New physics simulations at colliders

Benjamin Fuks

LPTHE / Sorbonne Université

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Outline

1. A comprehensive approach for Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
4. Parton showers, hadronisation & underlying event
5. Summary

Monte Carlo simulations for new physics

✦ Path towards the characterisation of new physics

- ✤ Fitting and interpreting deviations
- ✤ Predictions of associated signatures/signals

✦ Characterisation of new physics at the LHC

- ✤ Accurate measurements ⊕ precision predictions

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Monte Carlo simulations play a key role

✦ Characterisation of new physics at the LHC

✤ Accurate measurements ⊕ precision predictions

BSM simulations: where are we?

✦ New physics simulations - a challenge

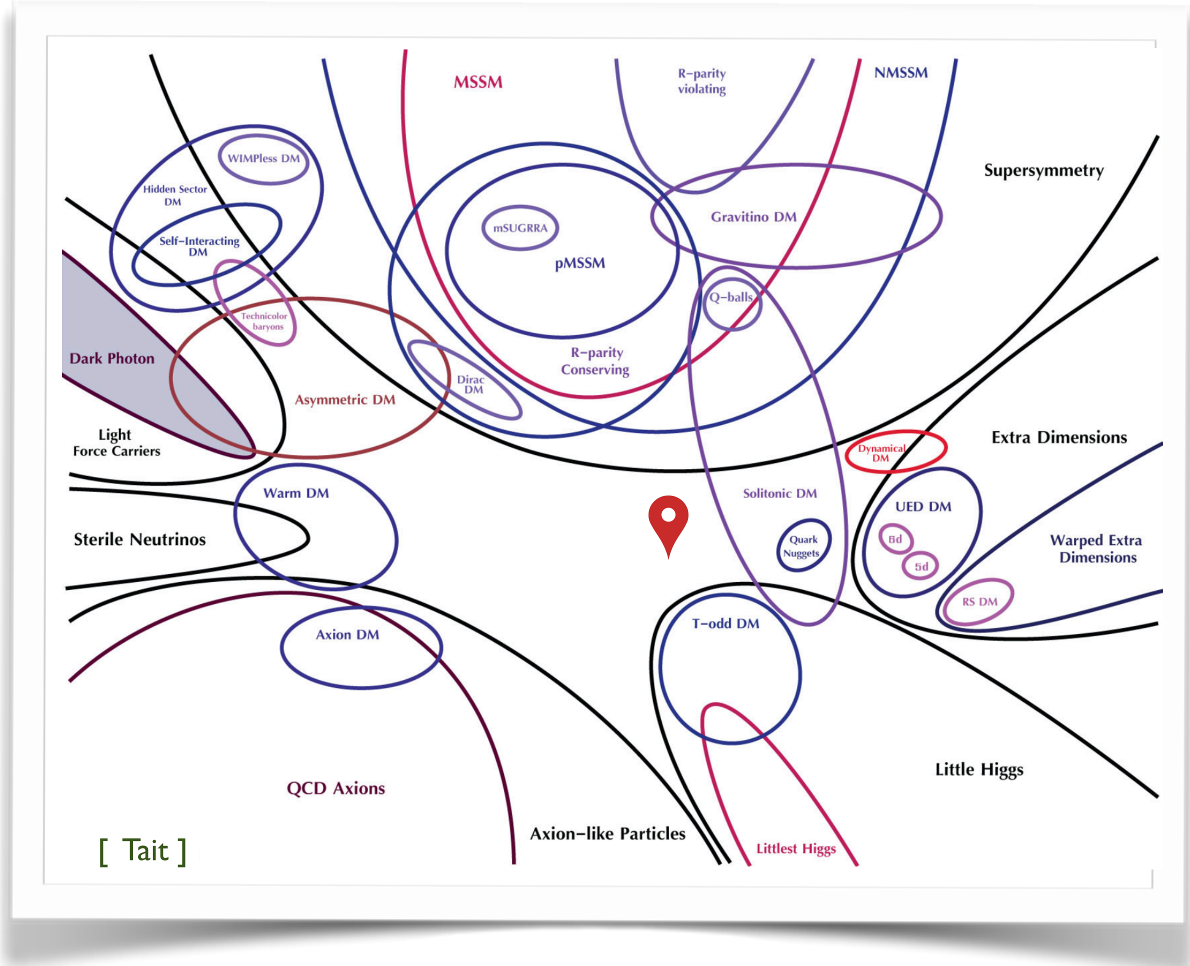
✤ No sign of new physics

✤ SM-like measurements

→ no leading candidate theory

✤ Plethora of models to consider

→ many implementations in tools



BSM simulations: where are we?

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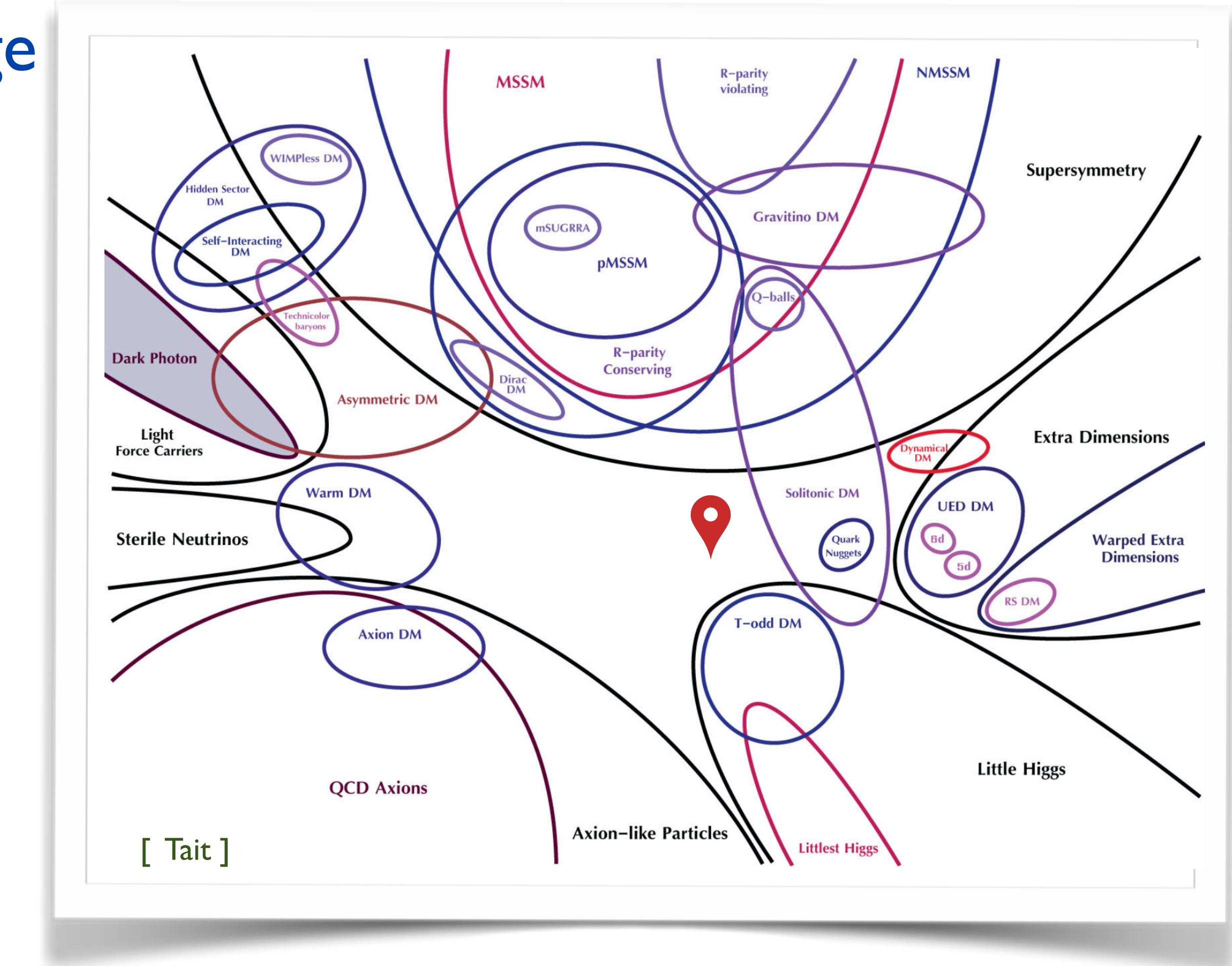
Despite of this, new physics is standard today



✦ New physics is standard

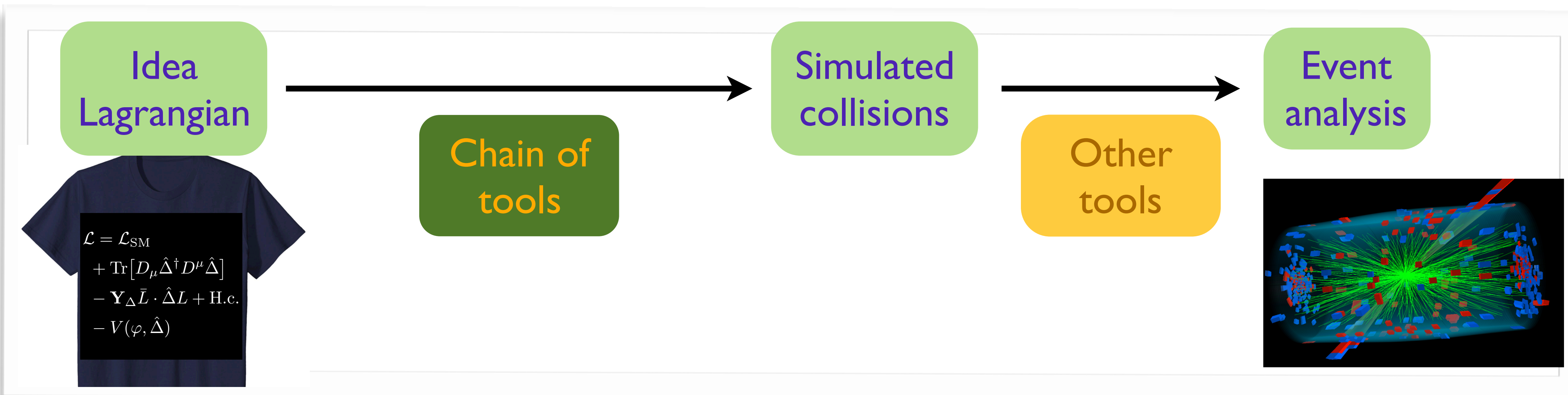
- ✤ 20-25 years of developments → LO simulations are bread and butter
- ✤ Simulations at the NLO QCD accuracy easily achieved
  - ★ For any model/process (→ MADGRAPH5_aMC@NLO)

From Lagrangians to events

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Streamlining the connection of a physics models to events

- Any new physics model can be implemented
- Easy to validate and maintain



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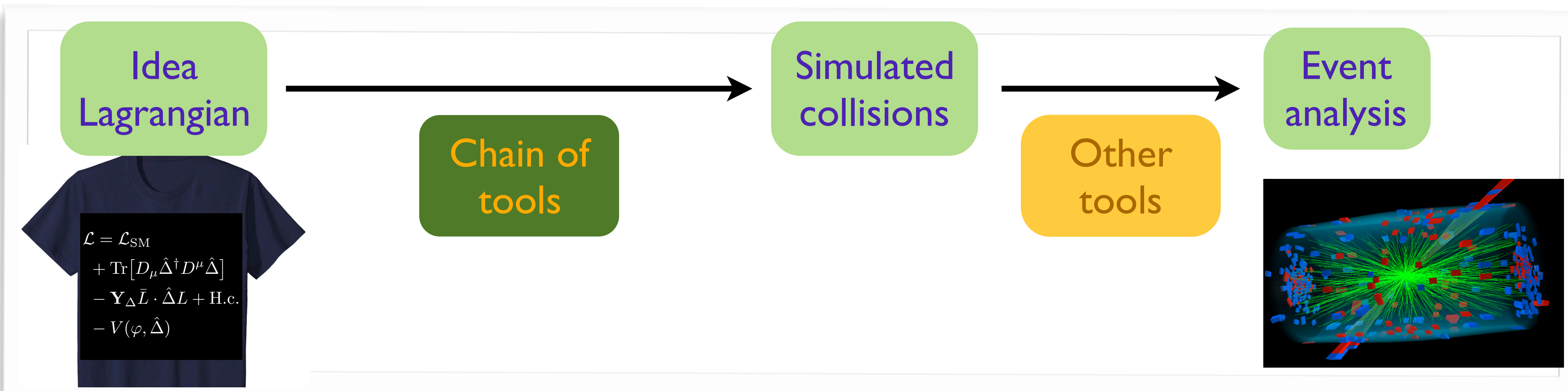
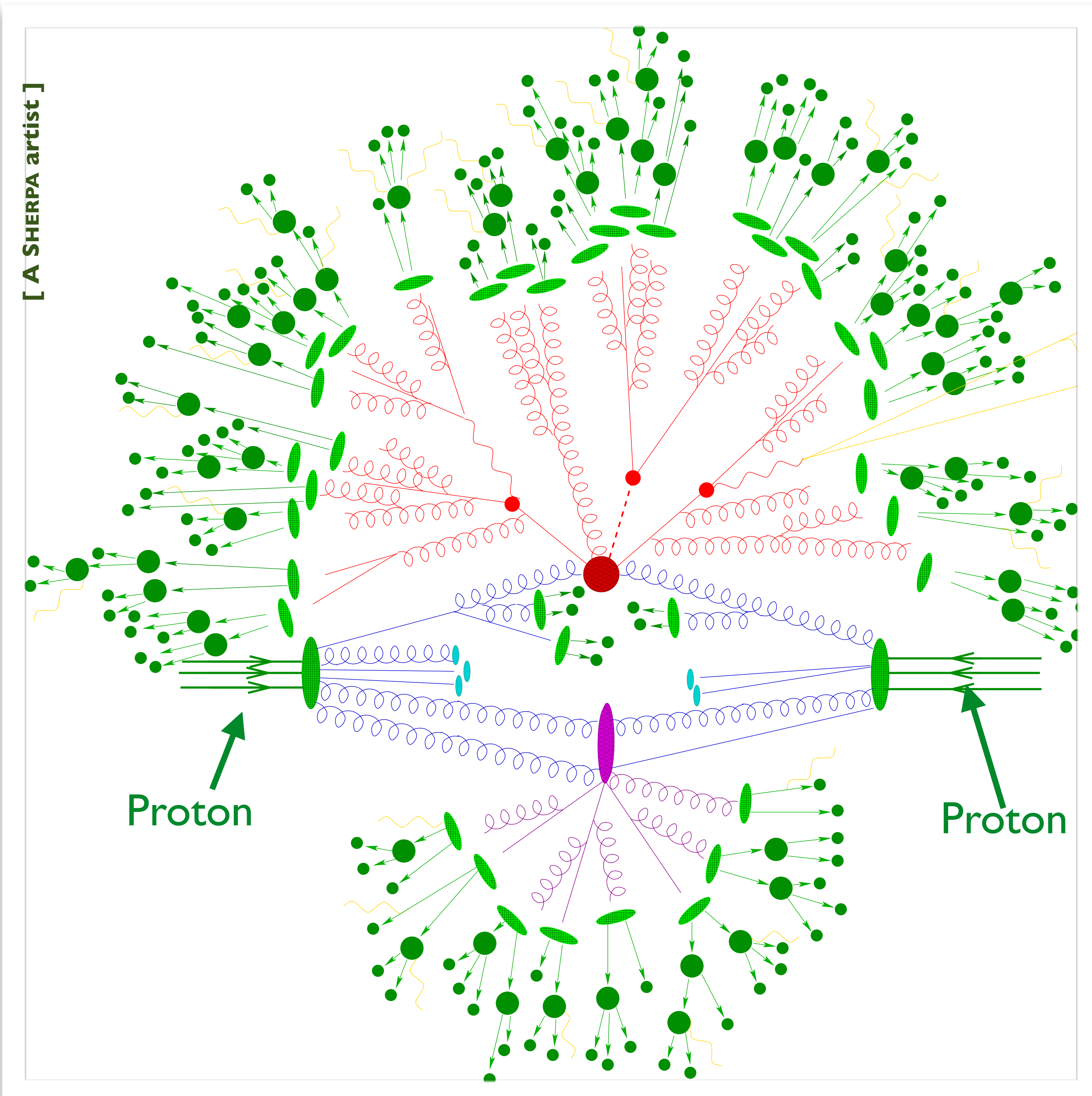


✦ Why a chain of several tools?

- ✤ Phenomena at colliders occur at different scales → factorisation

Deciphering a proton-proton collision



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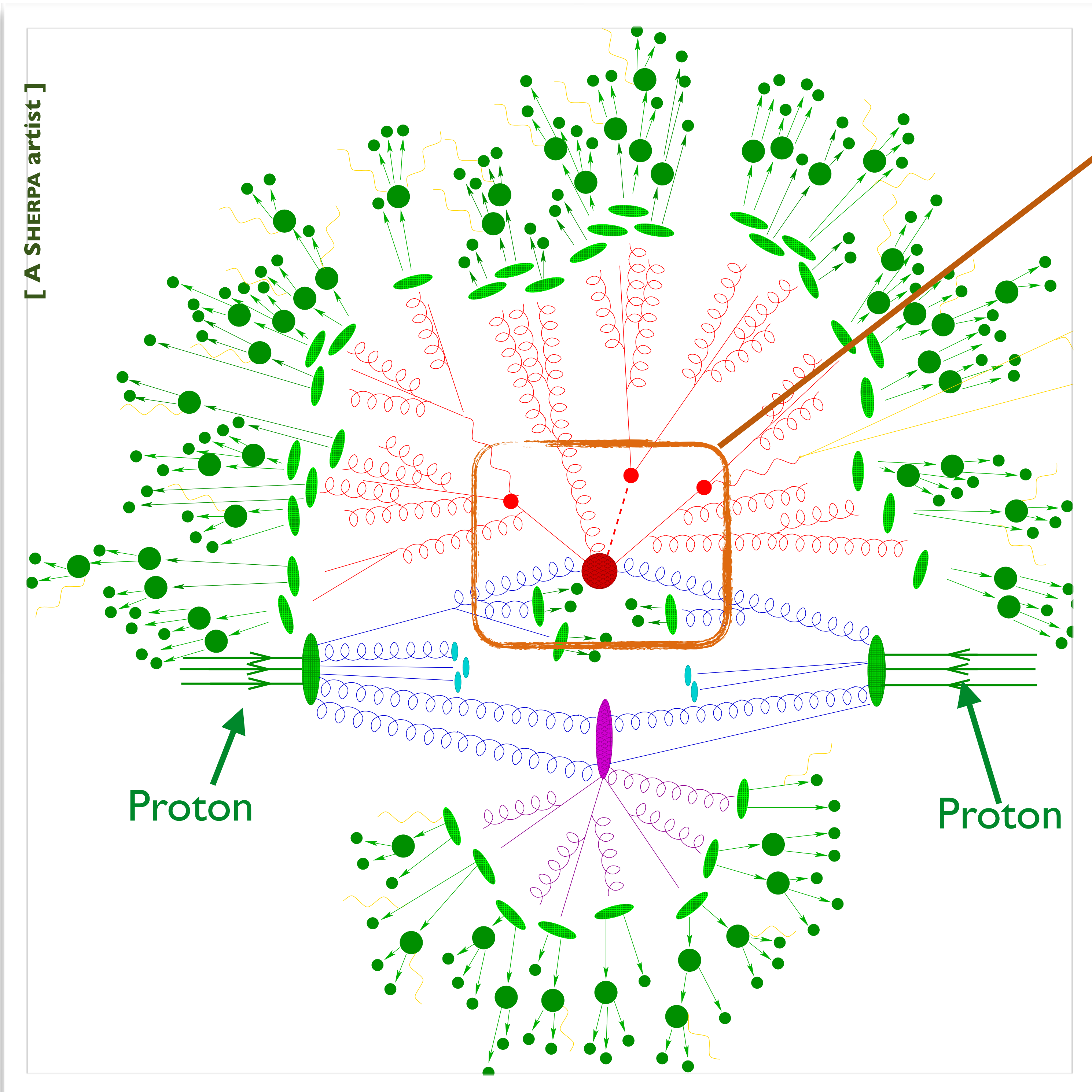

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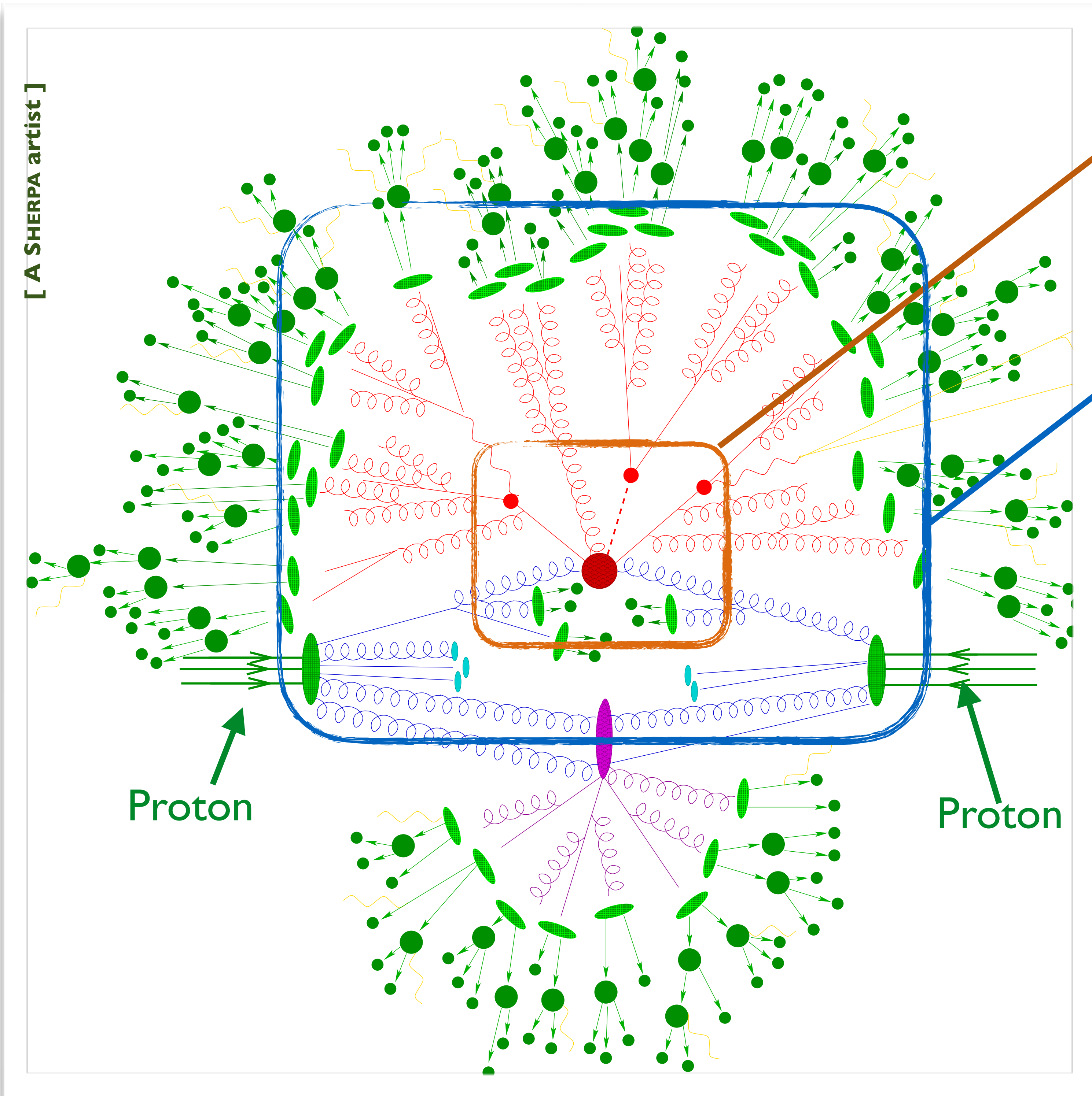

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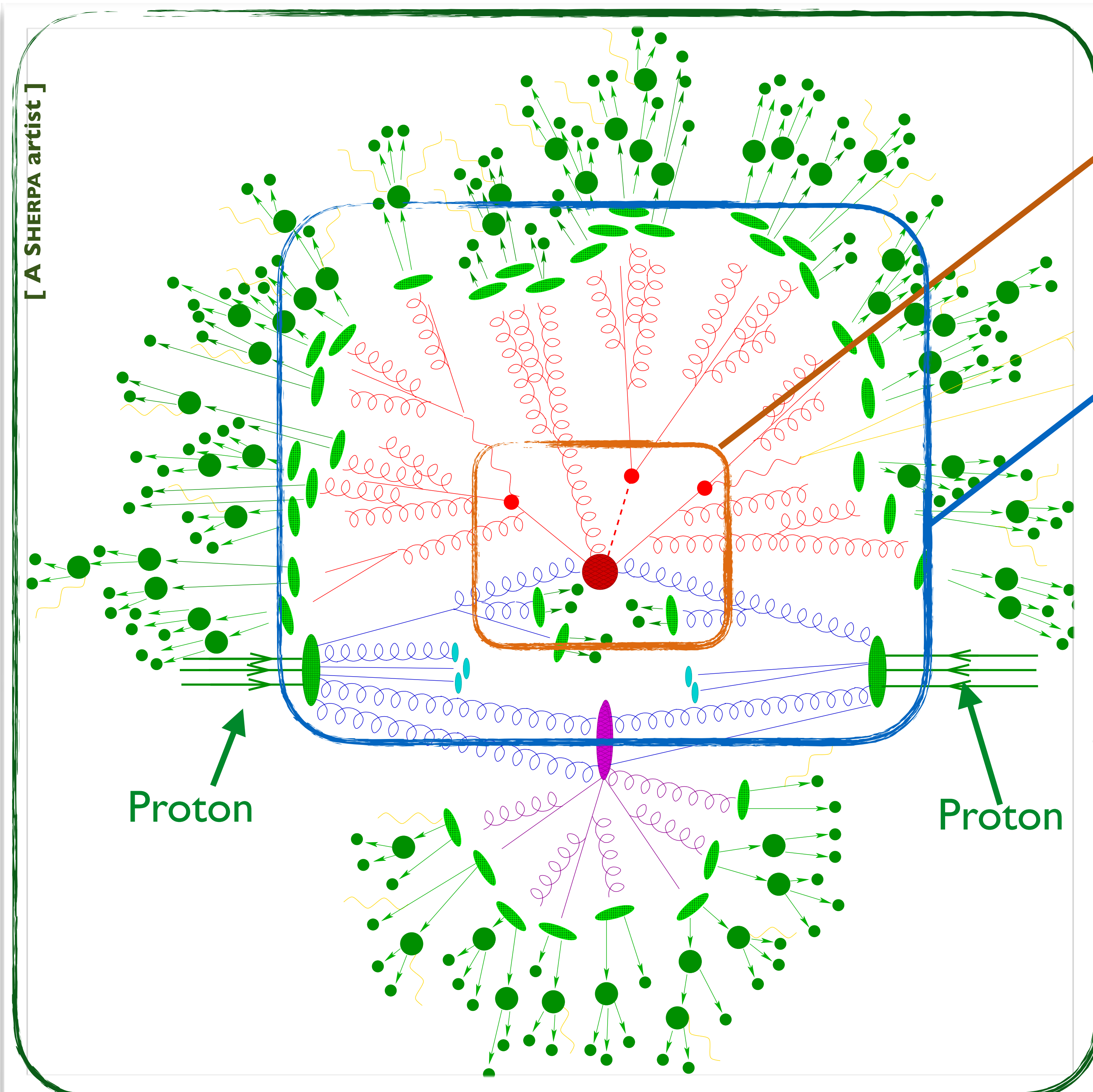

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- Model-based, universal

✦ Underlying event (sub-GeV)
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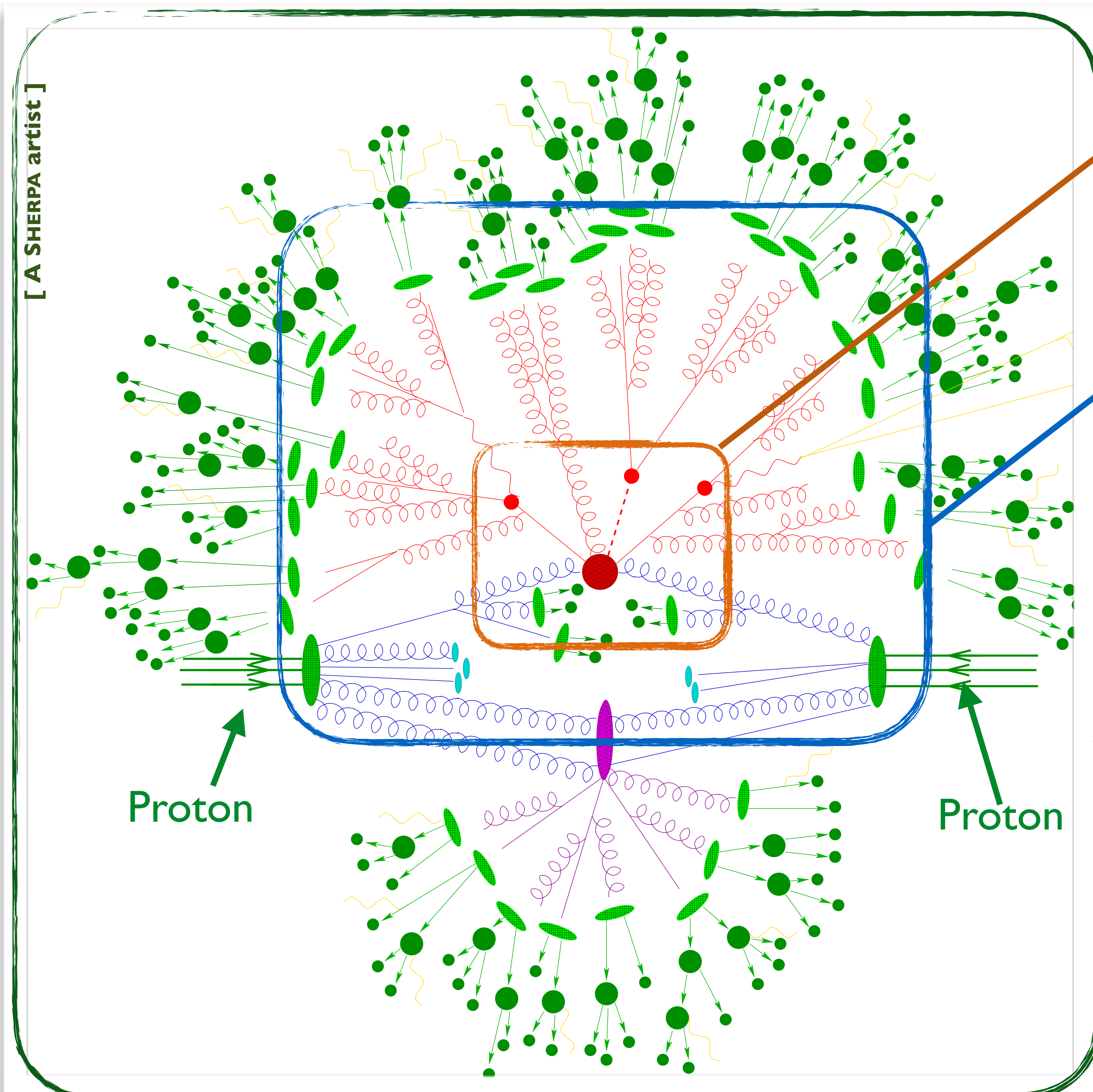

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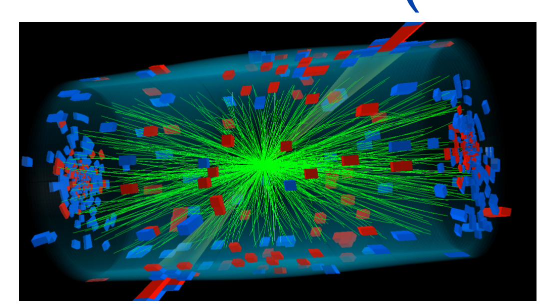
✦ Detector sim. (sub-MeV)

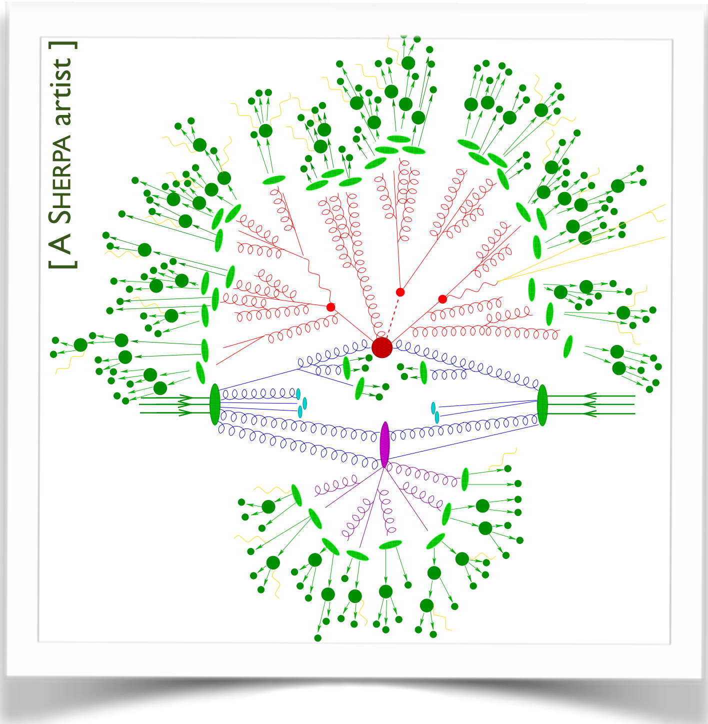


Monte Carlo simulations for proton collisions

✦ Multi-scale problem → factorisation

- ✤ TeV scale: hard scattering (new physics?)
- ✤ Down to Λ_{QCD}: QCD environment
- ✤ Down to sub-MeV: interactions with a detector

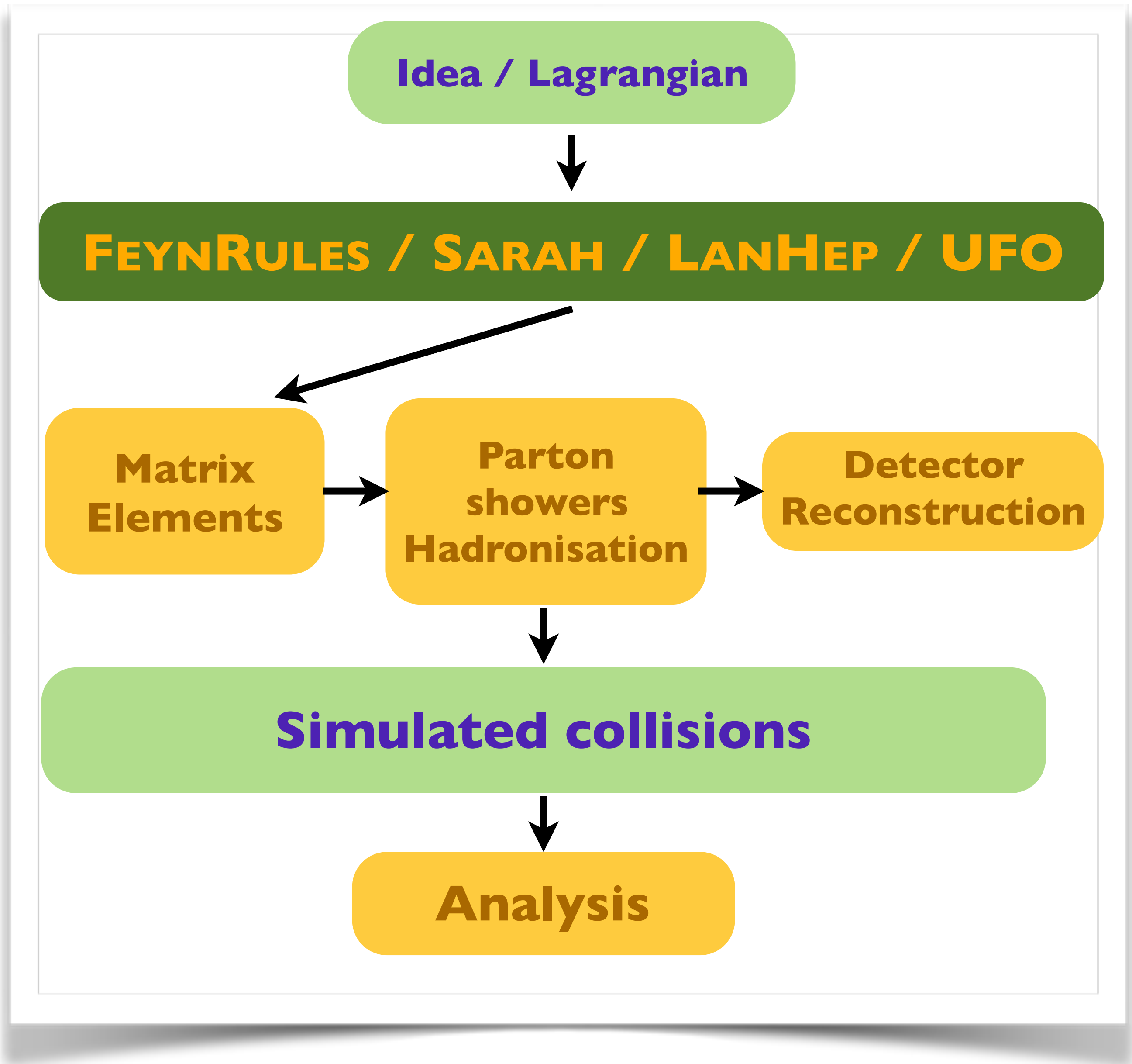
Tools and methods for each step



Making new physics a standard

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Tools connecting an idea to simulated collisions



