Quantum Computation II

Three families of algorithms

Gate circuits

Search - Grover
QFT - Shor
Deutsch

Annealing

Direct Annealing
Adiabatic Evolution

Variational

Autoencoders
Eigensolvers
Classifiers

Quantum Algorithms

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Adiabatic evolution (Farhi,Goldstone,Gutmann)

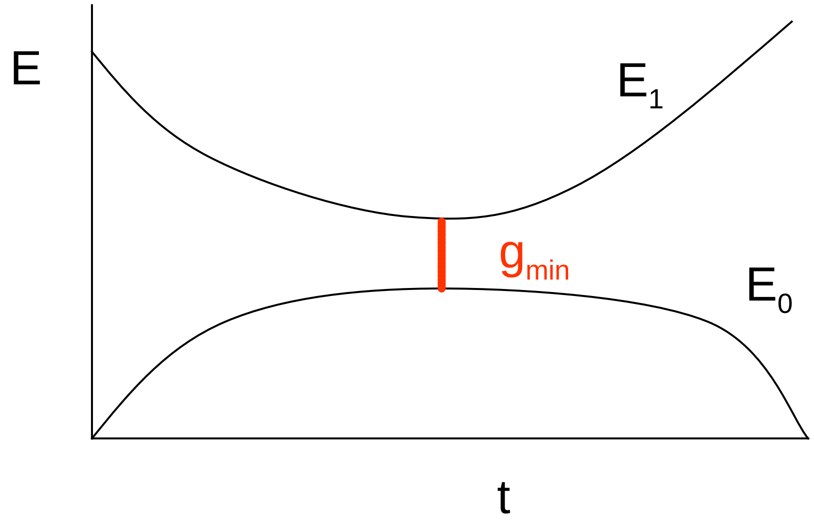
$$H(s(t)) = (1-s(t))\, H_0 + s(t)\, H_p$$

$s(0)=0 \xrightarrow{t} s(T)=1$

Inicial hamiltonian — Problem hamiltonian

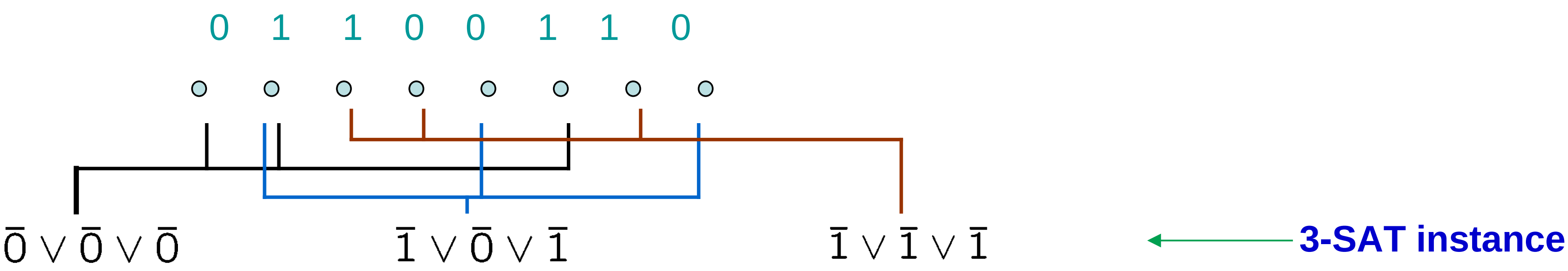
Adiabatic theorem:

$$|\langle E_0; T|\psi(T)\rangle|^2 \geq 1-\epsilon^2 \qquad \text{if} \qquad \frac{\max|\frac{dH_{1,0}}{dt}|}{g_{min}^2} \leq \epsilon$$



$$T \sim \frac{1}{g_{min}^2}$$

Exact Cover



For every clause, one out of eight options is rejected

3-SAT is NP-complete
k-SAT is hard for k > 2.41
3-SAT with m clauses: easy-hard-easy around m=4.2n

Exact Cover

A clause is accepted if 001 or 010 or 100

Exact Cover is NP-complete

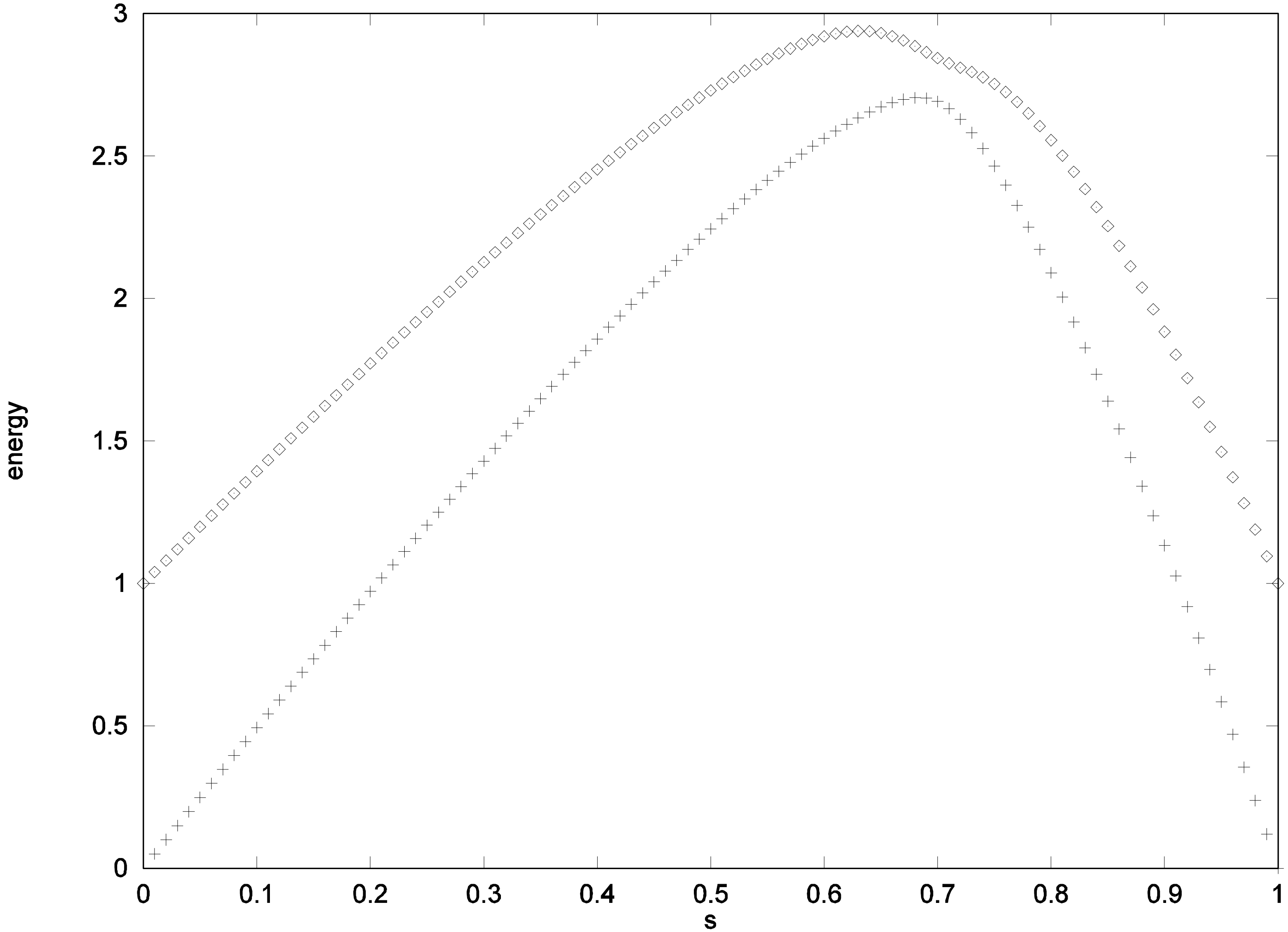
Mapping Exact Cover to a quantum computer

$$H_0 = \sum_i \frac{d_i}{2} \sigma_i^x$$

$$H_P = \sum_c H_{c(ijk)}$$

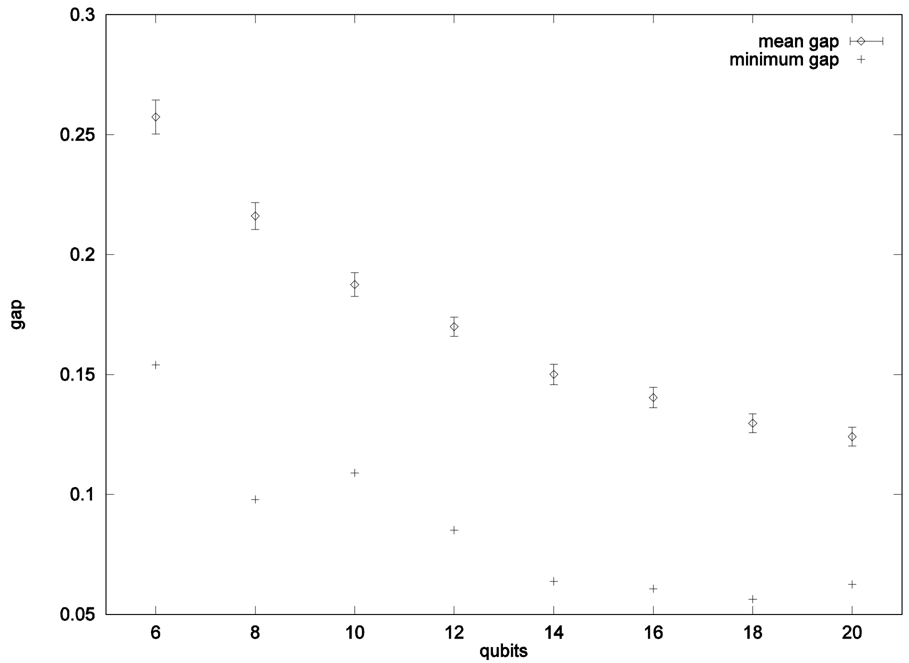
$$H_{c(ijk)} = \left(z_i + z_j + z_k - 1 \right)^2$$

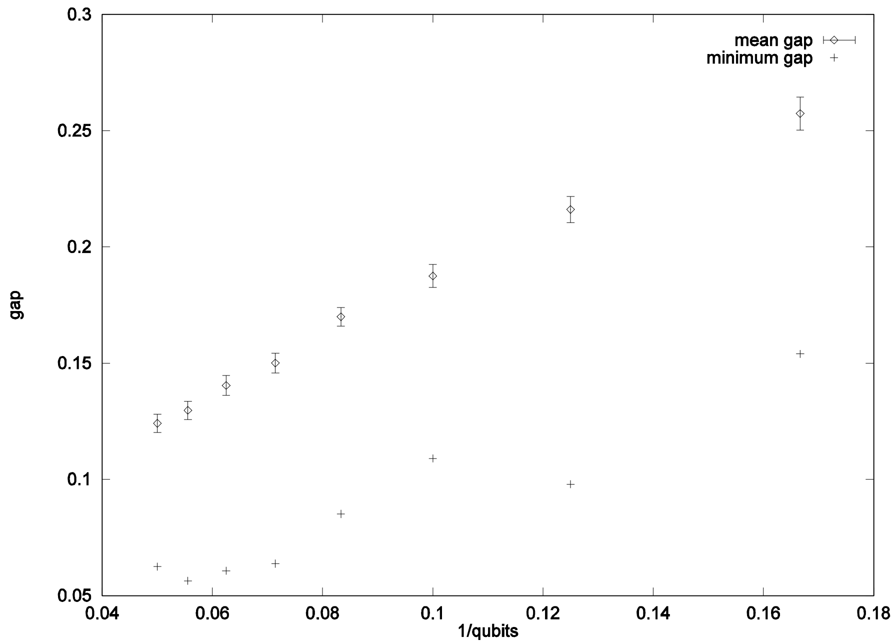
H_P is diagonal, quadratic, non nearest-neighbor (spin glass)



