Unification of gauge and Yukawa couplings

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Based on M. Khojali *et al.*, arXiv:1706.02313[hep-ph].

What do we know about the Higgs?

The discovery of a Higgs boson allows a direct probe of the EWSB sector of the SM.

Its mass has been precisely measured, and is close to the EW scale itself, which has materialised the issue of naturalness.

The couplings follow the SM expectations:
being proportional to mass.

The uncertainties are still large!

Note that coupling measurements are always subject to model assumptions.

On the theoretical side, recall that mass terms for scalar fields are not protected by any quantum symmetry, therefore any new physics sector that couples to it will feed into the value of the mass.

Recall that

$$\mathcal{L}_{Higgs} = (D_\mu\phi)^\dagger (D^\mu\phi) + \mu^2\phi^\dagger\phi - \lambda\left(\phi^\dagger\phi\right)^2$$

"wrong sign"

It well describes the symmetry breaking, but no dynamical insight!

Do we still need BSM?

We have a pretty good idea of the mechanism

But we don't know how to protect it:

$$\delta m_h^2 \sim \frac{g^2}{16\pi^2} M_{NPhys}^2$$

Having measured a light Higgs brings the naturalness problems to the real world!

A natural Higgs = A special type of scalar

Supersymmetry: All scalars are special, associated to a fermion by a space-time symmetry

$$\text{Scalar} \leftrightarrow \text{Fermion}$$

Compositeness: Special scalars are bound states of more fundamental fermions

$$\phi \sim \langle \psi \psi \rangle$$

Goldstones: Goldstone bosons are special massless scalars, where masses can be generated by symmetry breaking spurions.

Usually in association with compositeness: $\phi \sim \langle \psi\psi \rangle$

$V(h)^{tree} = 0$, no potential allowed at tree level!

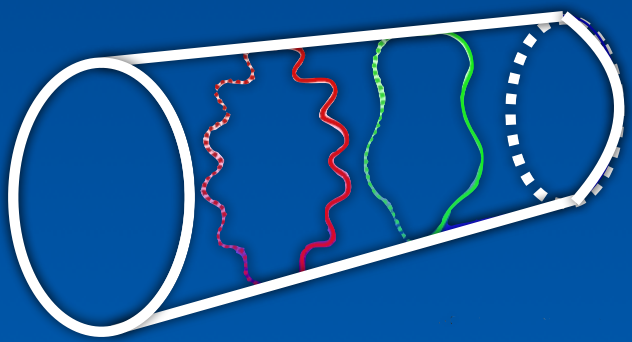
Scalars as gauge bosons: In extra dimensions, the extra polarisations of gauge bosons are special 4D scalars!

$$\phi \sim A_5\,,\; A_6\,,\; \ldots A_D$$

Again, no potential allowed at tree level:
$V(h)^{tree} = 0$

Extra dimensions in nut-shell

Think of an additional space dimension wrapped on itself.



XD fields → tower of KK states

Frequencies → KK masses

Geometry → KK parities

For us 4D beings the 5D fields appear as massive 4D fields.

$$\phi\left(x^{\mu}\,,\,y\right)$$

Periodicity condition: $\phi\left(x^{\nu}\,,\,2\pi R\right) = \phi\left(x^{\mu}\,,\,0\right)$

Fourier expansion: $\phi\left(x^{\mu}\,,\,y\right) = \sum_{n\in\mathbb{Z}} \tilde{\phi}_n\left(x^{\mu}\right) e^{in/Ry}$

5D EOM

$$\left[\partial_{\mu}\partial^{\mu} - \partial_y^2\right]\phi\left(x^{\mu}\,,\,y\right) =$$

$$\sum_{n\in\mathbb{Z}} e^{in/Ry}\left[\partial_{\mu}\partial^{\mu} - \frac{n^2}{R^2}\right]\tilde{\phi}_n\left(x^{\mu}\right) = 0$$

