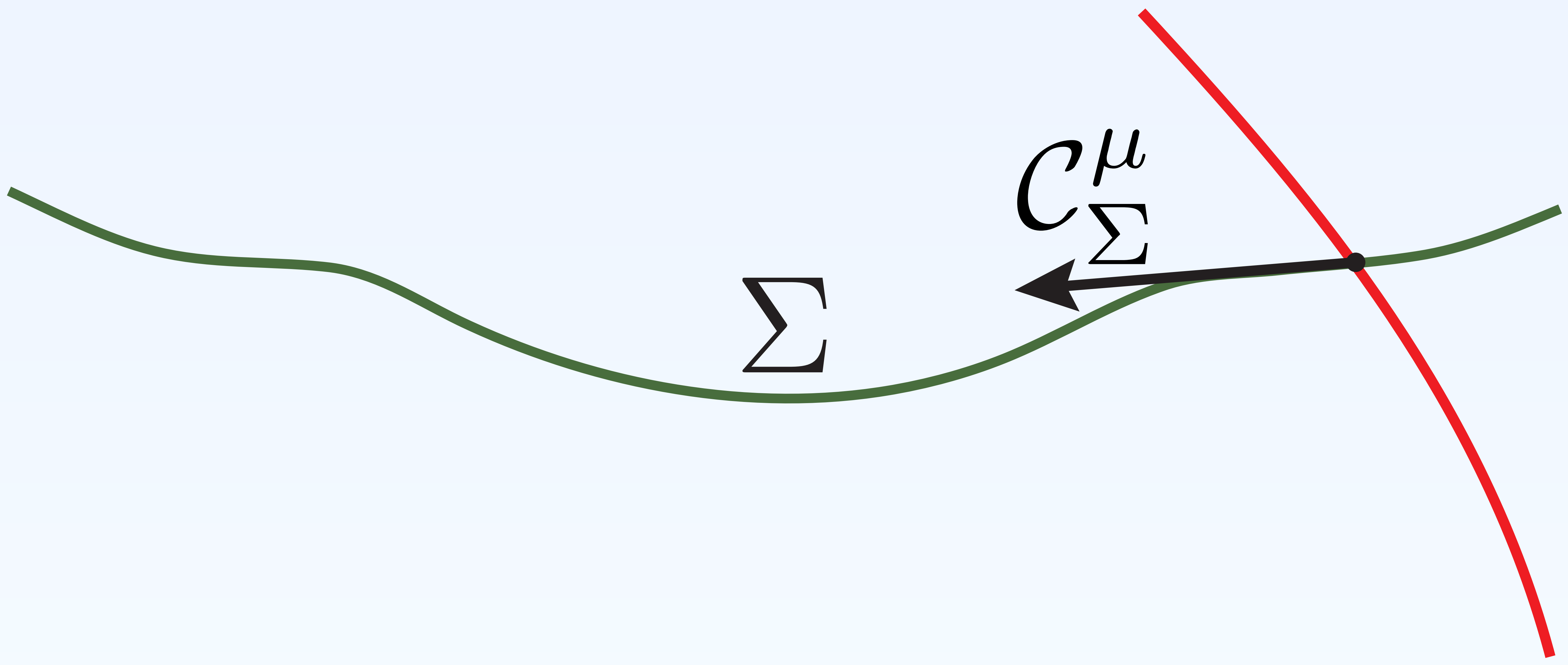


$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{C}_{\mathcal{O}} = \Pi_{\mathrm{R}} - \Pi_{\mathrm{L}}$$

Evaluation of the local integral

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{C}_{\mathcal{O}} = \Pi_{\mathrm{R}} - \Pi_{\mathrm{L}} = \int_{\mathbf{S}^{d-1}} \underbrace{\sigma\, u_\mu}_{\mathcal{P}_\mu} \mathcal{C}^\mu_\Sigma$$

where the complexity vector field $\mathcal{C}^\mu_\Sigma$



PC duality for thin shells

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{C}_{\mathcal{O}} = P_{\mathcal{C}}$$