Typical gap for an instance of Exact Cover

R. Orús, JIL

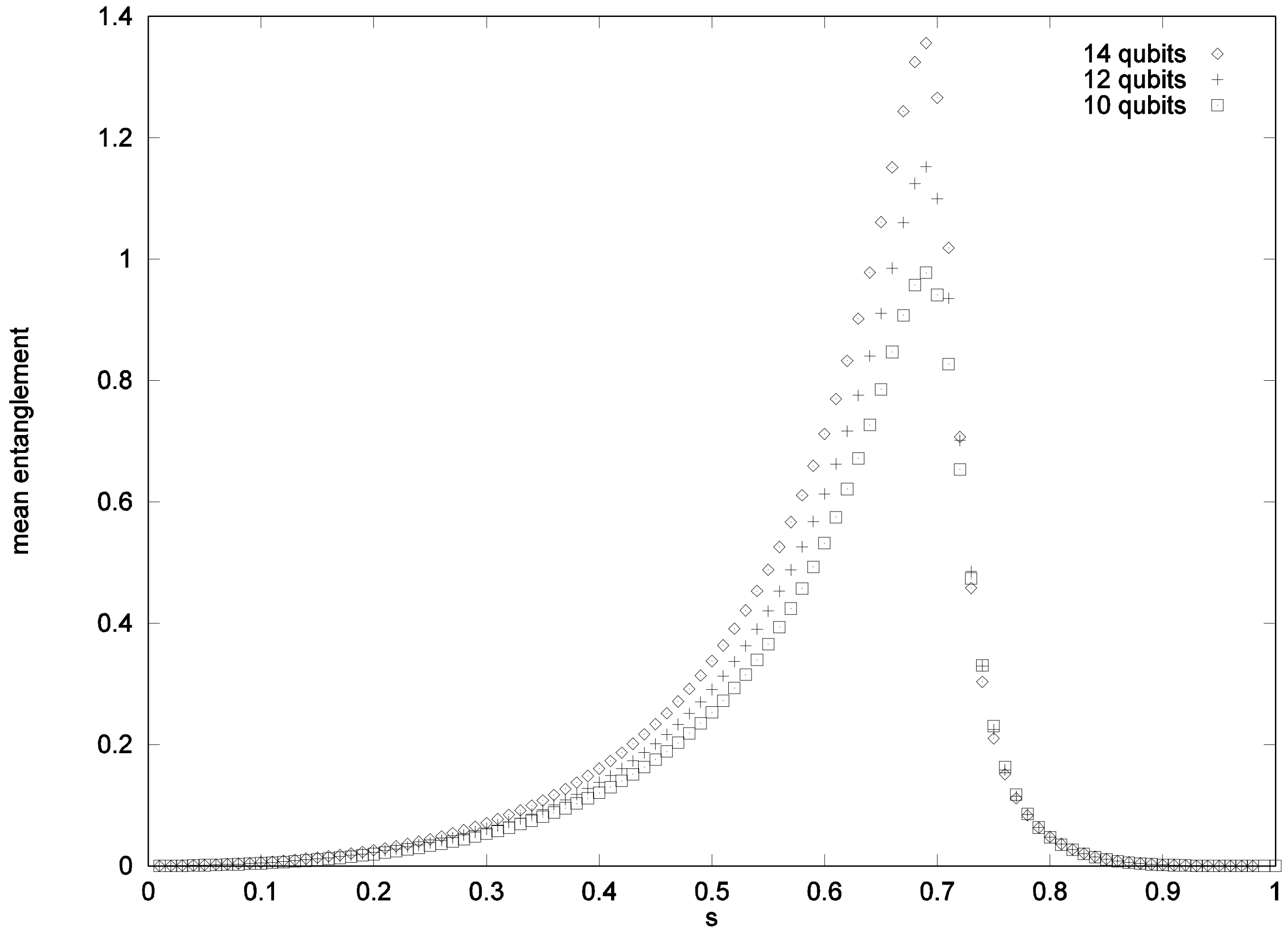




Scaling for averaged instances is consistent with gap ~ 1/n

NP is defined on the worst instance

Scaling of entropy for Exact Cover



A quantum computer passes nearby a dramatic quantum phase transition!

Adiabatic evolution can accommodate any optimization problem

Dwave can address QUBO problems

QUBO = Quadratic Unconstrained Binary Optimization problems

$$H_P = A_{ij} q_i q_j + B_i q_i$$

Ex: Qeens problem

$$H_P = \sum_{j \in cols} \left(1 - \sum_{i \in rows} q_{ij}\right)^2 + \sum_{i \in rows} \left(1 - \sum_{j \in cols} q_{ij}\right)^2 + \sum_{ij, kl\, diags} q_{ij} q_{kl}$$

Quantum Approximate Optimization Algorithm (QAOA)

Variational ansatz $\left|\psi(\beta,\gamma)\right\rangle$

$$\left|\psi(\beta,\gamma)\right\rangle = U(\beta_p)\,U(\gamma_p)\ldots U(\beta_1)\,U(\gamma_1)\left|\psi_0\right\rangle$$

$$U(\beta) = e^{i\beta H_0} \qquad U(\beta) = e^{i\gamma H_P}$$

Trotterization of adiabatic evolution

Quantum Algorithms

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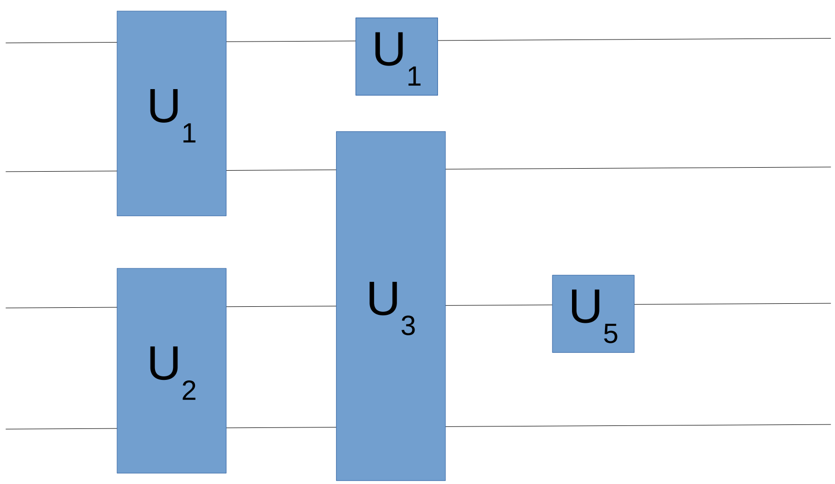
Learn from results

e.g. Supervised learning
Unsupervised learning
Reinforcement learning

Q learning

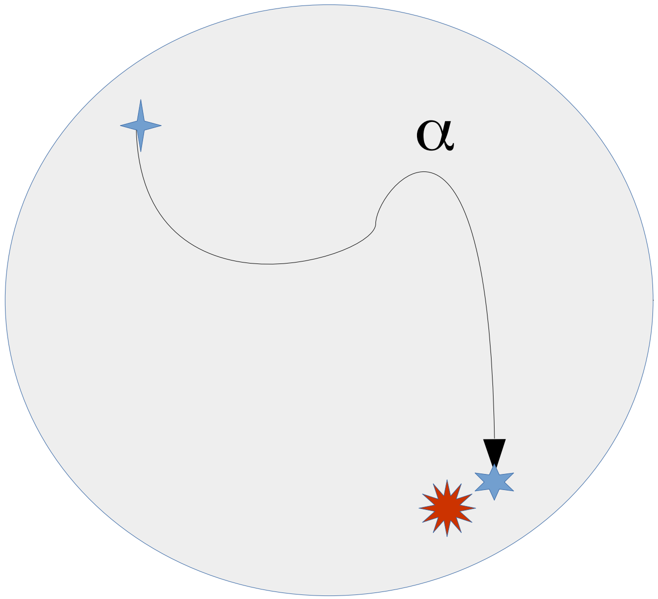
Variational circuits



$$U(\vec{\alpha}) = U_n \ldots U_2 U_1$$

Delivers quantum states

Explores a large (Hilbert) space



Classical characterization of a global unitary

Q Computer is a machine that generates variational states

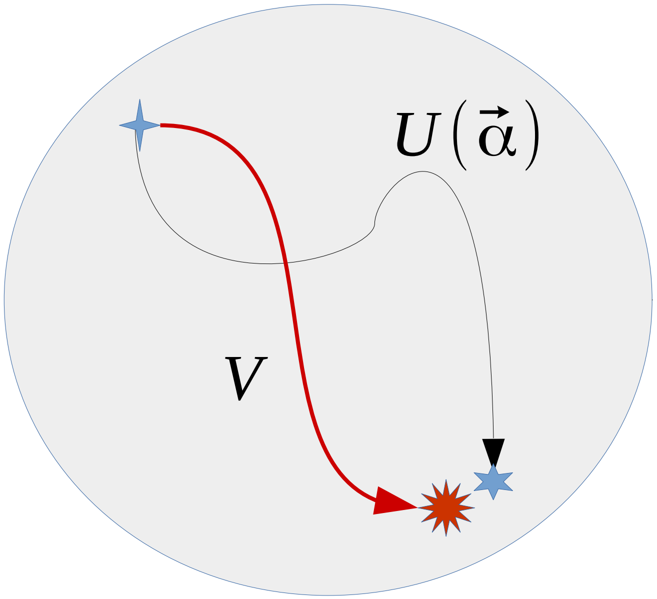
near optimal solution

Variational Quantum Computer!!

$\{U_i\}$ A dense set of unitaries

Find an approximation to *V*

$$|U_k \ldots U_2 U_1 - V| < \delta$$



Solovay-Kitaev Theorem

$k \sim O\left(\log^c \frac{1}{\delta}\right)$ operations

$$c < 4$$

optimal solution

Good news!