4D EOM

Geometric parity can be imposed:
Orbifold projection

$$\phi\,(x^{\nu}\,,\,-y) = \pm\phi\,(x^{\mu}\,,\,y)$$

$$\phi\,(x^{\mu}\,,\,y) = \sum_{n=0}^{\infty}\cos\left(\frac{n}{R}y\right)\tilde{\phi}_n^{+} + \sum_{n=1}^{\infty}\sin\left(\frac{n}{R}y\right)\tilde{\phi}_n^{-}$$

Parity even
Includes massless
$n = 0$ mode

Parity odd

Extra dimension gauge fields

$$A_M = (A_\mu \,,\, A_5)$$

Vector Scalar

$$S = \int d^4x \int_0^{2\pi R} dy \, -\frac{1}{4g_5^2} F_{MN}F^{MN}$$

dim 1 dim 4

$$\frac{2\pi R}{g_5^2} = \frac{1}{g^2}$$

For the zero mode

A generic loop factor will thus grow with the energy.

Using naive dimensional analysis:

$$\frac{g_5^2}{16\pi^2}\Lambda \sim 1 \Rightarrow \Lambda \sim \frac{8\pi}{g^2}\frac{1}{R}$$

An XD Higgs mechanism

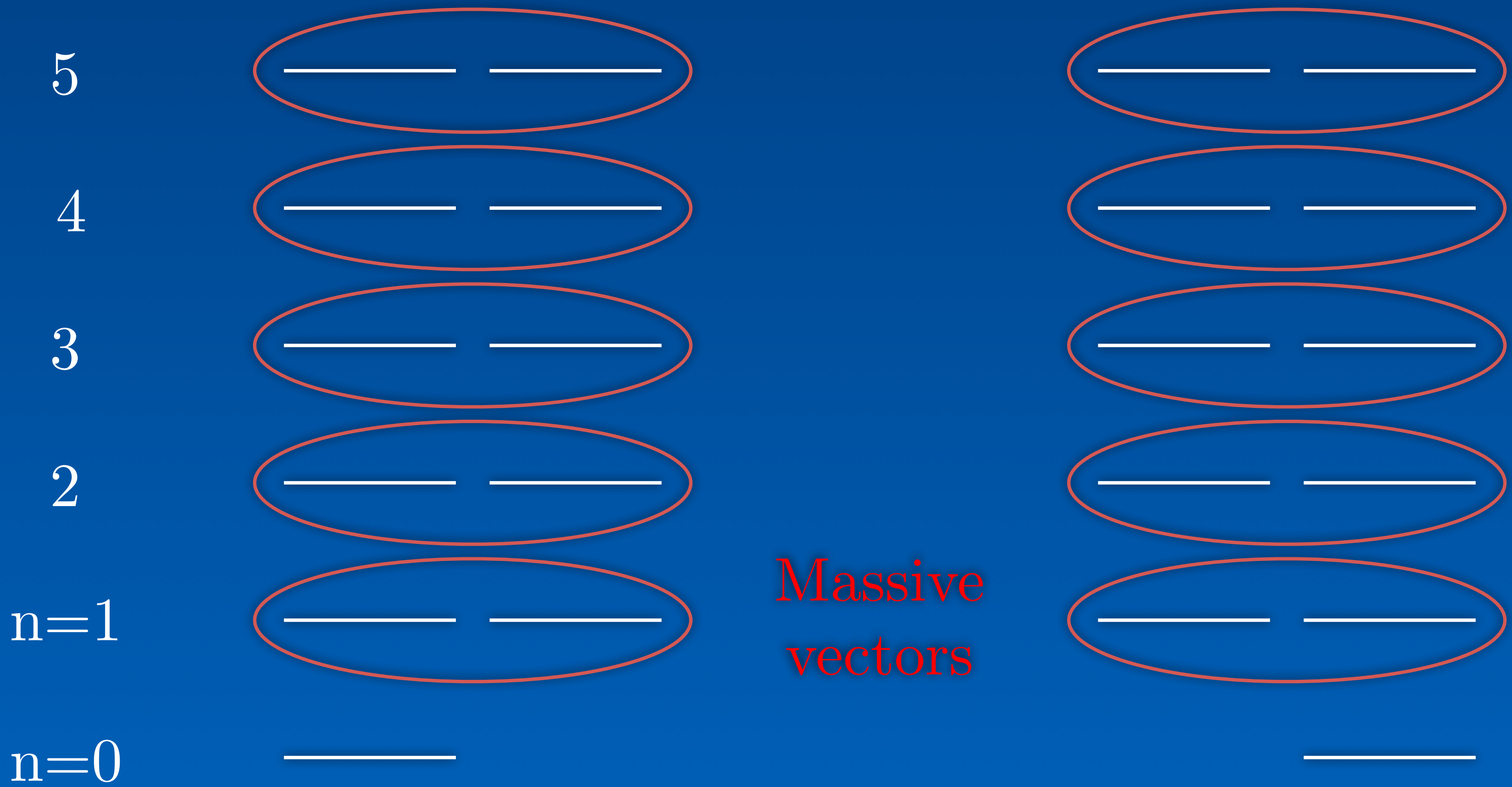
$$F_{MN}F^{MN} = F_{\mu\nu}F^{\mu\nu} - 2F_{5\mu}F^{5\mu}$$
$$= F_{\mu\nu}F^{\mu\nu} - 2\left(\partial_5 A_\mu\right)^2 - 2\left(\partial_\mu A_5\right)^2$$
$$+4\left(\partial_5 A_\mu\right)\left(\partial^\mu A_5\right)$$