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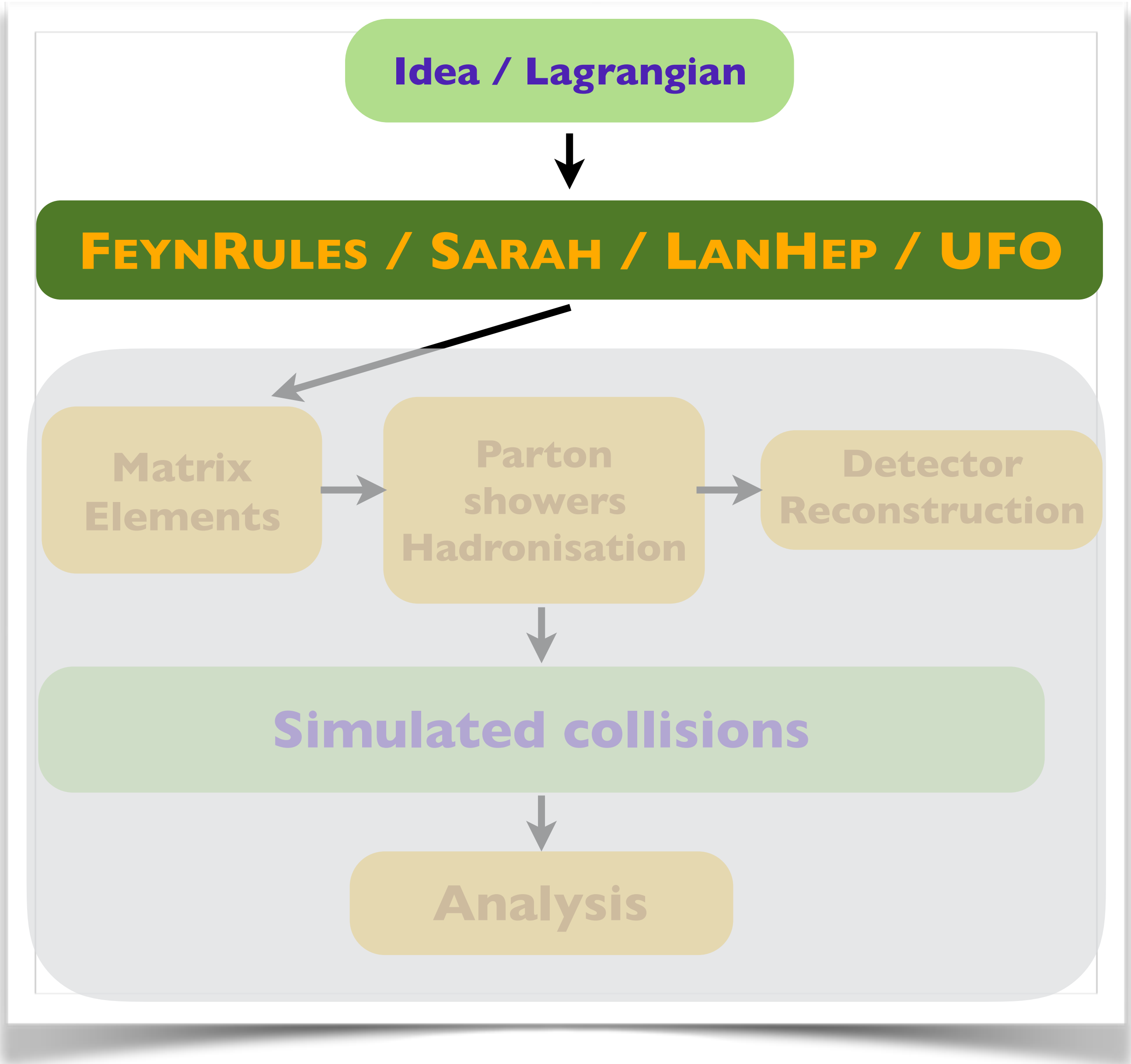

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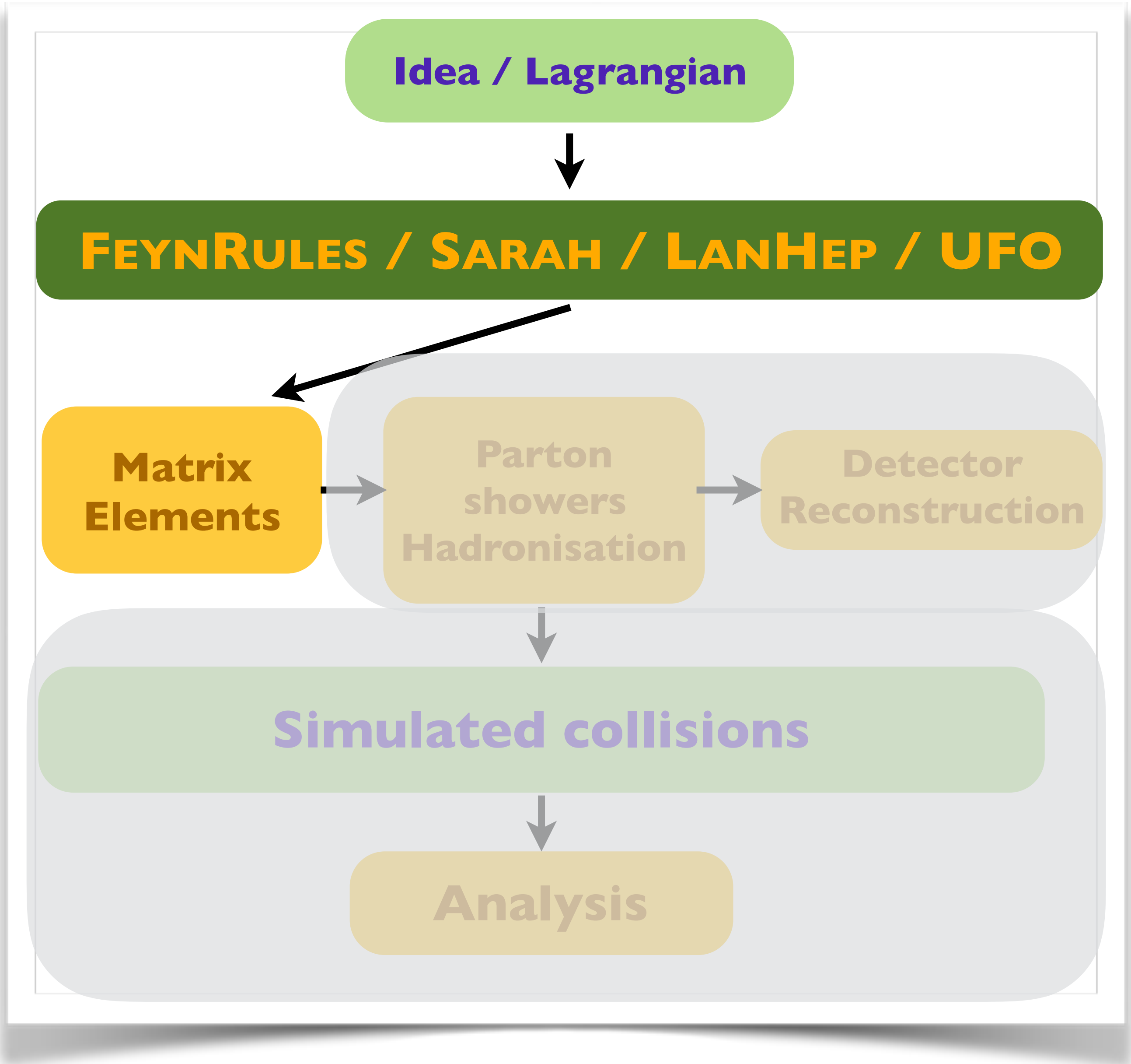

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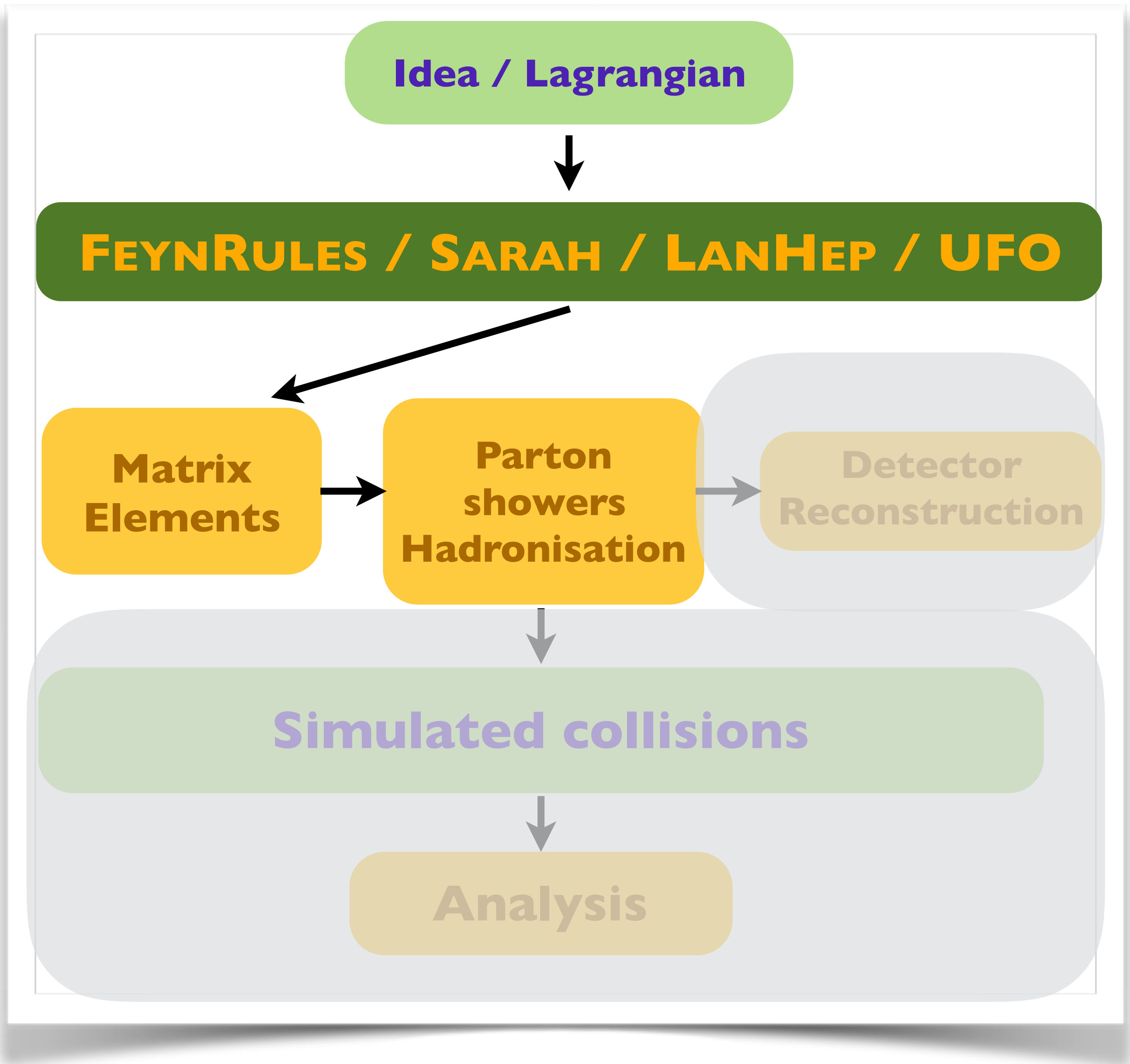

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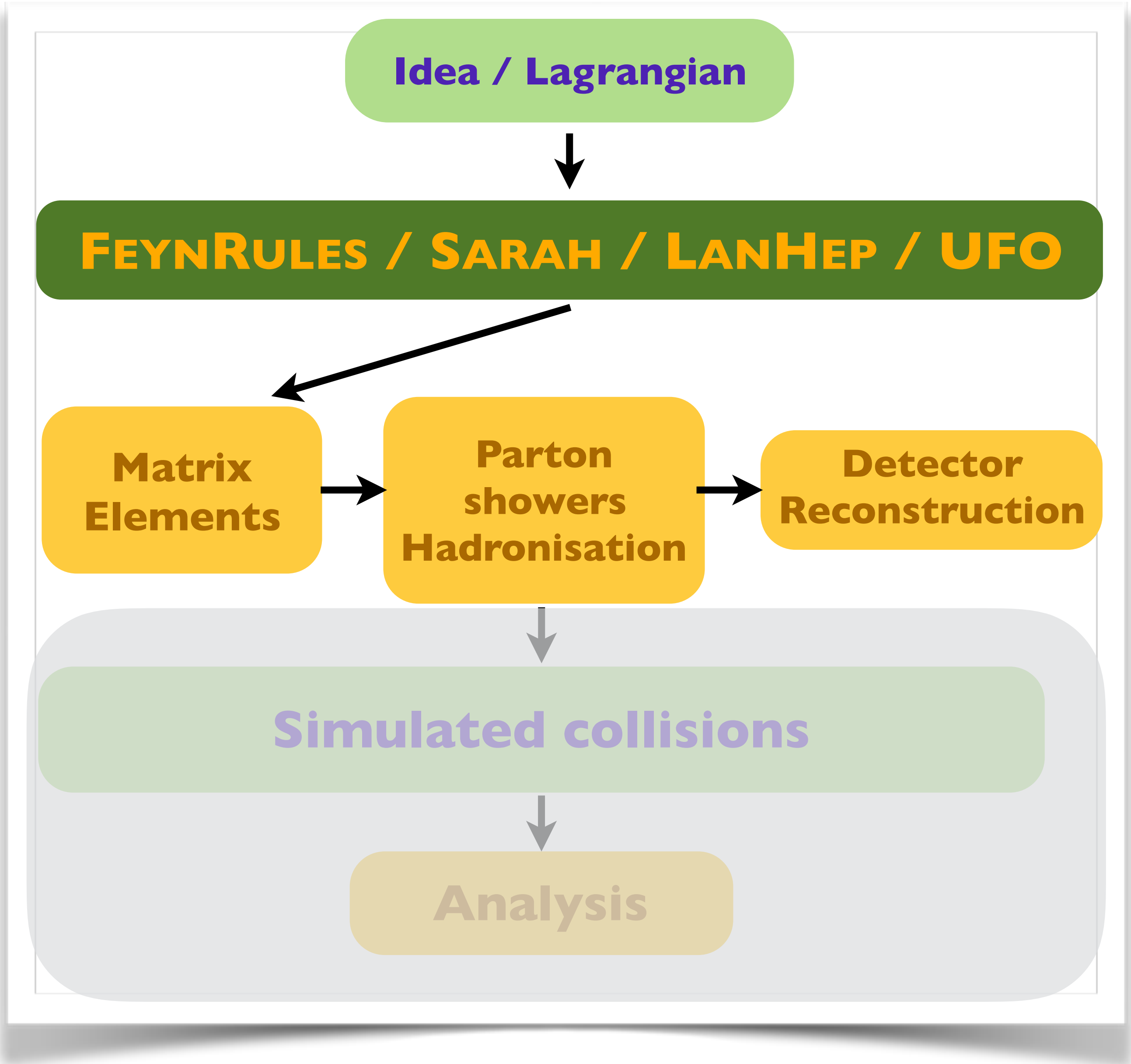

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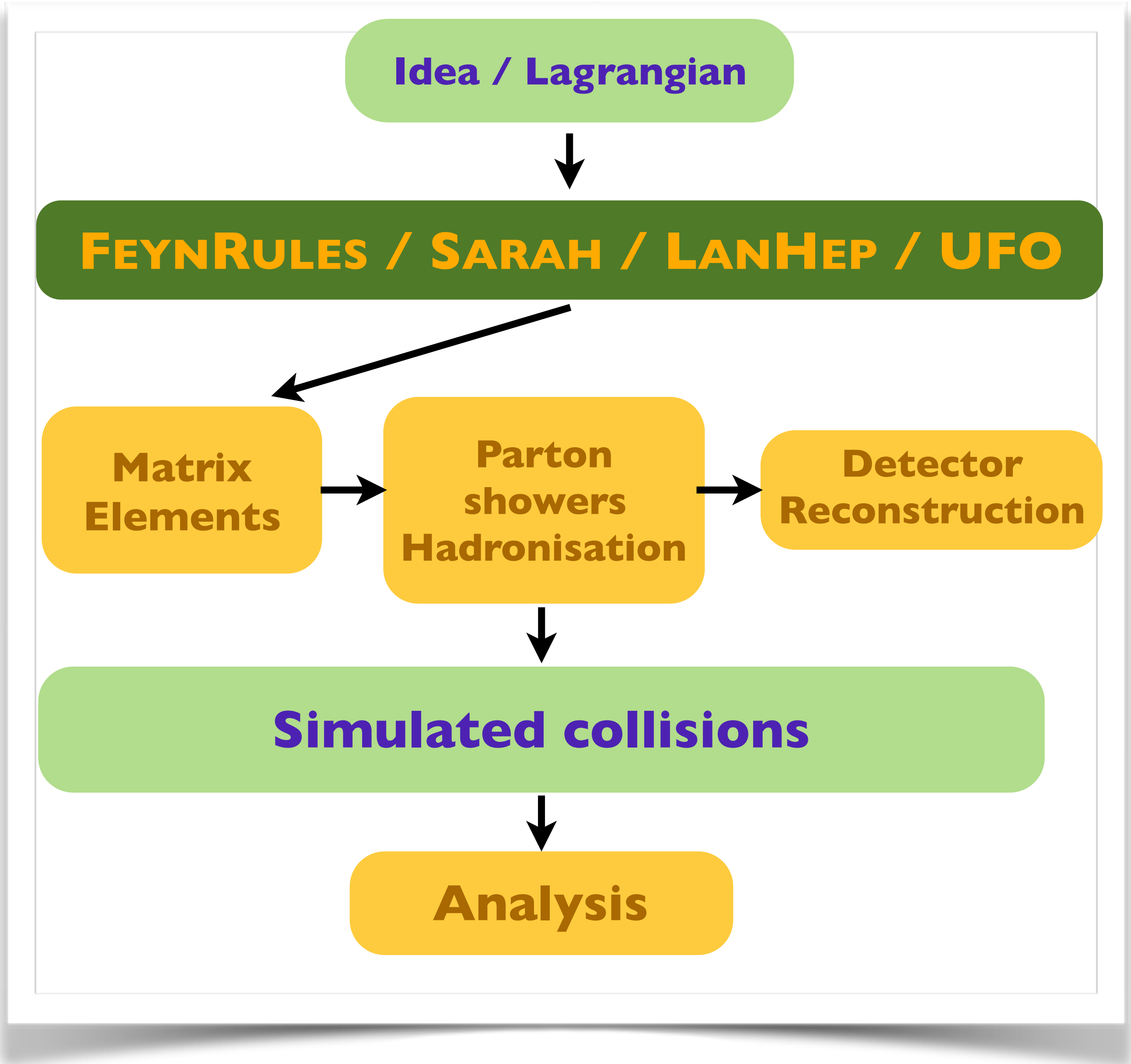

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- ✤ Event analysis
  - ★ Signal/background analysis
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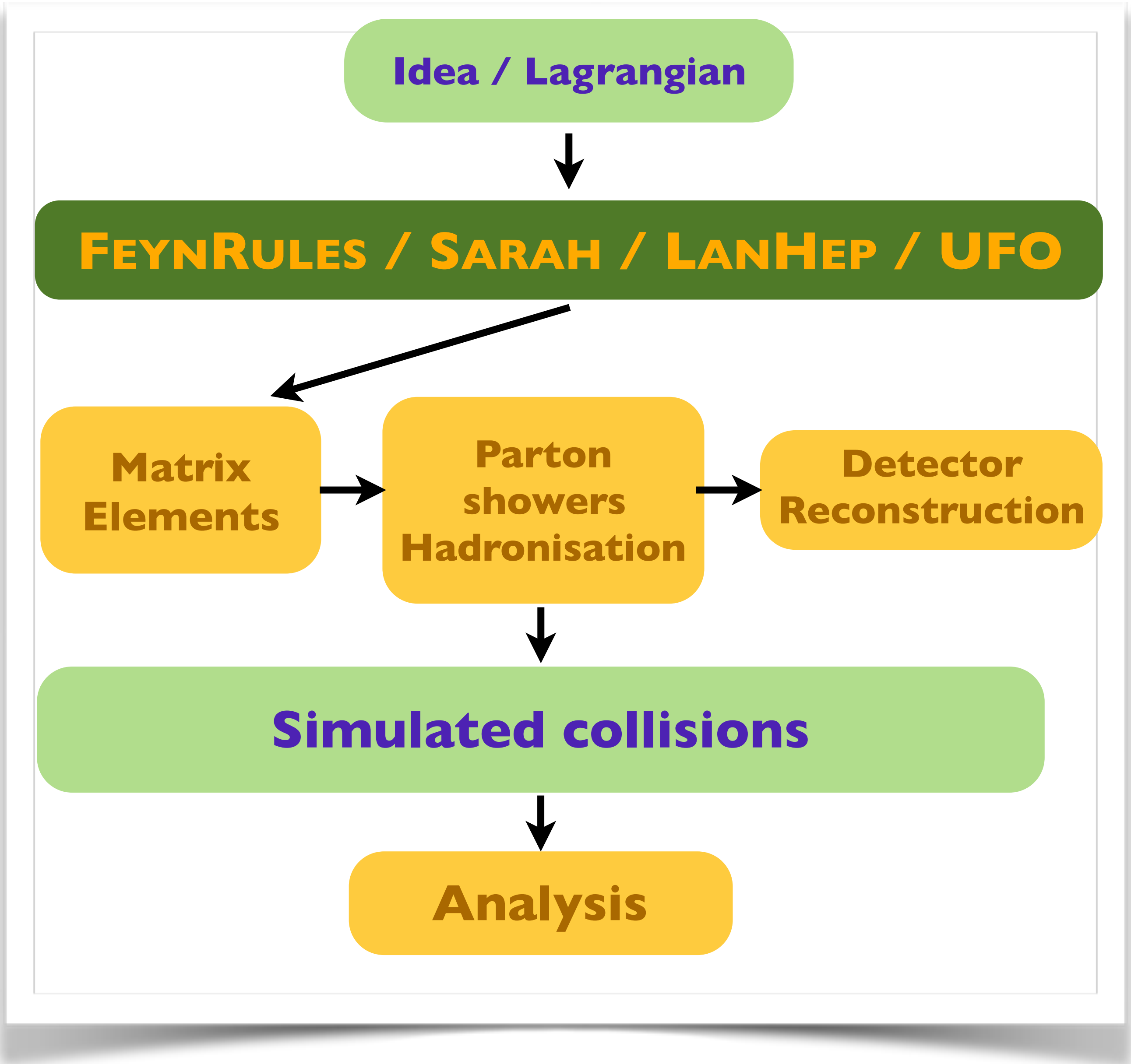

✤ Model building — Part 1

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- ★ Feynman diagram / amplitude generation
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Part 2

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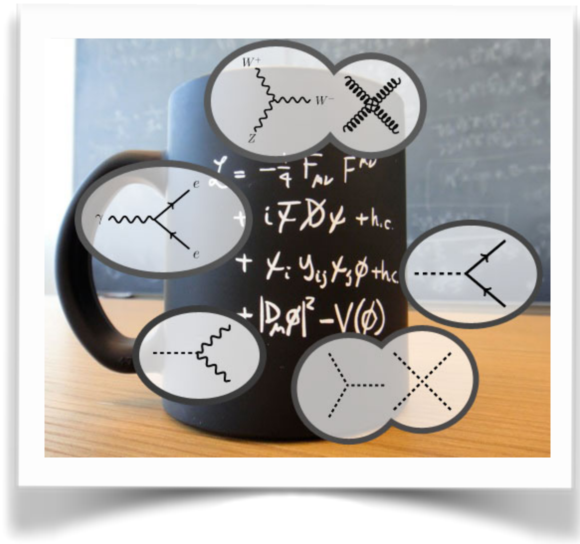
New physics simulations: the 'how-to'

✦ Implementing new physics in Monte Carlo programs

✤ Definition: particles, parameters & vertices (≡ Lagrangian)

→ translated in some programming language



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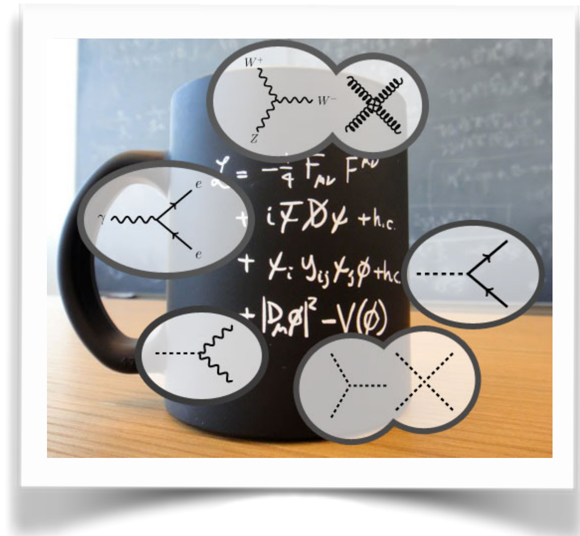

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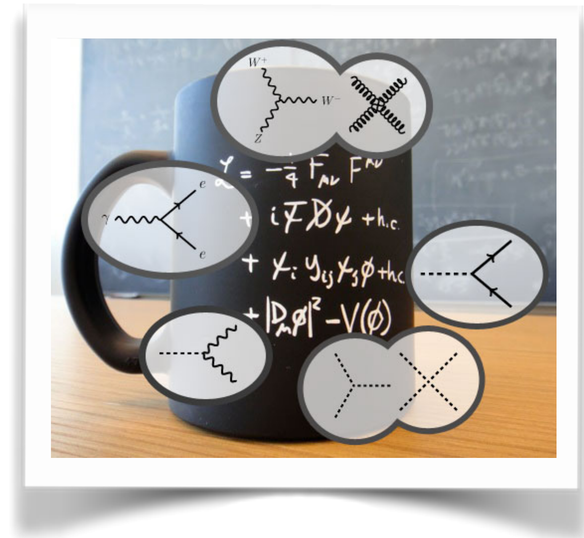

✦ Systematisation / automation

✤ Highly redundant (each tool, each model)

✤ No-brainer tasks (from Feynman rules to code)

From models to LO and NLO simulations

✦ The FEYNRULES platform (since 2009)

✤ From Lagrangians to files in a programming language

★ Few limitations (spin, colour representation, EFT)

★ Renormalisation in the on-shell scheme

[Christensen & Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr & BF (CPC'14)]

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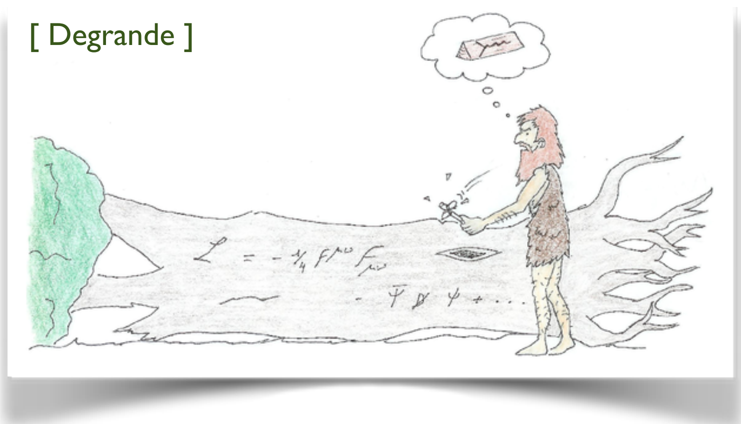

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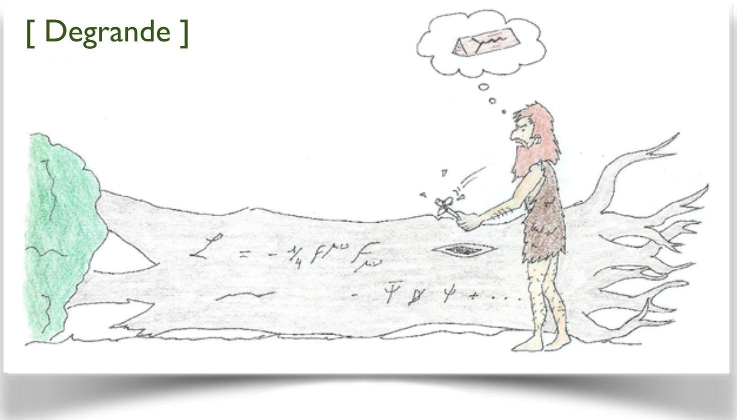

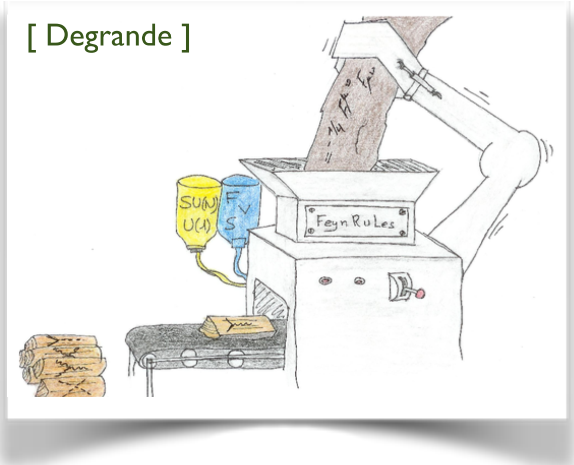

✦ Automation

✤ Working environment: MATHEMATICA

★ Flexibility, symbolic manipulations, design of new methods

★ Many built-in methods (superspace, spectrum, decays, NLO)

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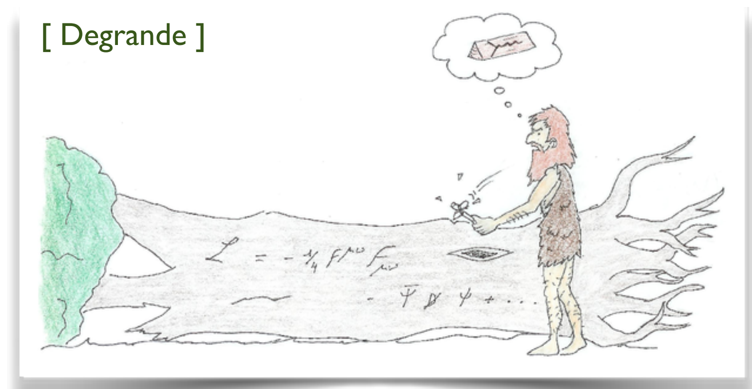

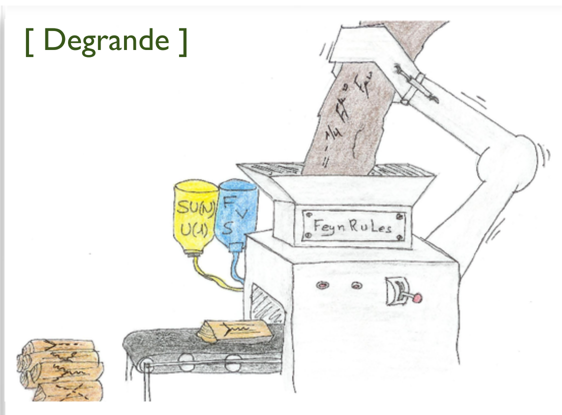

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✦ Interfaced to many tools

- ✤ CALCHEP, FEYNARTS, WHIZARD (more previously)
- ✤ UFO (HERWIG++, MG5AMC, SHERPA, WHIZARD, ...)

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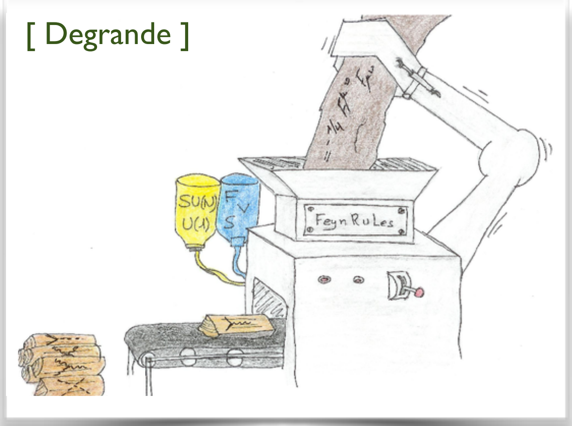

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✦ SARAH and LANHEP pursue a similar goal

✤ No NLO, different built-in methods, …

More about interfaces

- ✦ Each interface dedicated to a given tool is specific
  - ✤ Removal of vertices not compliant with the tool
    - ★ Colour structures
    - ★ Lorentz structures
  - ✤ Translation to a specific format and programming language

→ not efficient
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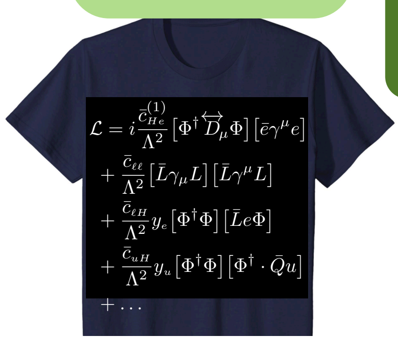
✦ One format to rule them all!

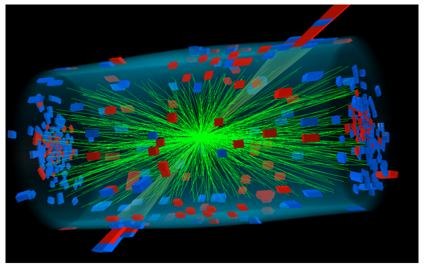
Idea Lagrangian



→ FeynRules →

UFO

→ Chain of tools → Simulated collisions → Other tools → Event analysis



The Universal Feynman Output

✦ The UFO in a nutshell

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]
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- UFO ≡ Universal FEYNRULES output → Universal Feynman Output
  - ★ Universal as not tied to any specific Monte Carlo program
- Set of PYTHON files to be linked to any code
- This module contains all the model information
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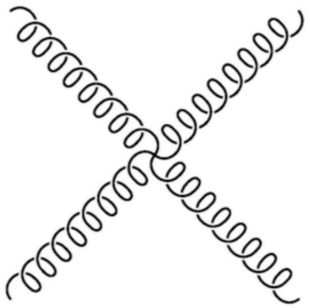
✦ The UFO is a standard

ALOHA | GOSAM | HERWIG ++ | MADANALYSIS 5 | SHERPA

MADGRAPH5_aMC@NLO | WHIZARD | LANHEP | SARAH

Interactions: the key strategy

- Decomposition in a spin x colour basis (coupling strengths ≡ coordinates)
  - Example: the quartic gluon vertex



$$
\begin{aligned}
& i g_s^2\, f^{a_1 a_2 b} f^{b a_3 a_4}\, \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}\right) \\
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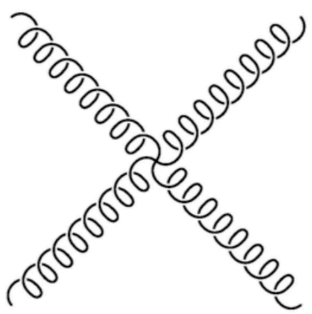

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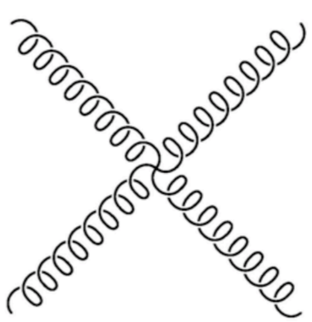
✤ UFO version



$$
\left(f^{a_1 a_2 b} f^{b a_3 a_4},\ f^{a_1 a_3 b} f^{b a_2 a_4},\ f^{a_1 a_4 b} f^{b a_2 a_3}\right)
\times \begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix}
\begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix}
$$

★ 3 elements for the colour basis
★ 3 elements for the spin (Lorentz structure) basis
★ 9 coordinates (6 are zero)

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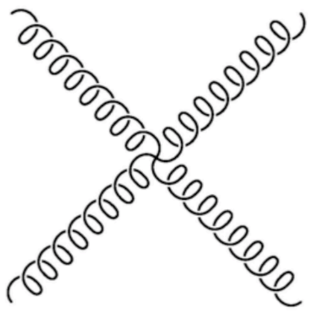

$$
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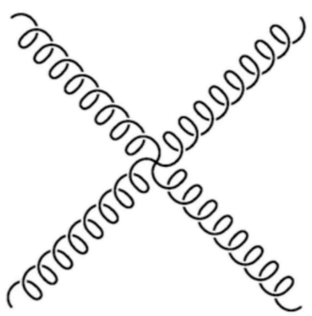
& i g_s^2\, f^{a_1 a_2 b} f^{b a_3 a_4}\ \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}\right)\\
&\quad + i g_s^2\, f^{a_1 a_3 b} f^{b a_2 a_4}\ \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right)\\
&\quad + i g_s^2\, f^{a_1 a_4 b} f^{b a_2 a_3}\ \left(\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right)
\end{aligned}
$$

✤ UFO version



$$
\left(f^{a_1 a_2 b} f^{b a_3 a_4},\ f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3}\right)
\times \begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix}
\begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix}
$$

Recycling the duplicates in the implementation

[across vertices]

★ 3 elements for the colour basis
★ 3 elements for the spin (Lorentz structure) basis
★ 9 coordinates (6 are zero)

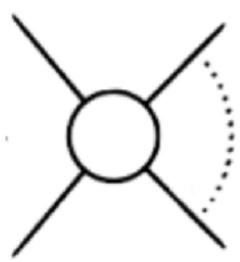
NLO cross sections

✦ Contributions to an NLO result in QCD

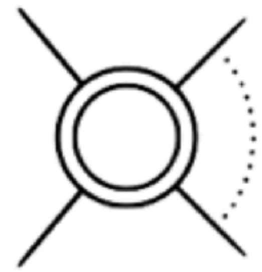
✤ Three ingredients: the Born, virtual loop and real emission contributions

$$\sigma_{NLO} = \int \mathrm{d}^4\Phi_n \mathcal{B} \quad + \quad \int \mathrm{d}^4\Phi_n \int_{\text{loop}} \mathrm{d}^d\ell \ \mathcal{V} \quad + \quad \int \mathrm{d}^4\Phi_{n+1} \ \mathcal{R}$$

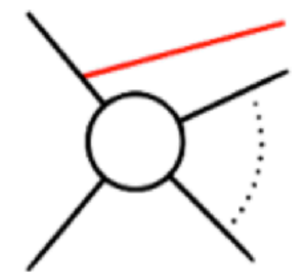
Born



Virtuals: one extra power of α_s and divergent



Reals: one extra power of α_s and divergent



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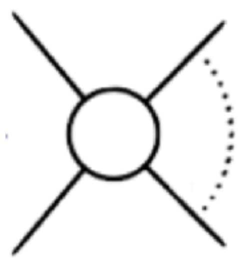
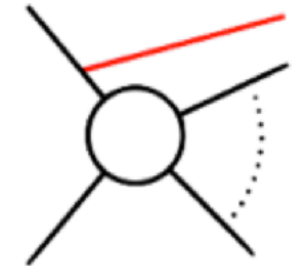