Note, the unitary we are looking for is not unique

$$H|\psi\rangle = E_0|0\rangle \qquad |\psi\rangle = U|0\rangle$$

$$|\widetilde{\psi}\rangle = \widetilde{U}|0\rangle$$

$$E_0 \leq \widetilde{E}_0 = min_{\widetilde{U}} \langle \widetilde{\psi}|H|\widetilde{\psi}\rangle$$

$$|U - \widetilde{U}| < \delta \qquad \rightarrow |E_0 - \widetilde{E}_0| \sim \delta^2$$

$$\rightarrow |S_0 - \widetilde{S}_0| \sim \delta$$

Only looking for a column

$$\widetilde{U} = \begin{pmatrix} U_{11} & .... \\ U_{21} & .... \\ . & .... \\ U_{2^n 1} & .... \end{pmatrix}$$

Exponential degeneracy
2^n effort, rather than 2^n x 2^n

Another piece of good news from Lieb-Robinson Theorem

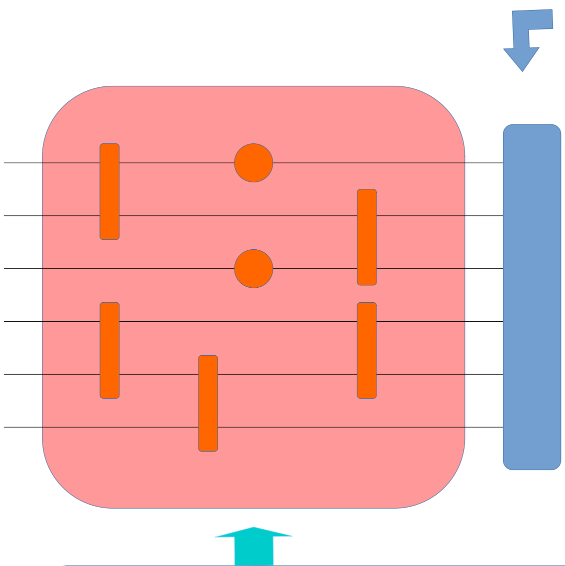
Ising interaction takes a product state of n qubits
into a maximally entangled one in