A mixing between massive vectors and scalars is present

In the orbifold projection vector and scalar have opposite parities.

$$A_M^+ = \left(A_\mu^+\,,\,A_5^-\right)$$
$$A_M^- = \left(A_\mu^-\,,\,A_5^+\right)$$

$$A_M^+ = \left(A_\mu^+\,,\ A_5^-\right) \qquad\qquad A_M^- = \left(A_\mu^-\,,\ A_5^+\right)$$



$$
\begin{aligned}
F_{MN}F^{MN} &= F_{\mu\nu}F^{\mu\nu} - 2F_{5\mu}F^{5\mu} \\
&= F_{\mu\nu}F^{\mu\nu} - 2\left(\partial_5 A_\mu\right)^2 - 2\left(\partial_\mu A_5\right)^2 \\
&\quad \boxed{+4\left(\partial_5 A_\mu\right)\left(\partial^\mu A_5\right)}
\end{aligned}
$$

A mixing between massive vectors and scalars is present

In the orbifold projection vector and scalar have opposite parities.

$A_M^+ = \left(A_\mu^+ \,,\, A_5^-\right)$ Massless vector + massive vectors

$A_M^- = \left(A_\mu^- \,,\, A_5^+\right)$ Massless scalar + massive vectors

XD gauge theories have a built-in Higgs mechanism via special scalars

Gauge symmetries can be broken by orbifold projection (parity assignments)

An $SU(3)$ toy model

The gauge group is $SU(3)_c \times SU(3)_w$ on an $S^1/\mathbb{Z}_2$ orbifold:

The enhanced weak symmetry allows the unification of the SM gauge bosons and the Higgs doublet.

In fact, the adjoint of $SU(3)$ decomposes into $(\mathbf{3}, 0) + (\mathbf{2}, 1/2) + (\mathbf{2}, -1/2) + (\mathbf{1}, 0)$.

The orbifold breaks $SU(3)_w$ to $SU(2)_L \times U(1)$ via the projection matrix:

$$P = \begin{pmatrix} -1 & & \\ & -1 & \\ & & 1 \end{pmatrix},$$

where the gauge fields transform as

$$A_\mu \to P A_\mu P^\dagger \text{ and } A_5 \to -P A_5 P^\dagger$$

With this choice only the SM gauge fields have a zero mode.

$$SU(2) \qquad\qquad\qquad\qquad U(1)$$

$$A_M^+ : \begin{pmatrix} \frac{1}{2}W_3 & \frac{1}{\sqrt{2}}W^+ & 0 \\ \frac{1}{\sqrt{2}}W^- & -\frac{1}{2}W_3 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \frac{1}{2\sqrt{3}}\begin{pmatrix} B & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & -2B \end{pmatrix}$$

$$A_M^- : \qquad \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H_0 \\ H^- & H_0^* & 0 \end{pmatrix}$$

In the scalar sector, the zero mode is a single complex $SU(2)$ doublet.