Extra information needed

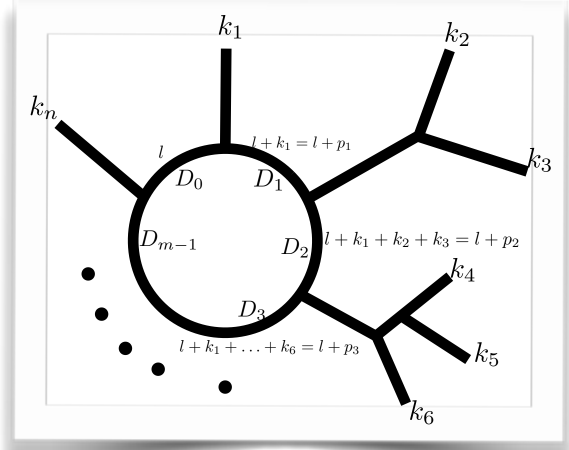
Loop calculations

✦ Dimensional regularisation: calculations in $d = 4 - 2\varepsilon$

- Divergences explicit ($1/\varepsilon^2$, $1/\varepsilon$)
- Numerical methods work in 4 dimensions → R_1 / R_2 terms

$$\int \mathrm{d}^d\ell \frac{N(\ell,\tilde{\ell})}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} \quad \text{with } \bar{\ell} = \ell + \tilde{\ell}$$

D-dim   4-dim   (-2ε)-dim



[Ossala, Papadopoulos, Pittau (NPB'07; JHEP'08)]

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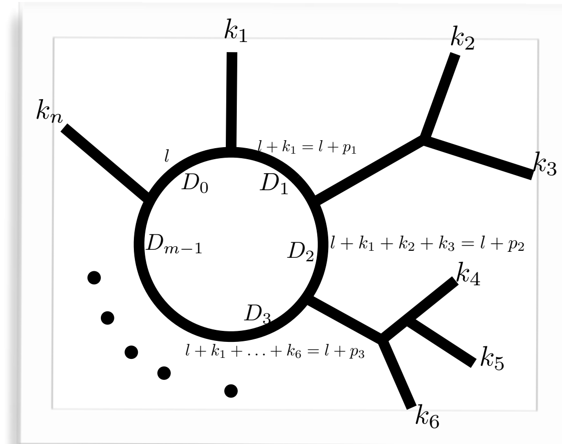
$\bar{\ell}$: D-dim; ℓ: 4-dim; $\tilde{\ell}$: (-2ε)-dim



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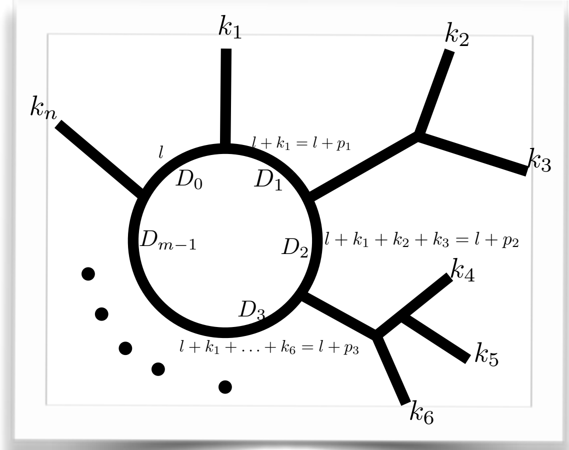

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★ Seen as extra diagrams with special Feynman rules (→ *R_2 Feynman rules*)

★ Connected to the UV structure of the integrals (like the UV counterterms)

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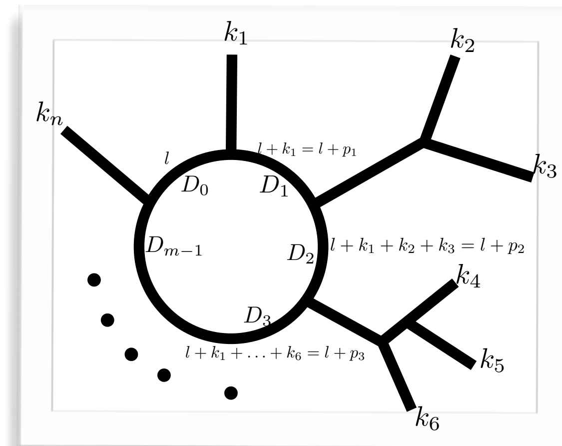

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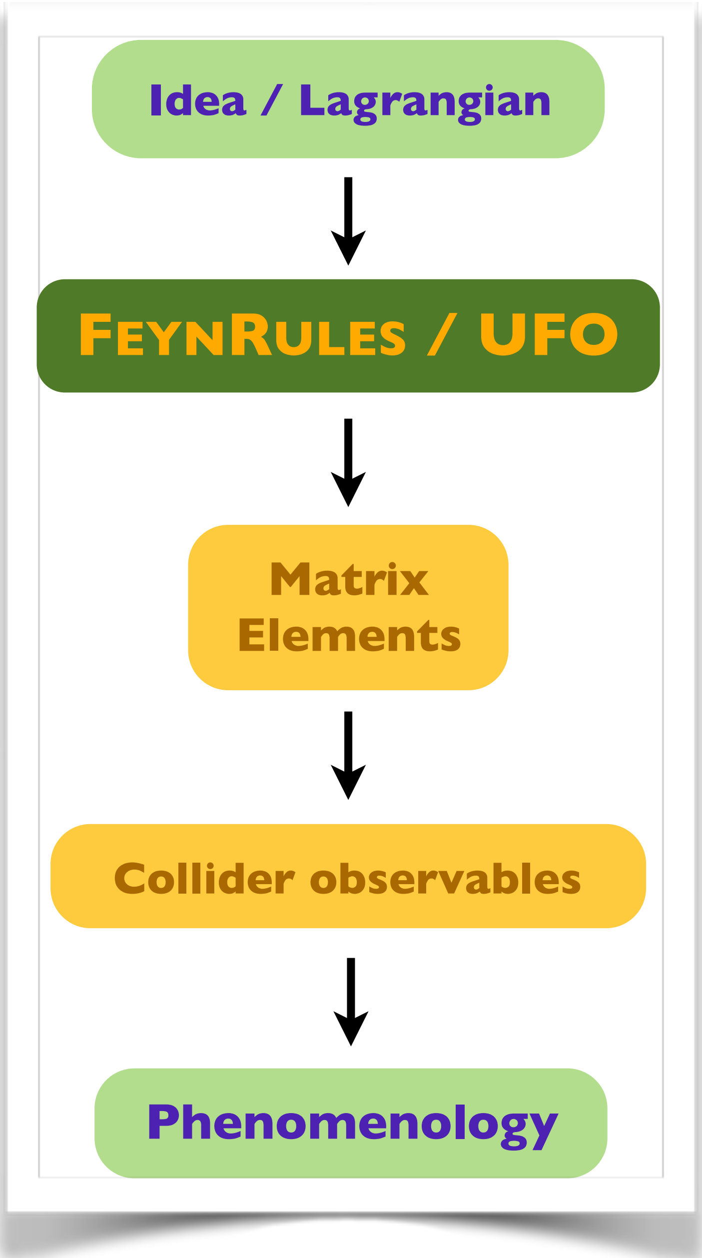
UFO @ NLO

Automated NLO simulations



✦ Model building: from Lagrangian to tools

✤ FeynRules ⊕ MoGRe ⊕ NLOCT → UFO @ NLO

✤ General on-shell renormalisation scheme

[Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ; Degrande (CPC'15)]
[Degrande, Duhr, BF, Mattelaer & Reither (CPC'12)]
[Frixione, BF, Hirschi, Mawatari, Shao, Sunder & Zaro (JHEP`19)]

✦ Hard scattering

✤ Feynman diagram, matrix elements

✤ MG5aMC → predictions at LO/NLO

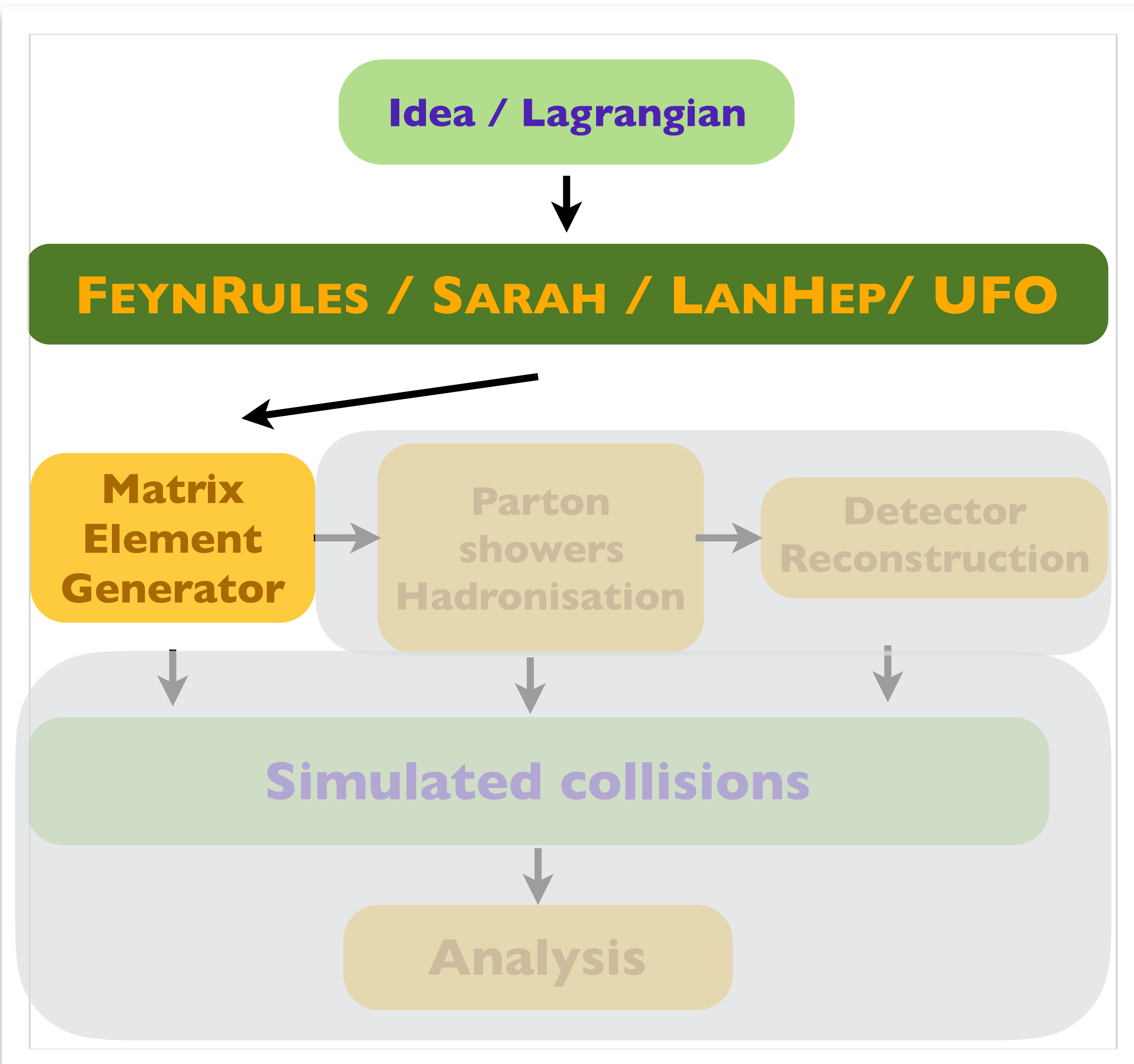
[Alwall *et al.* (JHEP'14)]

Outline

1. A comprehensive approach for Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. **From models to hard-scattering events**
4. Parton showers, hadronisation & underlying event
5. Summary

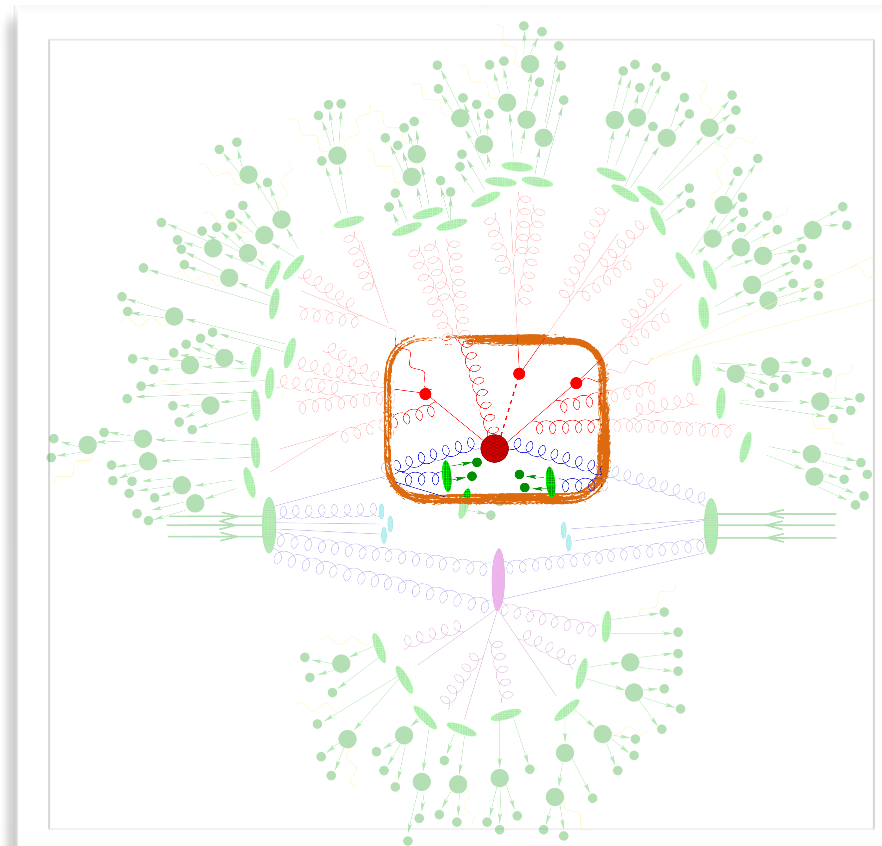
Back to the simulation chain

✦ Tools connecting an idea to simulated collisions



✤ Hard scattering process

- ★ Feynman diagram / amplitude generation
- ★ Monte Carlo integration
- ★ Event generation



QCD 101: predictions at the LHC

✦ Distribution of an observable ω: the QCD factorisation theorem

$$\frac{d\sigma}{d\omega} = \sum_{ab} \int dx_a \, dx_b \, \mathbf{f_{a/p_1}}(x_a; \mu_F) \, \mathbf{f_{b/p_2}}(x_b; \mu_F) \, \frac{d\sigma_{\mathbf{ab}}}{d\omega}(\ldots, \mu_F)$$

- Long distance physics: the parton densities
- Short distance physics: the differential parton cross section $d\sigma_{ab}$
- Separation of both regimes → the factorisation scale μ_F
  - ★ Choice of the scale → theoretical uncertainties

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✦ Short distance physics: the partonic cross section

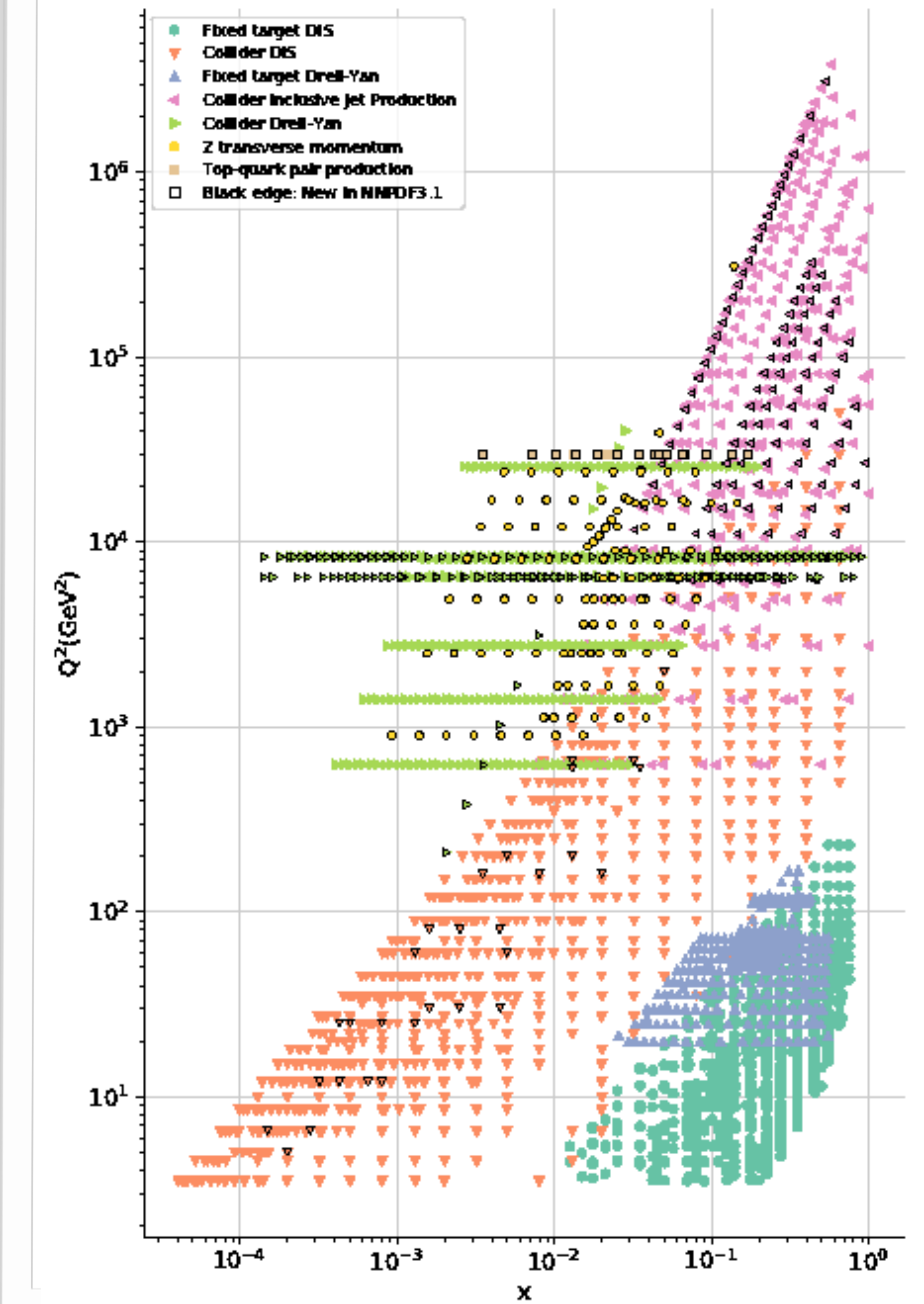
- Order by order in perturbative QCD: $d\sigma = d\sigma^{(0)} + \alpha_s\, d\sigma^{(1)} + \ldots$
  - ★ More orders → more precision
  - ★ Truncation of the series and α_s → theoretical uncertainties

Feynman diagrams (from UFOs)

Parton densities

✦ Long distance physics: parton densities
  ✤ Relate hadrons to their content

[NNPDF 3.0 (EPJC'17)]



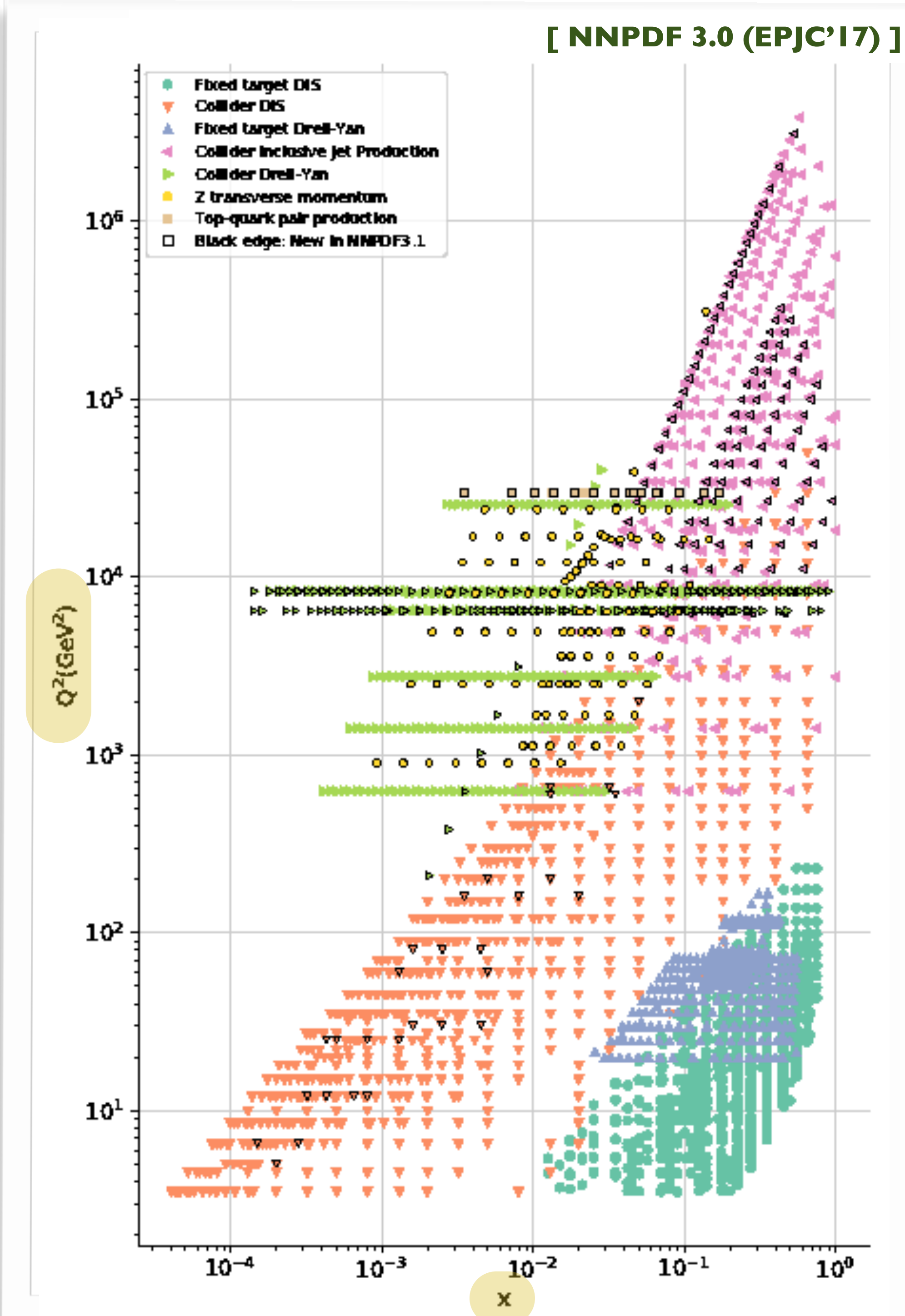
Parton densities

✦ Long distance physics: parton densities

✤ Relate hadrons to their content

✤ Depend on the momentum fraction *x* of the parton in the proton

✤ Depend on a scale Q

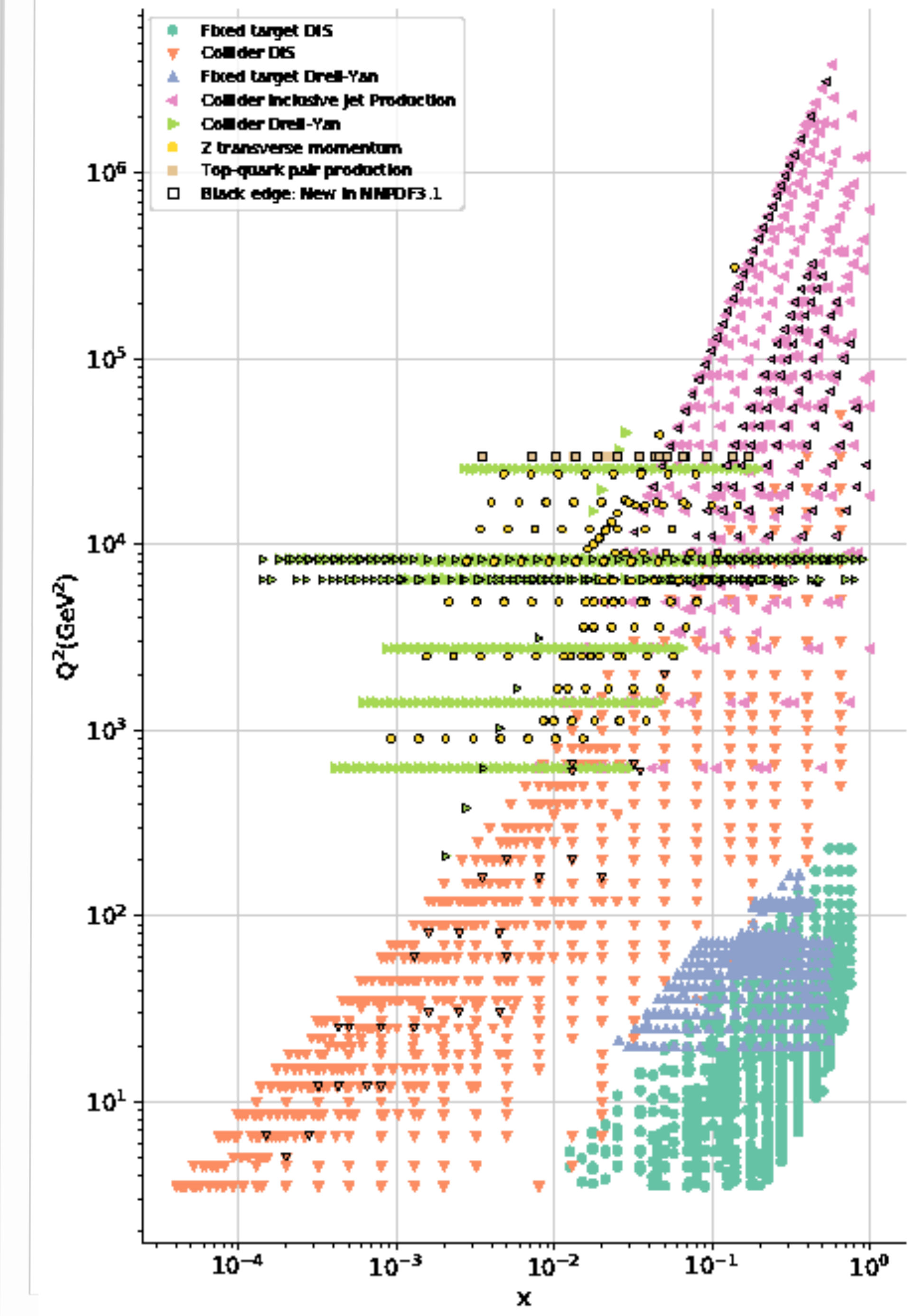


Parton densities

✦ Long distance physics: parton densities

- ✤ Relate hadrons to their content
- ✤ Depend on the momentum fraction *x* of the parton in the proton
- ✤ Depend on a scale Q
- ✤ Fitted from experimental data [in some kinematical regimes (x,Q)]
- ✤ Evolution driven by QCD (DGLAP/BFKL)

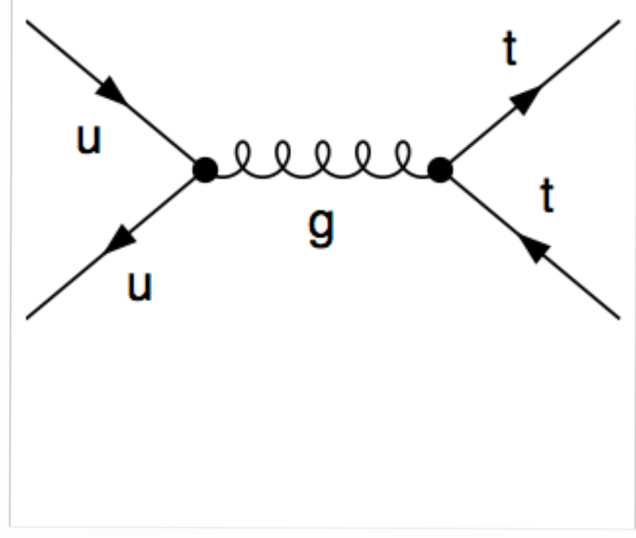
[NNPDF 3.0 (EPJC'17)]



Feynman diagram calculations

- Direct squared matrix element computations
  - Extraction of the amplitude from the Feynman rules

$$i\mathcal{M} = ig_s^2 \Big[\bar{v}_2\gamma^\mu u_1\Big] \frac{\eta_{\mu\nu}}{s} \Big[\bar{u}_3\gamma^\nu v_4\Big] T^a_{c_2c_1} T^a_{c_3c_4}$$



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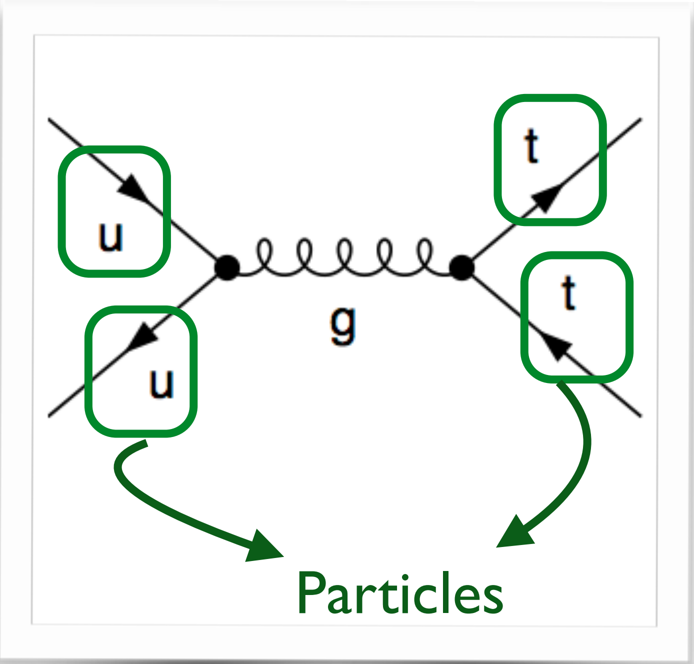

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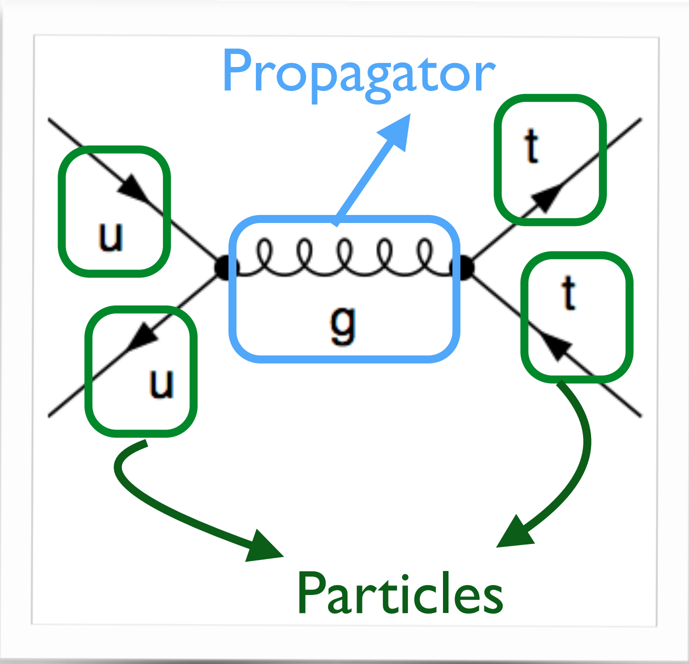

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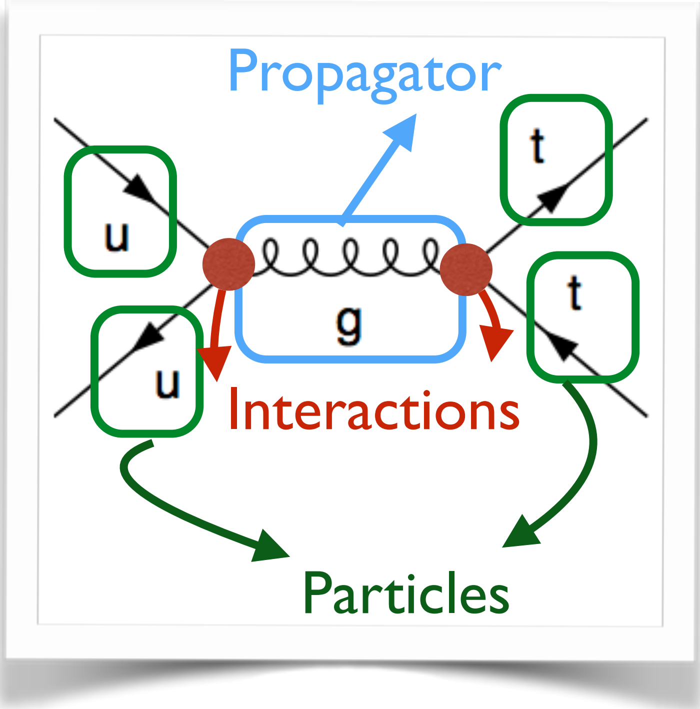

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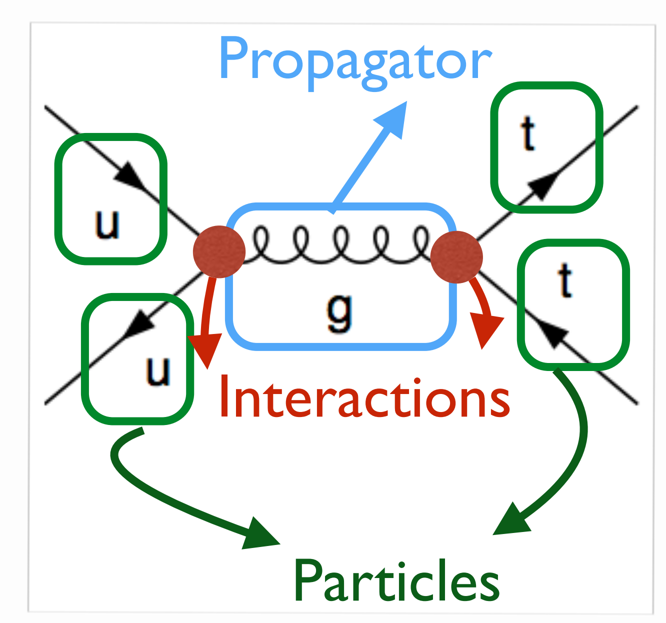
- Squaring with the conjugate amplitude
- Algebraic calculation (colour and Lorentz structures)
- Sum/average over the external states

$$\overline{|\mathcal{M}|^2} = \frac{1}{36}\frac{2g_s^4}{s^2}\mathrm{Tr}\left[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu\right]\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right]$$
$$= \frac{16g_s^4}{9s^2}\left[(p_1\cdot p_3)(p_2\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3)\right]$$



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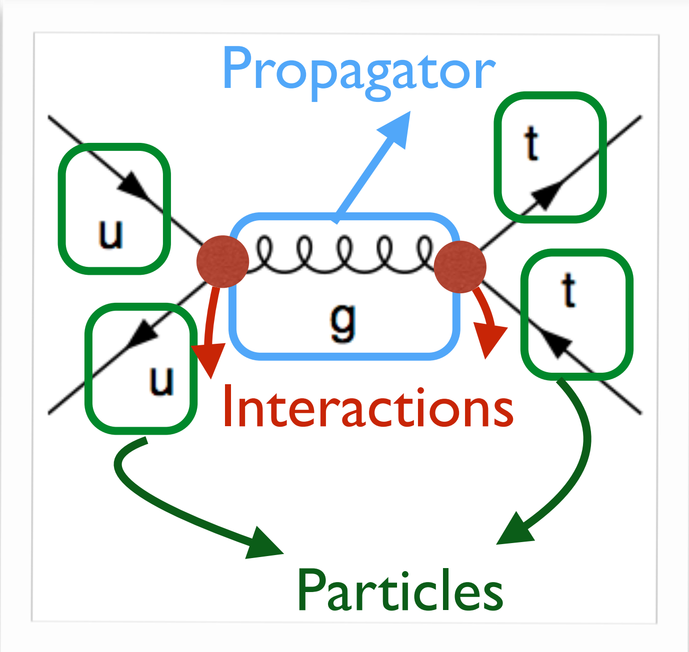

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$$= \frac{16 g_s^4}{9 s^2}\left[(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)\right]$$

✦ The number of diagrams increases with the number of final-state particles

✤ The complexity rises as N^2

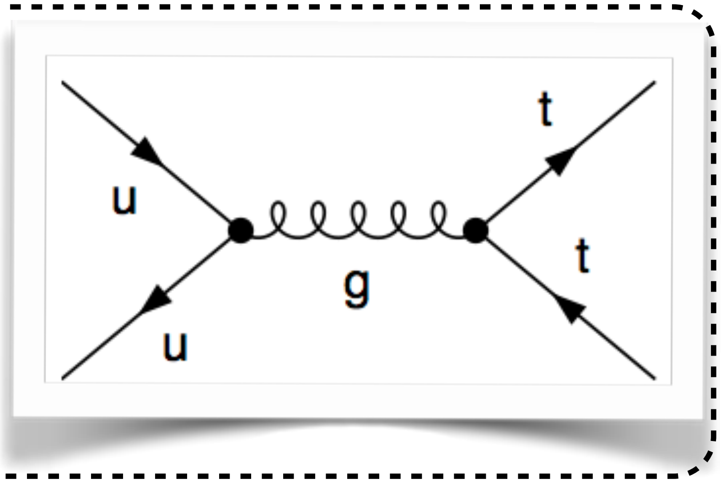
✤ Any calculation beyond 2-to-3 becomes a problem

➢ **Helicity amplitudes**

Helicity amplitudes

✦ Principle

- ✤ Evaluation of the amplitude for fixed external helicities
- ✤ Add all amplitudes (we get complex numbers)
- ✤ Squaring
- ✤ Sum/average over the external states