$$t \sim n$$

Variational Quantum Eigensolvers

Aspuru-Guzik et al.
IBM
Zapata
Blatt

any optimization problem

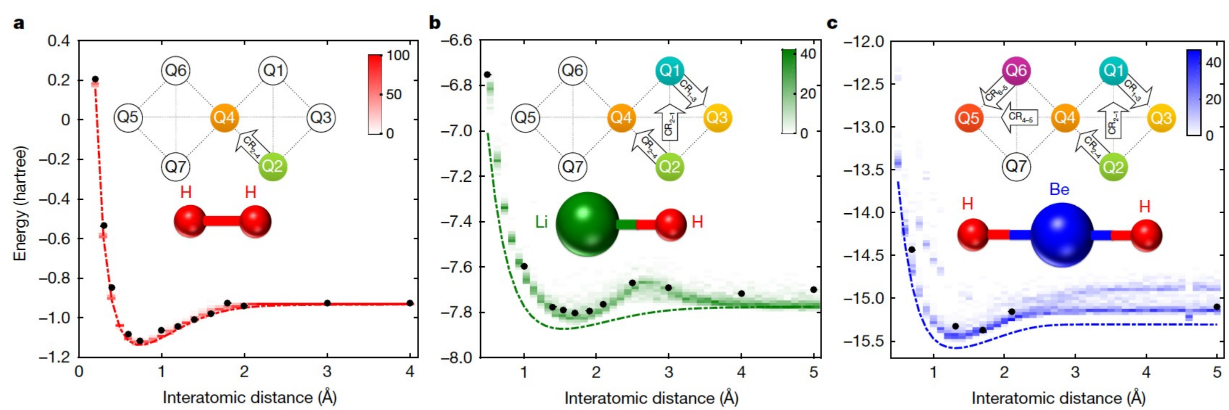


Read out → Energy
Correlations
Classical & Quantum

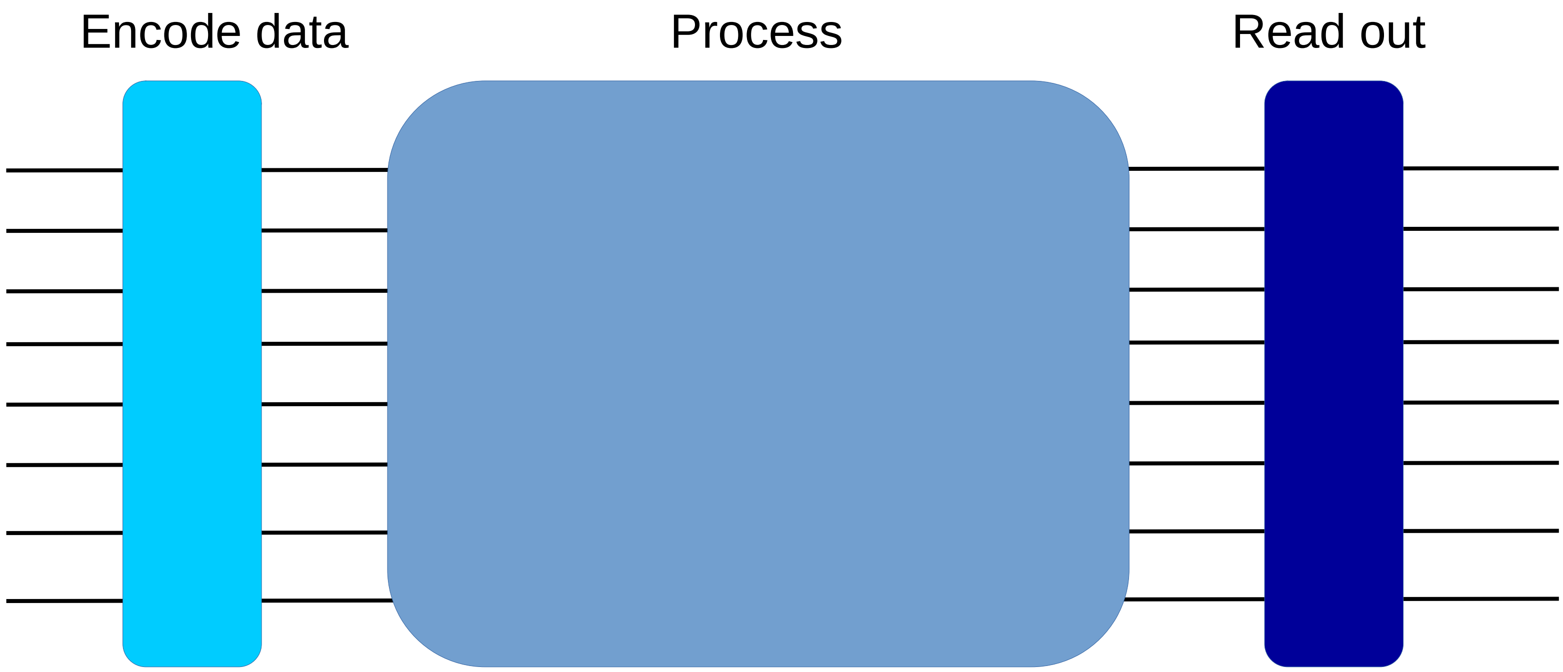
Machine learning
on circuit design

Hybrid classical/quantum algorithms

Quantum Chemistry

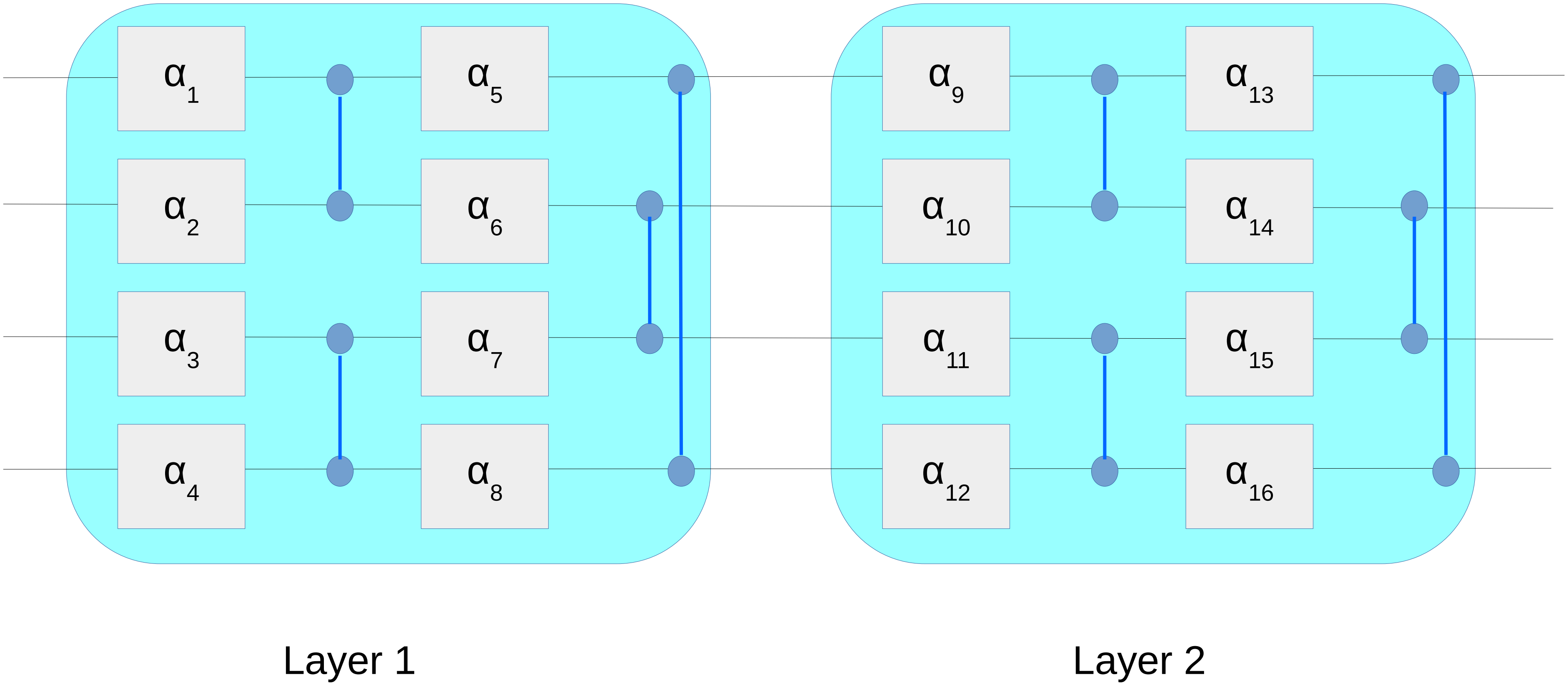


Many unexplored options

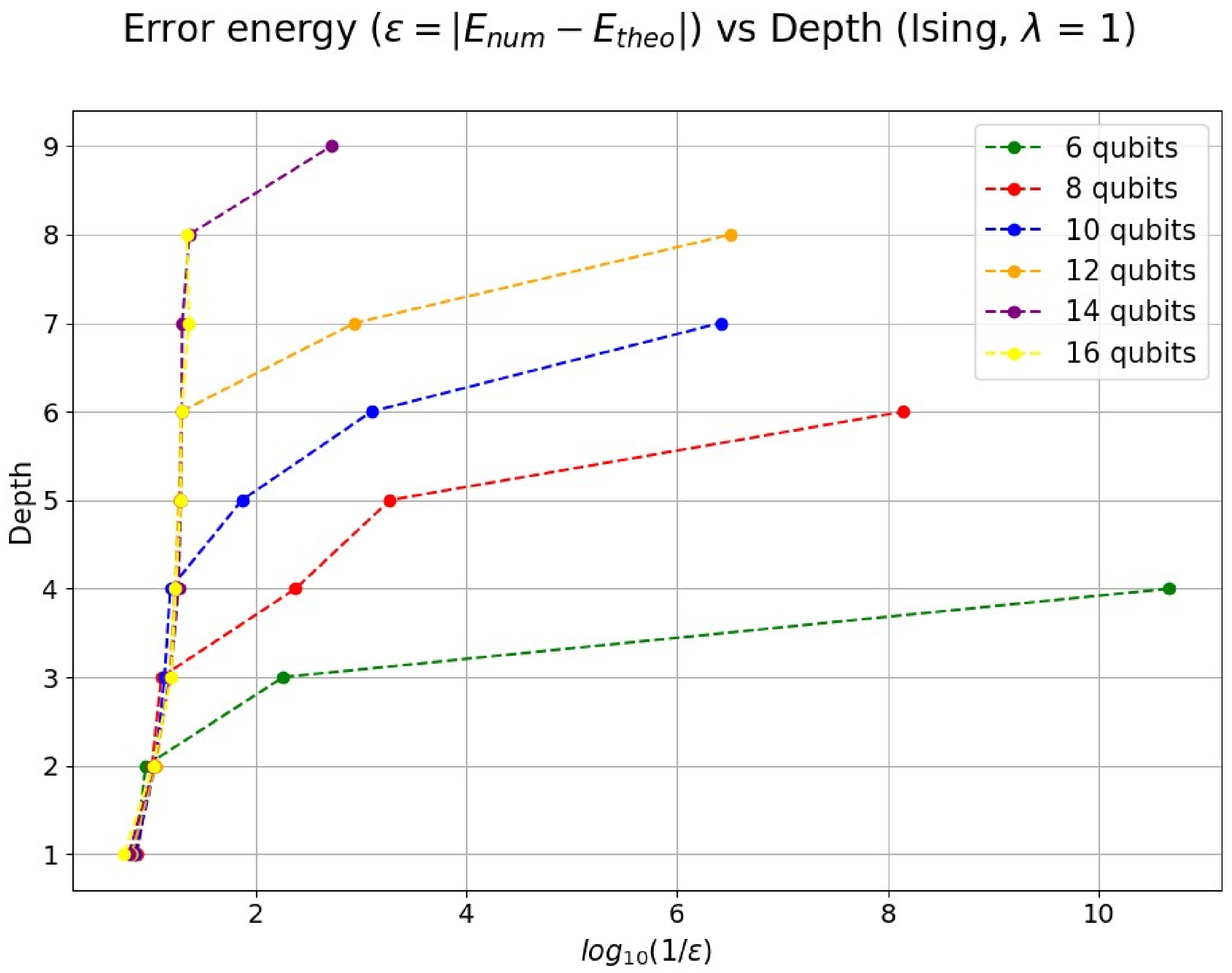


Efficient? Arquitecture? State discrimination?

Add data in the course of computation?



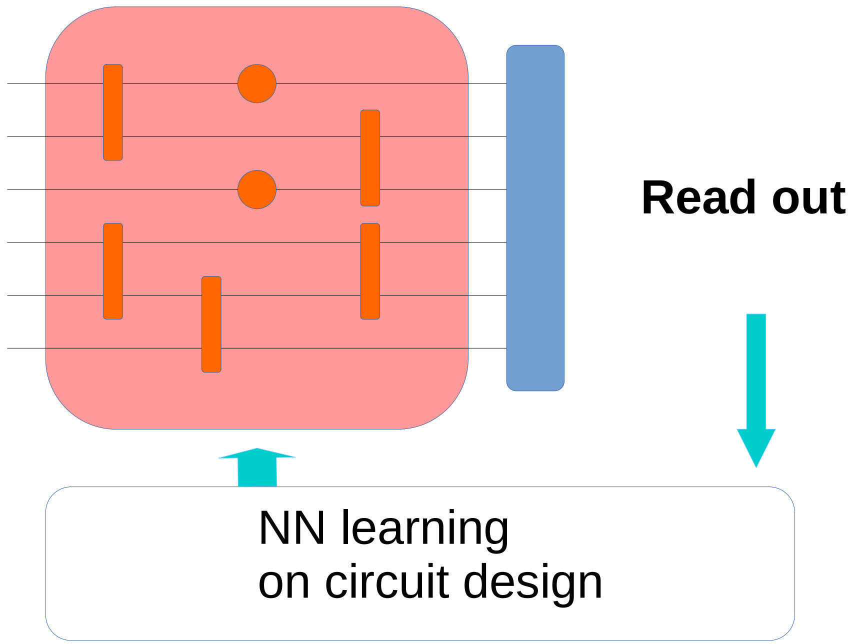
α1
α2
α3
α4
α5
α6
α7
α8
α9
α10
α11
α12
α13
α14
α15
α16
Layer 1
Layer 2



Simulation of CFT better than bounds from Solovay-Kitaev

C. Bravo, J. Lumbreras, JIL (2019)

A further AI step



Reinforcement learning for
a NN providing next QC circuit

(AlphaZero)

$$\vec{\theta}\,' = NN\left(\vec{\theta}\right)$$

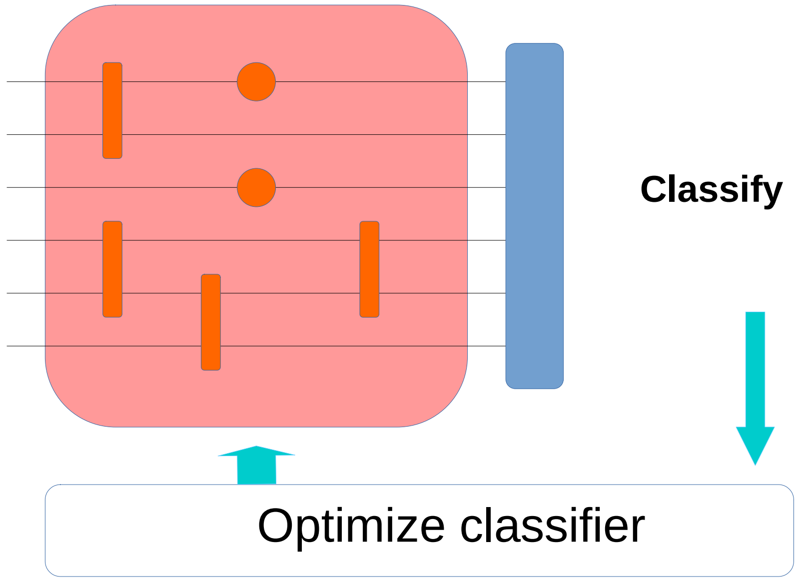
$$E = NN\left(\vec{\theta}\right)$$

(Results look great)

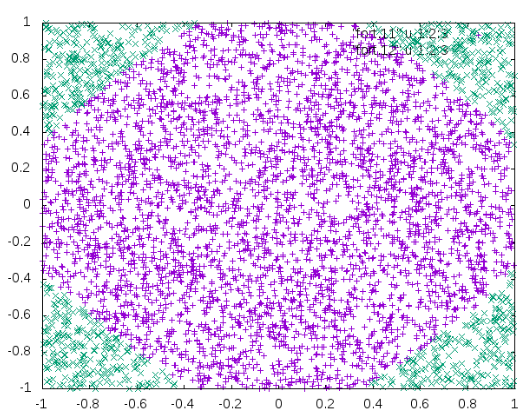
Variational quantum classifier

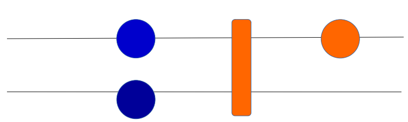
$$U(\vec{\alpha})$$

Codify data in gates



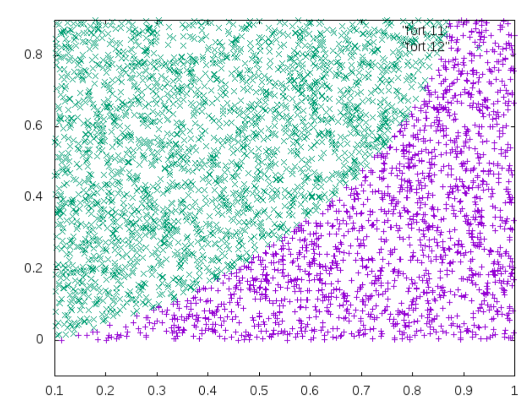
Classify points for a circle





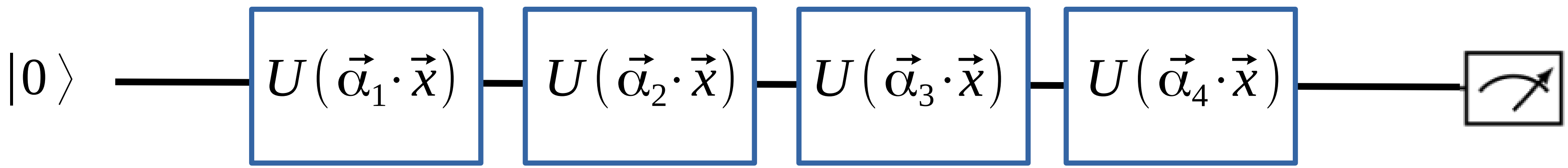
96.6% success
better if output 00 vs 11

Classify points for a parabola

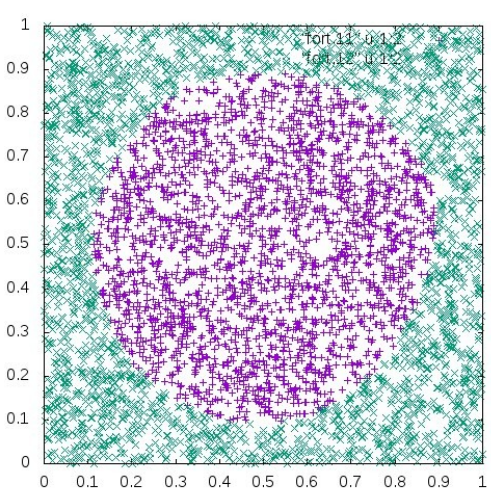


A. Perez, A Cervera-Lierta, E. Gil, JIL

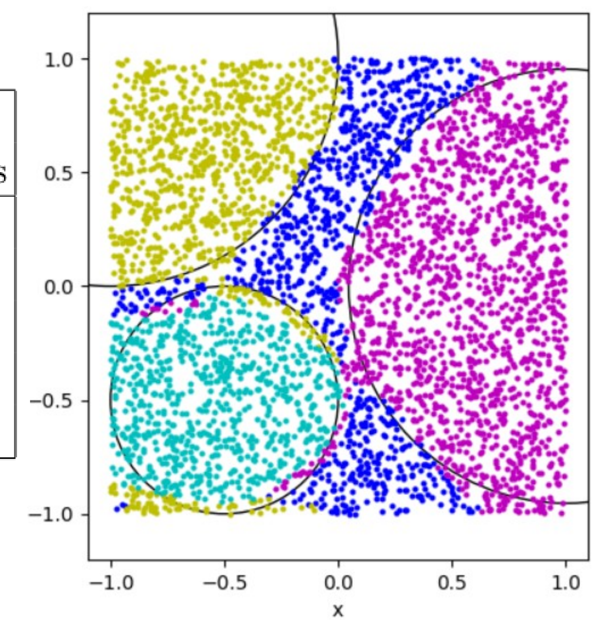
Re-uploading for a universal classifier with a single qubit!