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H_0 \\ H^- & H_0^* & 0 \end{pmatrix}$$

Can we identify this with the Higgs doublet?

$$U(1)\text{ charge:}\quad \frac{1}{2\sqrt{3}}(1-(-2)) = \frac{\sqrt{3}}{2}$$

$$g' = \sqrt{3}g \Rightarrow \sin^2\theta_W = \frac{3}{4}$$

Does not match the SM value

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & H^+ \\ 0 & 0 & H_0 \\ H^- & H_0^* & 0 \end{pmatrix}$$

Can we identify this with the Higgs doublet?

Easy solution: add a bulk $U(1)_X$ gauge symmetry and tune the g_X coupling

$U(1)$ charge: $$\frac{1}{2}g' = \frac{\sqrt{3}}{2}g + \frac{1}{2}g_X$$

Antoniadis, Benakli, Quiros: hep-th/0108005

Note also that all the massive modes in A_5 are eaten by the massive KK modes of the gauge bosons, and play the role of the longitudinal degrees of freedom, like in the usual Higgs mechanism:

the only physical scalar left in the spectrum is the zero mode.

The linearised gauge transformations in the bulk are:

$$A_\mu \to A_\mu + \partial_\mu \lambda(x, x_5) + i\,[\lambda(x, x_5), A_\mu]$$

$$A_5 \to A_5 + \partial_5 \lambda(x, x_5) + i\,[\lambda(x, x_5), A_5]$$

On the branes, $\lambda = 0$ for the broken generators, however, the gauge transformation will still impose on A_5 a shift coming from $\partial_5 \lambda$.

This is enough to forbid a tree level potential for A_5, also on the fixed points, and only loop contributions will generate a potential for the Higgs, that will be non-local from the 5D point of view, and finite.

The Hosotani mechanism

The Higgs vev can be geometrised

Hosotani, 1983

EWSB is induced by giving a vev to the Higgs, which is a gauge boson.

The vev can be removed by a suitable gauge transformation:

$$\Omega(y) = e^{i\alpha T_H y/R}$$

$$H_0 \to H_0 - \partial_y \left(\alpha \frac{y}{R} \right) = H_0 - \frac{\alpha}{R}$$

The periodicity condition is also affected:

$$\Omega(0)\phi(0) = \Omega(2\pi R)\phi(2\pi R)$$

$$\phi(0) = e^{i2\pi\alpha T_H}\phi(2\pi R)$$

$$\phi = \sum_n \tilde{\phi} e^{i(n+\alpha)y/R}$$

$$m_n = \frac{|n+\alpha|}{R}$$

The spectrum is shifted

$$m_0 = \frac{|\alpha|}{R}$$

Zero modes pick up a mass

How about fermions?

Consider a fundamental of $SU(3)$:

$$\psi = \begin{pmatrix} \psi_D \\ \psi_S \end{pmatrix} \qquad U(1)\text{ charges}: \begin{pmatrix} 1/6 \\ -1/3 \end{pmatrix}$$

$$\bar{\psi} D_5 \gamma^5 \psi \Rightarrow \frac{g}{\sqrt{2}} \bar{\psi}_D H \psi_S + h.c.$$

$y_f = \frac{g}{\sqrt{2}}$: Yukawa couplings are related to gauge couplings

The $SU(3)$ model fails

Incorrect prediction of the Weinberg angle

$$\sin^2\theta_W = \frac{3}{4}$$

Yukawas related to the gauge couplings:

$$m_f = m_W$$

Grossmann, Neubert: hep-ph/9912408

It is easy to reduce the fermion masses by use of localisation, but hard to enhance.

The value of the top mass is a serious issues.