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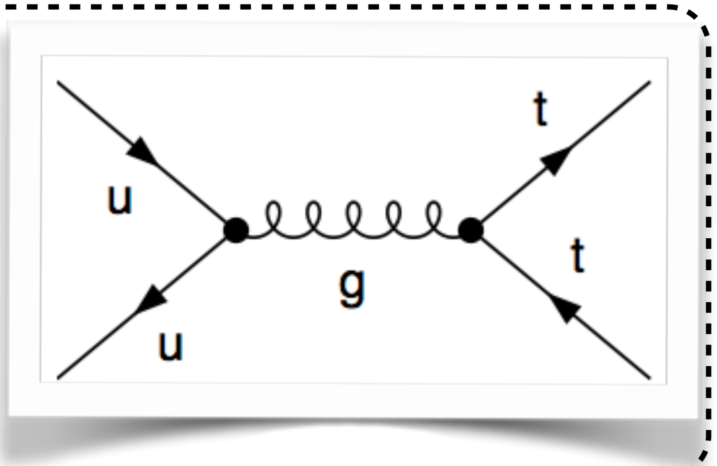
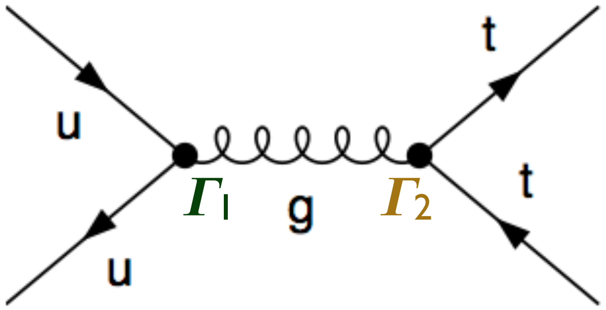


✦ Practical example



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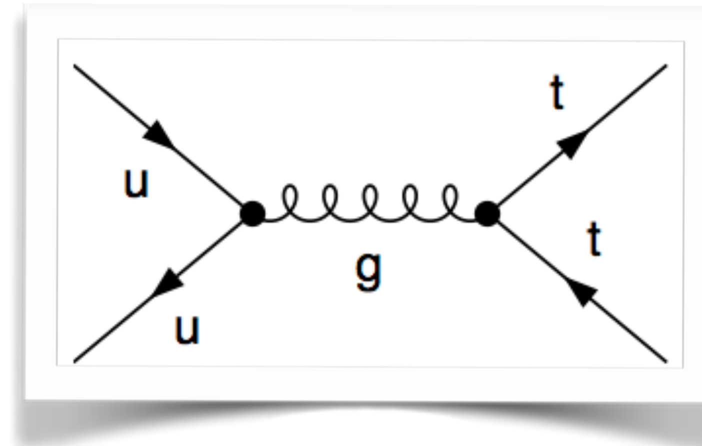

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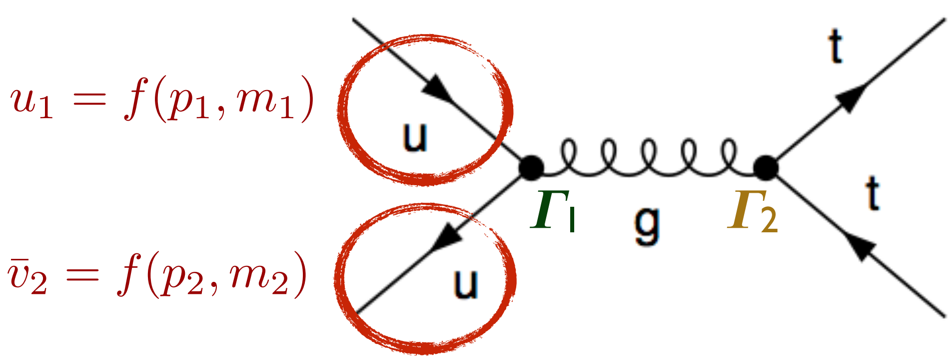

1. External incoming particles (numbers)
   - ★ For fixed helicity and momentum

Helicity amplitudes

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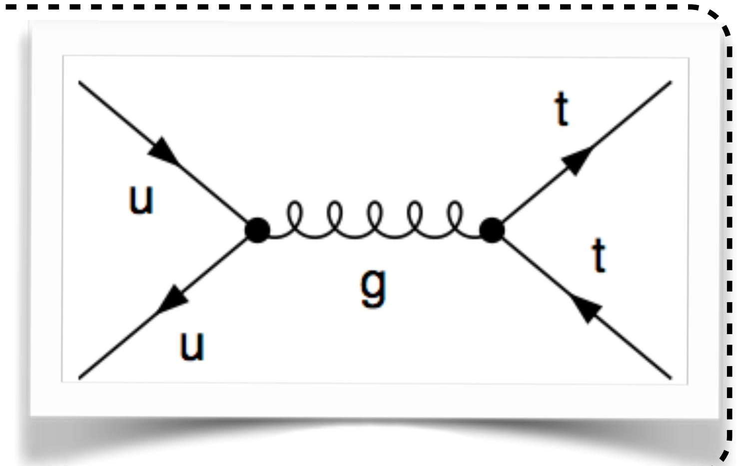

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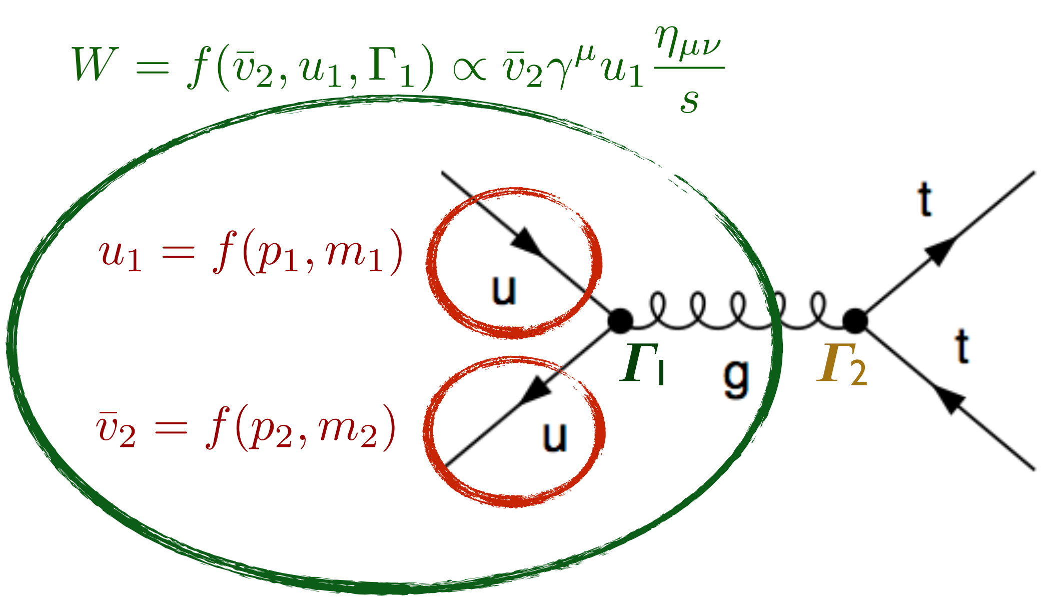

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2. Wave function of the gluon propagator

Helicity amplitudes

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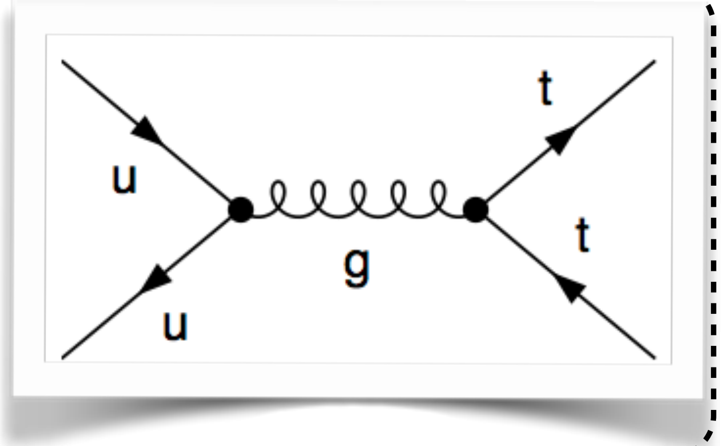

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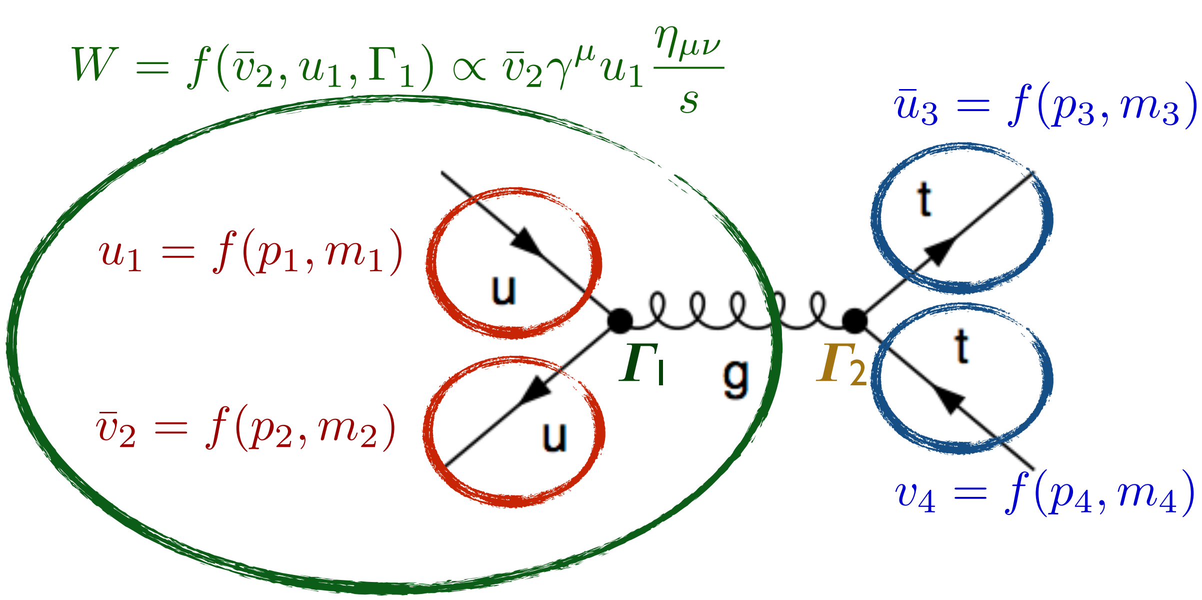

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2. Wave function of the gluon propagator
3. External outgoing particles

Helicity amplitudes

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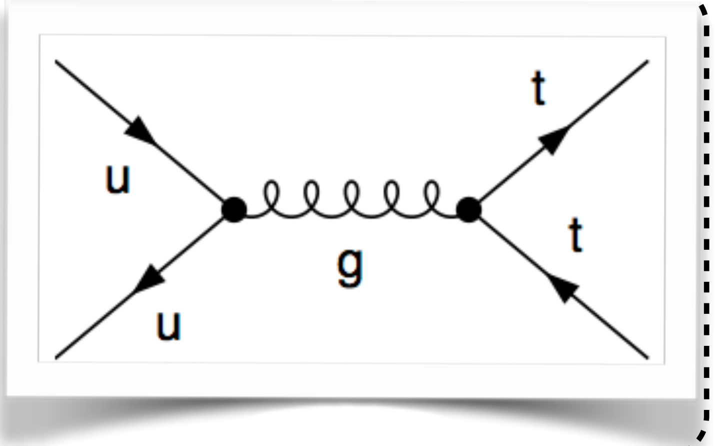

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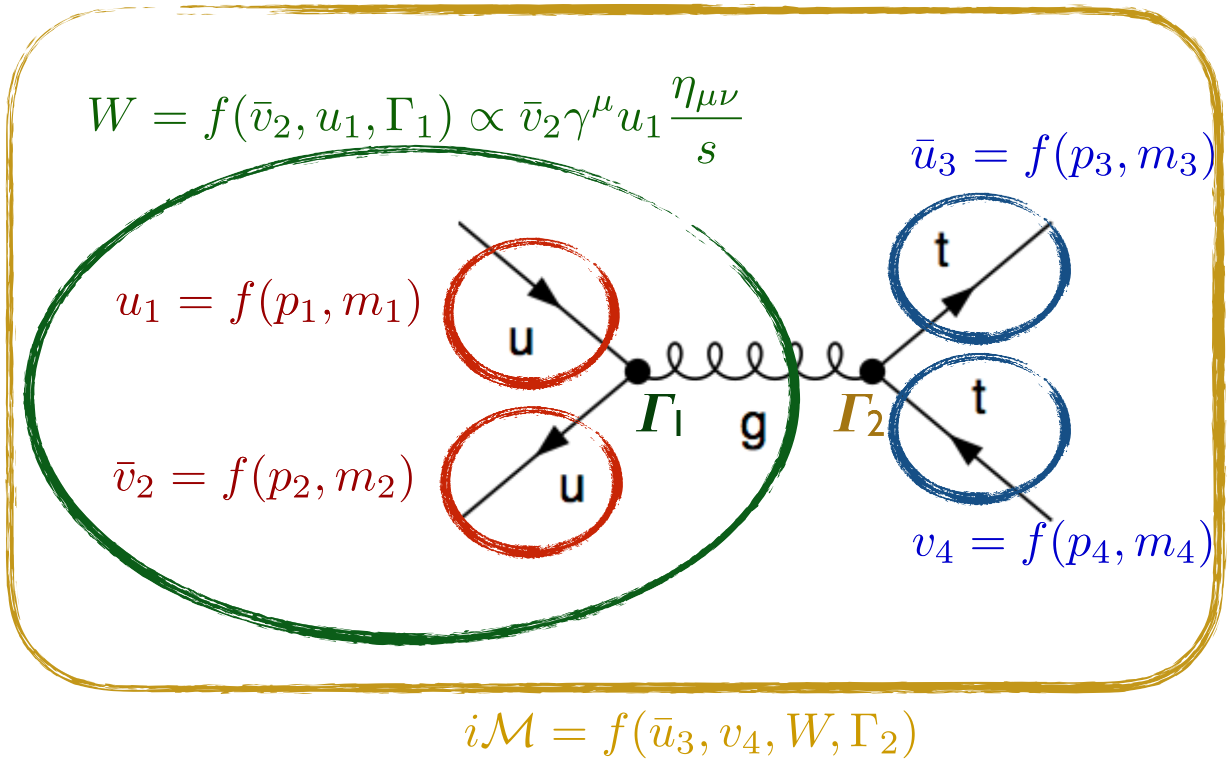
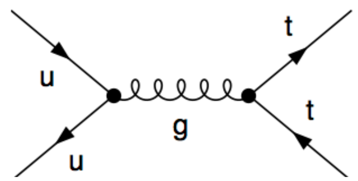


1. External incoming particles (numbers)
   ★ For fixed helicity and momentum
2. Wave function of the gluon propagator
3. External outgoing particles
4. Full amplitude (complex number)

✦ The building blocks of the amplitude are the so-called HELAS functions



$$u_1 = f(p_1, m_1)$$
$$\bar{v}_2 = f(p_2, m_2)$$
$$\bar{u}_3 = f(p_3, m_3)$$
$$v_4 = f(p_4, m_4)$$
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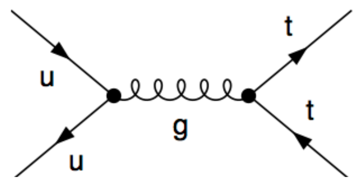
Sufficient for many models

HELAS

✦ The building blocks of the amplitude are the so-called HELAS functions



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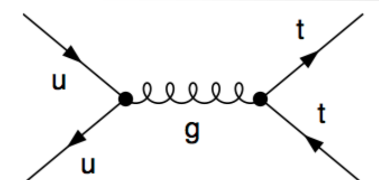

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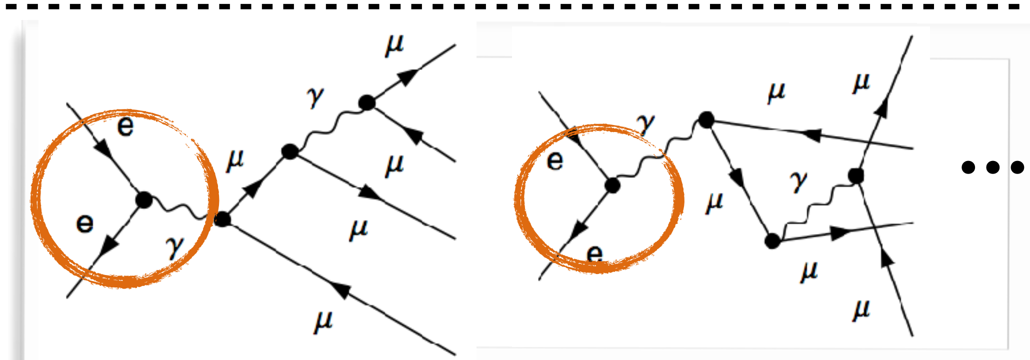
✦ Recycling: reusing pieces across diagrams

✤ Gain in computing time

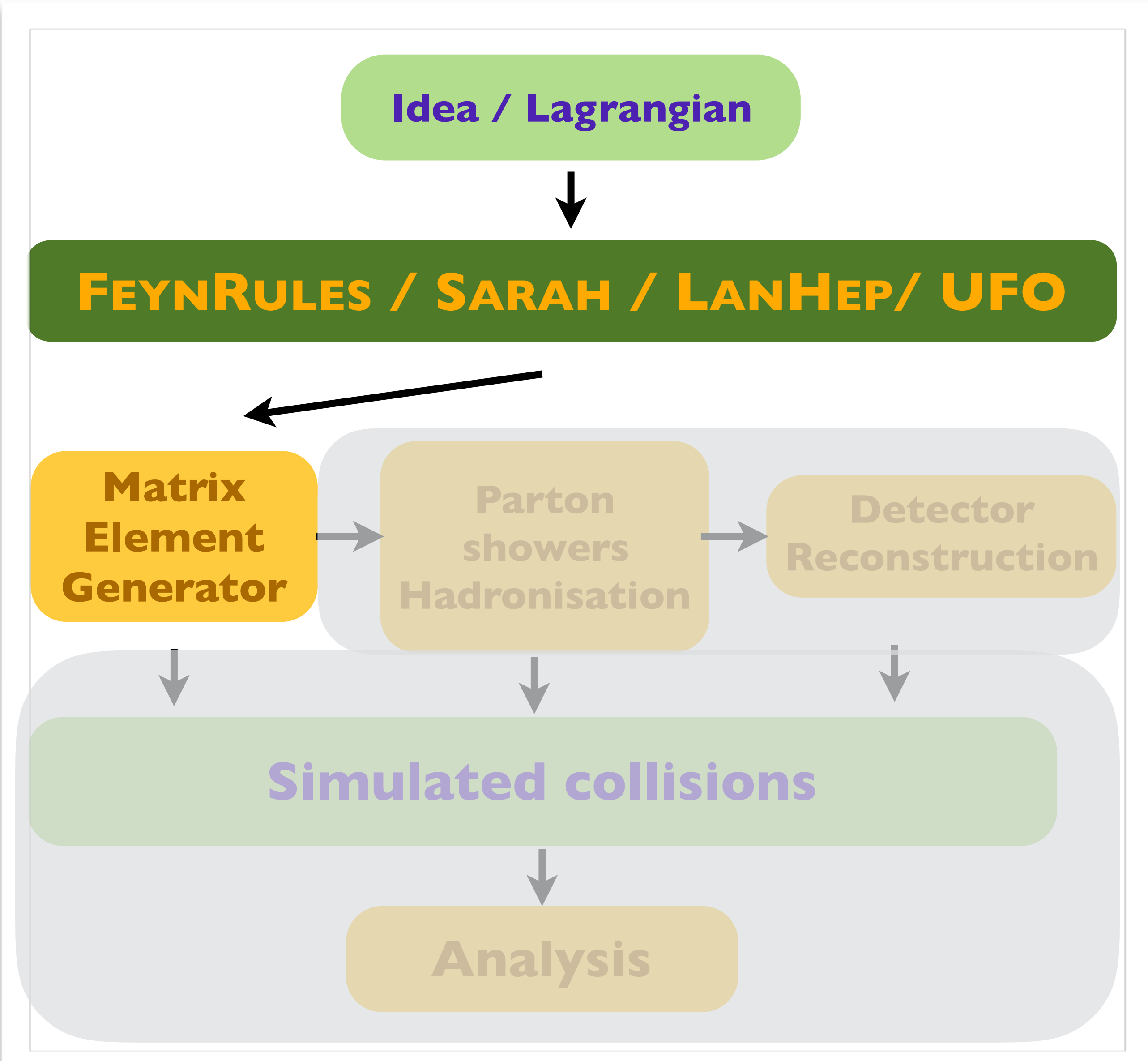


Comparison

	For M diags	For N particles	$2 \to 6$ example
Analytical	M^2	$(N!)^2$	10^9
Helicity	M	$N!\ 2^N$	10^7
Recycling	M	$(N-1)!\ 2^{N-1}$	5×10^5

Back to the simulation chain

✦ Tools connecting an idea to simulated collisions

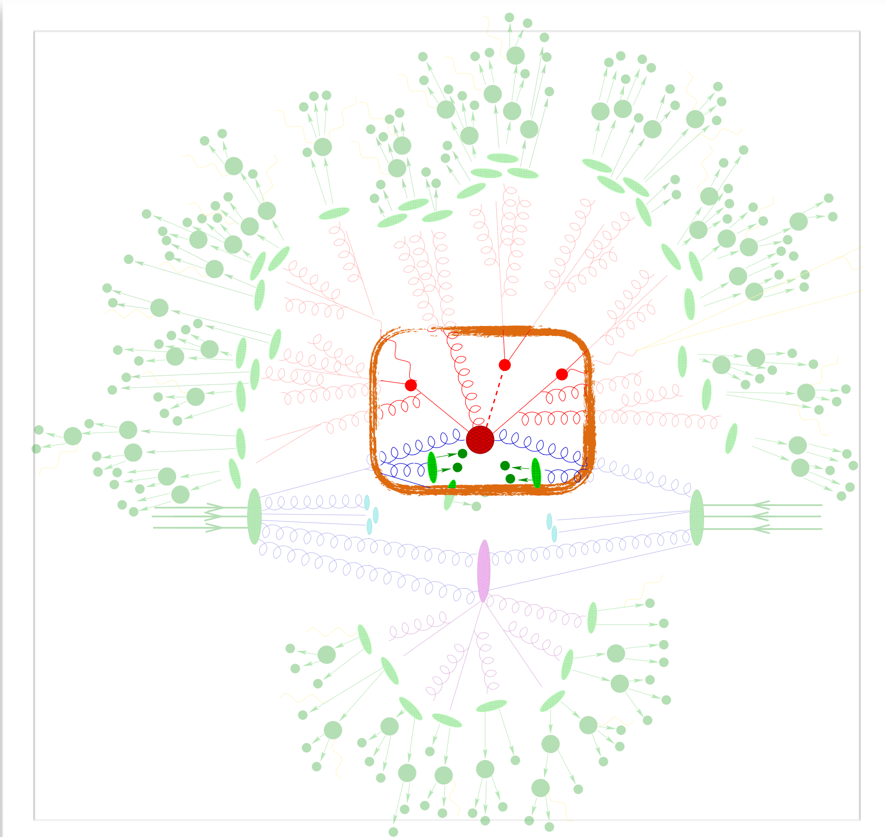


✤ Hard scattering process

★ Feynman diagram / amplitude generation

★ <u>Monte Carlo integration</u>

★ Event generation



Observable calculations

✦ The QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b \mathrm{d}\Phi_n \mathbf{f_{a/p_1}}(x_a, \mu_F)\ \mathbf{f_{b/p_2}}(x_b, \mu_F)\ \left|\mathcal{M}\right|^{\mathbf{2}} \mathcal{O}_\omega(\Phi_n)$$

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★ The phase space structure → analytical calculations hopeless

✤ The integrand is a very peaked function (propagators)

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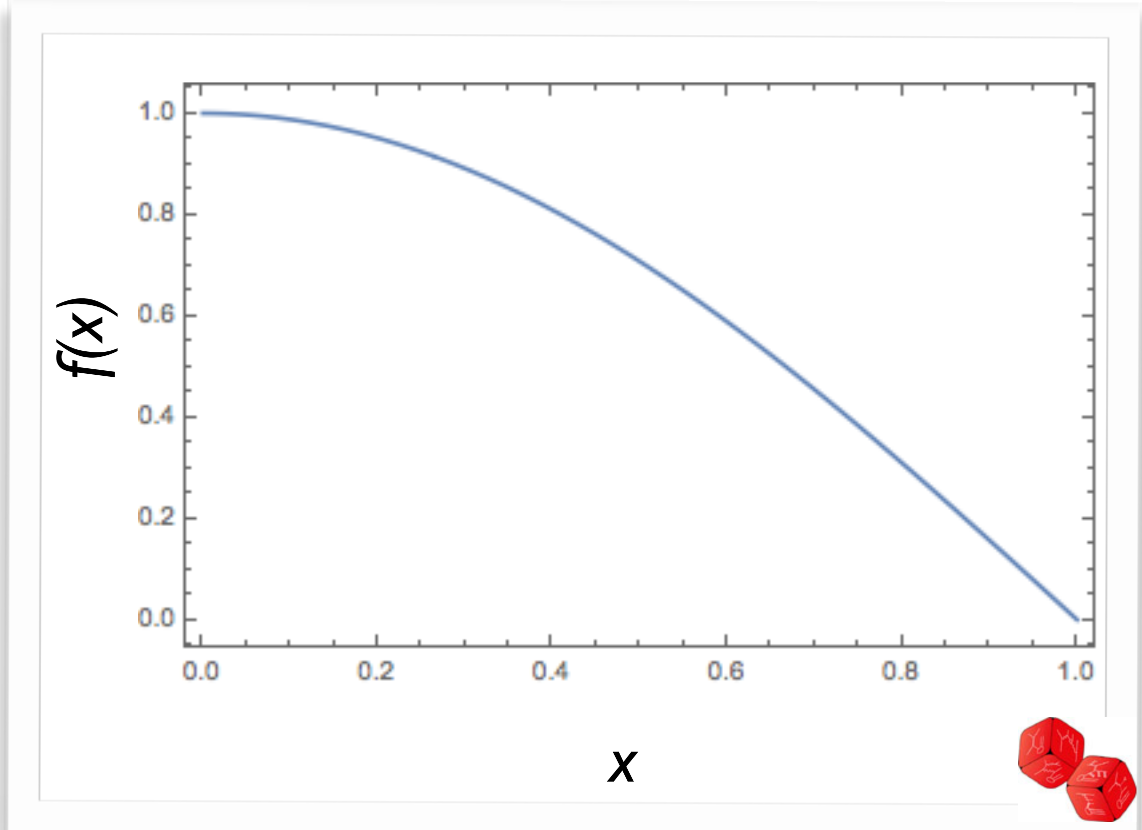
General and flexible numerical methods

Monte Carlo integration: the method

✦ The 1D example: evaluate the integral *I*

$$I = \int_a^b \mathrm{d}x \ f(x)$$



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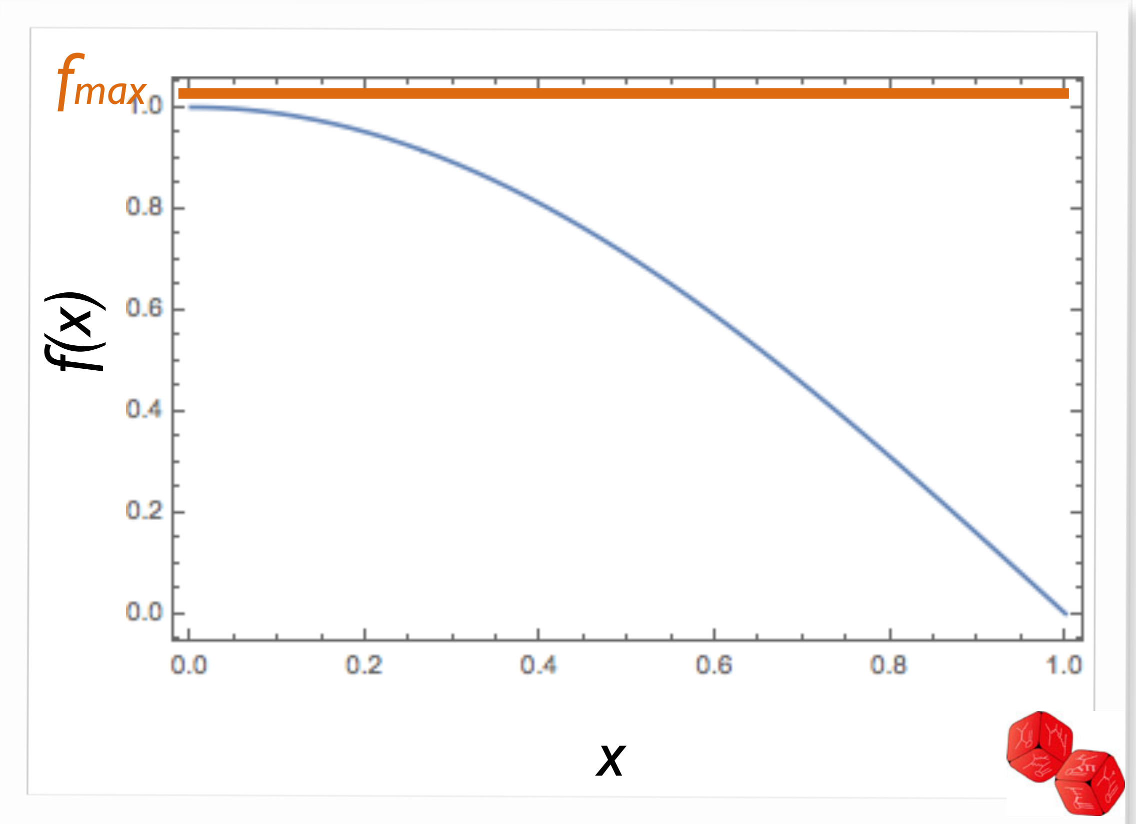

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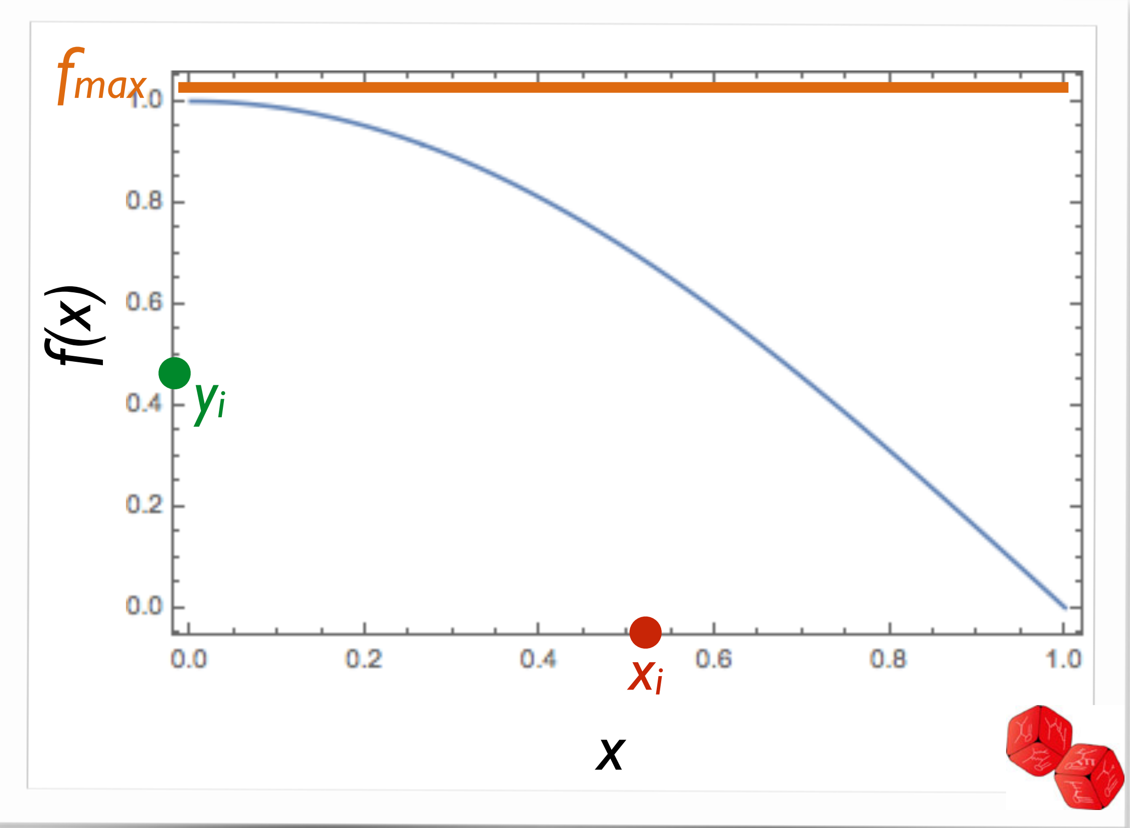

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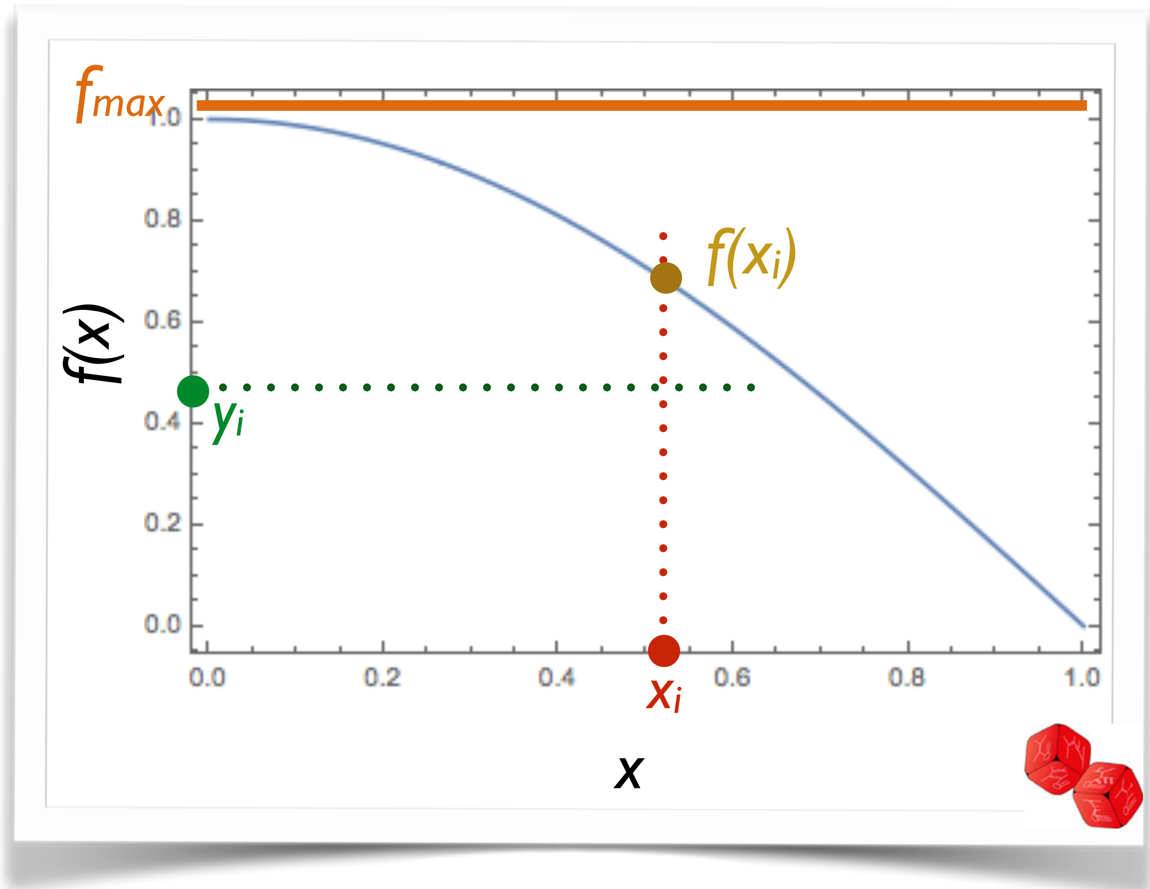

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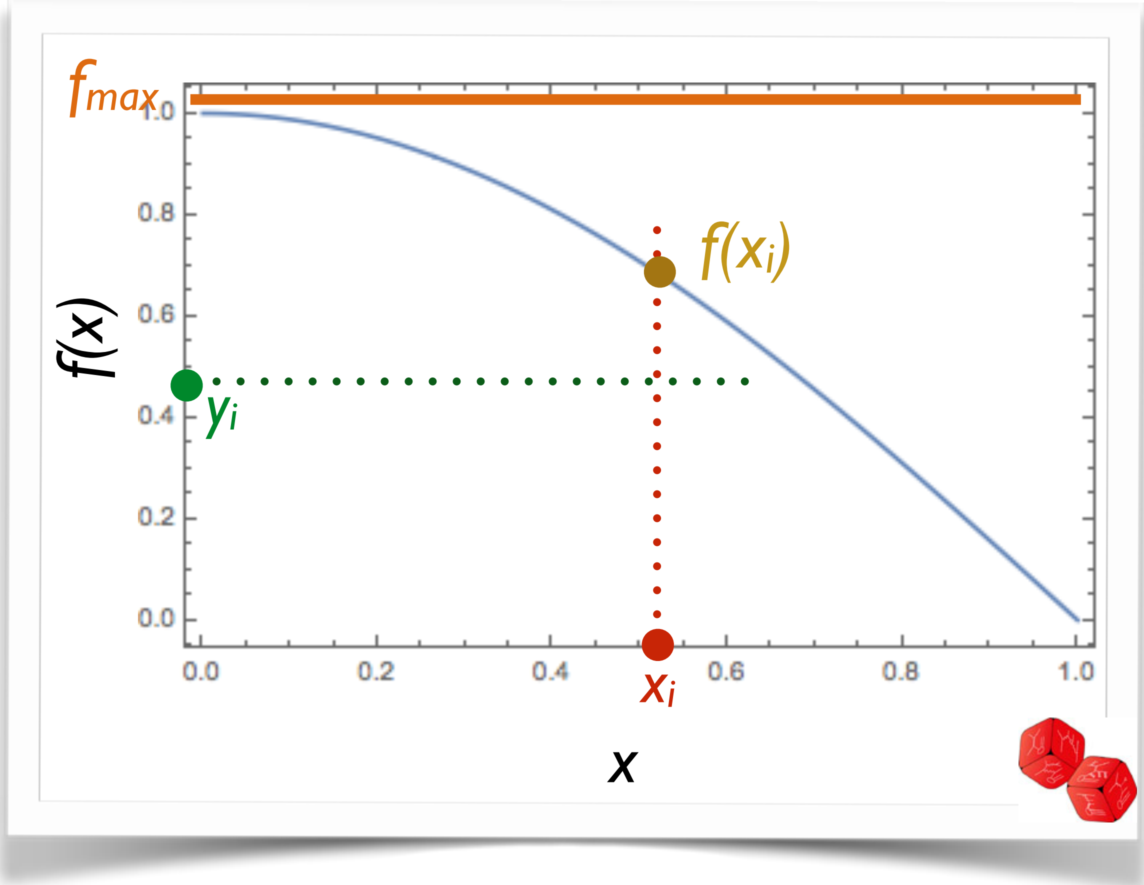
4. Evaluate the integral

$$I_N = \frac{N_{\text{accepted}}}{N_{\text{total}}} \; \mathcal{V}$$



Monte Carlo integration: the error

- ✦ The mean value theorem
  - ✤ If *f(x)* is continuous:

$$\exists \xi \in [a,b] : I = \int_a^b \mathrm{d}x \ f(x) = (b-a) f(\xi) = (b-a) \langle f \rangle$$