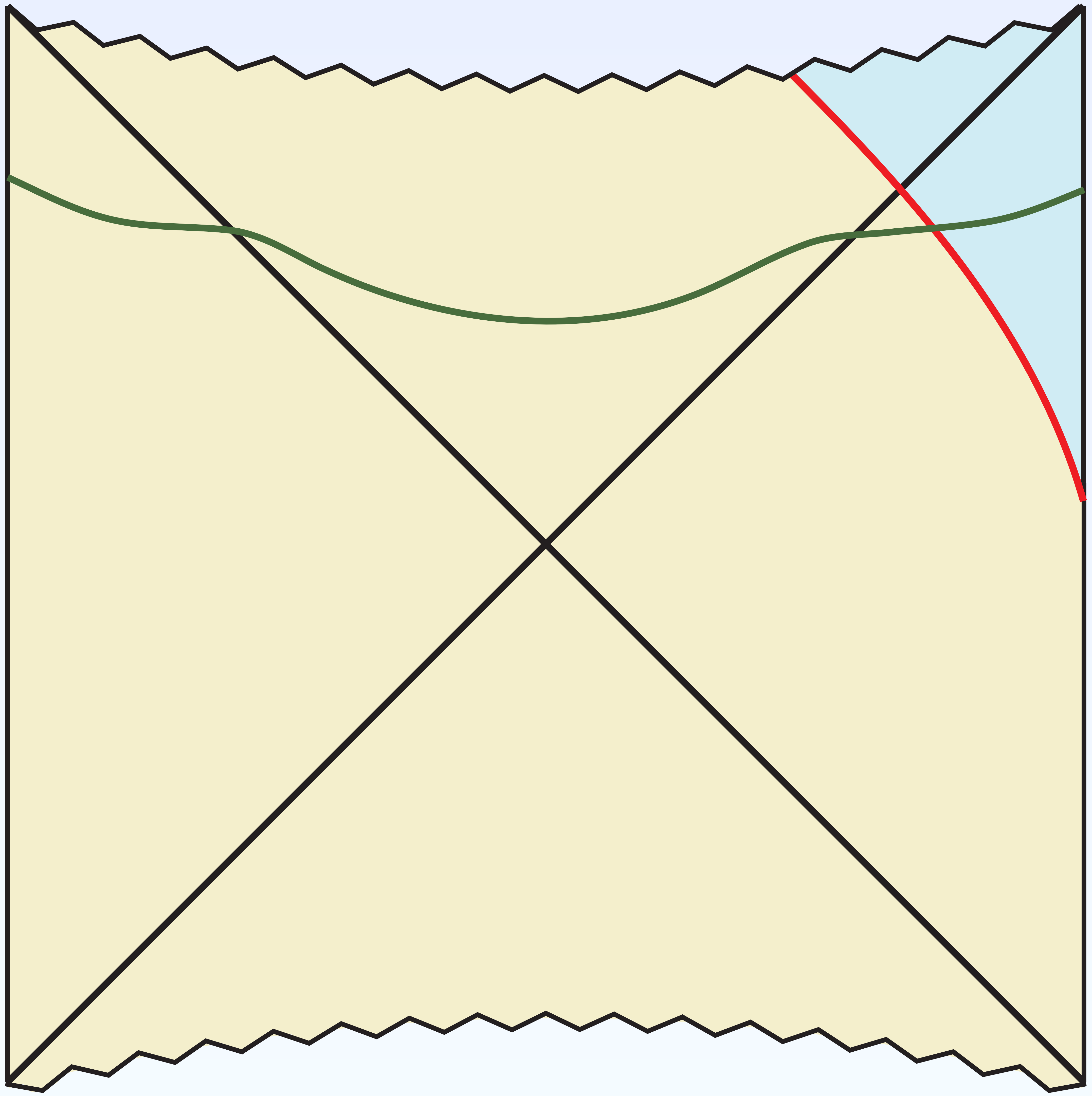
A more general reformulation of the complexity momentum is

$$P_{\mathcal{C}} = -\int_{\Sigma} N^{\mu}_{\Sigma}\, T_{\mu\nu}\, \mathcal{C}^{\nu}_{\Sigma}$$

4. Late time limit and the black hole interior

Extremal surfaces accumulate in the interior of the black hole

[Engelhardt, Wall '14]
[Stanford, Susskind '14]