Many solutions attempted:

- Looking for other gauge groups: $\mathbf{G}_2, \ldots$

  Csaki, Grojean, Murayama: hep-ph/0210133

- Embedding the top in higher representations.

  Cacciapaglia, Csaki, Park: hep-ph/0510366

- Adding localised couplings/fields.

  Scrucca, Serone, Silvestrini: hep-ph/0304220

- Changing the geometry of the space

  Contino, Nomura, Pomarol: hep-ph/0306259

- . . .

Geometry at work

In flat space gauge zero modes are constant:

$$\int dy H \bar{\psi}\psi = H \int dy \bar{\psi}\psi$$

In warped space gauge-scalar zero modes are NOT constant:

The overlap integral can give enhancement factors.

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(dx^2 - dz^2\right)$$

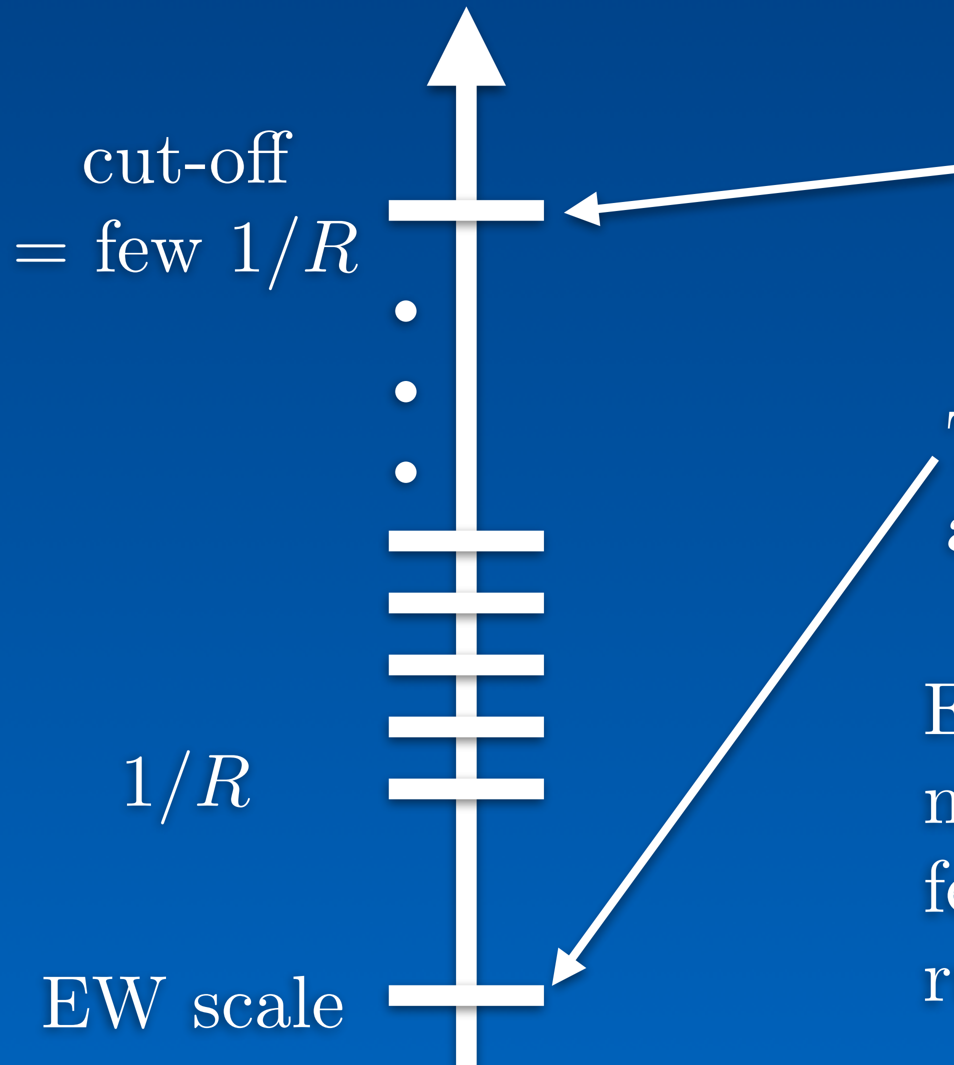
Conformally invariant if $x \to \epsilon x\,, \quad z \to \epsilon z$

Randall, Sundrum: hep-ph/9905221

The AdS/CFT duality (conjectured for supersymmetric models) suggests that the XD model may share the same features of (composite) conformal 4D models.