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- We can approximate $\langle f \rangle$ by an averaged sum, so that:

$$I = \int_a^b \mathrm{d}x\ f(x) \ \approx I_N = \frac{b-a}{N} \sum_{n=1}^N f(x_n)$$

  - ★ $\langle f \rangle$ is calculated by sampling the integrand at random points

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✦ The error is given by the variance (that can be calculated)

$$V = (b-a) \int_a^b \mathrm{d}x\ f^2(x) - I^2\ \approx V_N = \frac{(b-a)^2}{N} \sum_{n=1}^N f^2(x_n) - I_N^2$$

✤ Independent from the number of dimensions

Discretising an integral

✦ Integrals are evaluated as averaged sums over randomly chosen points

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✦ Result

$$I = I_N \pm \sqrt{\frac{V_N}{N}}$$

- ✤ The error can easily be estimated
- ✤ The error is independent from the number of dimensions
- ✤ Improvement possible by minimising V_N
- ✤ Ideal case: $f(x)$ = *cst* $(V=V_N=0)$
  - ★ Change of variables to flatten the integrand

Importance sampling: a practical example

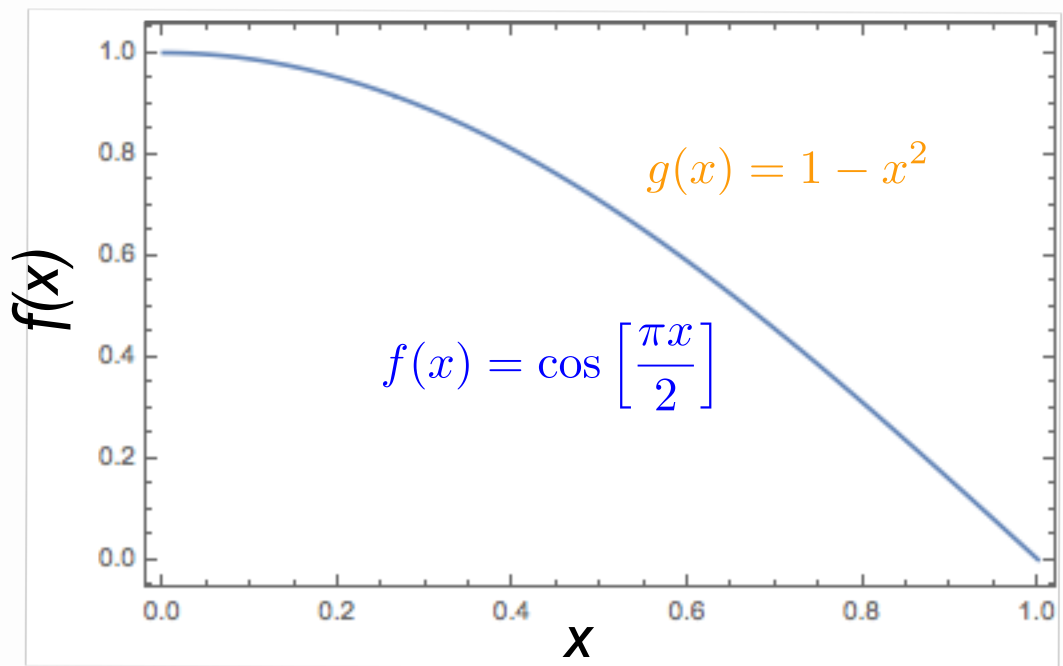
✦ Integral to calculate

$$I = \int_0^1 \mathrm{d}x \ \cos\left[\frac{\pi x}{2}\right] = \frac{2}{\pi} \approx 0.6366$$

$$I_N = 0.637 \pm \frac{0.307}{\sqrt{N}}$$

✦ Remarks

- Convergence is slow
- Precision ⇒ large N
- Strength: scalability with n_{dim}



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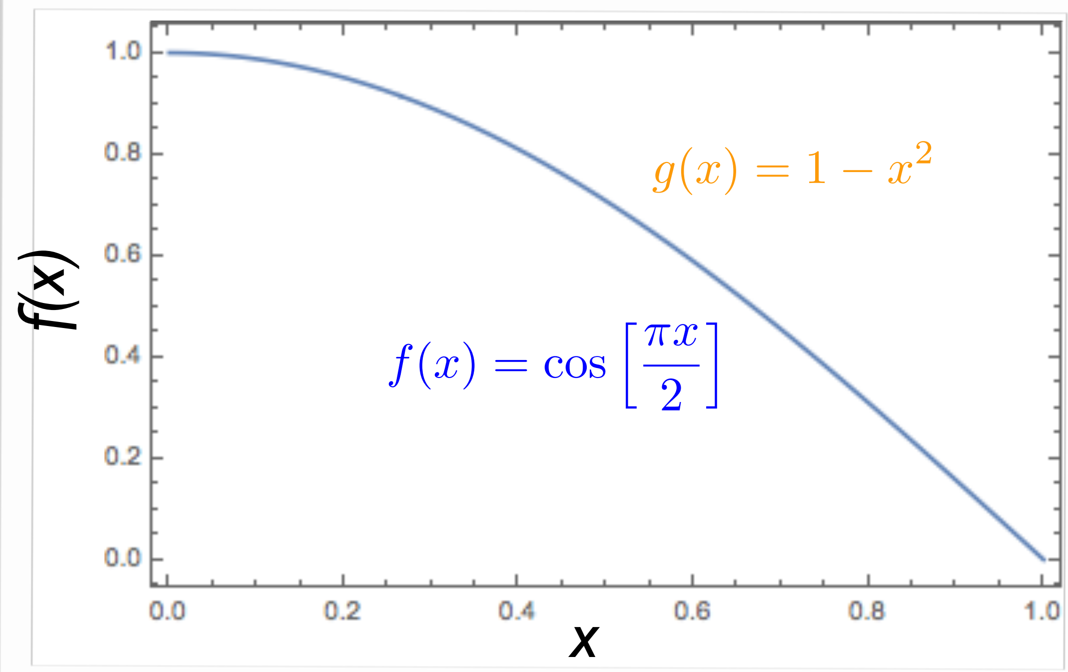

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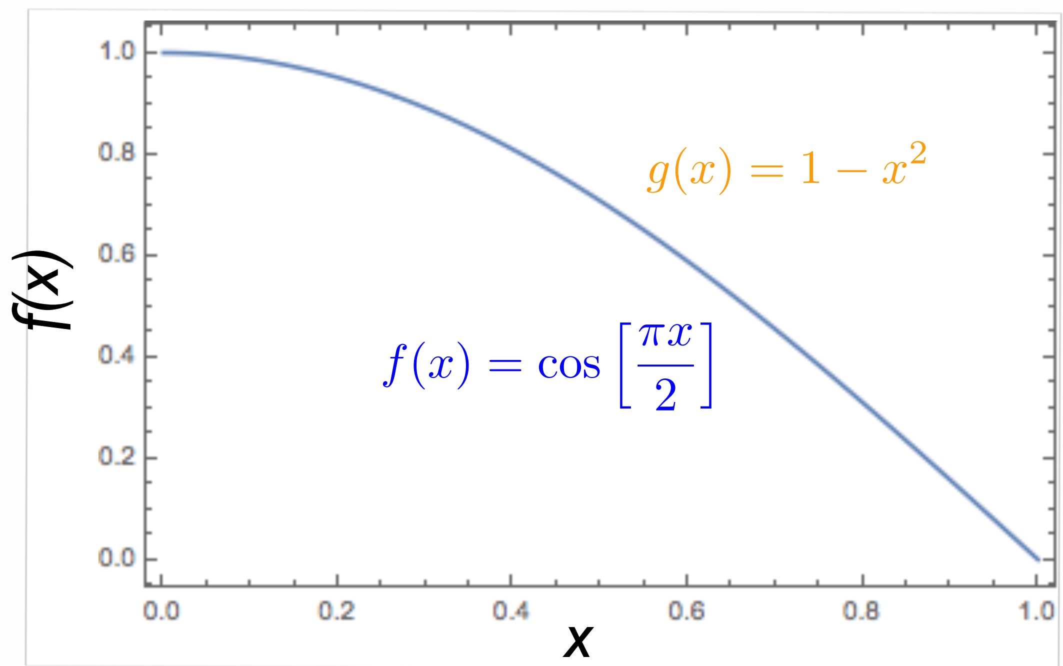

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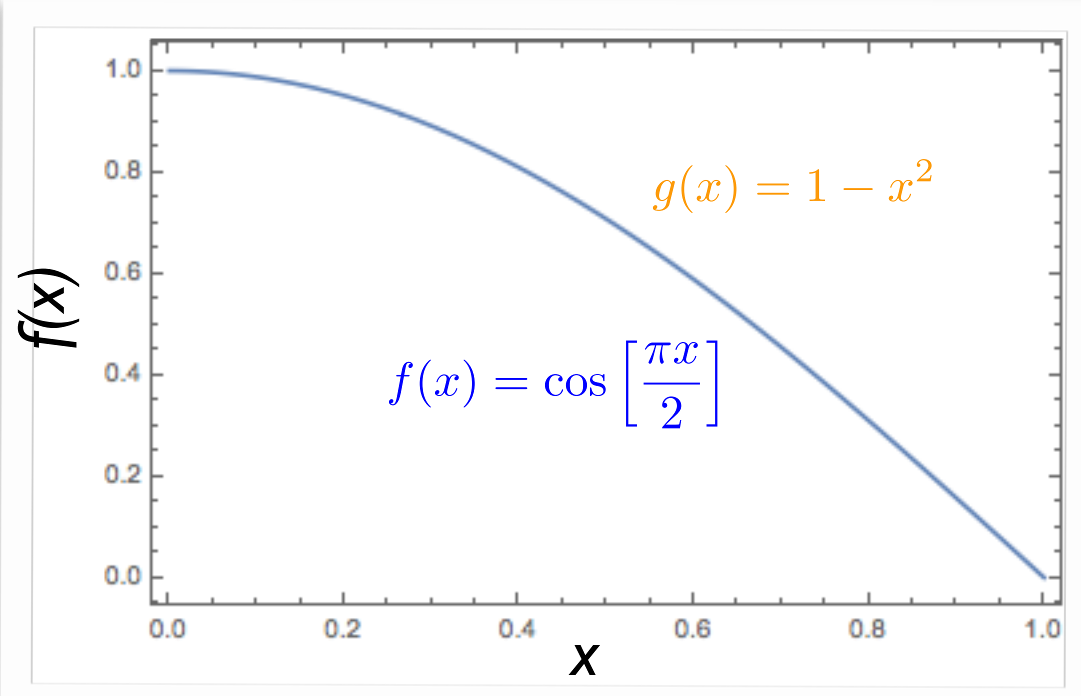

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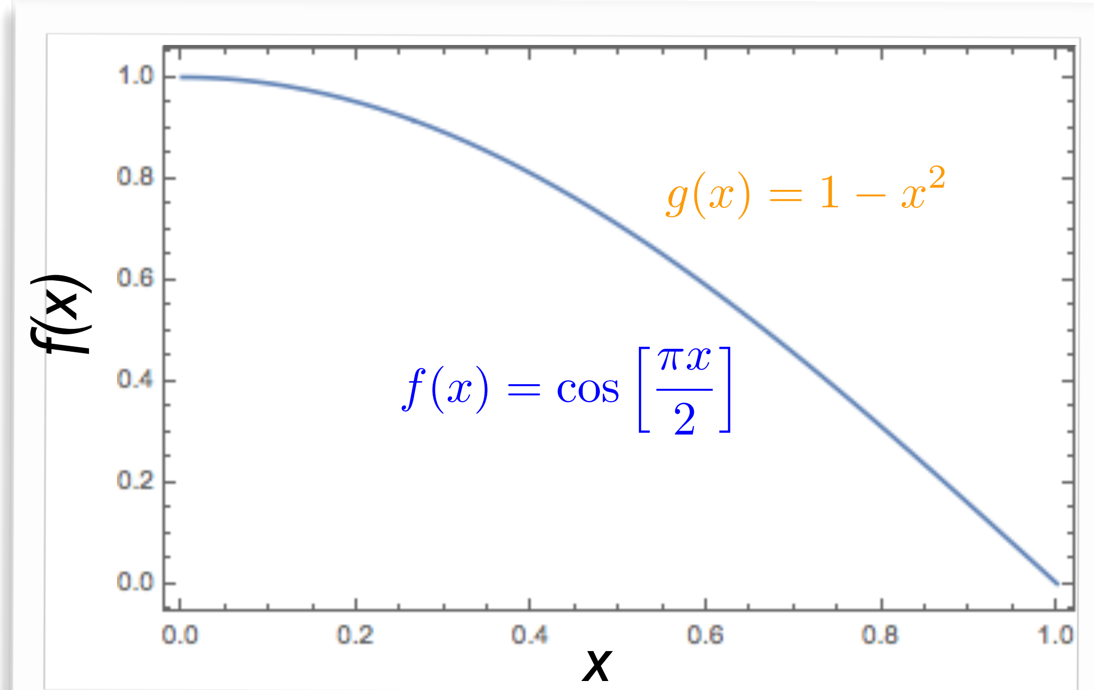

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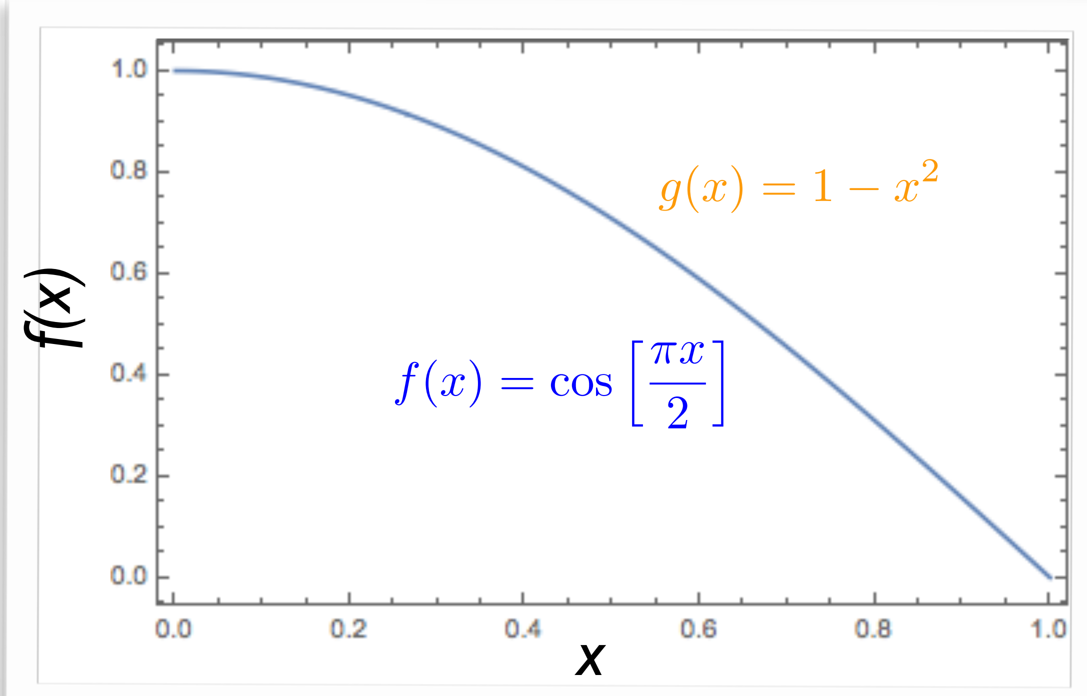

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➢ **Faster convergence**

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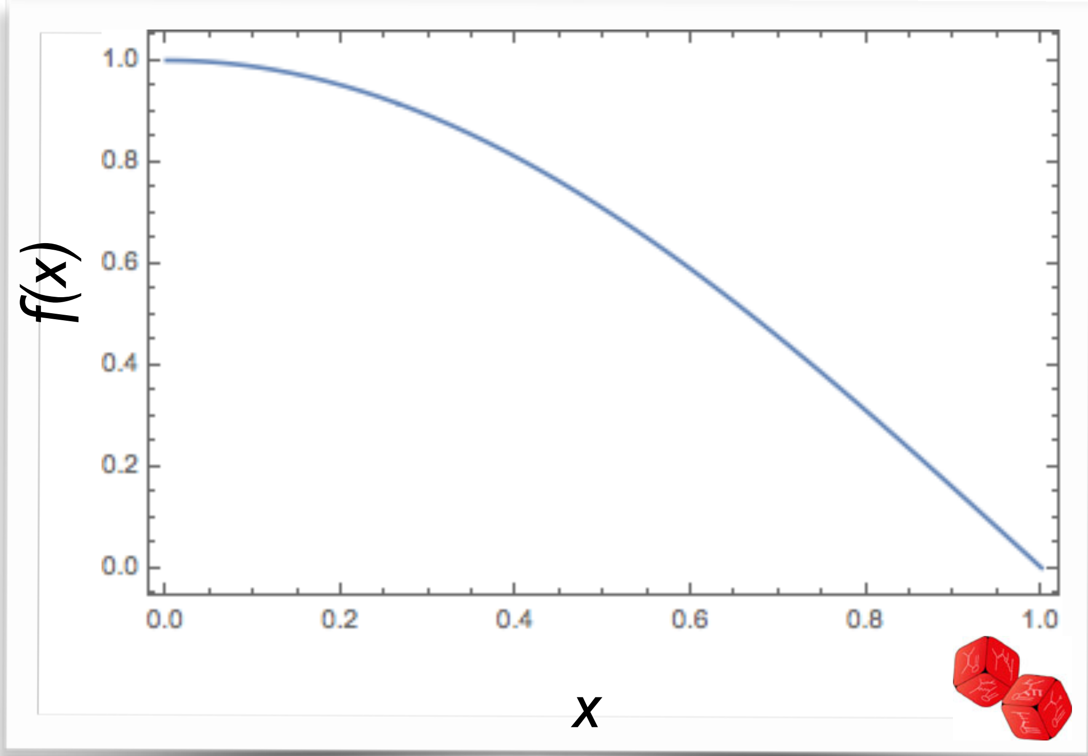

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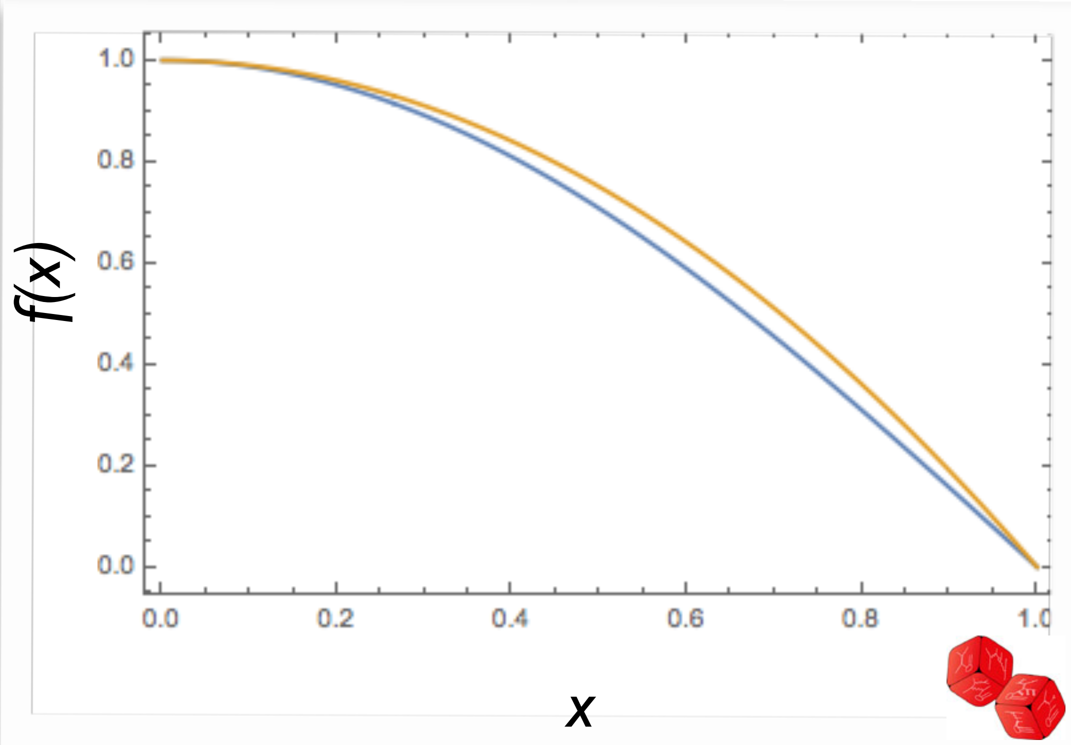

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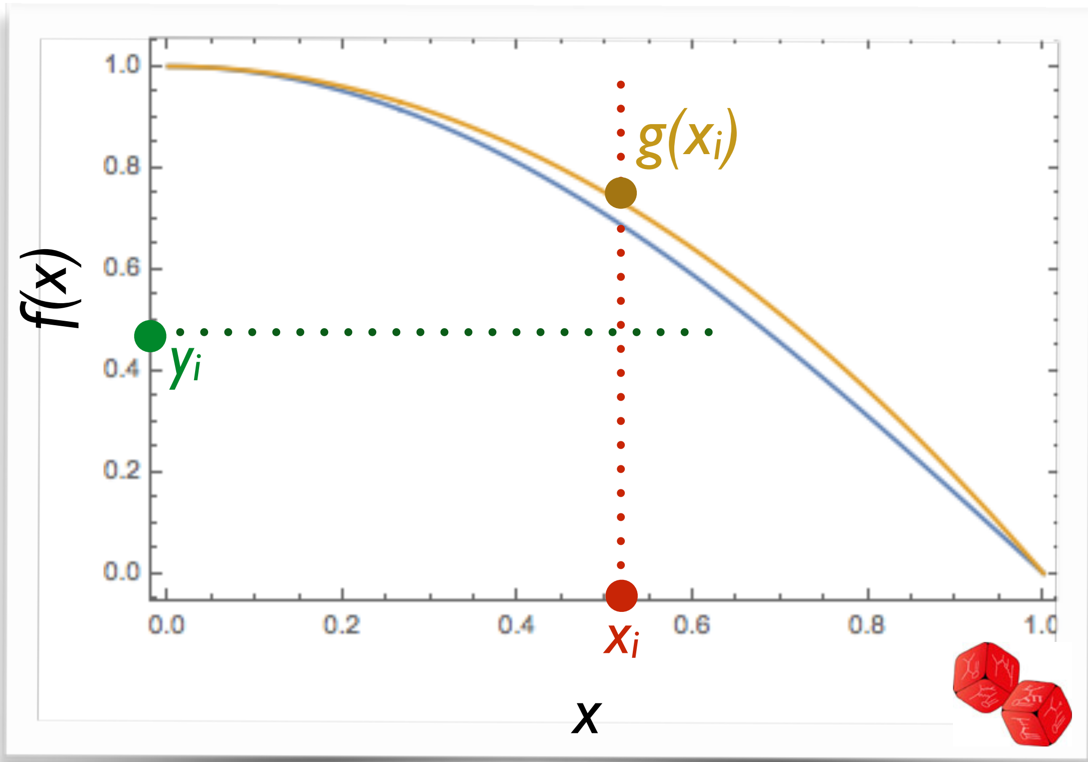

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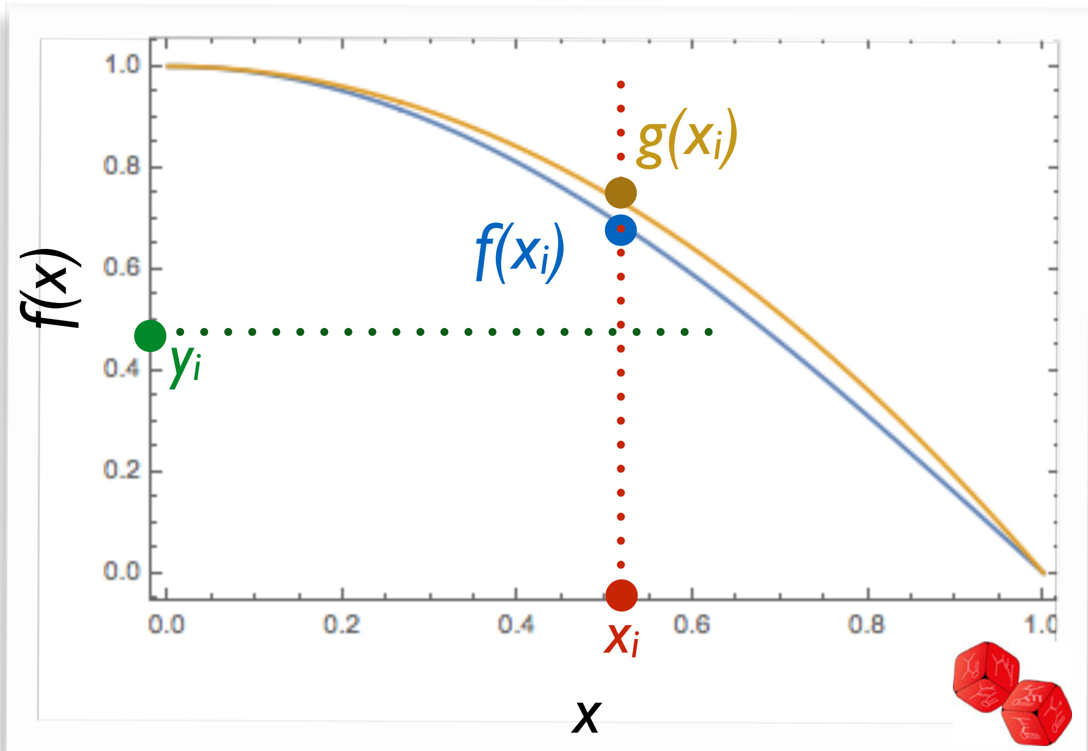

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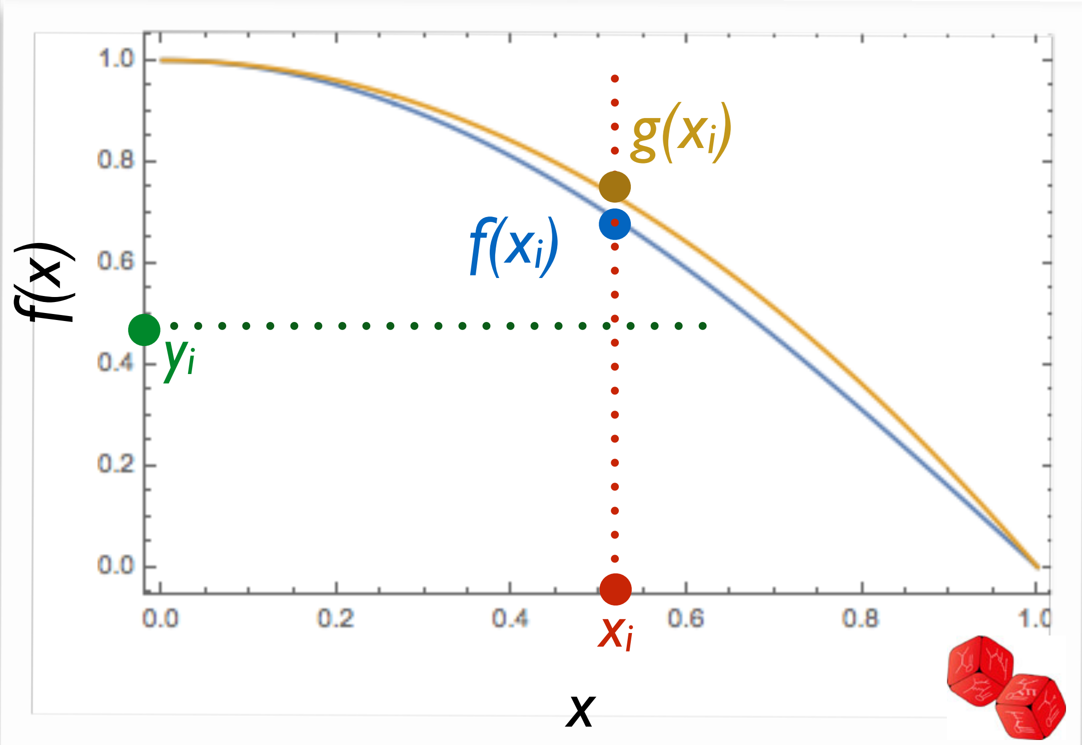

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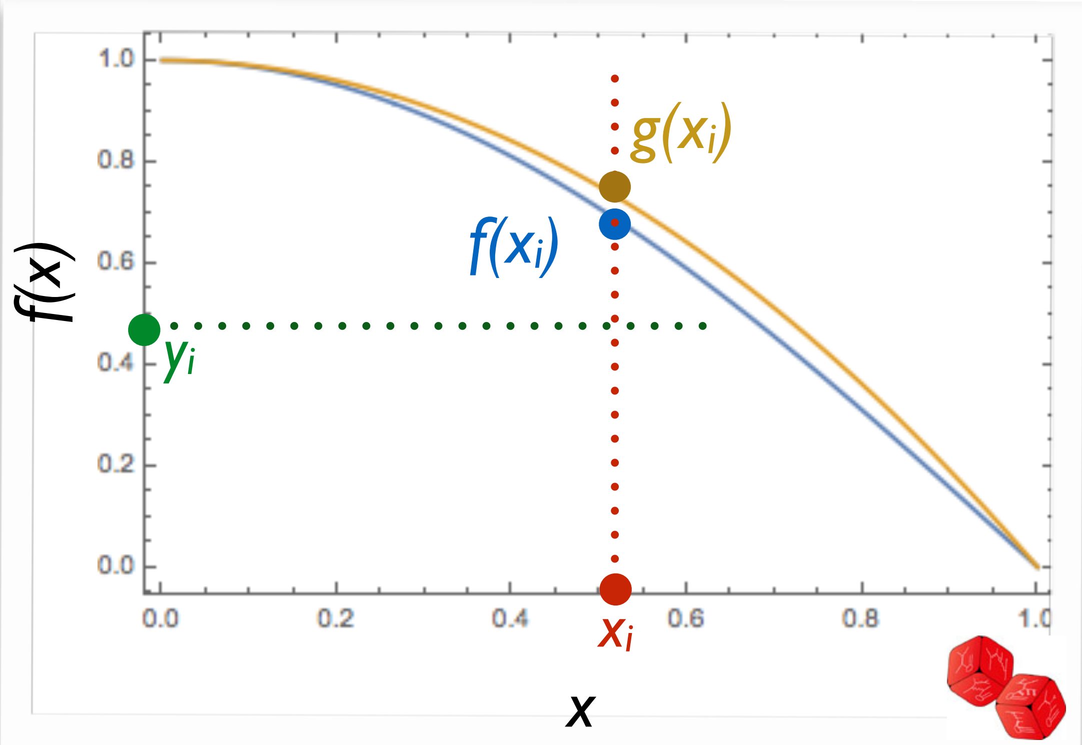

Improved efficiency

Problem of a peaked integrand

✦ QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b \mathrm{d}\Phi_n \mathbf{f_{a/p_1}}(x_a, \mu_F)\ \mathbf{f_{b/p_2}}(x_b, \mu_F)\ \left|\mathcal{M}\right|^{\mathbf{2}} \mathcal{O}_\omega(\Phi_n)$$

✤ For each point, we have a weight given by $\mathbf{f_{a/p_1}}(x_a, \mu_F)\ \mathbf{f_{b/p_2}}(x_b, \mu_F)\ \left|\mathcal{M}\right|^{\mathbf{2}}$

✤ Interpretation: each momentum configuration yields a weight

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✦ Problem: the integral is peaked (→ propagators)

- Random phase space points: very little chance to contribute
  - ★ Few points carry the bulk of the integral
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  - ★ Need for some knowledge about the integrand

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✦ Construction of an approximative function of the integrand

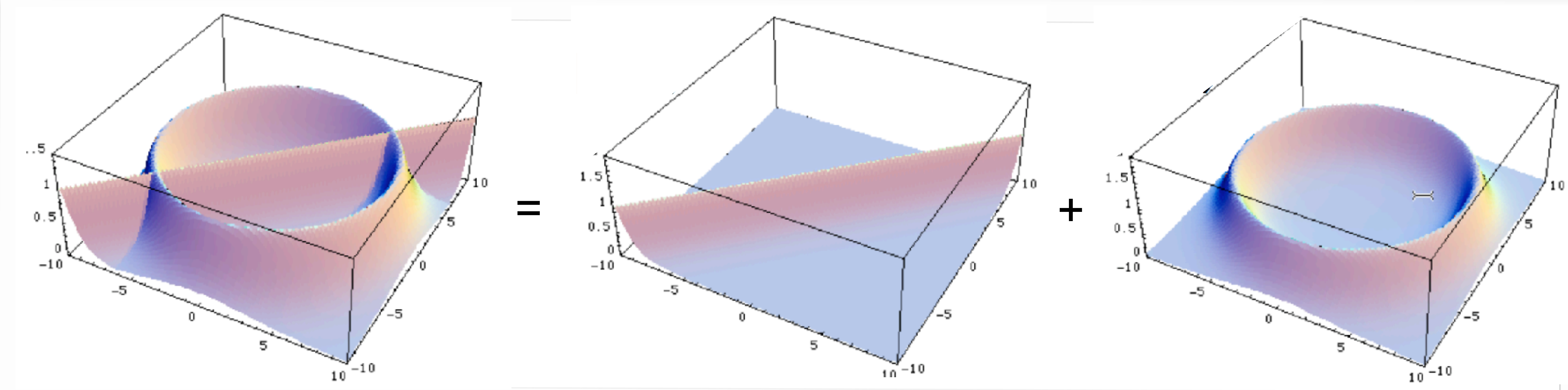
- Division of the integration domain in sub-domains (variable bin-size)
  - ★ Adjustment: identical variance in each bin
  - ★ Minimisation of the overall variance
- More bins where the integrand fluctuates more
  - ★ This binned function provides an approximation of the integrand *g(x)*

Multi-channel integration

✦ Separation of the integral among different channels

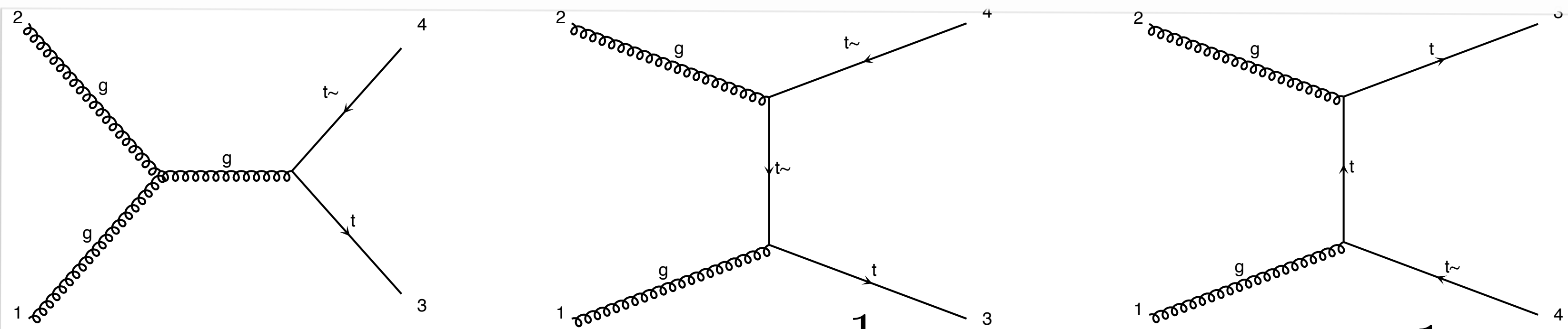
$$g(x) = \sum_{\text{channels}} \alpha_i g_i(x) \quad \text{with} \quad \sum_{\text{channels}} \alpha_i = 1$$

✤ Each channel takes care of one peak of the integrand at a time



Multi-channel integration: an example

✦ Top-antitop production: 3 diagrams



$$\mathcal{A}_s \propto \frac{1}{s} \qquad \mathcal{A}_t \propto \frac{1}{t - m_t^2} \qquad \mathcal{A}_u \propto \frac{1}{u - m_t^2}$$

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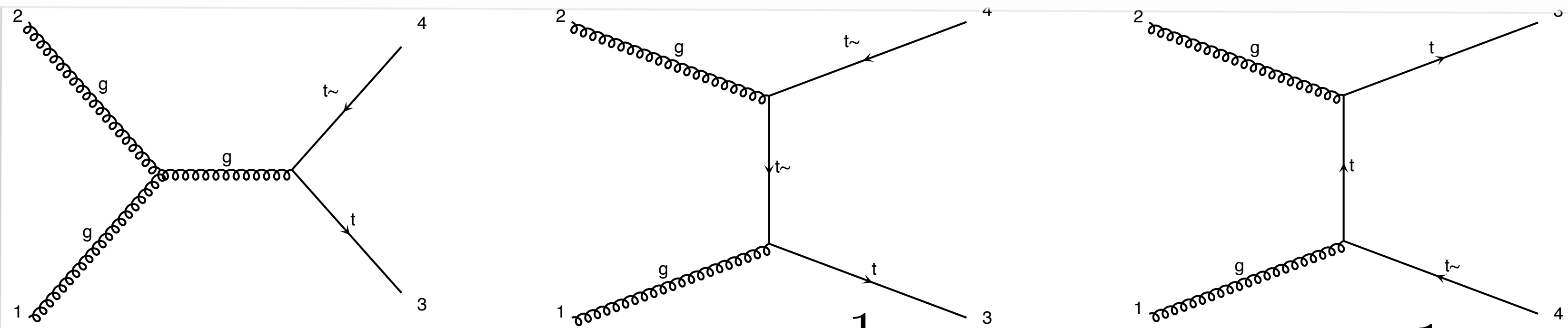