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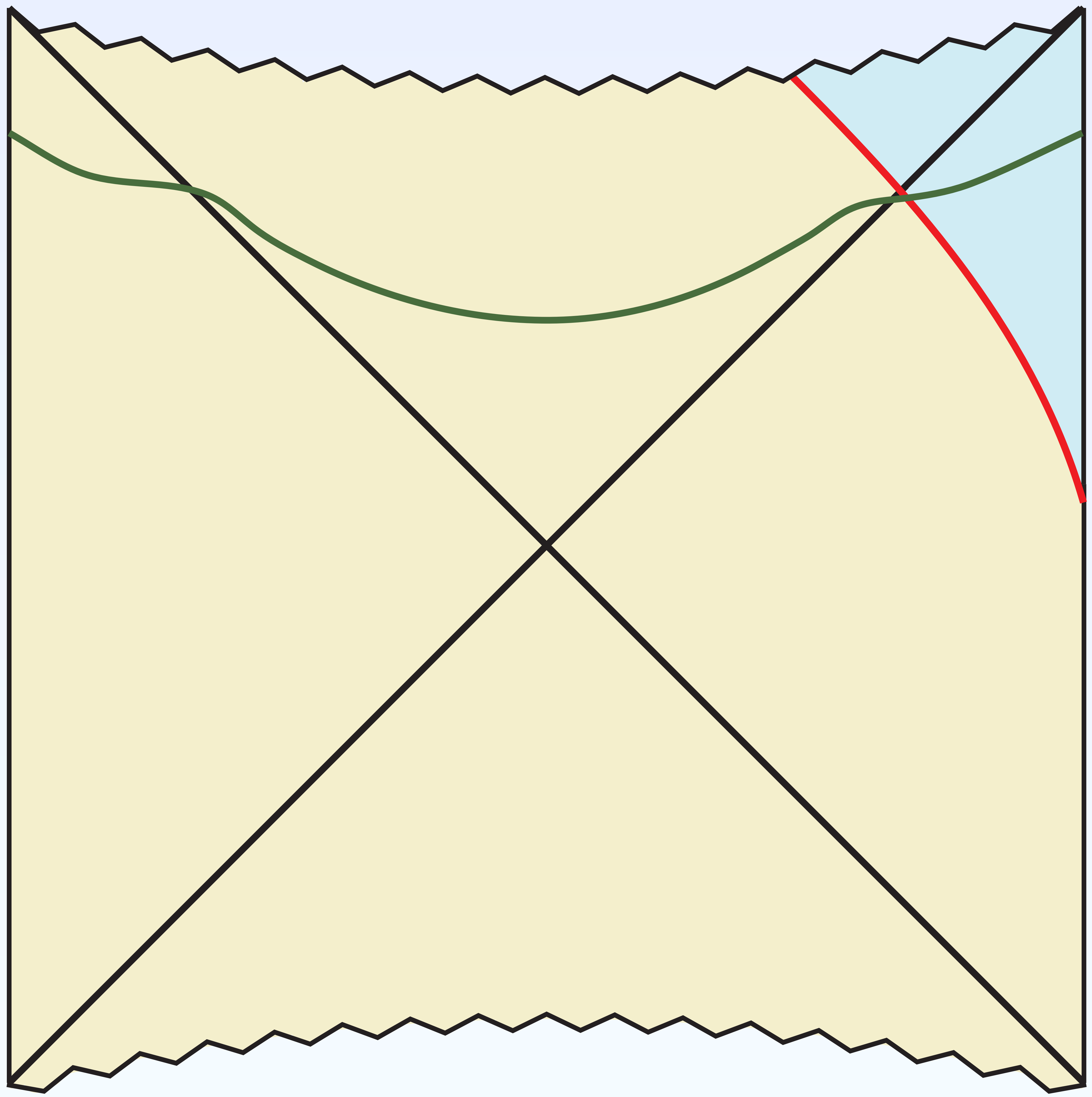