This observation lead to a revival of the composite Higgs models.

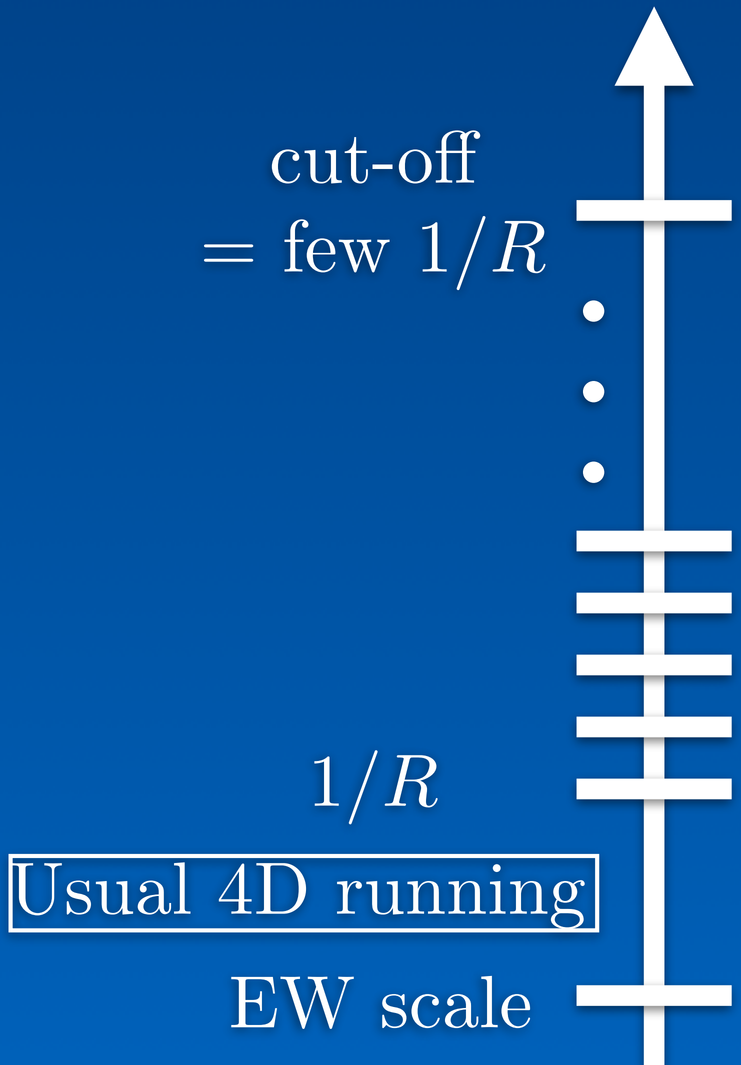
One fact has been overlooked so far: couplings run!



The Unified predictions are valid at this scale.

They need to be matched at the EW scale.

Even though the two scales may not be that far, XDs features an accelerated running.



At each threshold
new states enter
the running.

The collective effect
of the resonances
produces a fast linear running
(cf. 4D log running)

This fact is well known
for the gauge couplings
(see XD GUT models).

But the Yukawas
are also gauge couplings here!