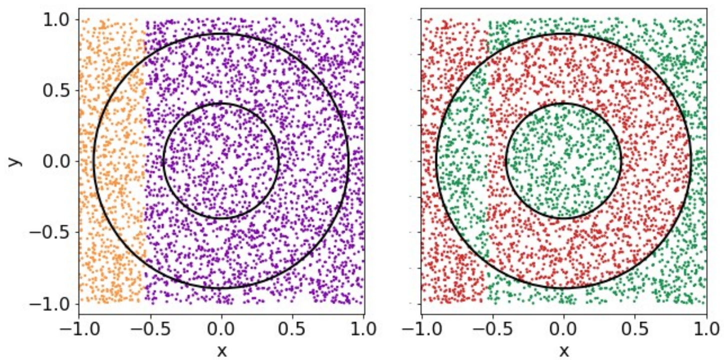
D dimensional via re-uploading
K categories via final measurement



Layers	2 classes			4 classes	
	Circle	Sphere	Hypersphere	Wavy-lines	3-circles
1	75.2%	70.2%	68.0%	70.4%	74.5%
2	89.7%	75.0%	72.6%	88.2%	83.0%
6	92.8%	86.5%	93.2%	89.8%	83.8%
10	96.1%	91.7%	85.5%	90.0%	91.6%
20	96.9%	93.0%	89.2%	89.4%	92.3%

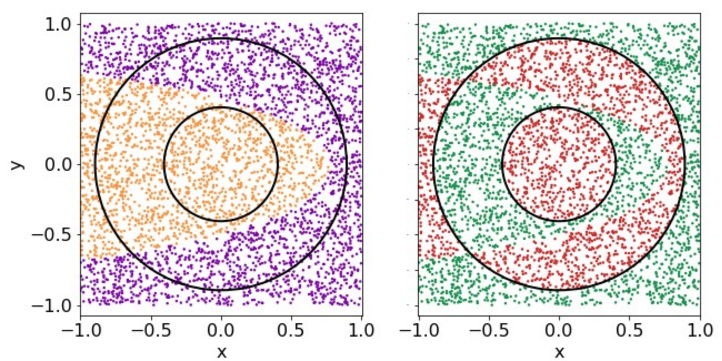


L 1



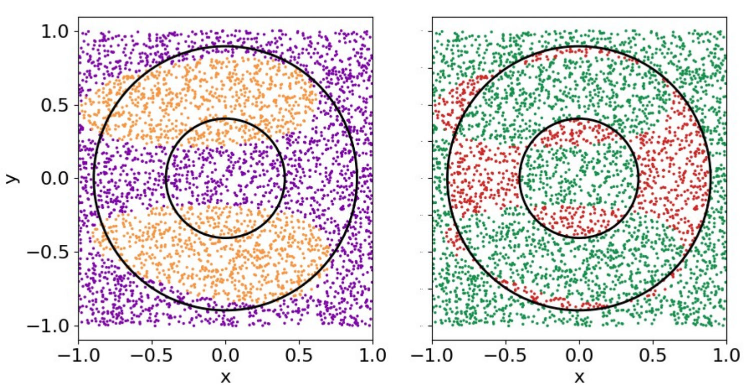
1.0
0.5
0.0
−0.5
−1.0
y
−1.0 −0.5 0.0 0.5 1.0
x
−1.0 −0.5 0.0 0.5 1.0
x

L 2



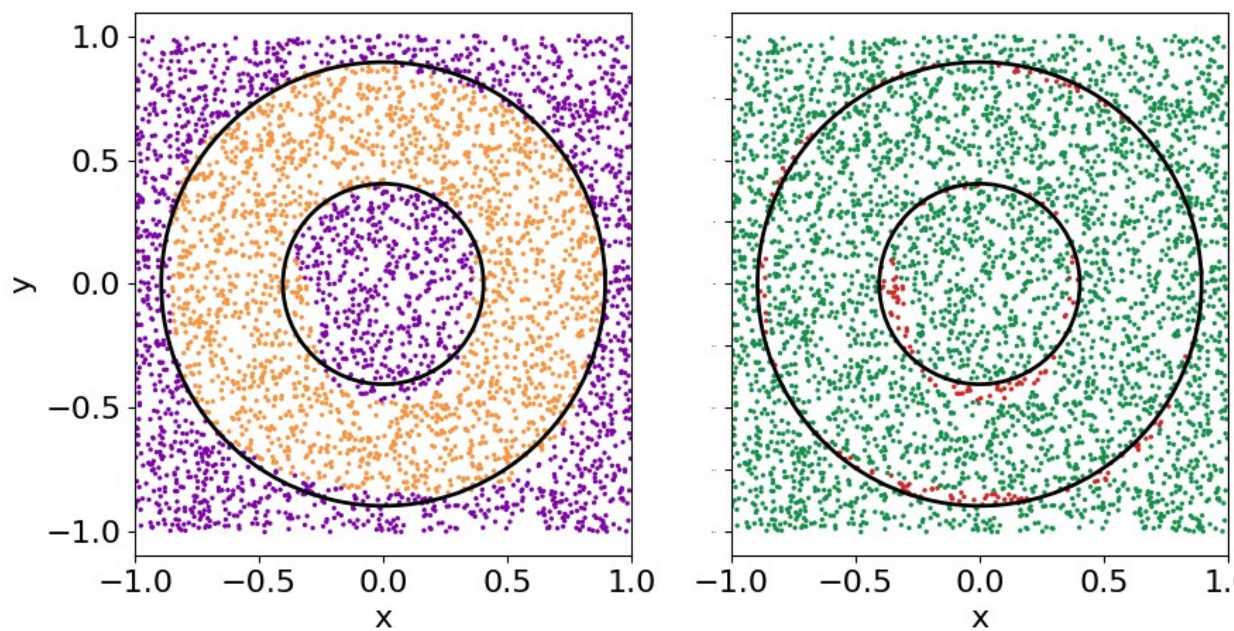
1.0
0.5
0.0
−0.5
−1.0
y
−1.0 −0.5 0.0 0.5 1.0
x
−1.0 −0.5 0.0 0.5 1.0
x

L 3



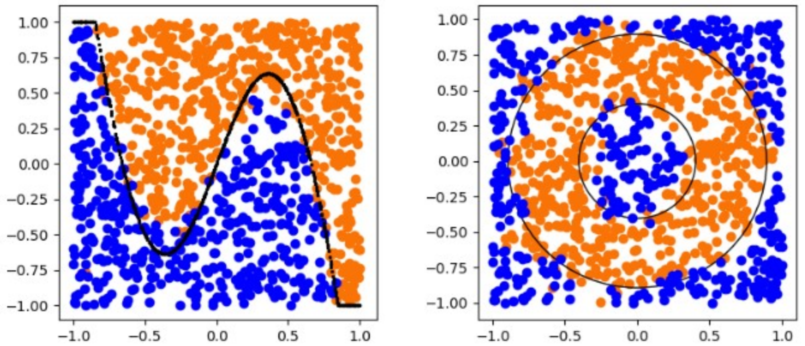
1.0
0.5
0.0
−0.5
−1.0
y
−1.0 −0.5 0.0 0.5 1.0
x
−1.0 −0.5 0.0 0.5 1.0
x

L 6



1.0
0.5
0.0
−0.5
−1.0
y
−1.0 −0.5 0.0 0.5 1.0
x
−1.0 −0.5 0.0 0.5 1.0
x

And now, the experiment...



4 layers

Problem	Classical classifiers		Quantum classifier	QPU
	NN	SVC	χ^2_{wf}	χ^2_{wf}
Circle	0.96	0.97	0.97	*0.96(0.93) ± 0.01
3 circles	0.88	0.66	0.90	0.85 ± 0.04
Hypersphere	0.98	0.95	0.78	*0.76(0.64)±0.01
Sphere	0.97	0.95	0.72	0.59±0.06
Squares	0.98	0.96	0.97	0.92±0.04
Non-Convex	0.99	0.77	0.95	0.91±0.03
Wavy Lines	0.95	0.82	0.94	0.90±0.04
Binary annulus	0.94	0.79	0.92	0.84 ±0.03
Annulus	0.96	0.77	0.94	0.89 ± 0.05

An Ion based single qubit quantum classifier in NISQ era (2021)