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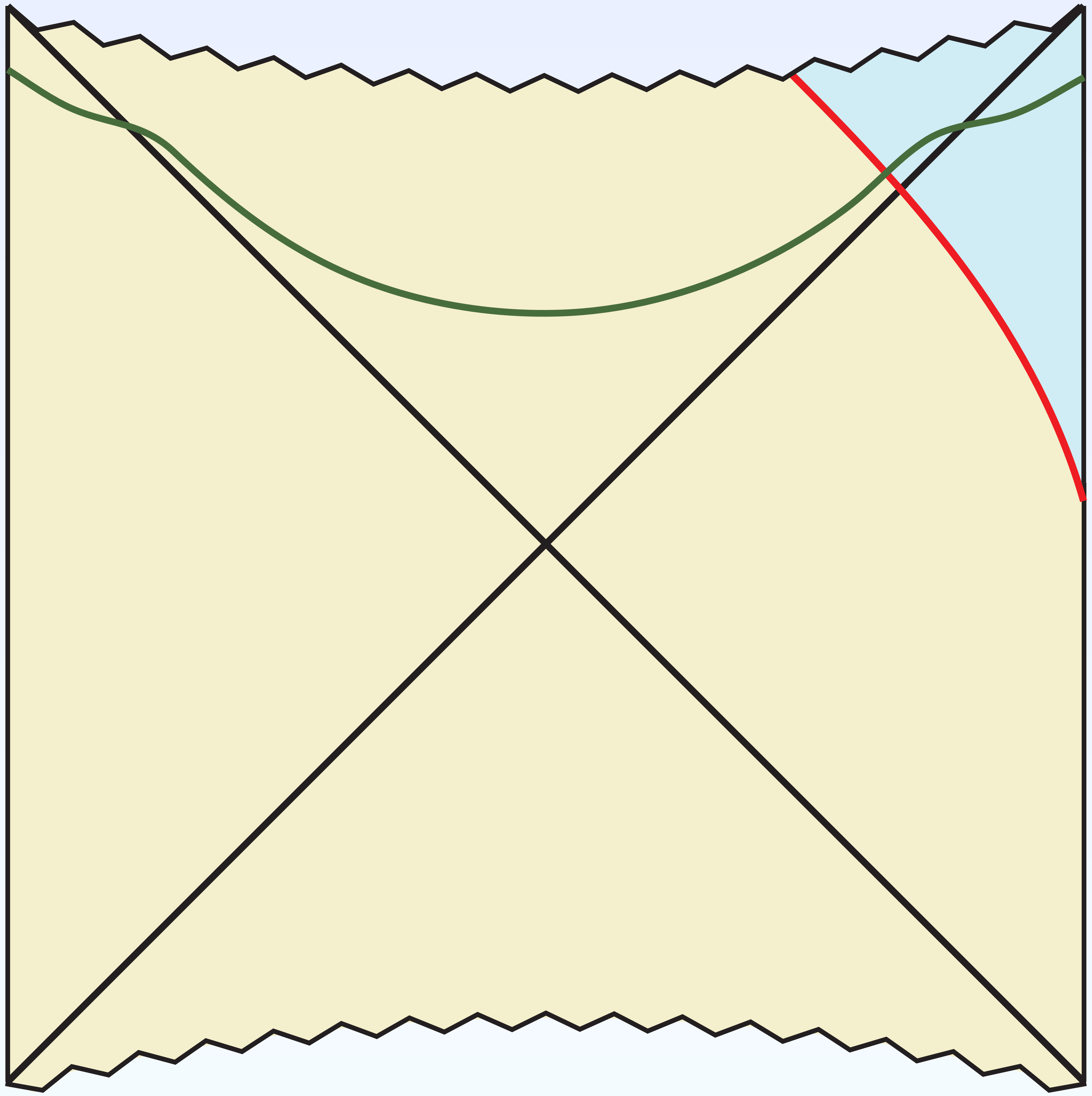