Reconsider the simple $SU(3)$ model

Cacciapaglia, AC, Deandrea, Khojali: hep-ph/1706.02313

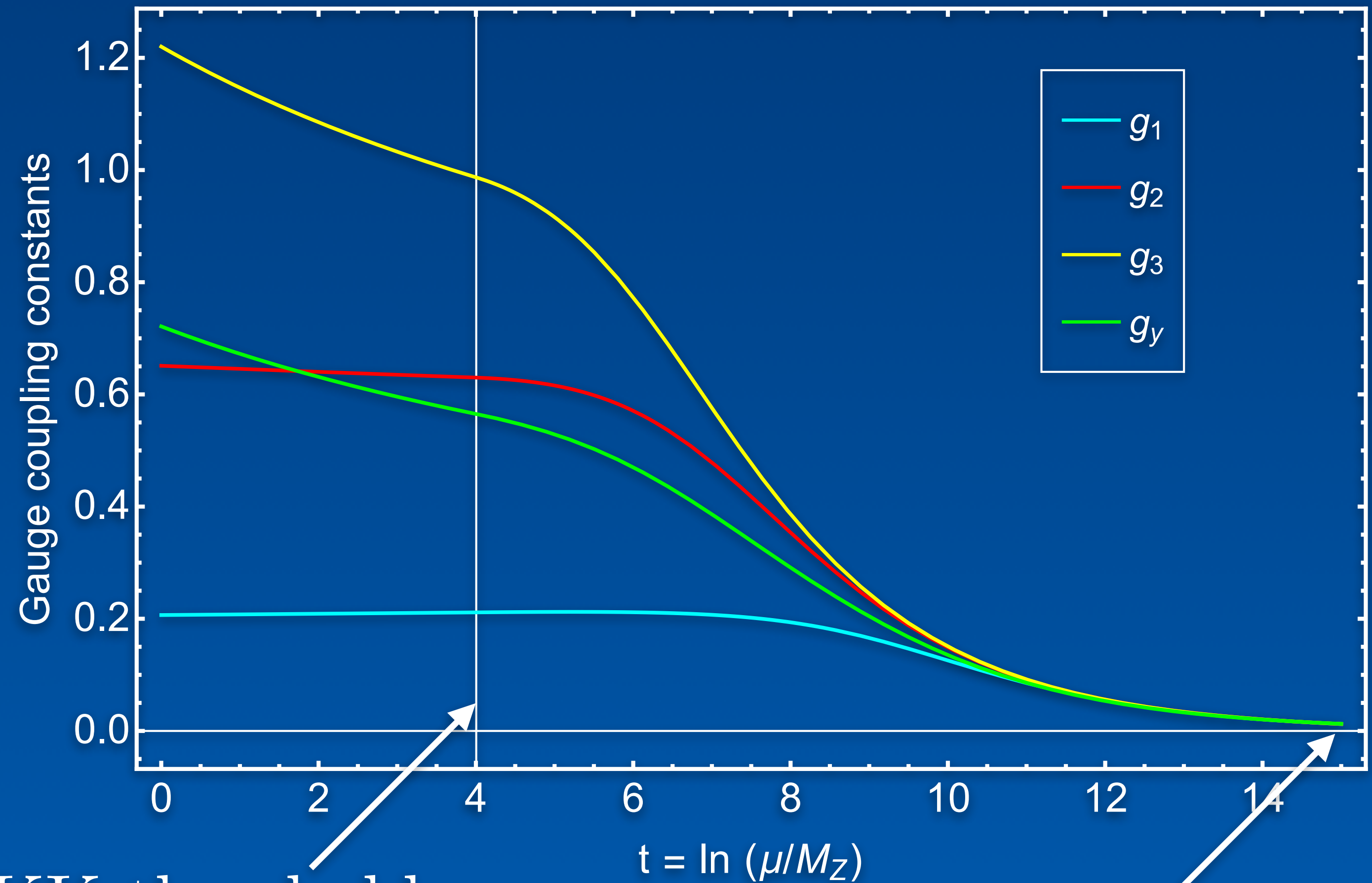
	$SU(2)_L$	$U(1)_Y$	Yuk.	$SU(3)_c$
	g	g'	y_t	g_s
$SU(3)$	g_{GHU}	$\sqrt{3}\, g_{\mathrm{GHU}}$	$g_{\mathrm{GHU}}/\sqrt{2}$	-
SM	0.66	0.35	1.0	1.2

Gauge and top Yukawa couplings in the $SU(3)$ GHU model compared to the SM values at the m_Z scale. We also include for completeness the QCD coupling.