T. Dutta, A. Perez-Salinas, J. Phua Sing Cheng, JIL, Manas Mukherjee

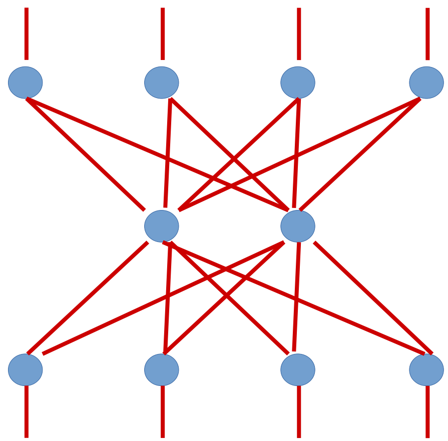
Variational quantum autoencoder

Detect a relevant subspace in data

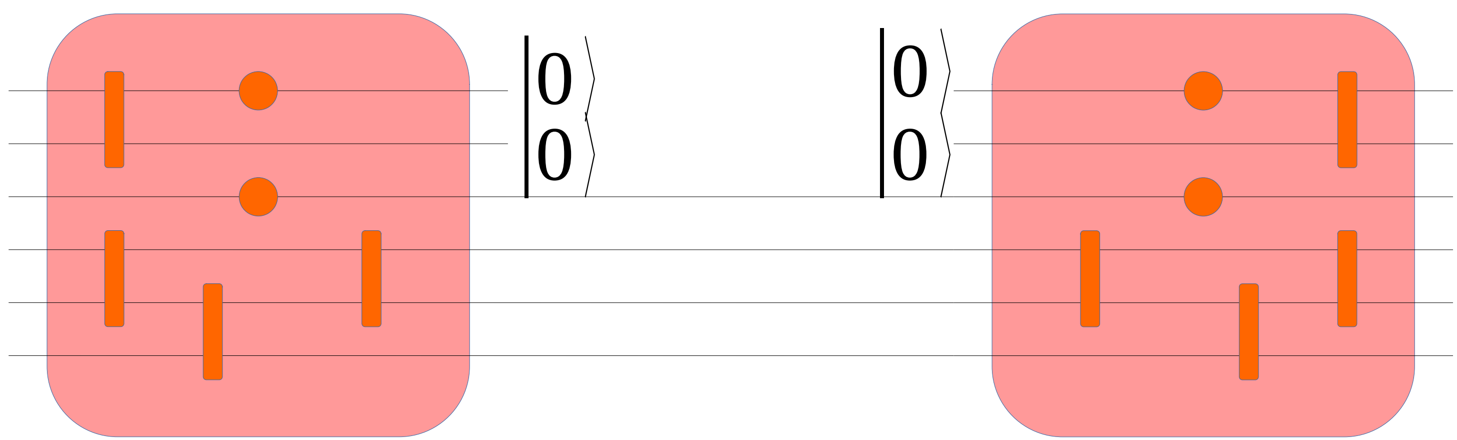
Compressor

Generate patterns

$$\mathrm{I} =$$

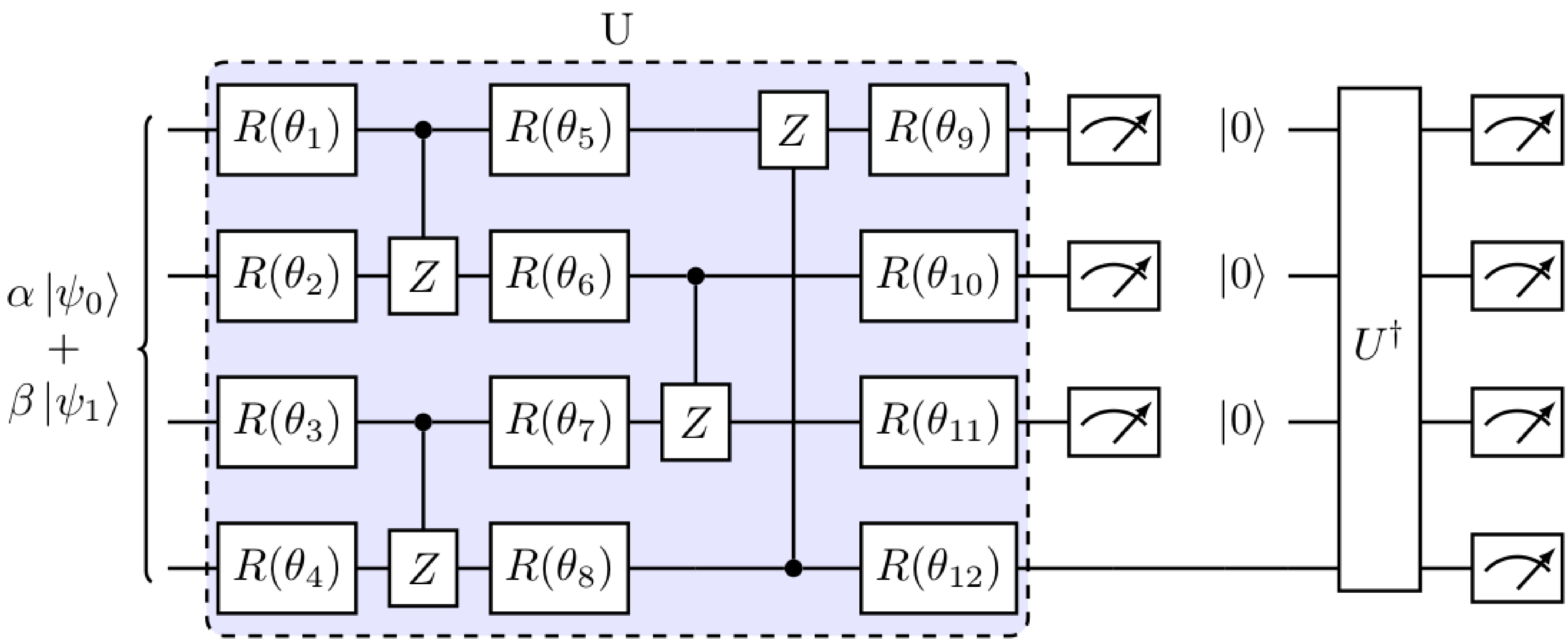


$$S \subset H$$



$$S$$

Autoencoder for a superposition of E_0 and E_1, Ising model



1 layer F=.78
5 layers F=.89
11 layers F=.97

Mimic MERA training!

Singular Value Decomposition

$$|\psi\rangle_{AB} = \sum_{i}^{\dim H_A} \sum_{j}^{\dim H_B} t^{ij} |i\rangle_A |j\rangle_B$$

$$|\psi\rangle_{AB} = \sum_{i}^{\dim H_A} \sum_{j}^{\dim H_B} U_{ik}^{(A)} \lambda_k V_{kj}^{+B} |i\rangle_A |j\rangle_B$$

$$|\psi\rangle_{AB} = \sum_{k}^{\chi \le (\dim H_A, \dim H_B)} \lambda_k |u_k\rangle_A |v_k\rangle_B$$

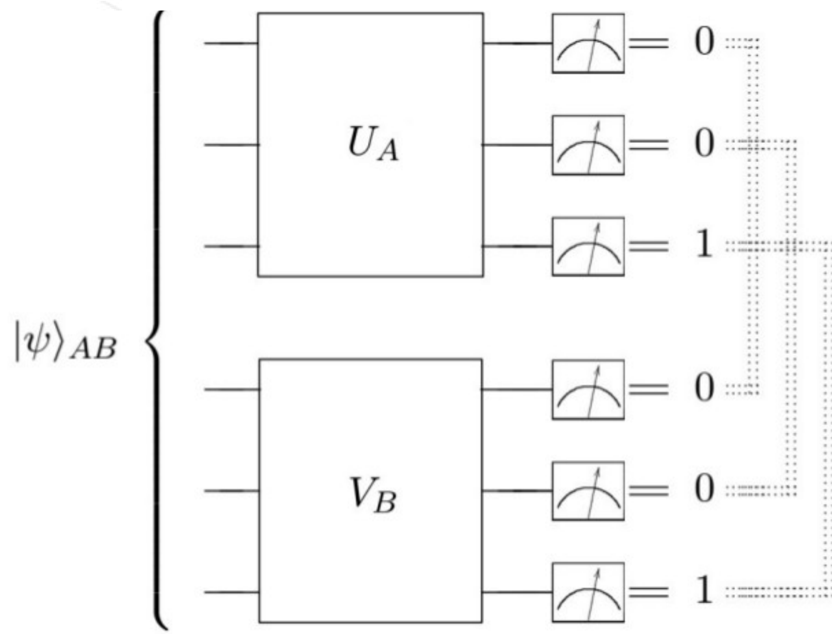
$$\rho_A = tr_B |\psi\rangle_{AB} \langle\psi|$$

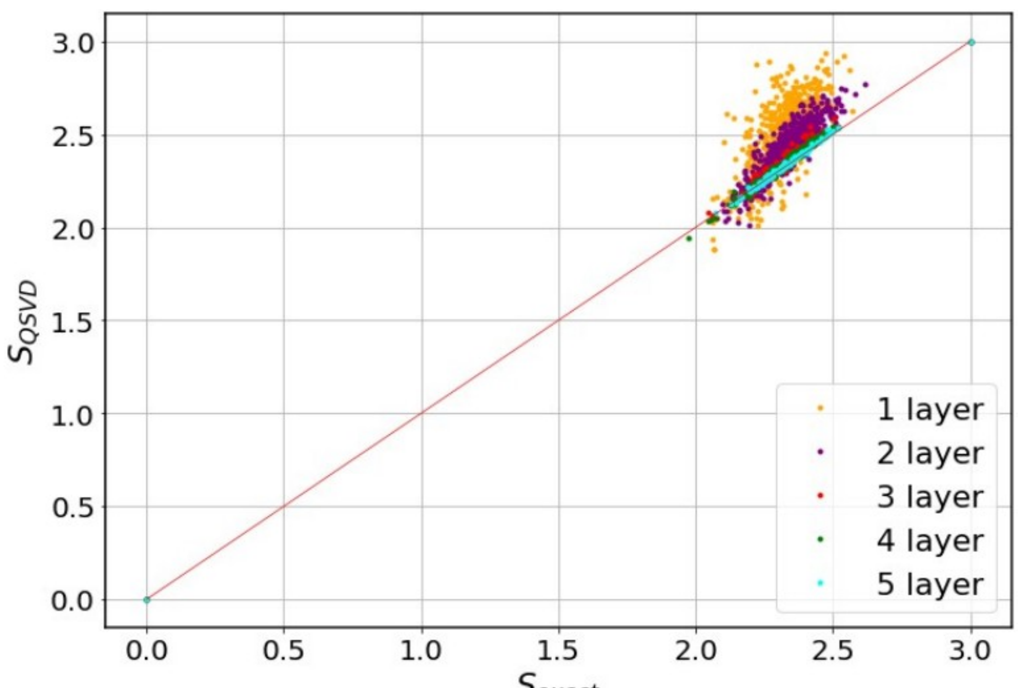
$$S_A = -\sum_{k=1}^{\chi} \lambda_k \log_2 \lambda_k$$

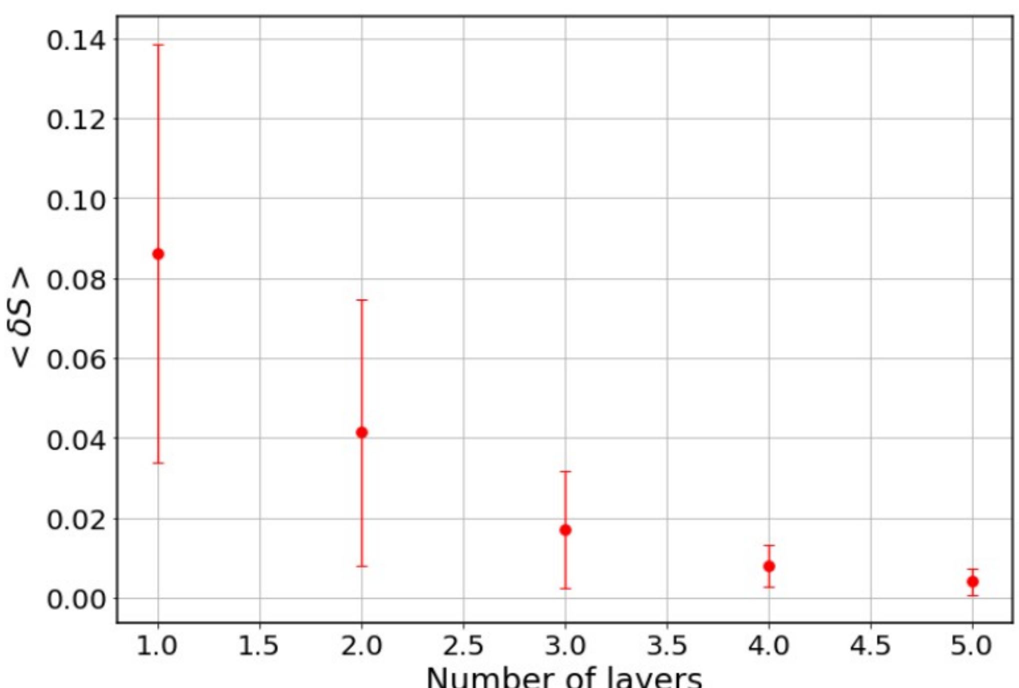
$$S_A = -tr_A \rho_A \log_2 \rho_A$$

$$|\psi\rangle_{AB} \xrightarrow{QSVD} U_A(\vec{\Theta})\otimes V_B(\vec{\Omega})\,|\psi\rangle_{AB} = \sum_{i=1}^{\chi} \lambda_i\, e^{i\alpha_i}|e_i\rangle_A|e_i\rangle_B$$