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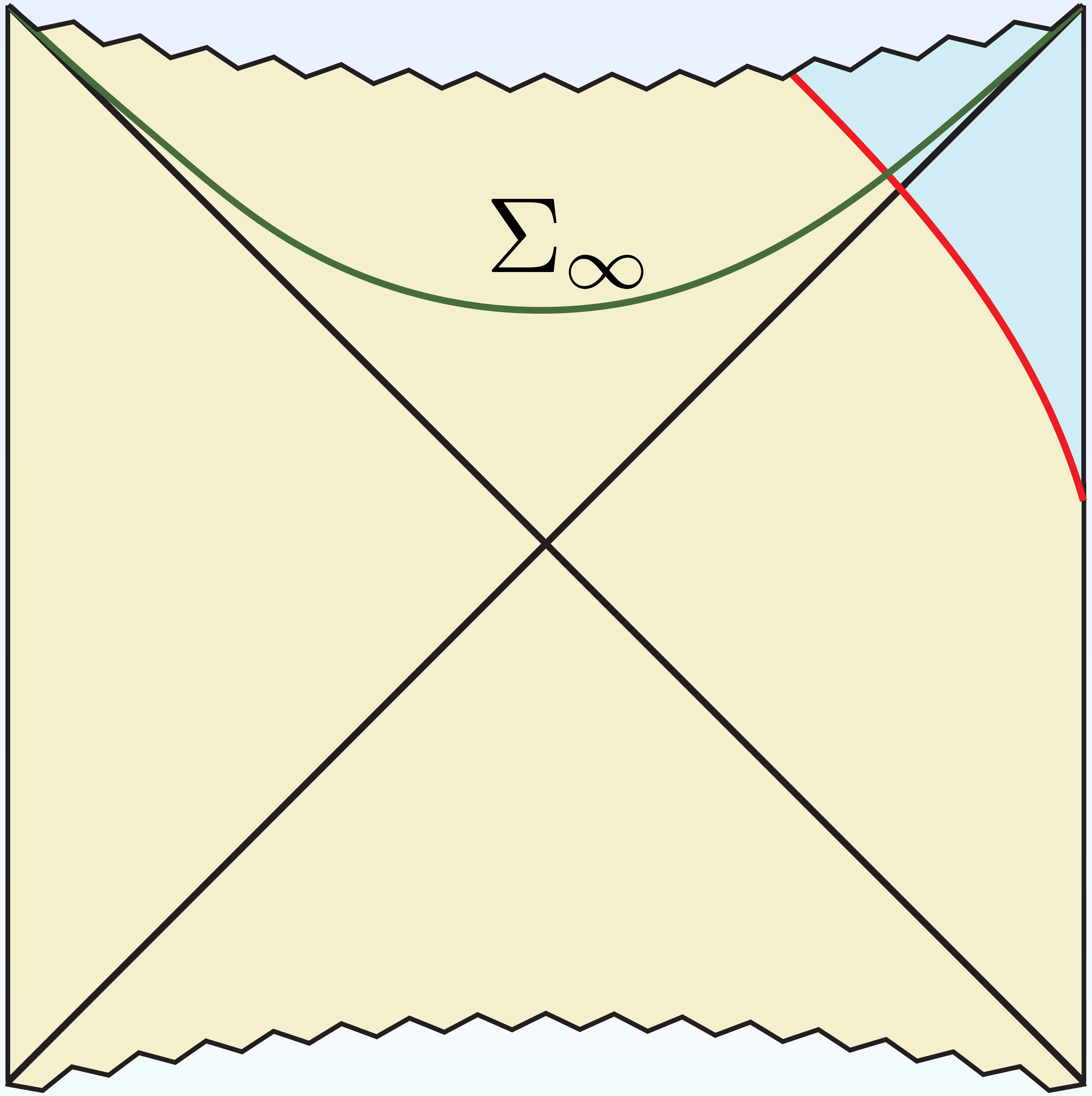