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✤ Three different pole structures

$$I = \int \mathrm{d}\Phi_2 \, \big|\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u\big|^2 = \sum_{i=s,t,u} \int \mathrm{d}\Phi_2 \, \frac{\big|\mathcal{A}_i\big|^2}{\big|\mathcal{A}_s\big|^2 + \big|\mathcal{A}_t\big|^2 + \big|\mathcal{A}_u\big|^2} \big|\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u\big|^2$$

$g_i(\Phi)$ — $g(\Phi)$ — $f(\Phi)$

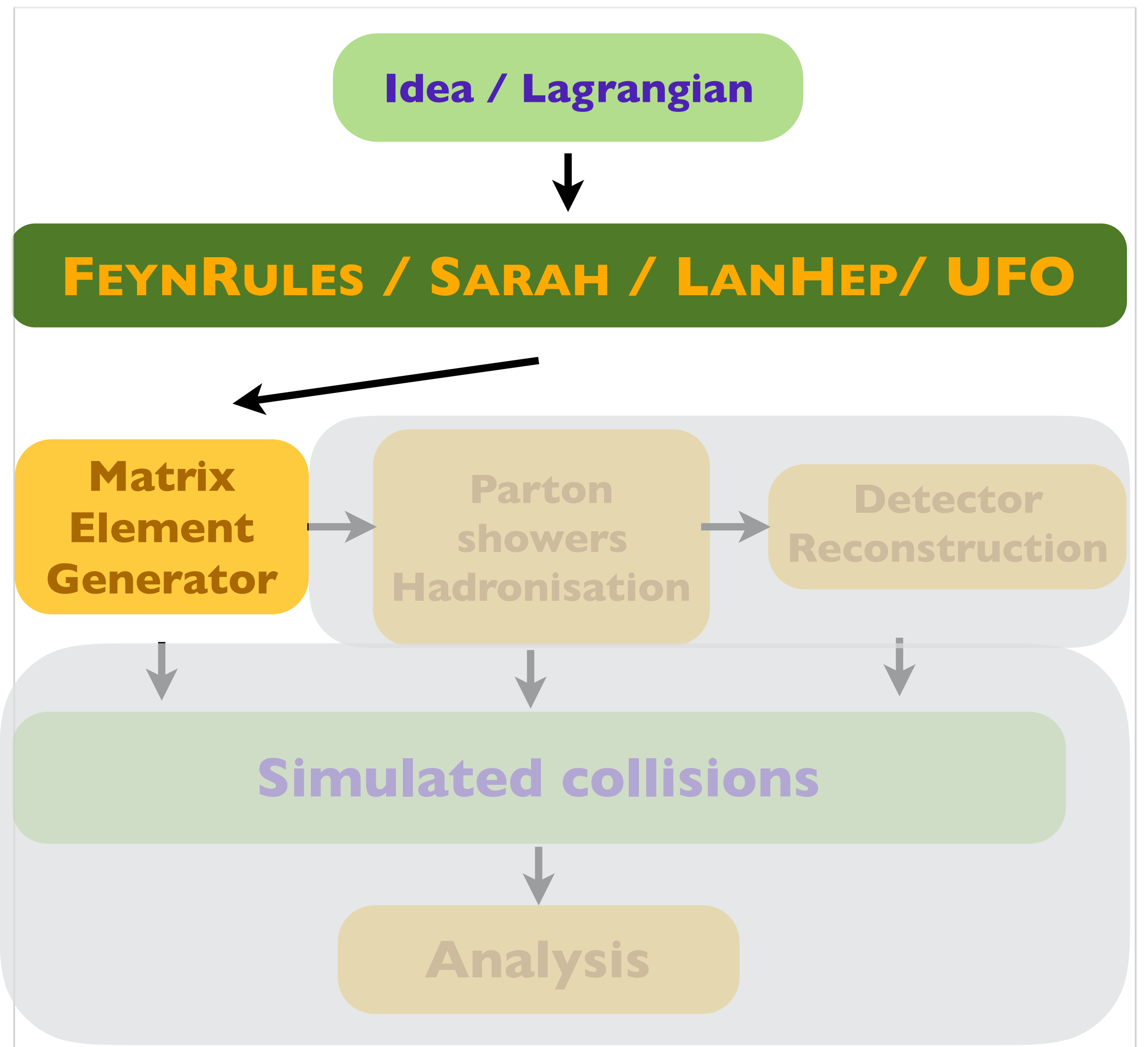
★ $f(\Phi) / g(\Phi) \simeq 1$

★ The integration of one single diagram is easy (the pole structure is known)

★ Multi-channeling on the basis of the different diagrams

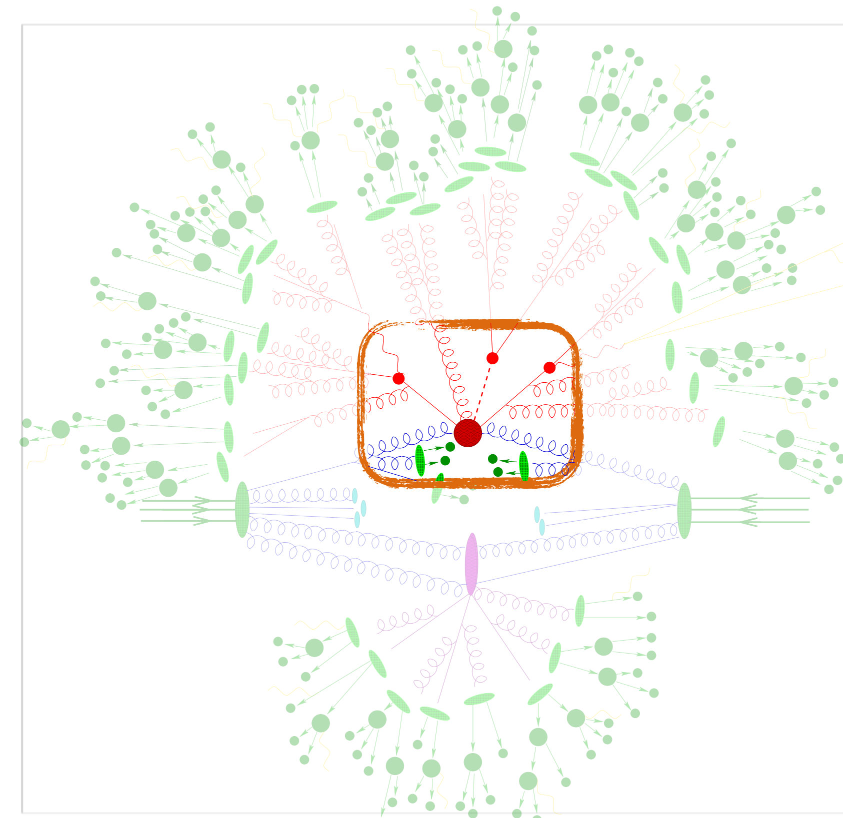
Back to the simulation chain

✦ Tools connecting an idea to simulated collisions



✤ Hard scattering process

- ★ Feynman diagram / amplitude generation
- ★ Monte Carlo integration
- ★ Event generation



Weighted and unweighted events

- ✦ Accepted points → event generation
  - ✤ One point ≡ one event
  - ✤ Integrand value → event weight
  - ✤ Not all events are equal

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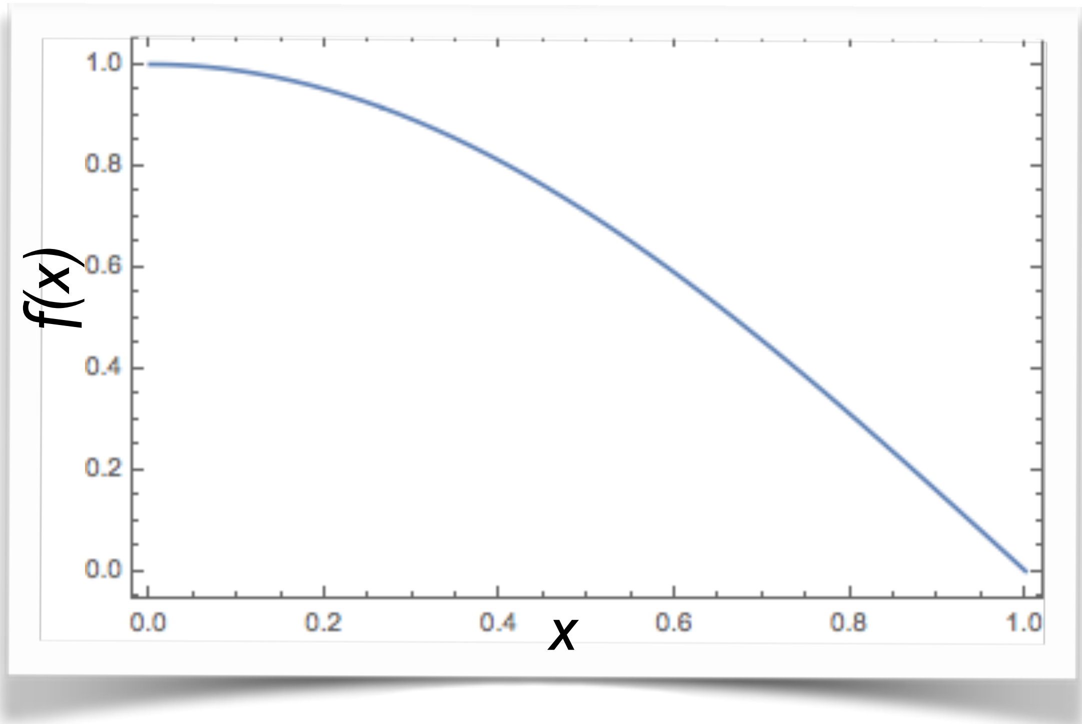

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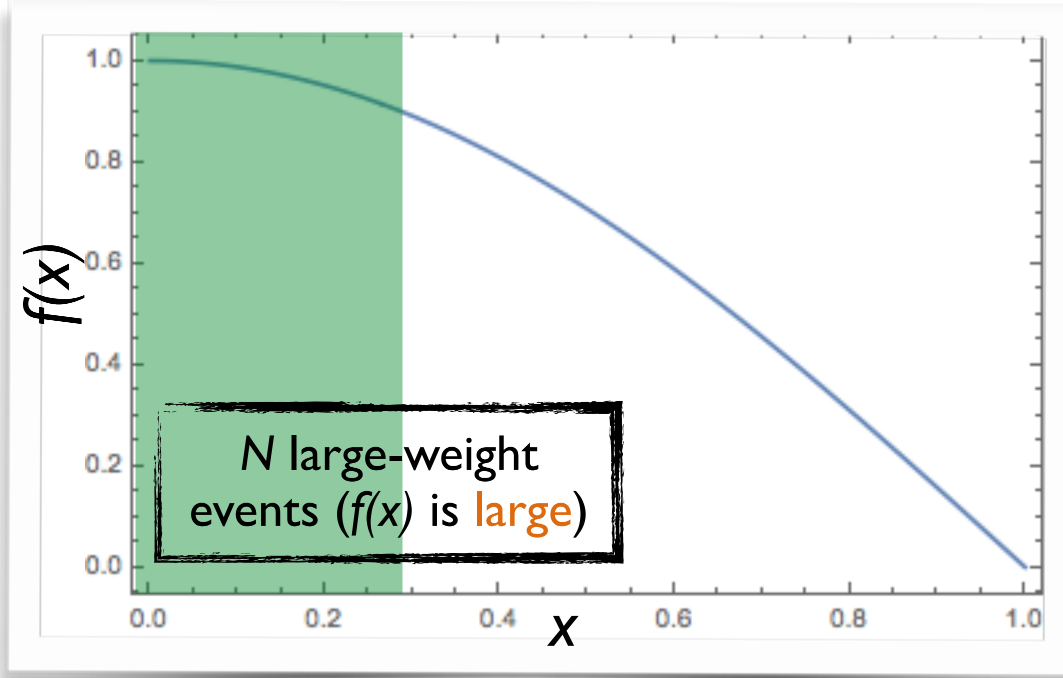

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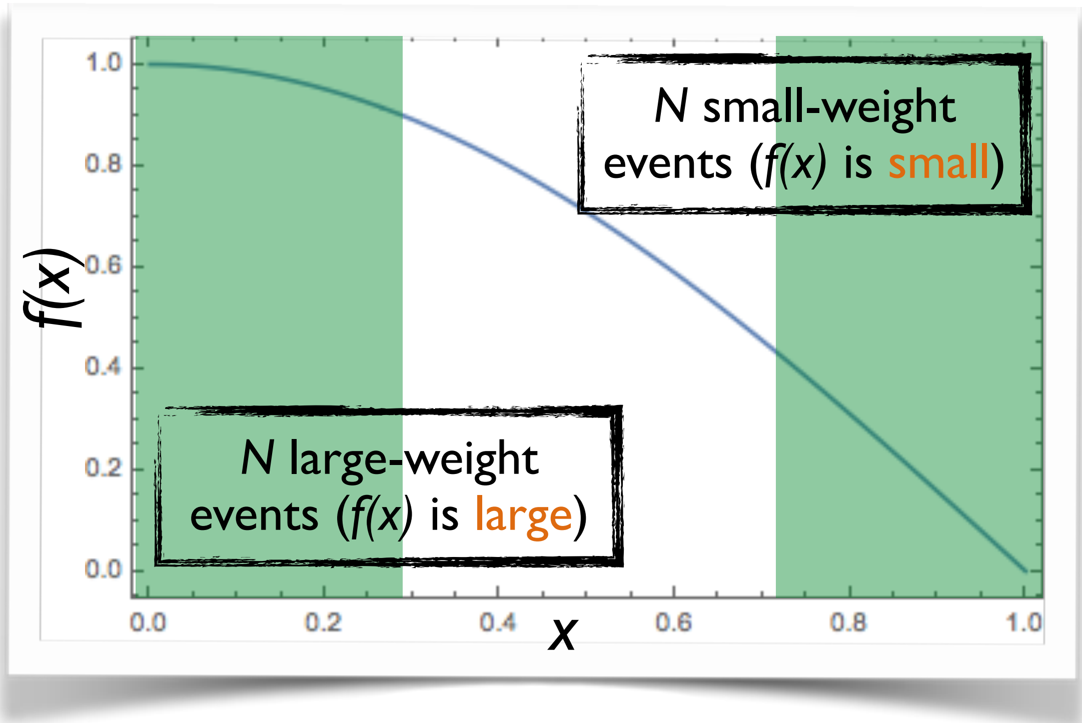

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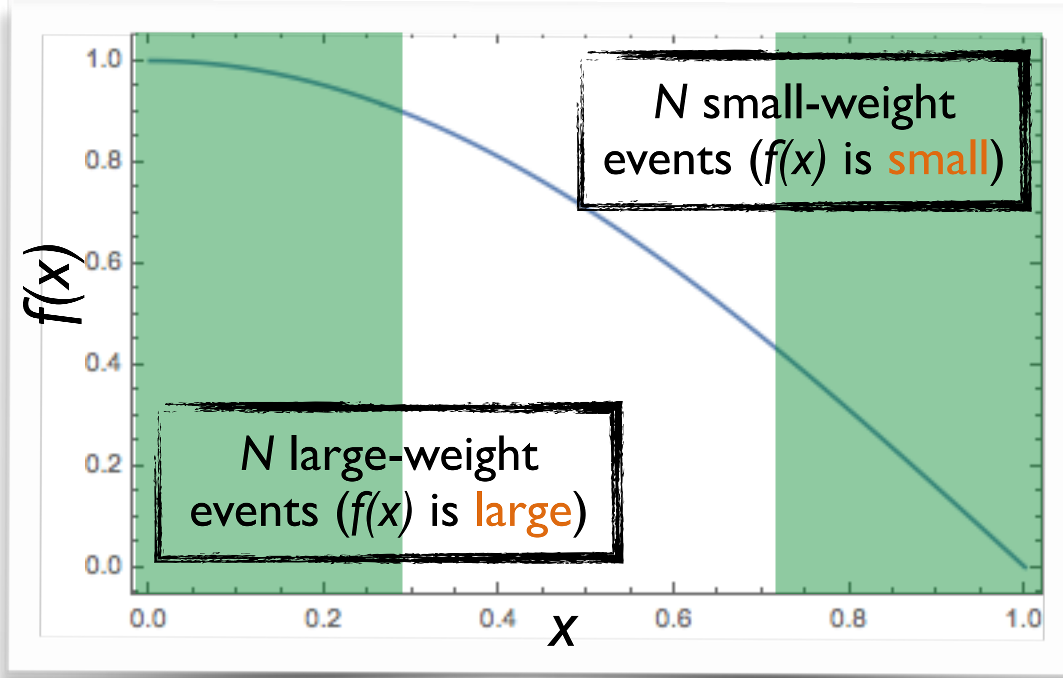

✦ Enforcing equal-weight events
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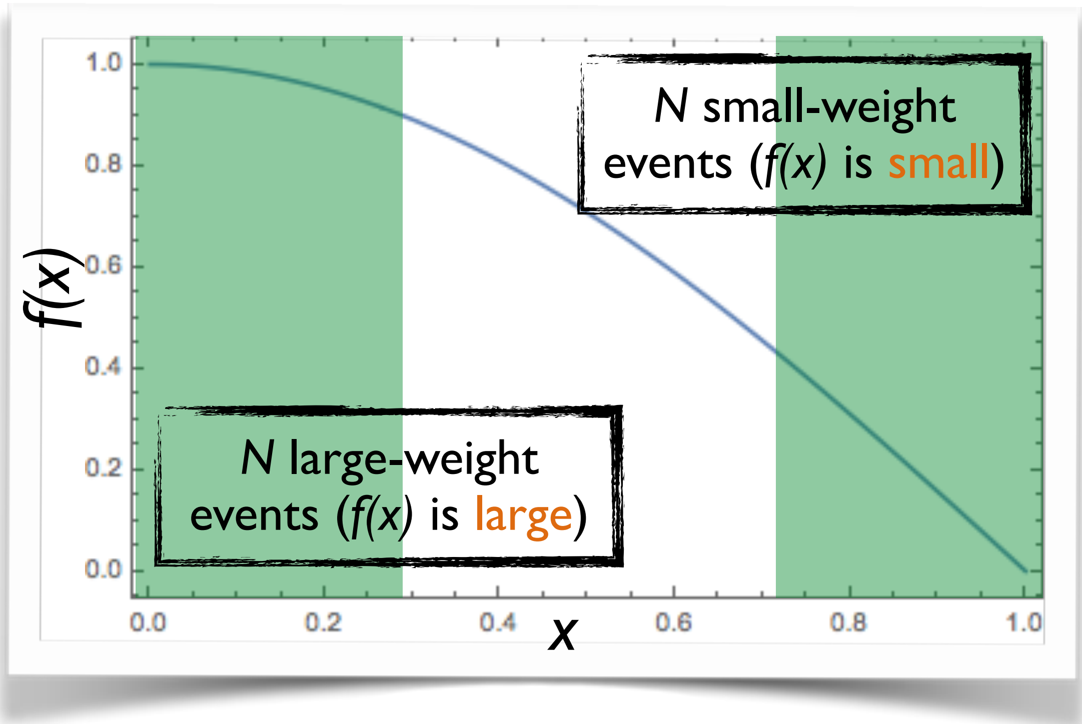

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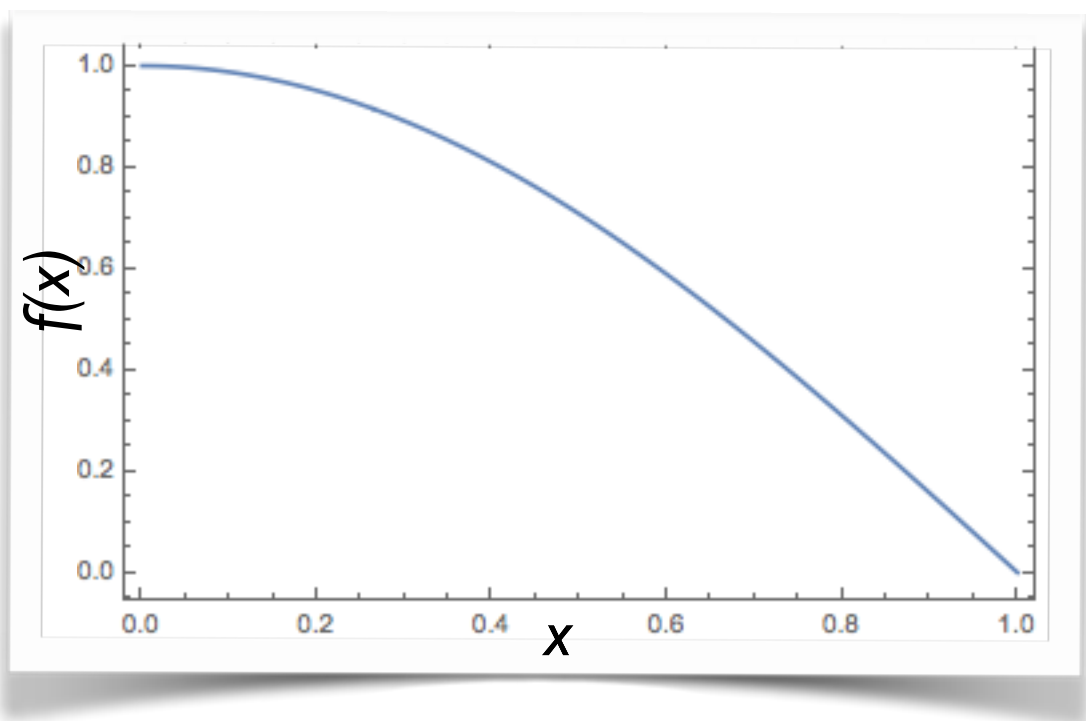

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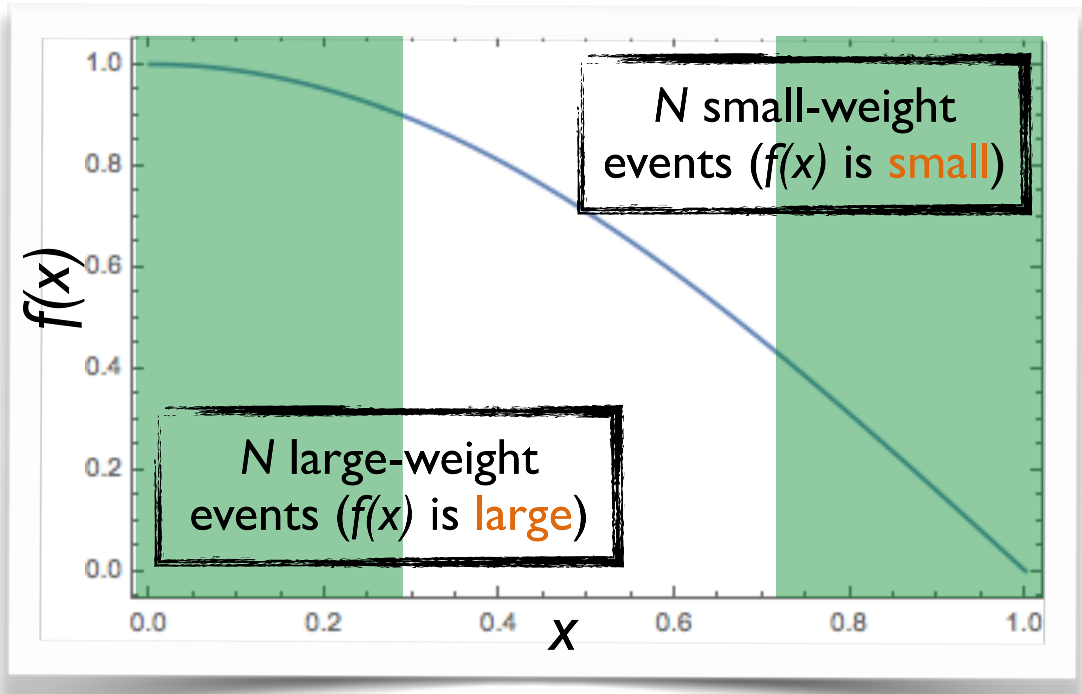

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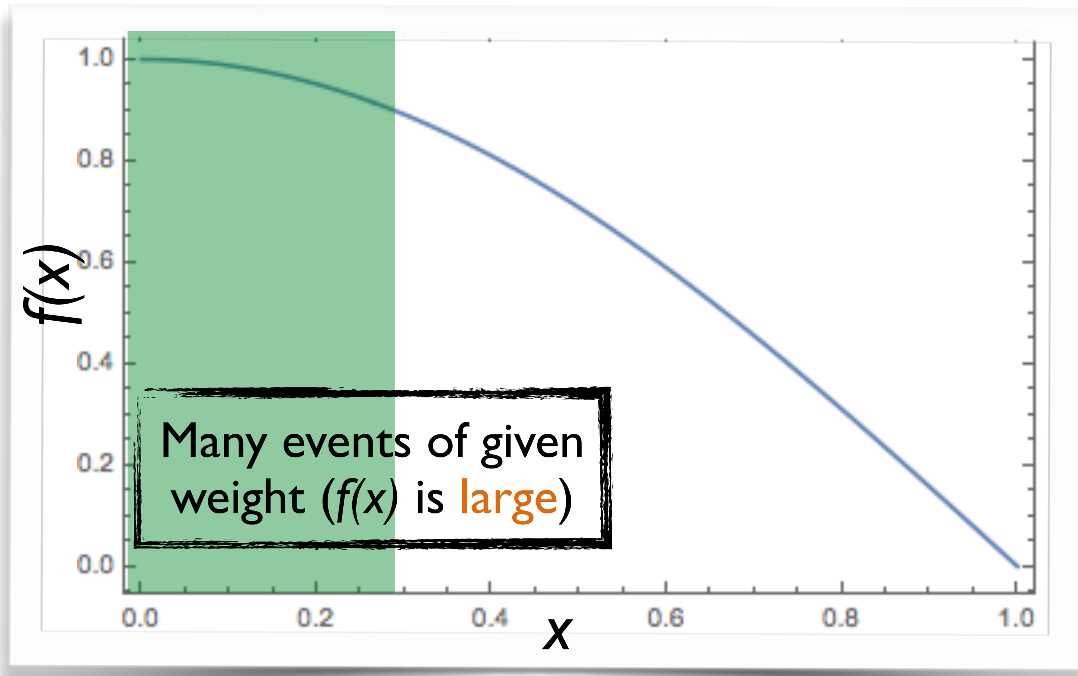

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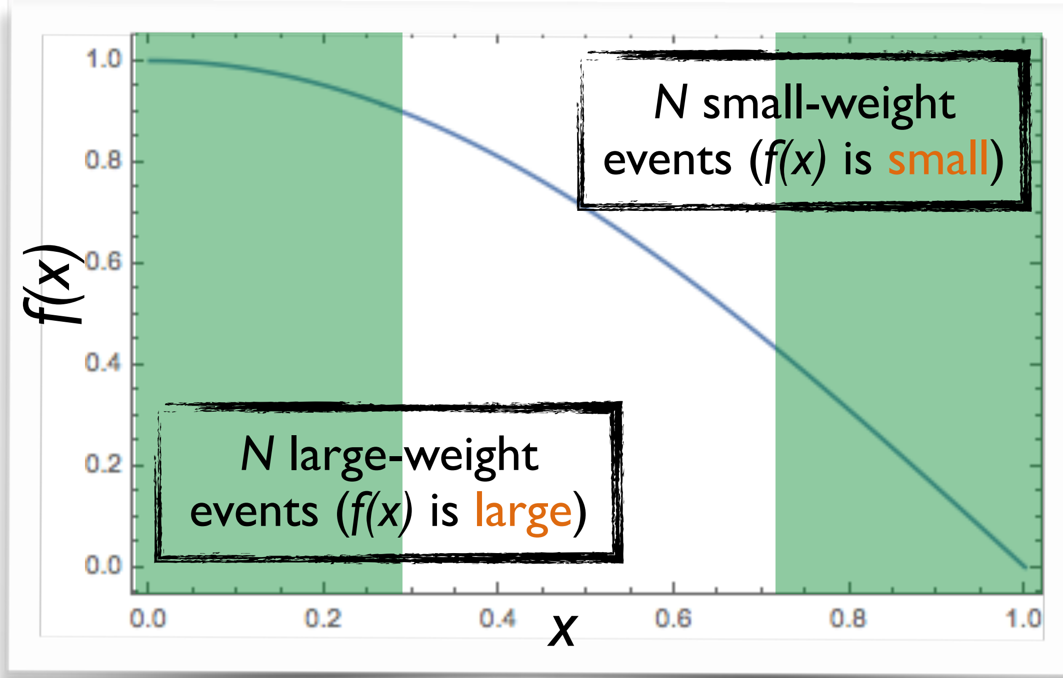

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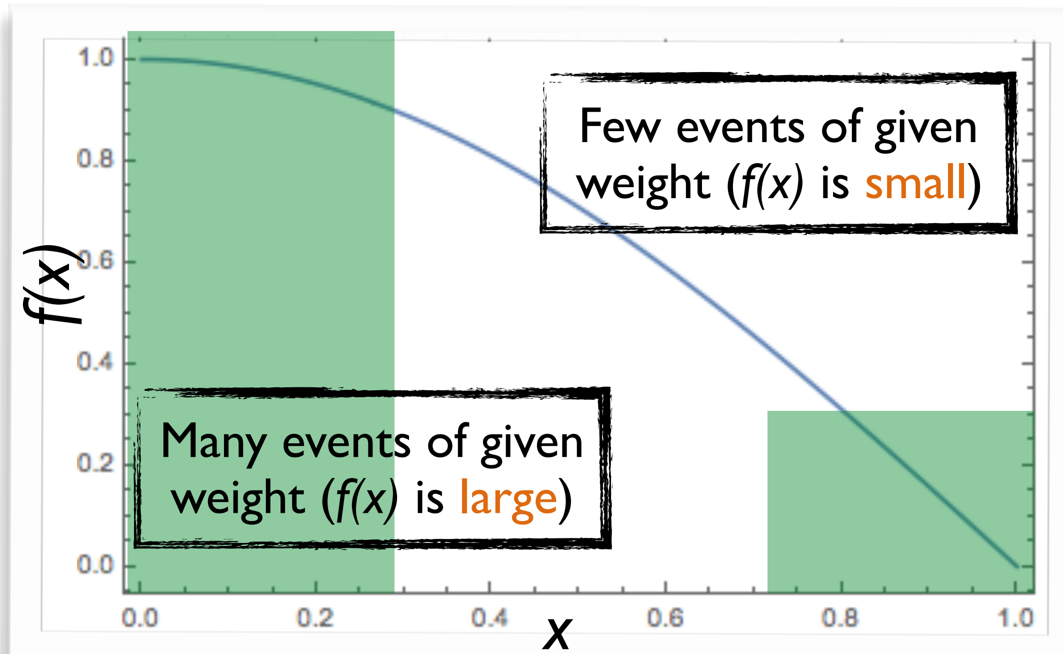
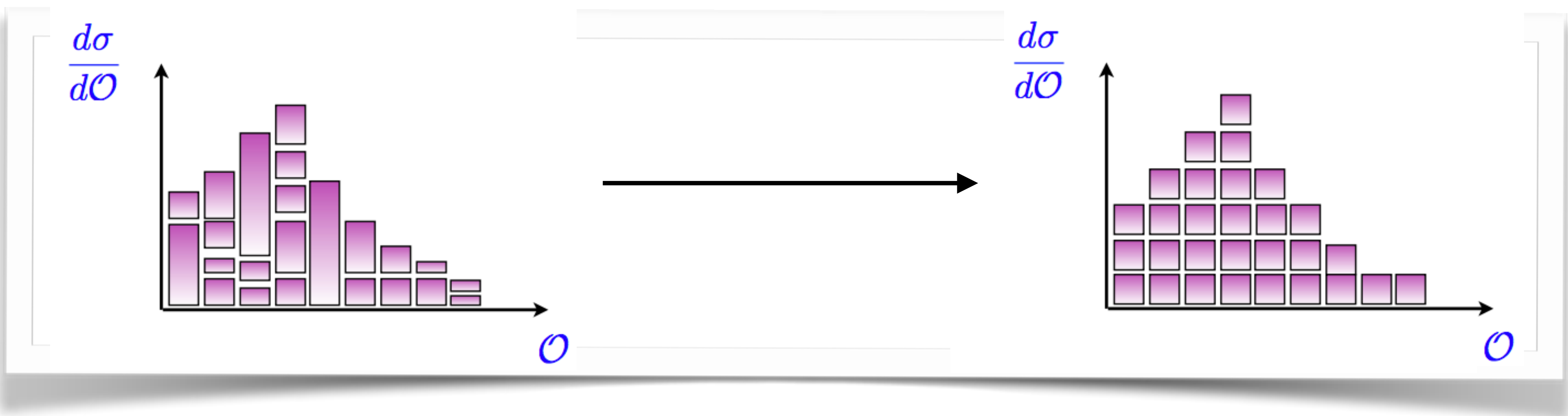


Unweighted events in practice

✦ Principle

- We calculate the maximum weight during the MC integration phase ω_{max}
- The desired number of events and the total rate yield the average weight $\langle\omega\rangle$
- Each generated event is accepted with a probability $\omega(\Phi)$ / ω_{max}
- Each event is assigned the weight $\langle\omega\rangle$



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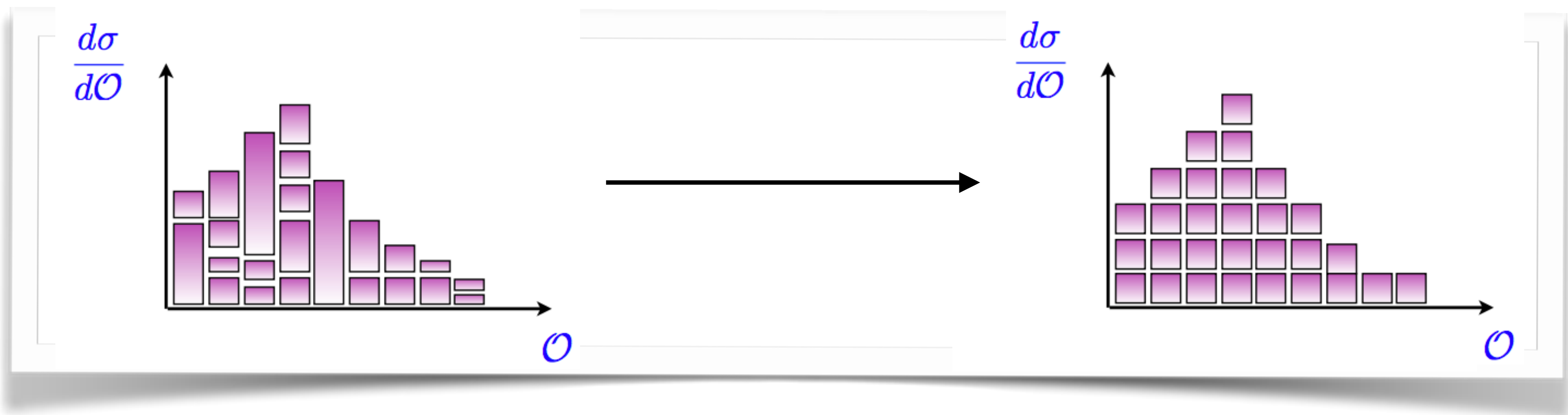
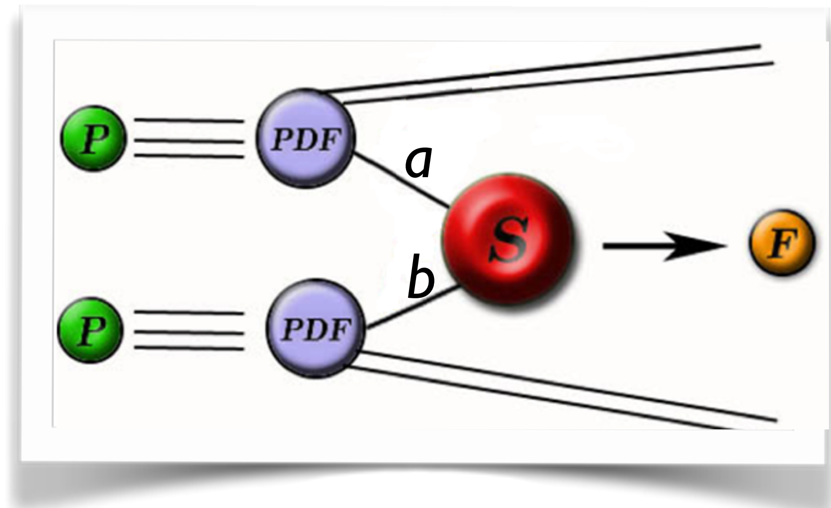


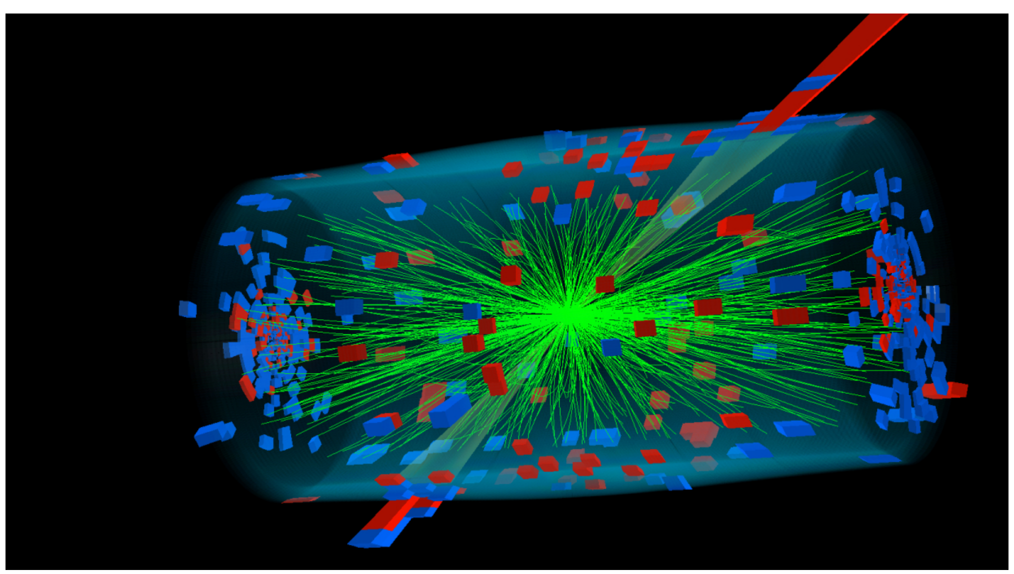
✦ Requirements

- The integrand has to be bounded from above (ω_{max} must exist)
- The integrand has to be positive-definite (can however be by-passed)

Summary so far

- ✦ Matrix elements can be generated automatically for any model
- ✦ Monte Carlo integration
  - ✤ Cross section
  - ✤ Unweighted events distributed as in nature
  - ✤ Any observable can be extracted
  - ✤ Selection cuts can be imposed
- ✦ Automated codes exist
  - ✤ MADGRAPH5_aMC@NLO, SHERPA, WHIZARD, *etc.*



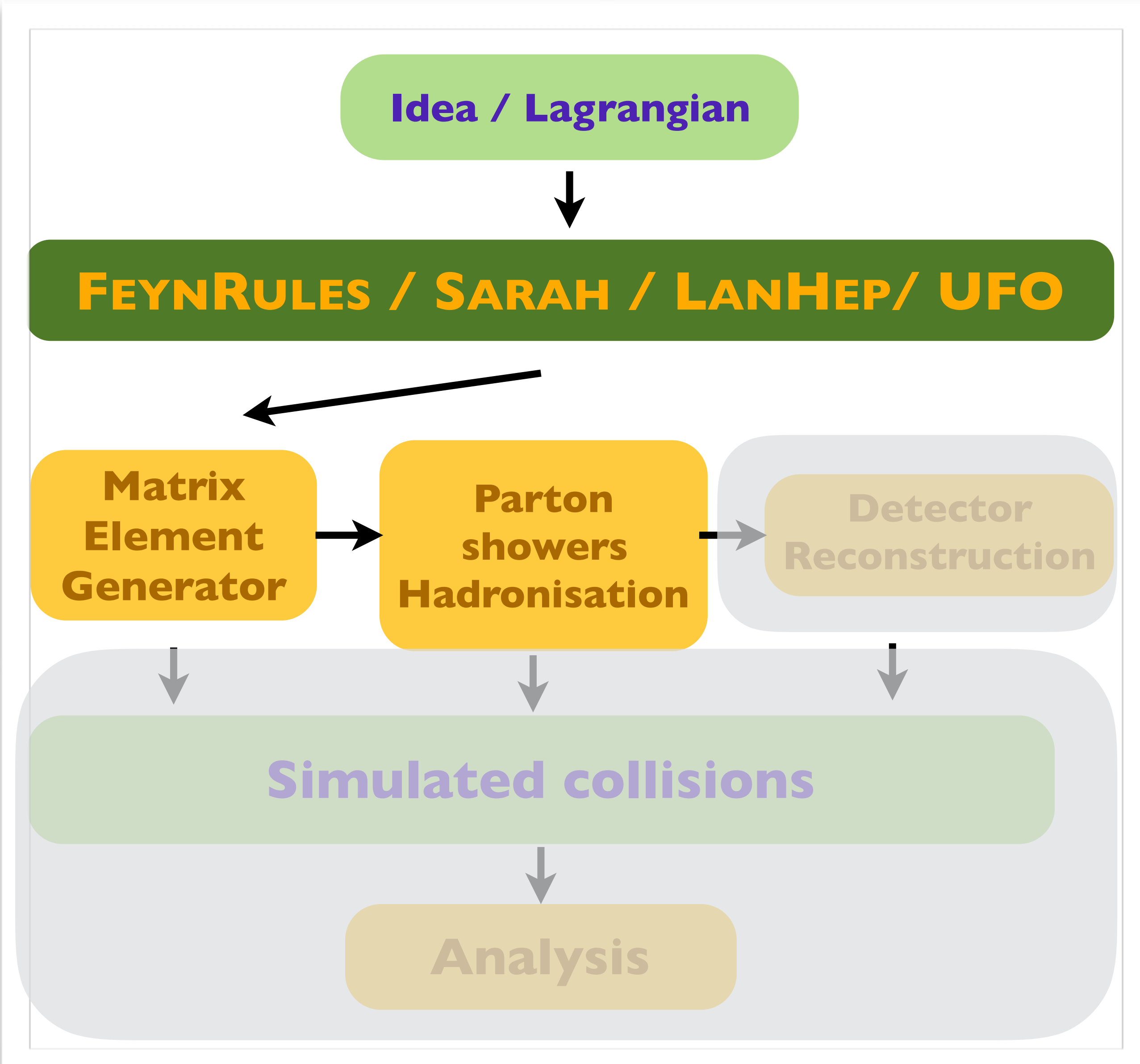


Still far from a realistic LHC collision...