Bravo-Prieto, García, JIL

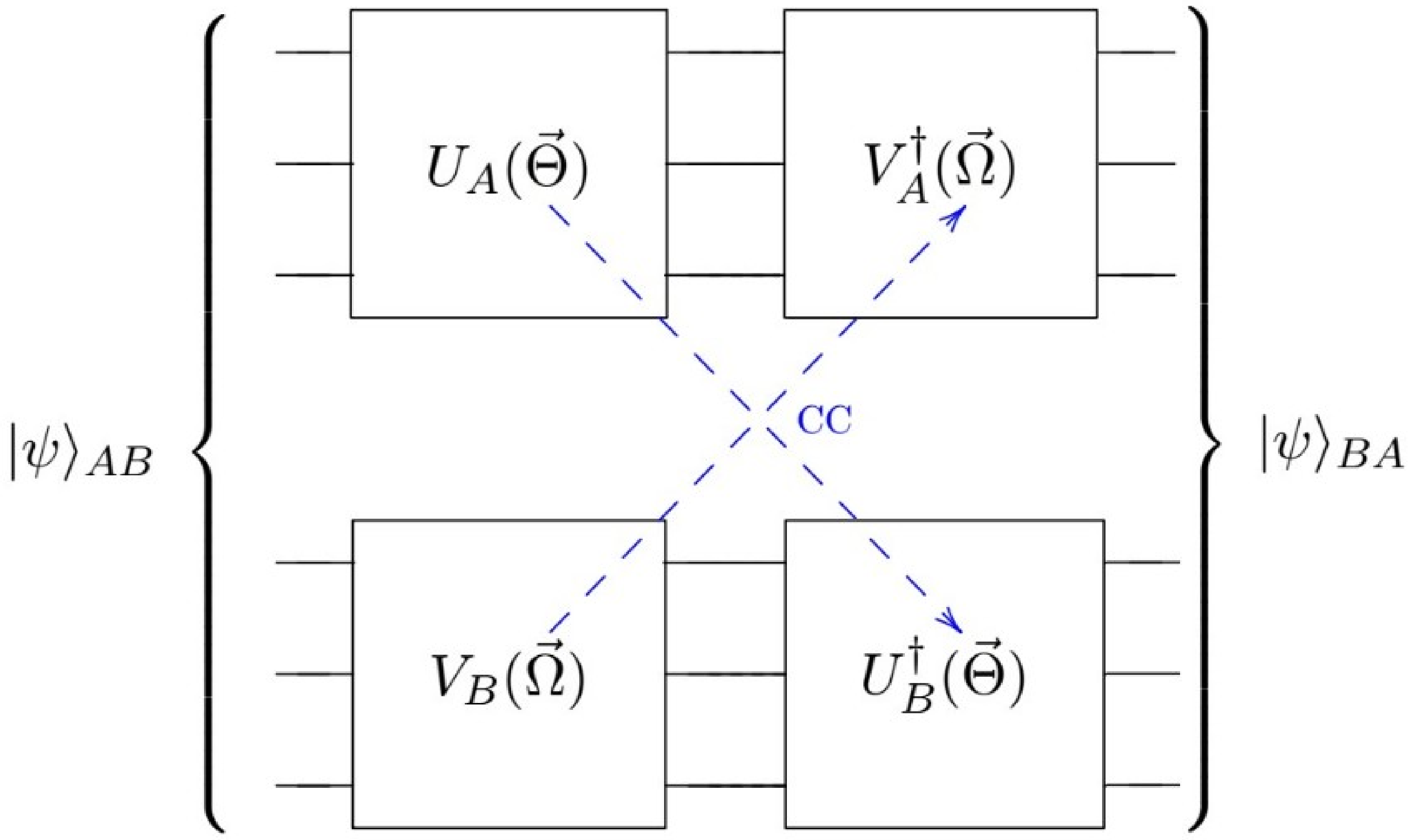






Once trained, SWAP with classical communication

$$(V_A^\dagger \otimes U_B^\dagger)\,(U_A \otimes V_B)\,|\psi\rangle_{AB} = |\psi\rangle_{BA}$$



CONCLUSION

A lot of work ahead!!!

Build a variational quantum machine?

Cloud access to pulses?

- no error correction
- fast hybrid classical-quantum operation
- many-body interactions are welcome
- every experimental action is subject to machine learning

ThanQs!

Known circuits

Search - Grover
QFT - Shor
Deutsch

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Direct Annealing
Adiabatic Evolution

Variational

Autoencoders
Eigensolvers
Classifiers

Design quantum circuits which implement an exact algorithm

e.g. expert system

Remote benchmark of a quantum computer

D. Alsina, JIL (2016)

Mermin inequalities

$$M_3 = (a_1' a_2 a_3 + a_1 a_2' a_3 + a_1 a_2 a_3') - (a_1' a_2' a_3')$$

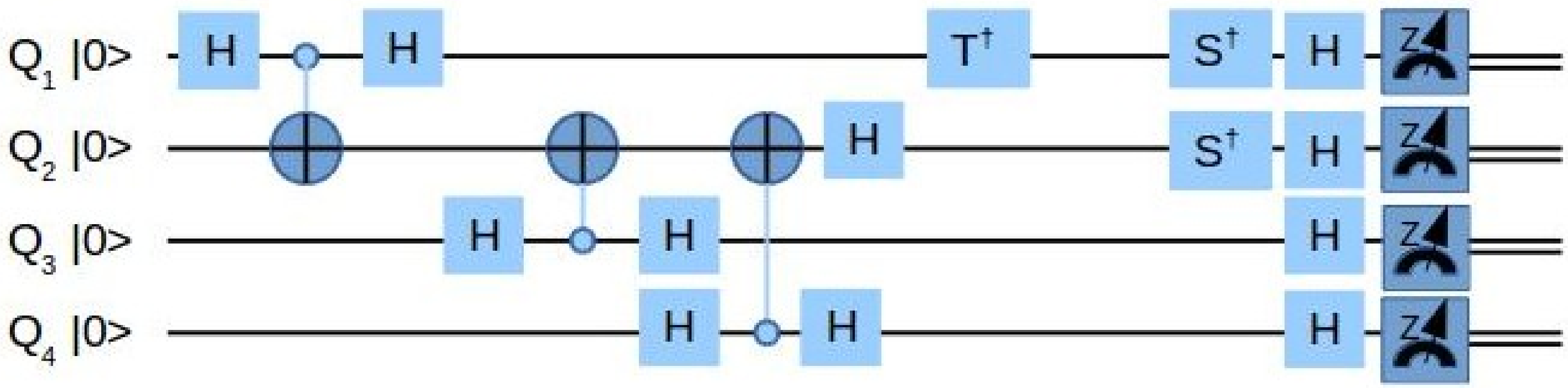
$$M_3^2 = 4I - [a_1, a_1'][a_2, a_2'] - [a_1, a_1'][a_3, a_3'] - [a_2, a_2'] - [a_3, a_3']$$

$$\langle M_3 \rangle^{LR} \leq 2 \qquad \langle M_3 \rangle^{QM} \leq 4$$

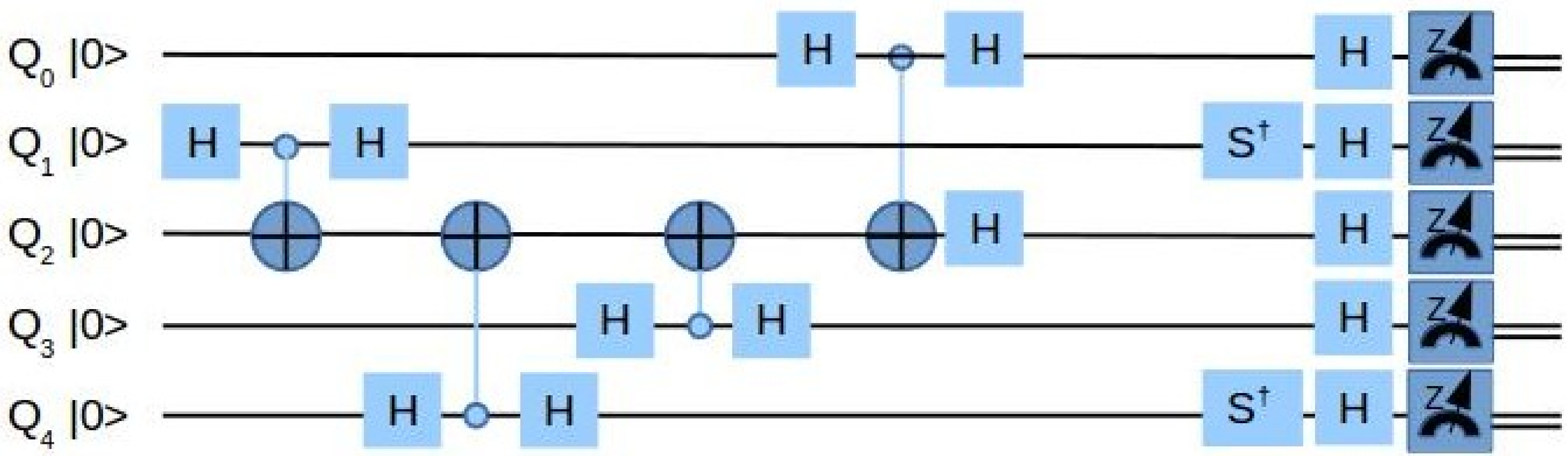
$$|\psi\rangle = \frac{1}{\sqrt{2}} (|000\rangle + i|111\rangle)$$

4- and 5-qubit maximally violating GHZ-like states

4



5



First remote benchmark of a quantum computer (IBM)

Result XXY	**000**	*001*	*010*	**011**	*100*	**101**	**110**	*111*
Probability	0.229	0.042	0.024	0.194	0.043	0.203	0.231	0.033
Result YYY	**000**	*001*	*010*	**011**	*100*	**101**	**110**	*111*
Probability	0.050	0.188	0.188	0.028	0.258	0.026	0.041	0.221

	LR	QM	EXP
3 qubits	2	4	**2.85± 0.02**
4 qubits	4	$8\sqrt{2}$	**4.81± 0.06**
5 qubits	4	16	**4.05± 0.06**

Violation of 3- and 4-qubit

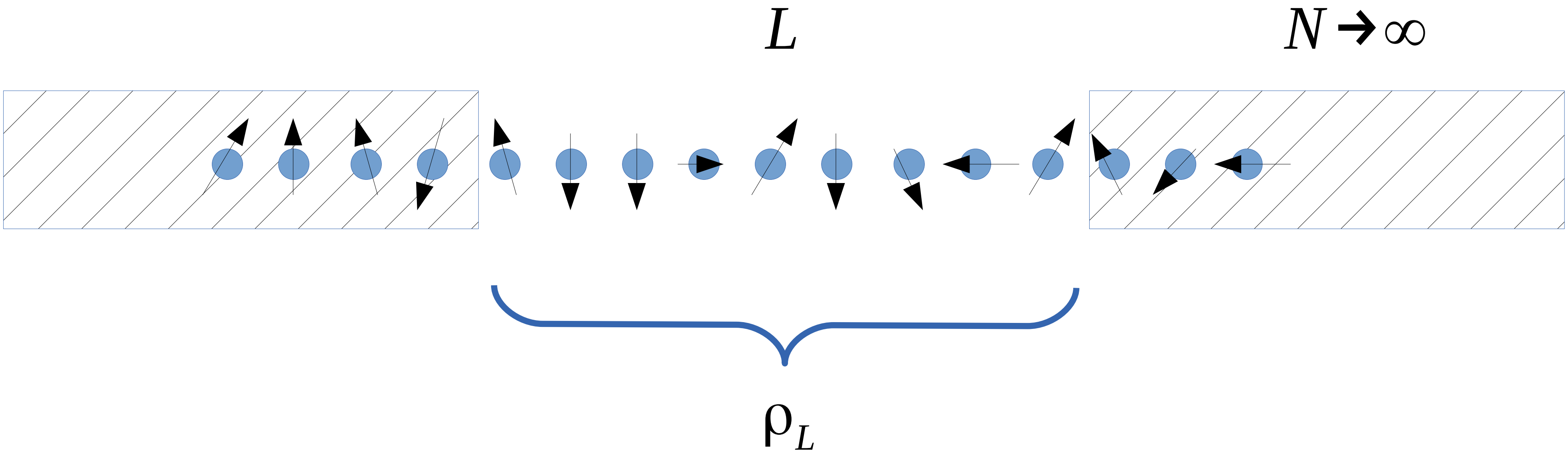
5-qubit remains very poor

Exact circuits for quantum phase transitions

F. Verstraete, I. Cirac, JIL (2008)
M. Hebenstreit, D. Alsina, JIL, B. Kraus (2017)
A. Cervera-Lierta (2018)

How much entangled is the ground state of the 1D Ising model?

$$H_{Ising} = \sum_i \sigma_i^x \sigma_{i+1}^x + \lambda \sum_i \sigma_i^z$$



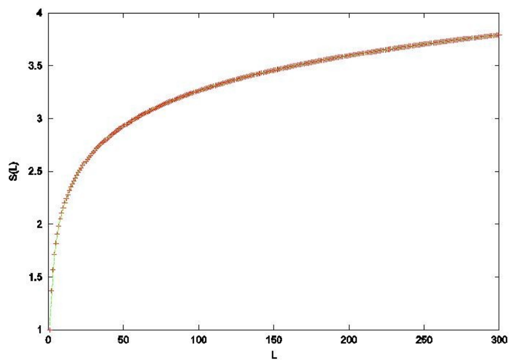
$S(\rho_L)$ as a function of the coupling ?