Rescaled couplings:

$$\{g_1 \,,\, g_2 \,,\, g_3 \,,\, g_t\} = \left\{ \frac{g'}{\sqrt{3}} \,,\, g \,,\, g_s \,,\, \sqrt{2} y_t \right\}$$

4D running



First KK threshold

$SU(2)$ and $U(1)$ unify

Running of the normalised gauges and Yukawas for the SU(3) GHU model, for $1/R = 5$ *TeV. The first KK mode enters at* $t_{\mathrm{KK}} \sim 4.0$*, while the Unification scale corresponds to* $t \sim 9.3$.

Note that the value of the couplings at Unification:

$$g_2 \sim 0.1$$

Number of KK modes below Unification:

$$n_{KK} \sim m_Z R e^{10} \sim 400$$

Naive dimensional analysis cut-off:

$$\Lambda R \sim \frac{8\pi}{g_{Unif}^2} \sim 2000$$

“top” Yukawa at the EW scale:

$$y_t \sim 0.6$$

For $g_i = \{g', g, g_s\}$:

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^2 + (S(t) - 1) b_i^{GHU} g_i^3$$

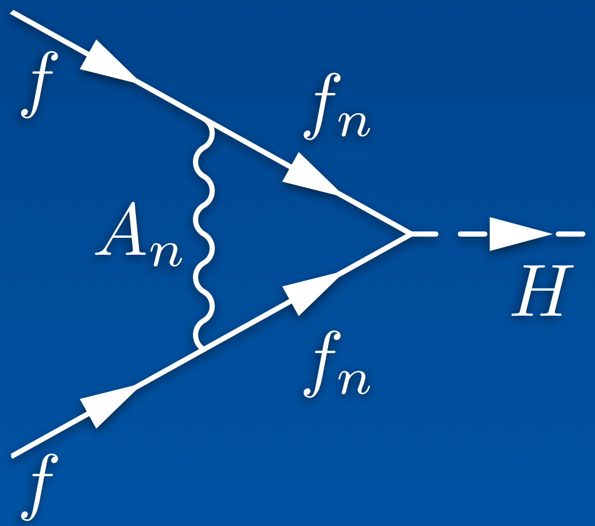
Encodes linear 5D runnings:

$$S(t) = \begin{cases} \mu R = m_Z R e^t & \text{for} \quad \mu > 1/R \\ 1 & \text{for} \quad m_Z < \mu < 1/R \end{cases}$$

$$b_i^{SM} = \left[\frac{41}{6}, -\frac{19}{6}, -7\right] \qquad b_i^{GHU} = \left[-\frac{17}{6}, -\frac{17}{2}, -\frac{17}{2}\right]$$

Running of the Yukawa

Complication: it depends on other couplings



We simplify by assuming that couplings of the KK modes follow the running of the gauge/Yukawa couplings

$$16\pi^2 \frac{dy_t}{dt} = \beta_t^{SM} + (S(t) - 1)\, \beta_t^{GHU}$$

$$\beta_t = y_t \left[c_t y_t^2 + \sum_i d_i g_i^2 \right]$$

The numerical values are:

$$c_t^{SM} = \frac{9}{2} \qquad d_i^{SM} = \left[-\frac{5}{12}\,,\ -\frac{9}{4}\,,\ -8\right]$$

$$c_t^{GHU} = \frac{21}{2} \qquad d_i^{GHU} = \left[-\frac{35}{24}\,,\ -\frac{39}{8}\,,\ -4\right]$$

Imposing unification relations:

$$g' = \sqrt{3}g\ ,\quad y_t = \frac{g}{\sqrt{2}}$$

$$\beta_t = \left(-4g^2 - 4g_s^2\right)\frac{g}{\sqrt{2}}$$

Conclusions

- Gauge-Higgs Unification models have been prematurely abandoned
- Running of the gauge and Yukawa couplings needed to properly match the theory to the EW scale
- We show in a $SU(3)$ toy model that the tree level tensions can be softened by the running
- More work needs to be done towards a realistic model: Higgs mass?

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