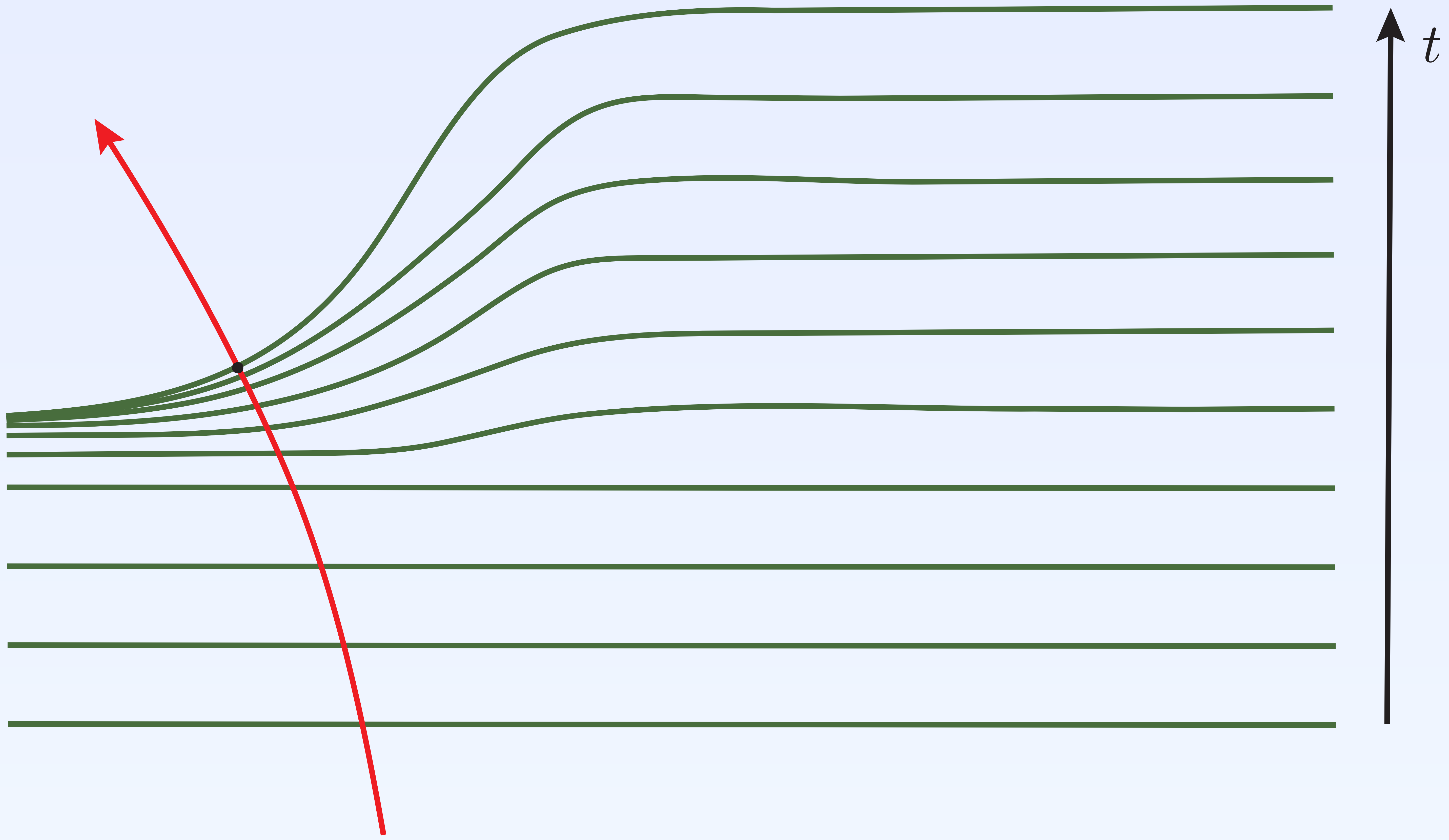

Freezing of time in this foliation makes the momentum asymptote to a constant

$$P_{\mathcal{C}} \sim m_{\text{shell}} \quad \longrightarrow$$

Linear growth of complexity
(up to Heisenberg time)

6. Conclusions

Geometric derivation of the PC duality for spherical thin shells operators. Same derivation holds for any equation of state, including branes.

At late times, the phenomenon of accumulation of maximal slices causes the complexity momentum to become constant $\longrightarrow$ linear growth of the complexity

Spherical symmetry is a strong assumption in our derivation, non-trivial to get around

It would be nice to have a local definition of the complexity vector field $\mathcal{C}^{\mu}_{\Sigma}$ intrinsic to Σ as well as a generic derivation (work in progress)

MOITAS GRAZAS!