Outline

The simulation chain - step 3

✦ Tools connecting an idea to simulated collisions

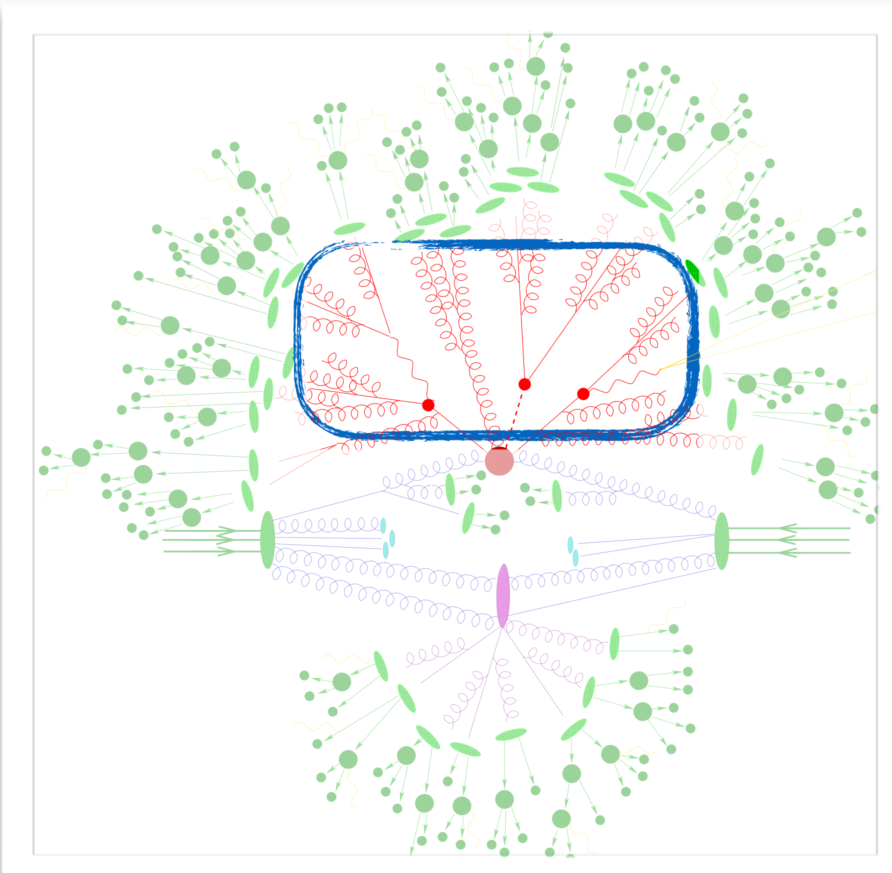


✤ QCD environment

★ Parton showering

★ Hadronisation

★ Underlying event



Parton evolution - generalities

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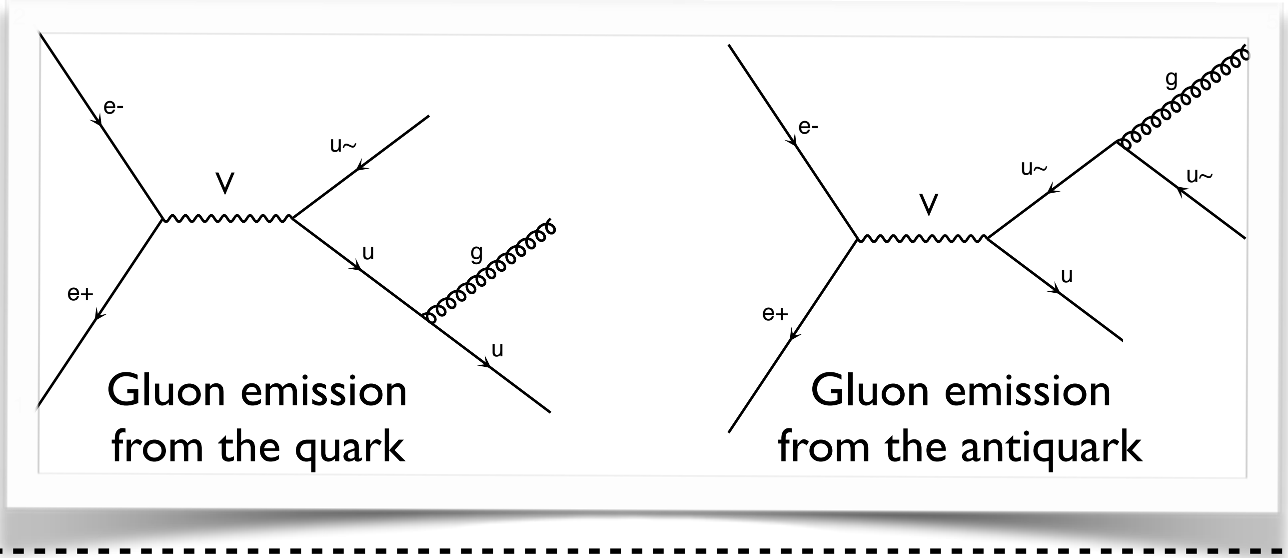
- ✦ Highly energetic coloured particles radiate
  - ✤ Each parton is dressed with an arbitrary number of partons (multiple radiation)
    - ➢ Radiated partons also radiate
  - ✤ One ends up with a cascade of radiations ➢ parton showers

QCD factorisation in the infrared

✦ Toy process: $e^+ e^- \to u \bar{u} g$



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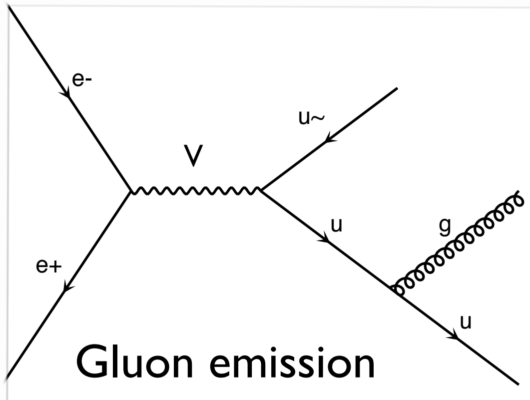
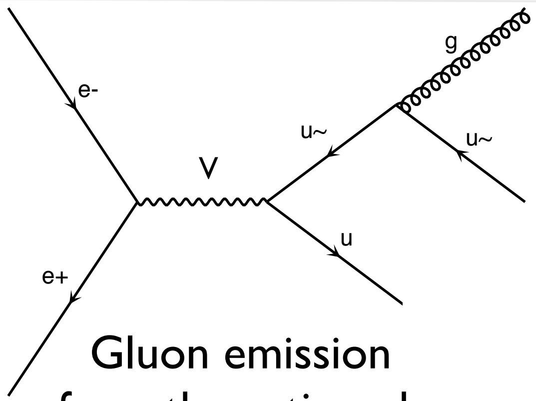


Gluon emission from the quark



Gluon emission from the antiquark

$$\mathrm{d}\sigma_{2\to 3} \propto C_F \frac{\alpha_s}{2\pi} \frac{2}{\sin^2\theta} \frac{1+z^2}{1-z} \,\mathrm{d}\cos\theta \,\mathrm{d}z$$

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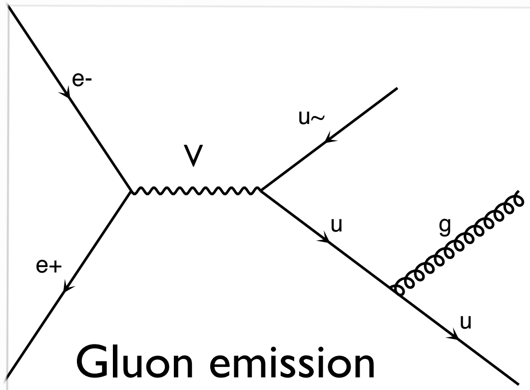

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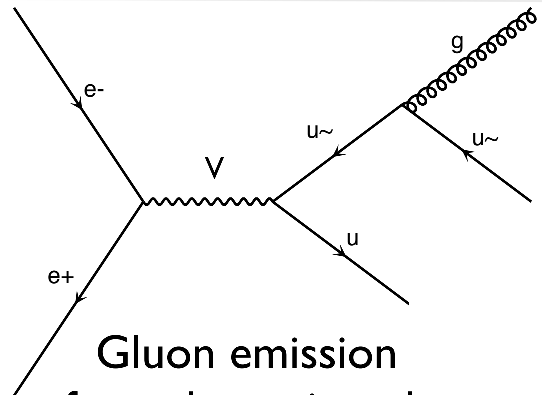

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  - → Divergent in the soft limit (z ~1)

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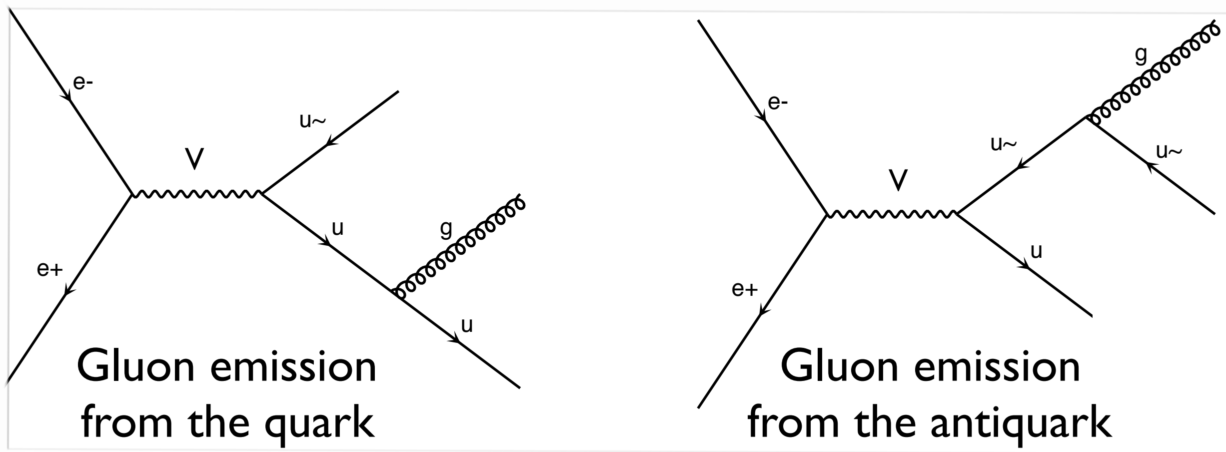

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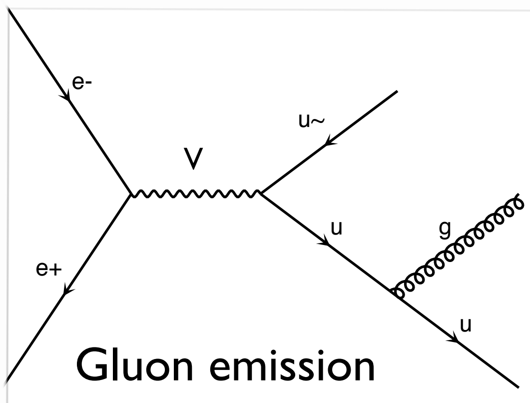

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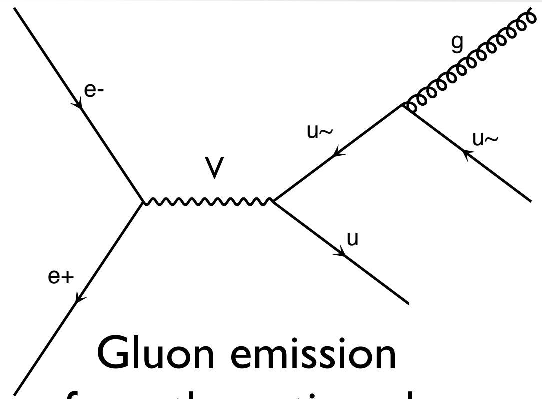

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$$= \frac{d\theta^2}{\theta^2} + \frac{d\bar{\theta}^2}{\bar{\theta}^2}$$

Separation of q/q̄ collinear emissions

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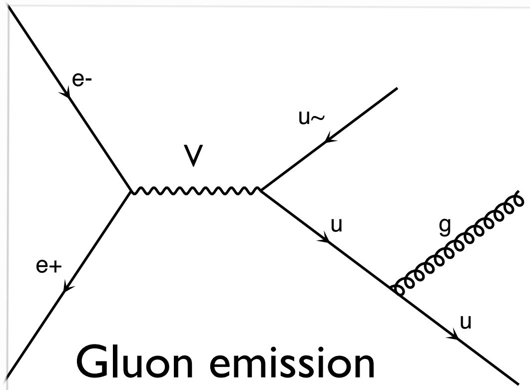

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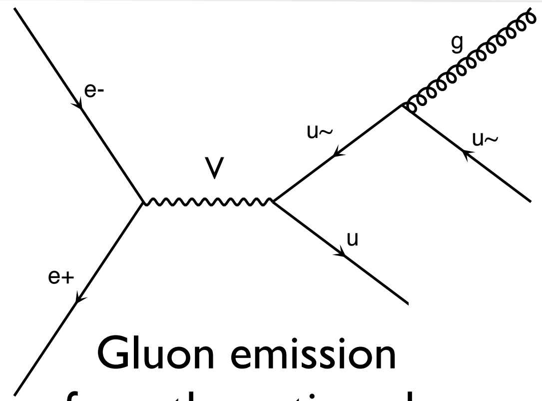

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✦ QCD radiation can be factorised (in the soft and collinear limit)

$$d\sigma_{2\to 3} \propto \sigma_{2\to 2} \sum_{i=q,\bar{q}} C_F \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} \frac{1+z^2}{1-z} \, dz$$

Generalisation: factorisation formula

- In the collinear limit, QCD emission factorises and is universal
  - Small angle emission unresolved (long time scales)
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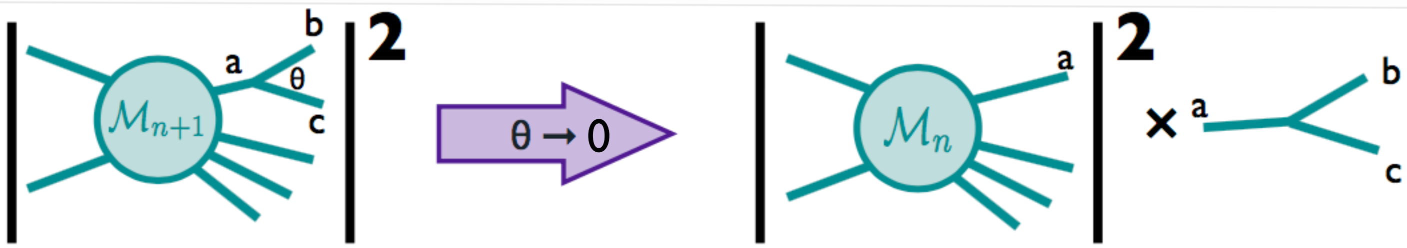

$$\mathrm{d}\sigma_{n+1} \propto \sigma_n \sum_{\text{partons}} \frac{\mathrm{d}t}{t}\, \mathrm{d}z\, \frac{\alpha_s(t)}{2\pi}\, P_{ab}(z) \qquad \text{with} \qquad \frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}p_T}{p_T^2} = \frac{\mathrm{d}q^2}{q^2} \equiv \frac{\mathrm{d}t}{t}$$

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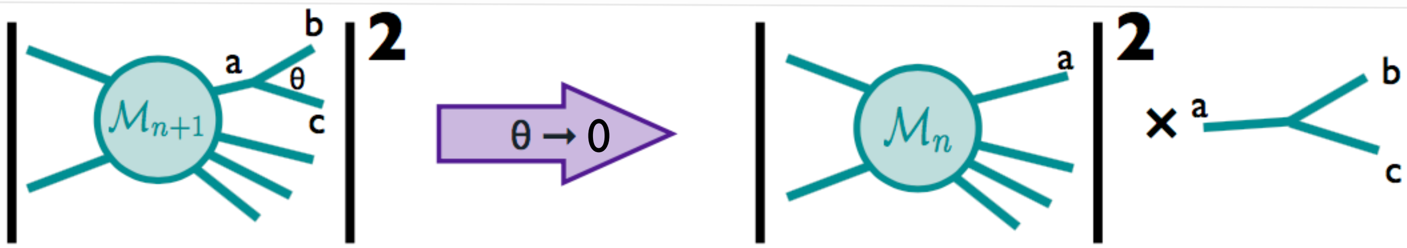

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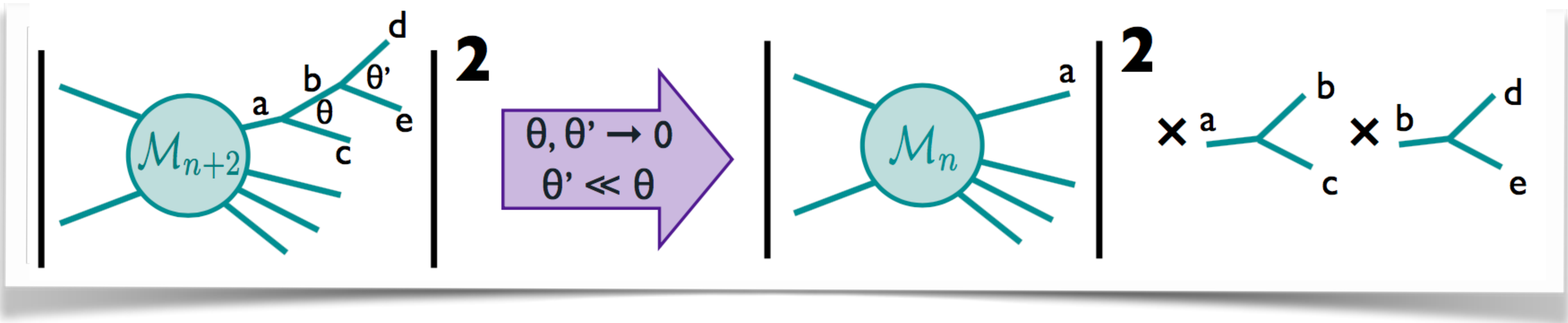
- ✤ The strong coupling is evaluated at the scale *t*
  - ★ *t* is the evolution variable (hardness of the branching, vanishes in the collinear limit)
  - ★ *t* controls the collinear behaviour
- ✤ $P_{ab}(z)$ consists in the QCD splitting kernels
  - ★ z controls the soft behaviour
  - ★ Universal resummation of their higher-order corrections

Further generalisation: multiple emission

✦ We iteratively add 2 emissions



✤ $\theta' \ll \theta$: successive emission are ordered (or $t \ll t'$)

Further generalisation: multiple emission

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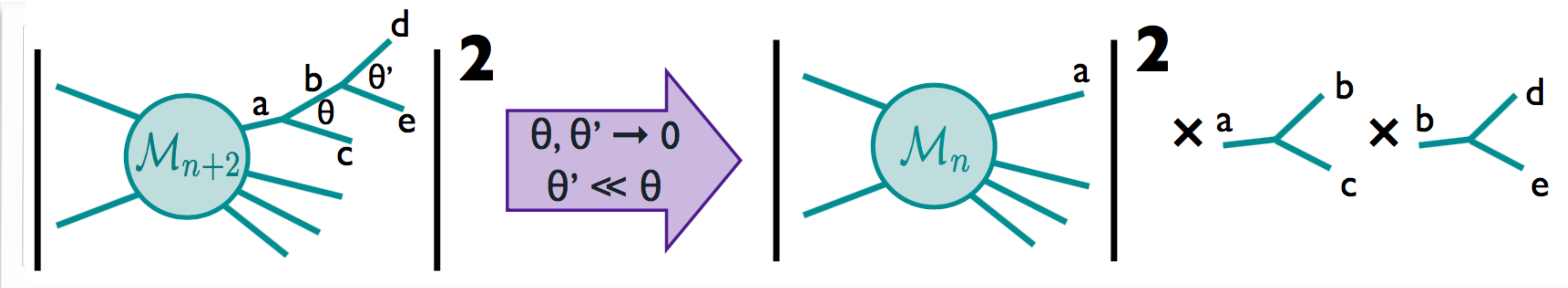

✤ $\theta' \ll \theta$: successive emission are ordered (or $t \ll t'$)

✤ Leading contribution to the $(n+2)$-body matrix element

$$\begin{aligned} \mathrm{d}\sigma_{n+2} &\propto \sigma_{n+1} \sum_{a,b} \frac{\mathrm{d}t}{t}\ \mathrm{d}z\ \frac{\alpha_s(t)}{2\pi}\ P_{ab}(z) \\ &\propto \sigma_n \sum_{a,b,b'} \left[\frac{\mathrm{d}t}{t}\ \mathrm{d}z\ \frac{\alpha_s(t)}{2\pi}\ P_{ab}(z)\right] \times \left[\frac{\mathrm{d}t'}{t'}\ \mathrm{d}z'\ \frac{\alpha_s(t')}{2\pi}\ P_{bb'}(z')\right] \end{aligned}$$

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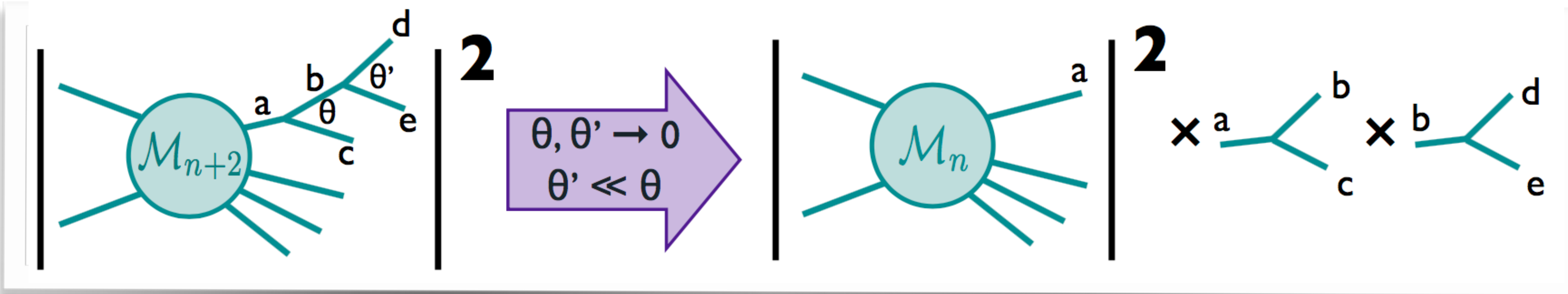

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✦ Iterative sequence of ordered emissions

- The $n+1$ emission independent of the history → Markov chain (no interferences)
- Leading contribution to the (n+k)-emission configuration: $\theta_1 \gg \theta_2 \gg \theta_3 \gg \ldots$

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- ✦ Parton showers: building a radiation history
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1 &= P_{\text{no emission}}(t+\mathrm{d}t) + P_{\text{emission}}(t+\mathrm{d}t) \\
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- ✤ Solution (probability a parton does not radiate between t_1 and t_2):

$$
\Delta_a(t_1, t_2) \equiv P_{\text{no emission}}(t_1, t_2) = \exp\left[- \int_{t_1}^{t_2} \frac{\mathrm{d}t}{t} \sum_b \int \mathrm{d}z \frac{\alpha_s(t)}{2\pi} P_{ab}(z) \right]
$$

Parton showers: the algorithm

✦ Splitting kernels and the Sudakov yield an evolution equation

$$\phi_a(t,t_0) = \Delta_a(t,t_0) + \sum_b \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \mathrm{d}z \; \Delta(t,t') \frac{\alpha_s(t')}{2\pi} P_{ab}(z) \; \phi_b(t', z t_0) \phi_c(t', (1-z)t_0)$$

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✦ The parton shower algorithm

- Start: a parton a at a scale t_0
- We generate an emission scale t_I according to the Sudakov probability $\Delta_a(t_0,t_I)$
  - ★ If $t_I < t_{\rm cut}$, the algorithm stops ($t_{\rm cut}$ ≡ breaking down of perturbative QCD)
  - ★ If $t_I > t_{\rm cut}$, we generate z_I according to $P_{ab}(z)$ → one extra final-state parton
- Iteration until stops for all partons

Limitations and improvements

✦ Limitations / improvements

- ✤ Parton showers ≡ collinear approximation of the leading corrections
  - ★ Matching with the hard-scattering matrix elements
  - ★ Multiparton matrix element merging

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★ Matrix exponentiation ≡ resummation (improved Sudakov)

★ Higher-order corrections (not trivial)

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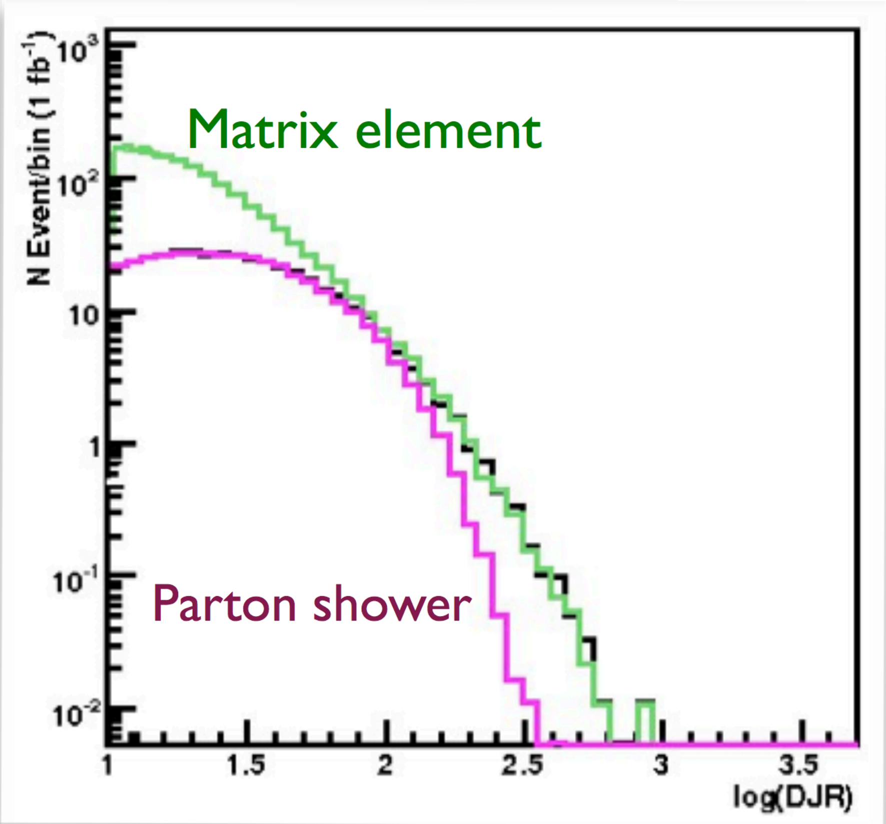
- Fixed-order calculations
- Full treatment of spin and colour
- Technical limit on the multiplicity
- Valid for hard and well-separated partons



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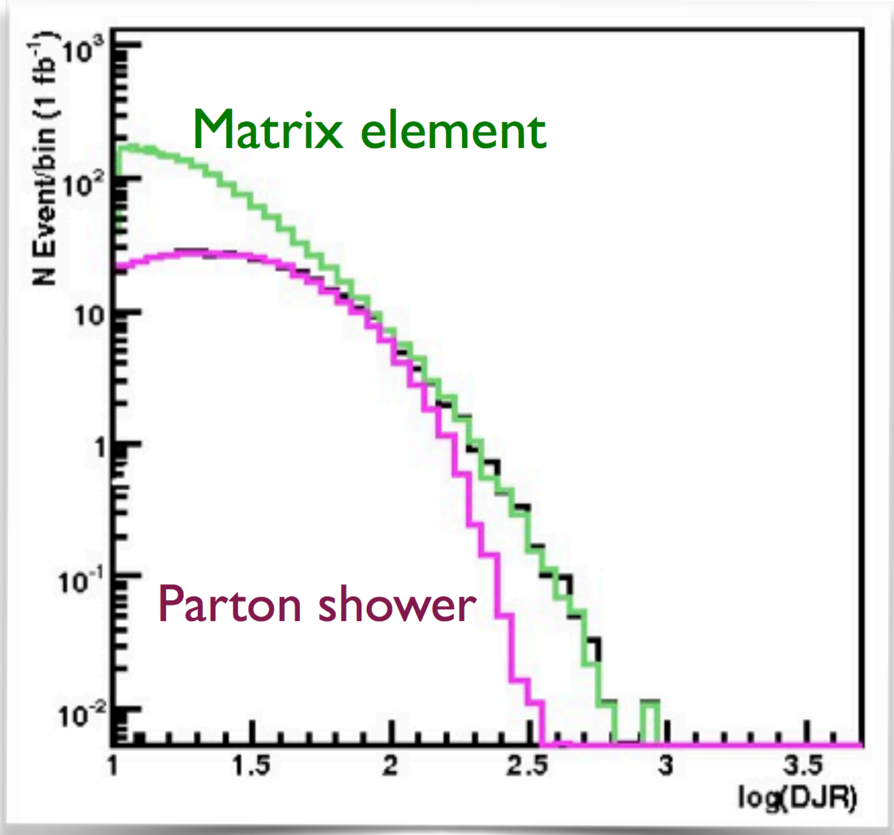

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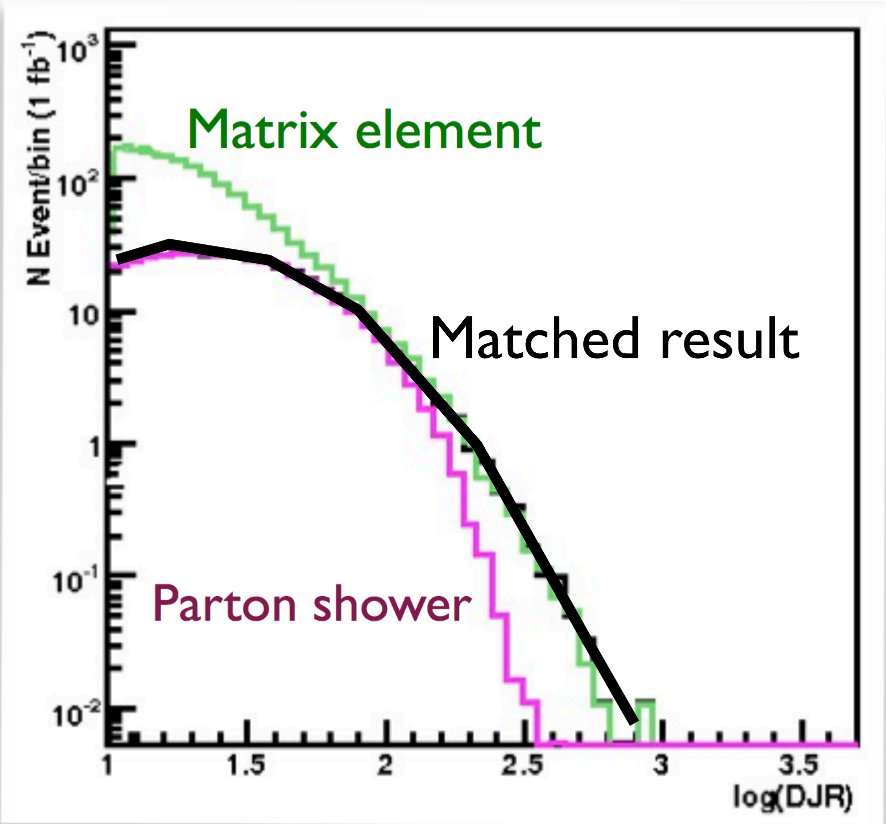

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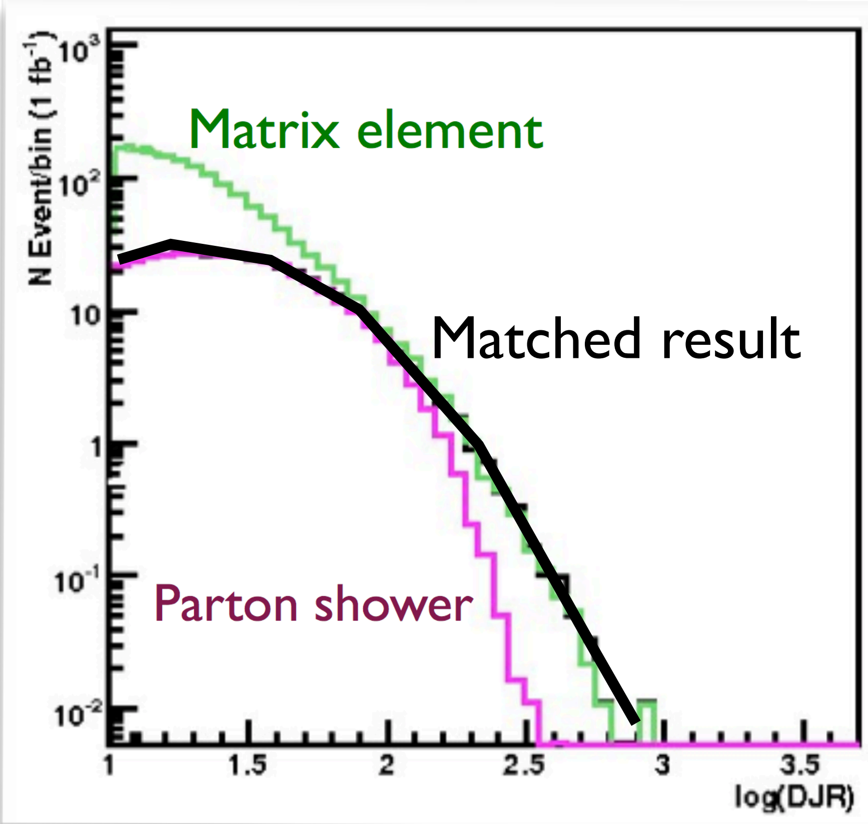

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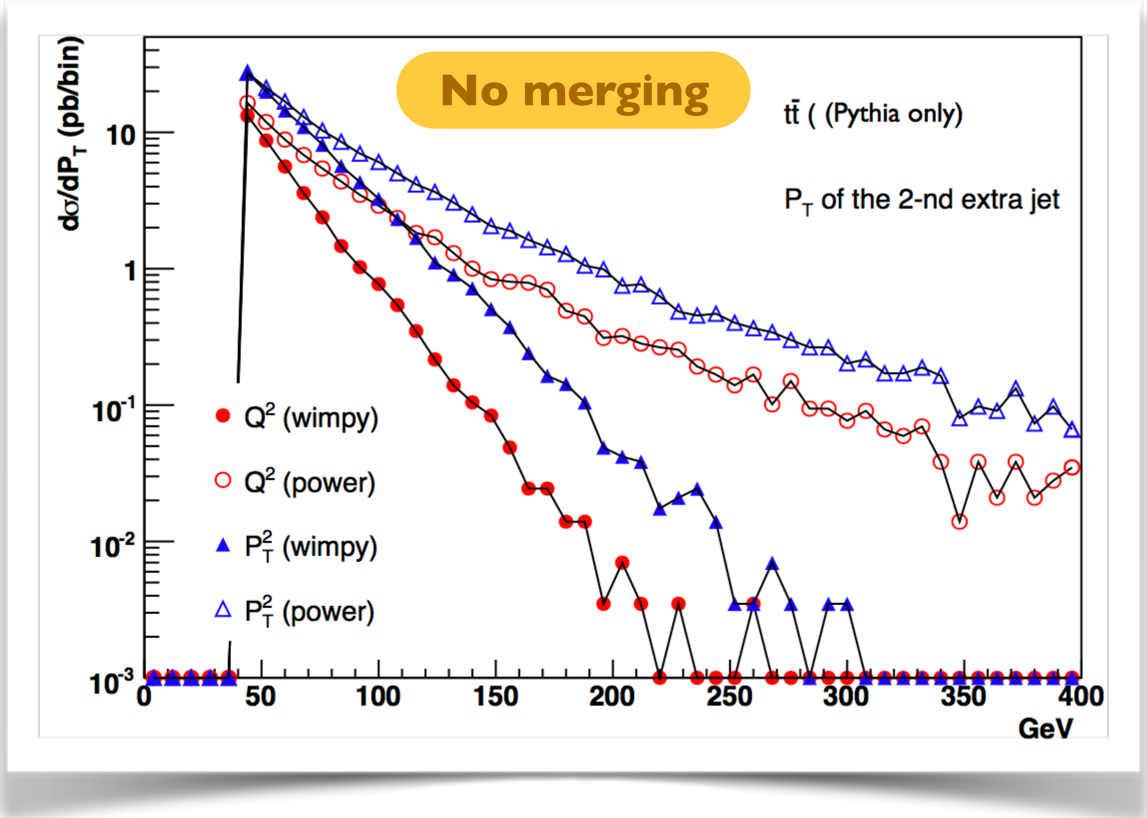
✦ Multipartonic matrix element merging (prescription)

- ✤Matrix elements containing 0, 1, 2, … *N* extra partons
- ✤Parton showering of each event
- ✤Removal of any double counting

Matching / merging at work

✦ Different shower configuration leads to different predictions

✤ Loss of predictive power (especially in the hard regime)



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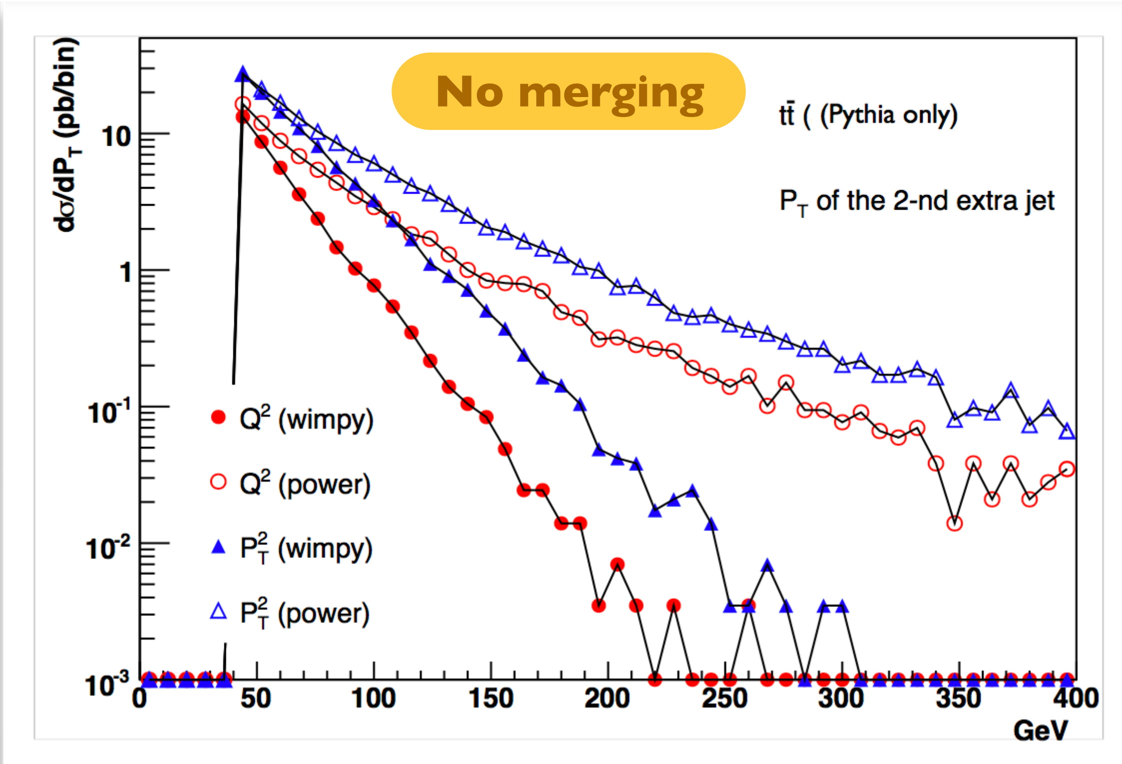
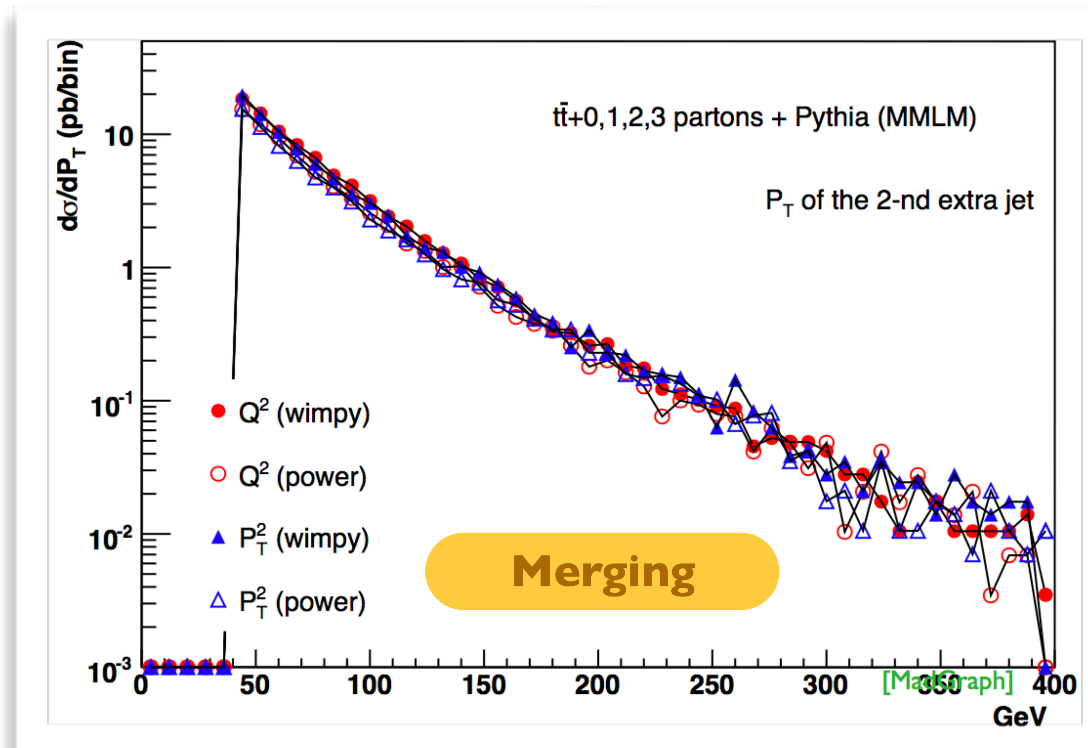
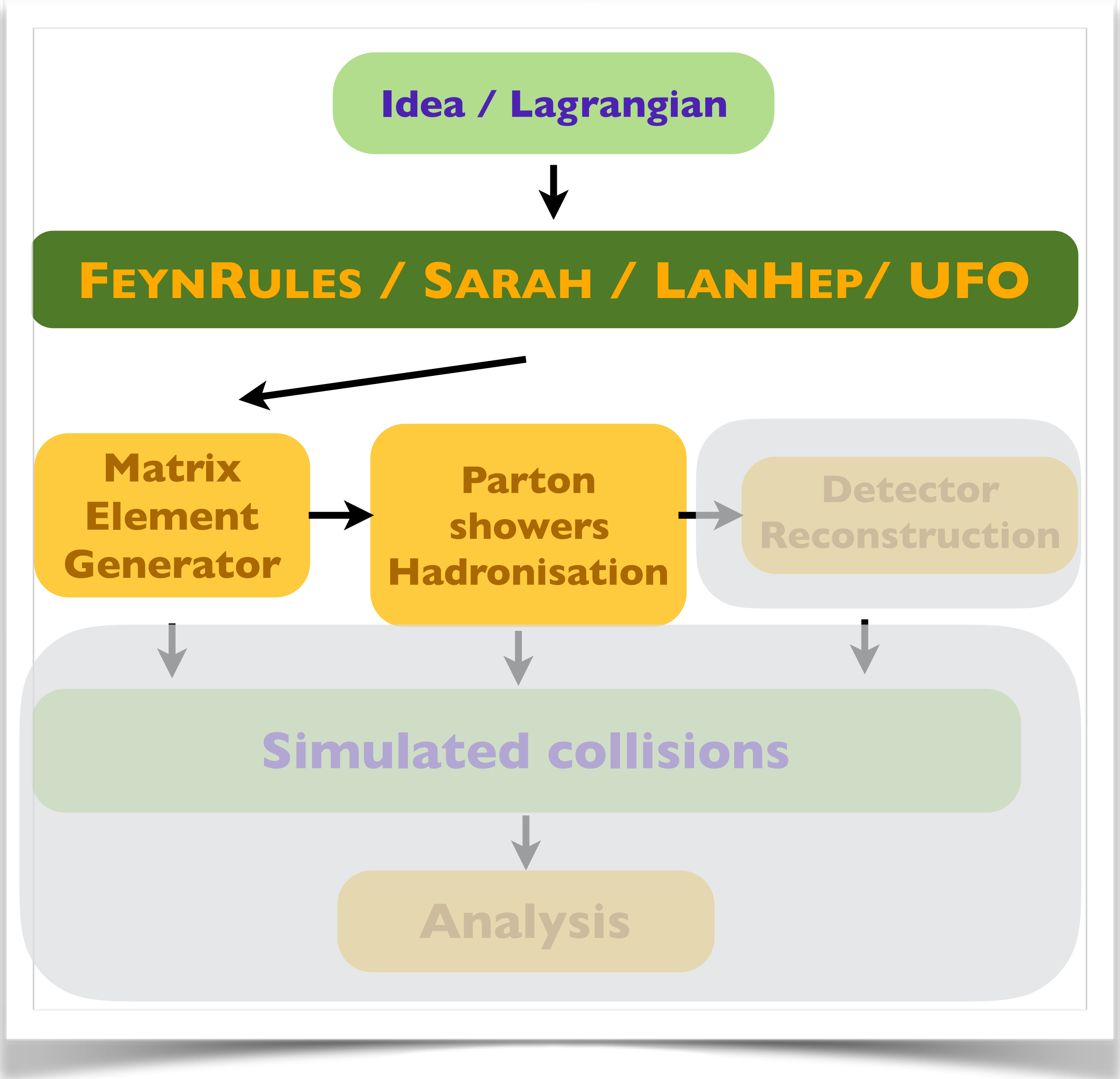


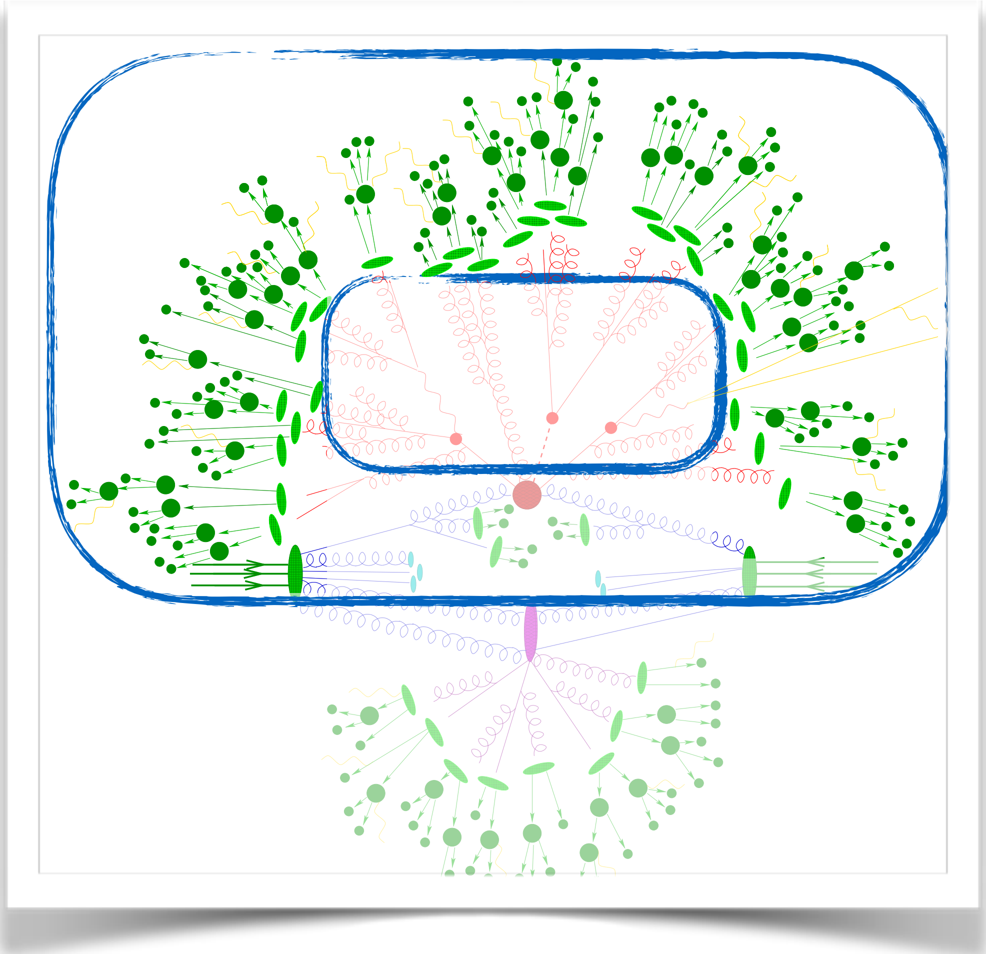


The simulation chain - step 4

✦ Tools connecting an idea to simulated collisions



✤ QCD environment
- ★ Parton showering
- ★ Hadronisation (and hadron decays)
- ★ Underlying event



Hadronisation

✦ Generalities

✤ Perturbative QCD breaks down at scales around 1 GeV

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- ✤ The Lund string model **[Andersson, Gustafson, Ingelmanm & Sjöstrand (PR'83)]**
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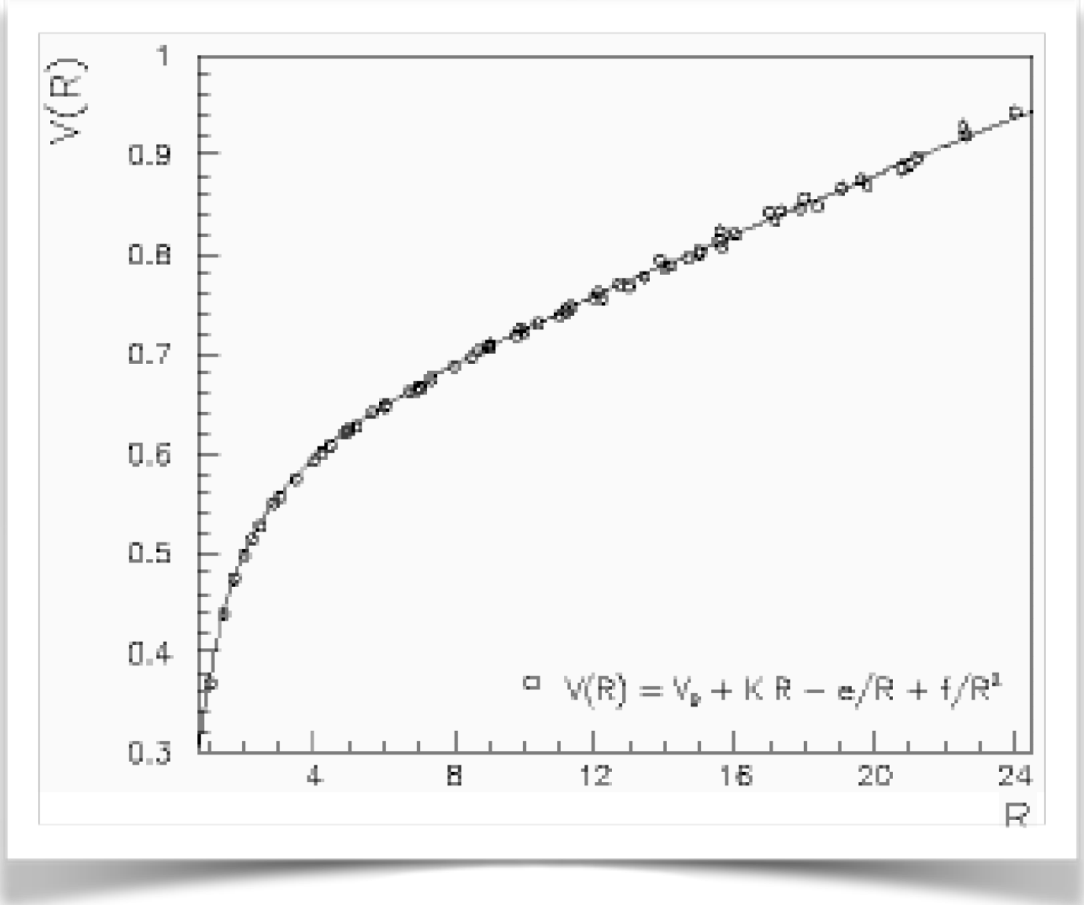
✦ Hadron decays

- ✤ Thousands of different channels
- ✤ Based on form factors
- ✤ Large uncertainties (the sum of the branching fractions may not be 1)
- ✤ Significant impact on the event shape

The Lund string model

✦ The QCD potential of a $q\bar{q}$ pair grows linearly with the distance

- ✤ As calculated by lattice QCD
- ✤ As calculated from quarkonia masses



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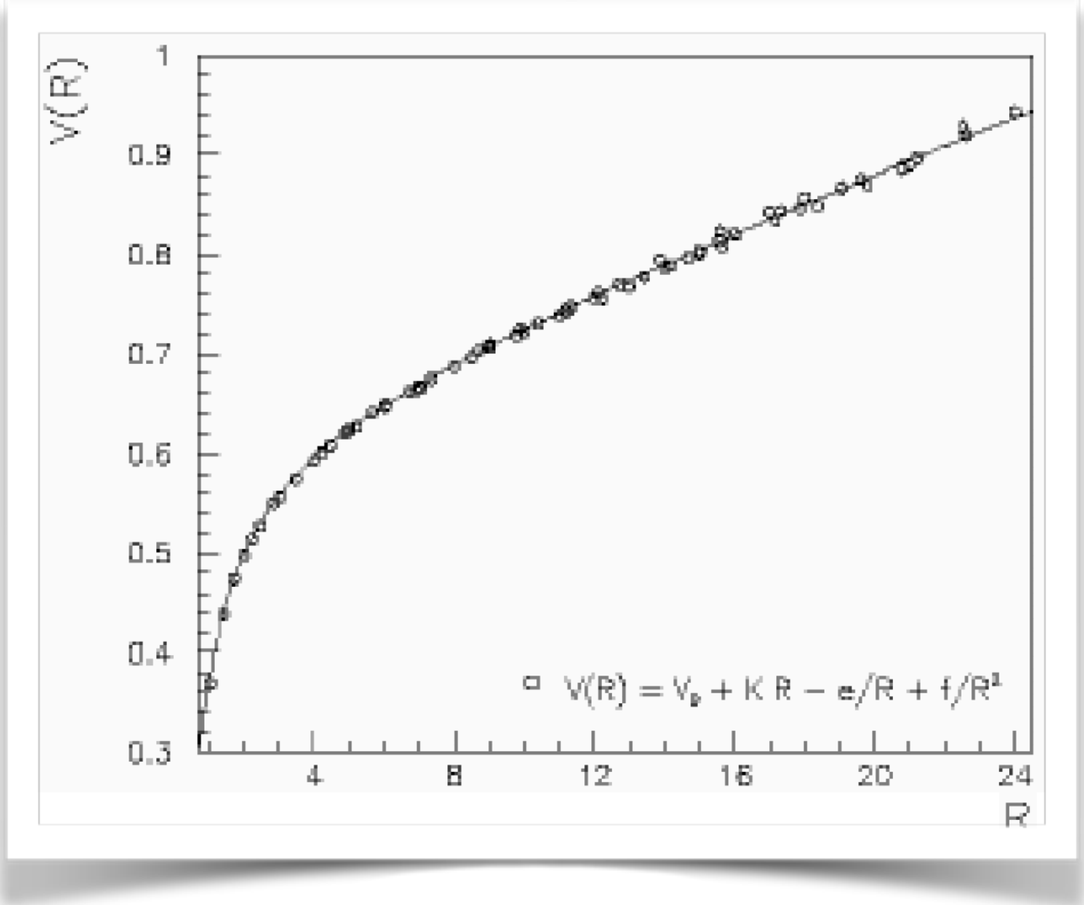
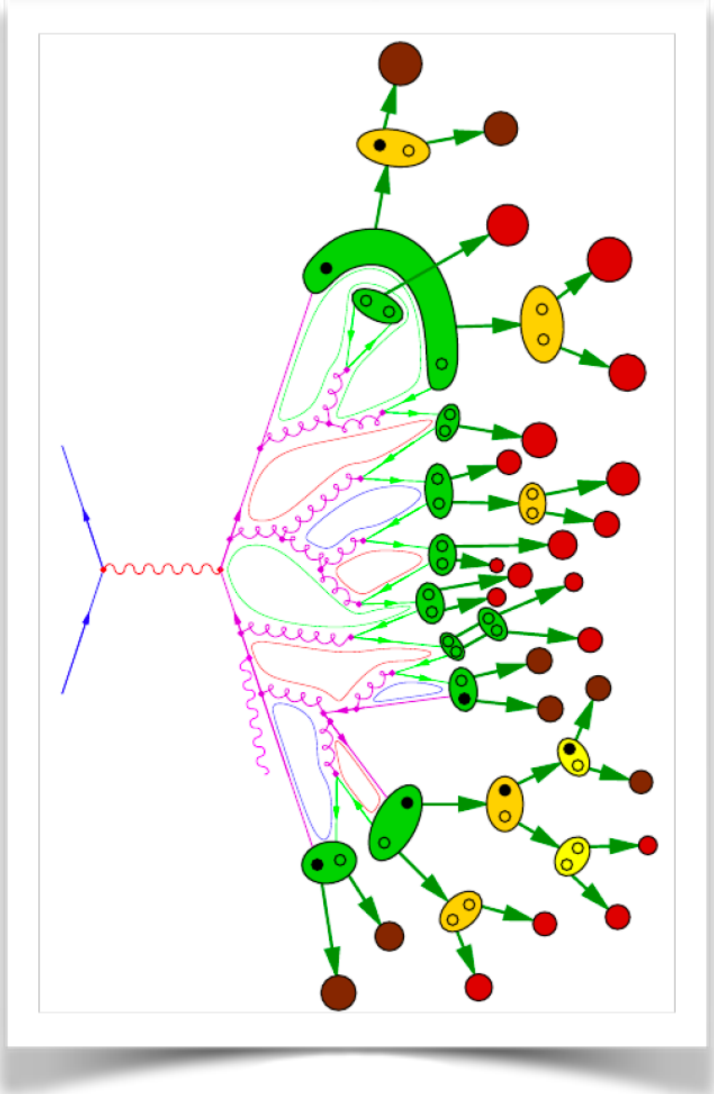


- Linear growth → uniform string tension
  - Connects the colour charges
  - At large distance
    - More favorable to create a new pair
    - The string is broken into hadrons

The cluster model

✦ Relies on pre-confinement

- ✤ Colour singlets formation
  - → From partons close in phase space
  - → Non-perturbative gluons splitting in $q\bar{q}$ pairs
- ✤ Clustering of colour-adjacent partons



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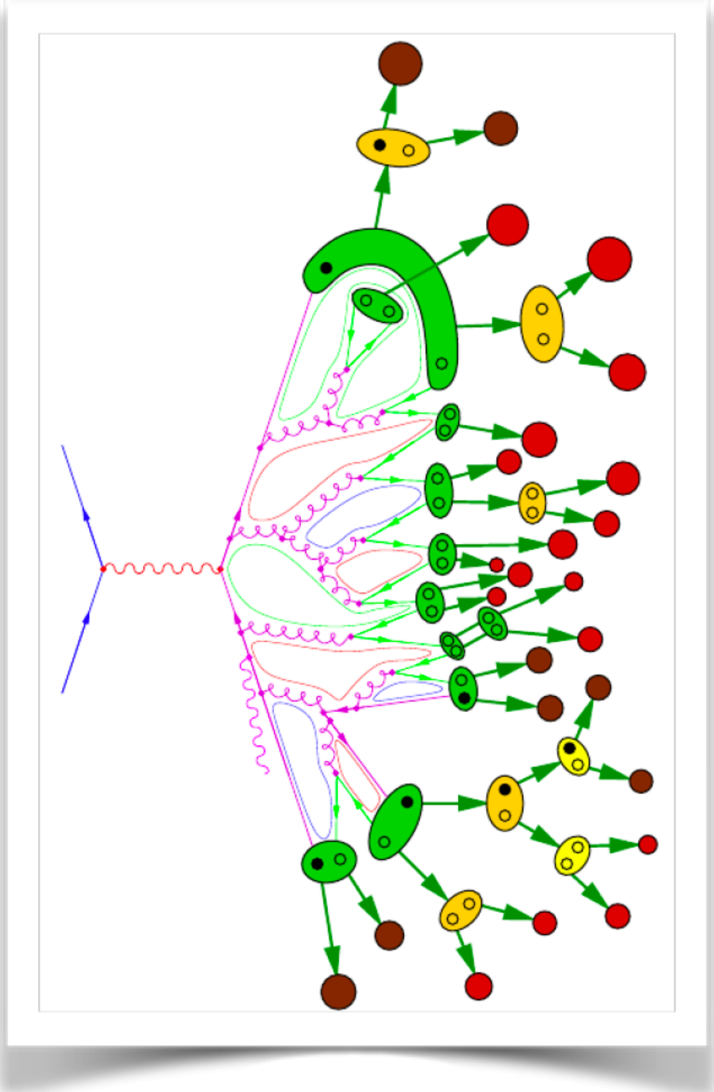
- Heavy clusters decay into light clusters
- Clusters decay into / radiate hadrons



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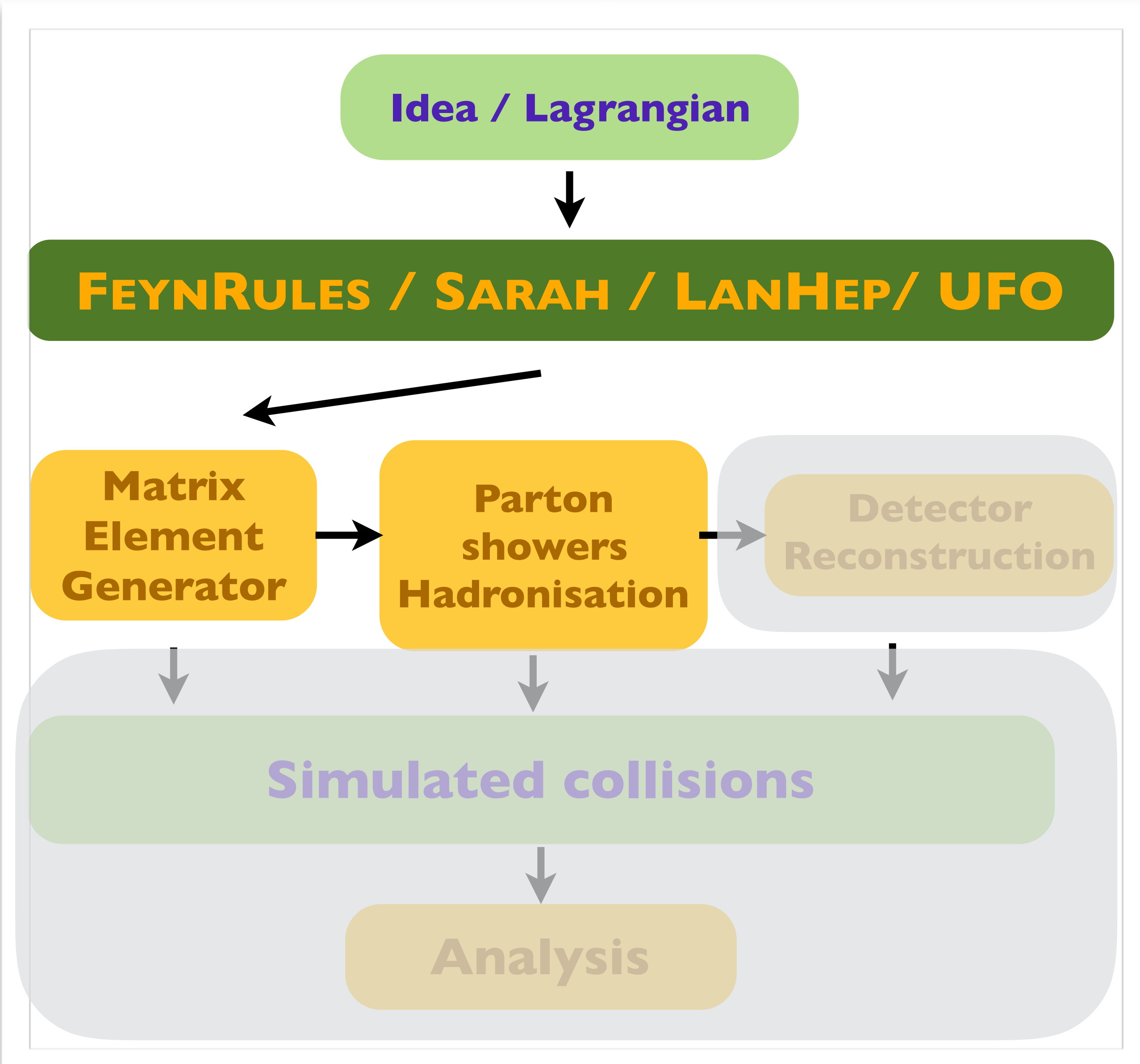

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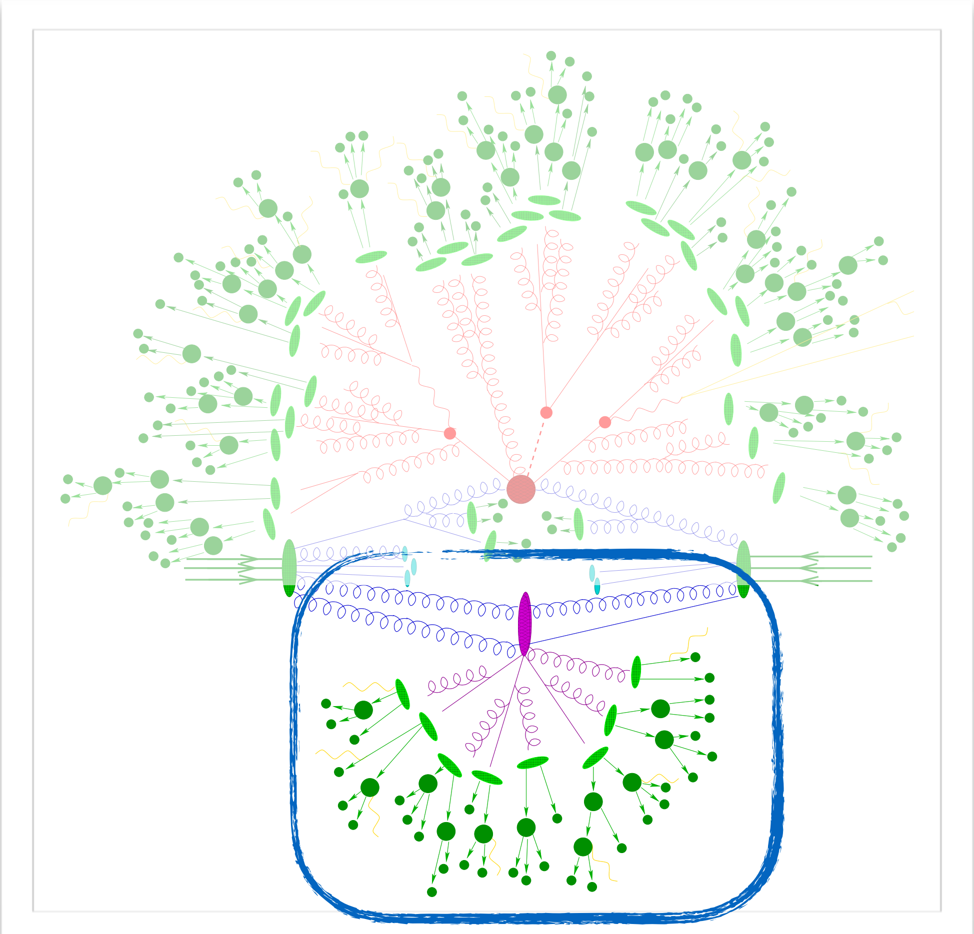

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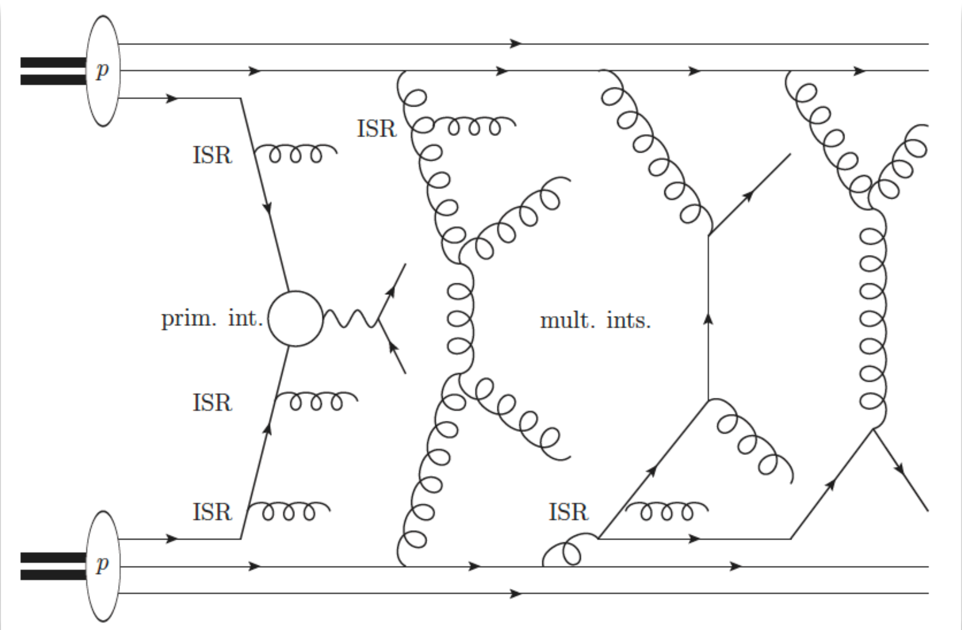
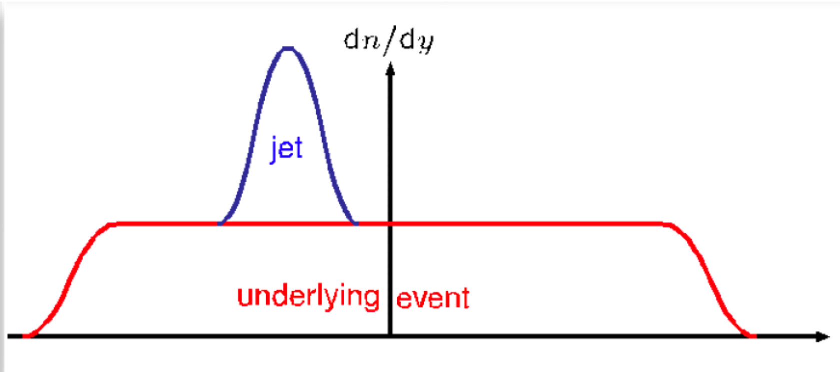


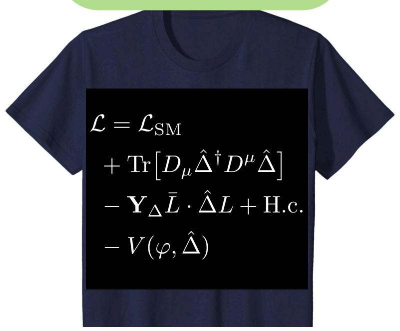


Outline

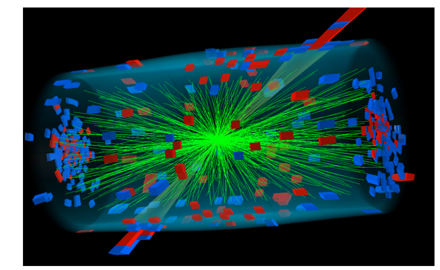
1. A comprehensive approach for Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
4. Parton showers, hadronisation & underlying event
5. **Summary**

Summary

Idea
Lagrangian



→ Chain of tools → Simulated collisions → Other tools → Event analysis



- ✦ Collider event simulation is a complex process
  - ✤ Nature allows us to break it into factorisable pieces → simulations step-by-step
    - ★ Hard scattering
    - ★ Parton showers
    - ★ Hadronisation
    - ★ Underlying event
    - ★ Detector simulation (not covered here)

Automated tools to handle each piece, for any model