Entanglement detects quantum phase transitions

Entanglement scales at critical points, saturates away from criticality

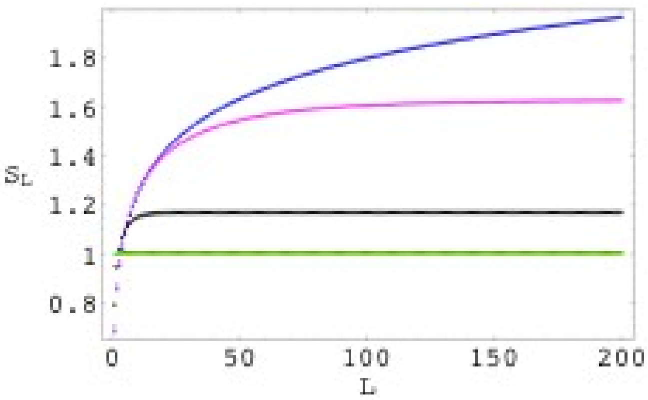
Entanglemen is characterized by the central charge defining the universality class

At Quantum Phase Transition



$$S_L \big|_{L\to\infty} \to \frac{c}{3}\log_2 L$$

Away from Quantum Phase Transition



$$S_{L=N/2\to\infty} = \frac{c}{6}\log_2 |1-\lambda|$$

Vidal, Latorre, Rico, Kitaev ; Wilzeck et al ; Cardy-Calabrese

Conformal Field Theory

- A theory is defined through the Operator Product Expansion

$$O_i(x)O_j(y) \approx \frac{\delta_{ij}}{|x-y|^{h_i+h_j}} + \frac{C_{ij}^{\ k}}{|x-y|^{h_i+h_j-h_k}} O_k(y) + \dots$$

Scaling dimensions Structure constants

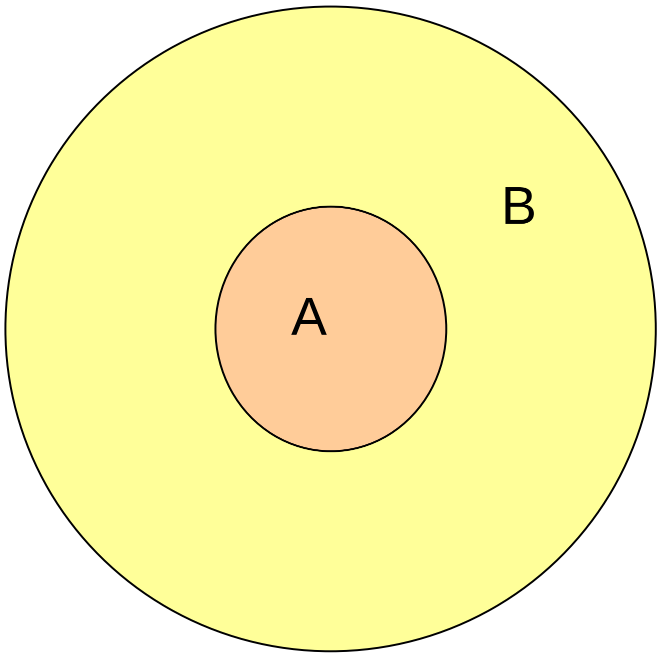
- In d=1+1, the conformal group is infinite dimensional:
  the structure of “descendants” is fixed
  the theory is defined by C_{ijk} and h_i

$$T(z)T(w) \approx \frac{c}{|z-w|^4} + \frac{1}{|z-w|^2} T(w) + \dots$$

Stress tensor

Central charge ⟺ **Entanglement**

Scaling of entanglement in higher dimensions



Srednicki 93 Free field theory

$$S(\rho_A) \propto \left(\frac{R}{\epsilon}\right)^{d-1}$$ Area Law

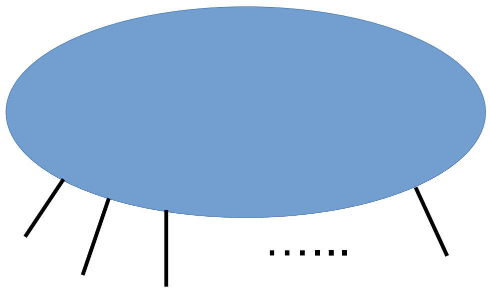
Rigorous proof on lattices
Fermions and gauge theories

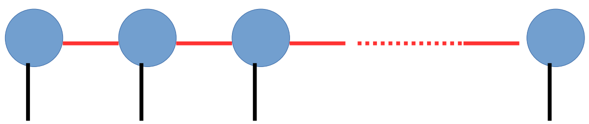
Link to Black Hole physics via holography

Area law emerges from local interactions

Tensor networks: Matrix Product States, PEPS, MERA

Matrix Product State (DMRG)





$$|\psi\rangle = \sum_{i_1 i_2 \dots i_n} t^{i_1 i_2 \dots i_n} |i_1 i_2 \dots i_n\rangle$$

$$t^{i_1 i_2 \dots i_n} = Tr\left(A^{i_1} A^{i_2} \dots A^{i_n}\right)$$

$$2^n$$

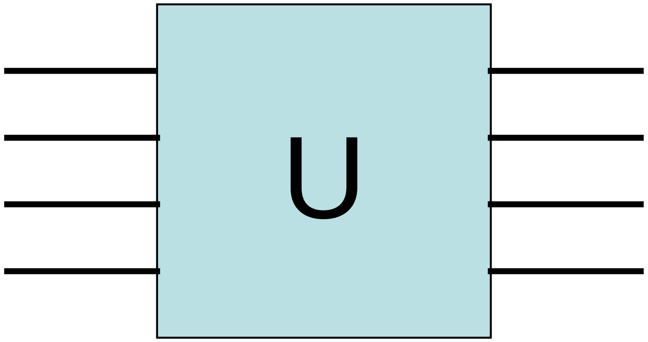
$$2 n \chi^2$$

Tensor networks allow for the efficient representation and manipulation of entanglement for slightly entangled states

$$S = \frac{c}{6} \log \chi^{\kappa}$$

Quantum Simulation of a Quantum Phase Transition on a quantum computer

Prepare
a product state



Obtain
the ground state Ising

$$|\psi\rangle_{ISING} = U|\psi\rangle_{trivial}$$

$$e^{-\beta H_{ISING}} = U\, e^{-\beta H_{ISING}}\, U^{+}$$

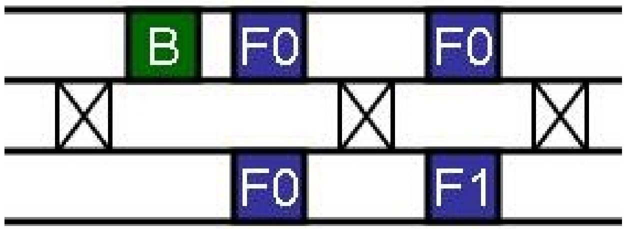
$$e^{-itH_{ISING}} = U\, e^{-itH_{ISING}}\, U^{+}$$

Quantum circuit for 4-qubit Ising

$$H_{QI} = \sum_i \sigma_i^x \sigma_{i+1}^x + \lambda \sum_i \sigma_i^z$$

$$U(F0) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{\alpha}{\sqrt{2}} & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{\alpha}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & -\alpha \end{pmatrix}$$

Bogoliubov Fast Fourier transform



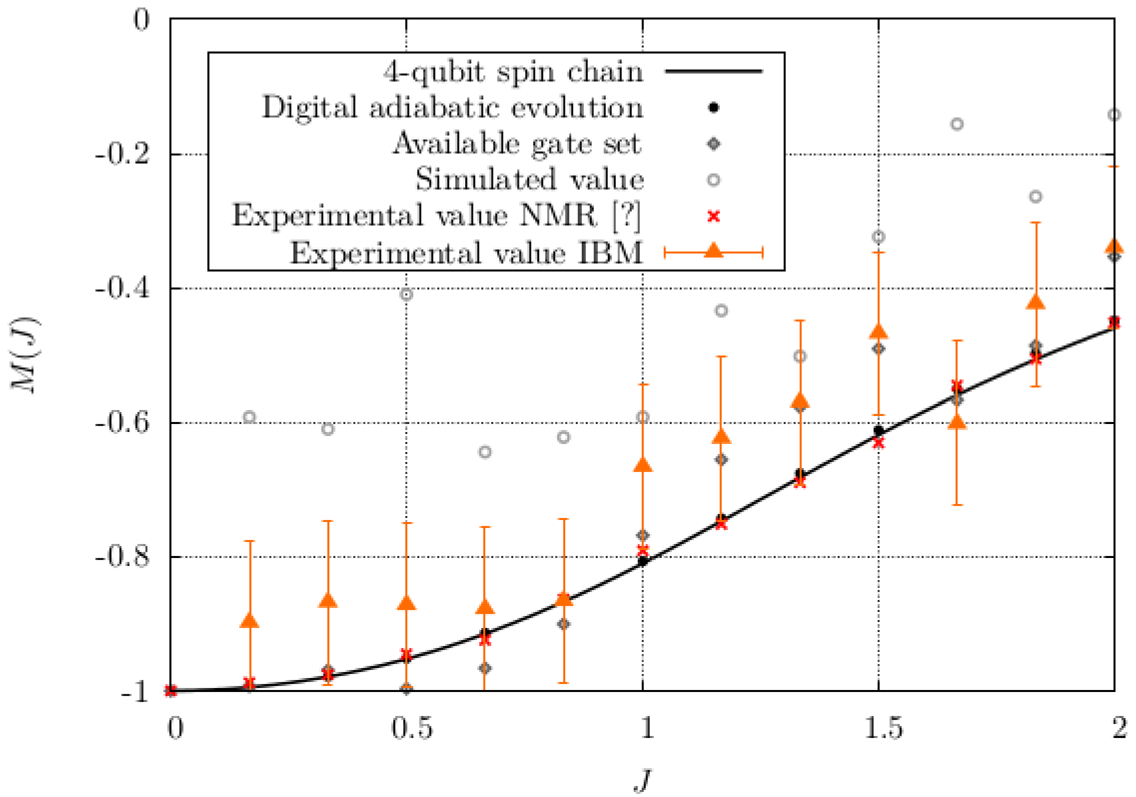
$$U(B) = \begin{pmatrix} \cos(\vartheta(\lambda)) & 0 & 0 & i\sin(\vartheta(\lambda)) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i\sin(\vartheta(\lambda)) & 0 & 0 & \cos(\vartheta(\lambda)) \end{pmatrix}$$

$$U(fSWAP) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

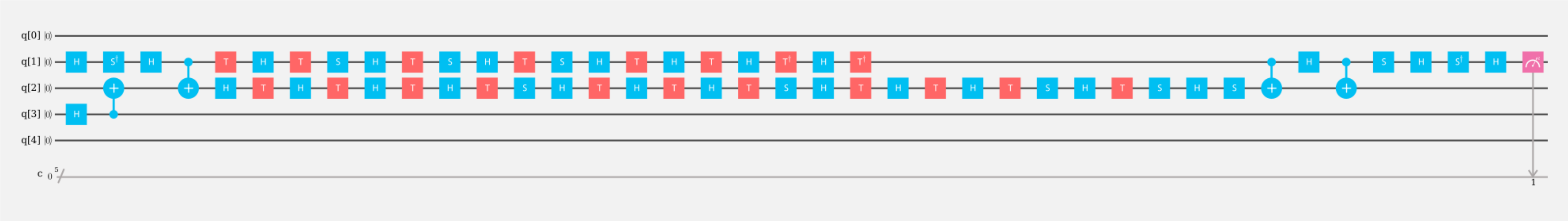
Cirac, Verstraete, JIL (2008)

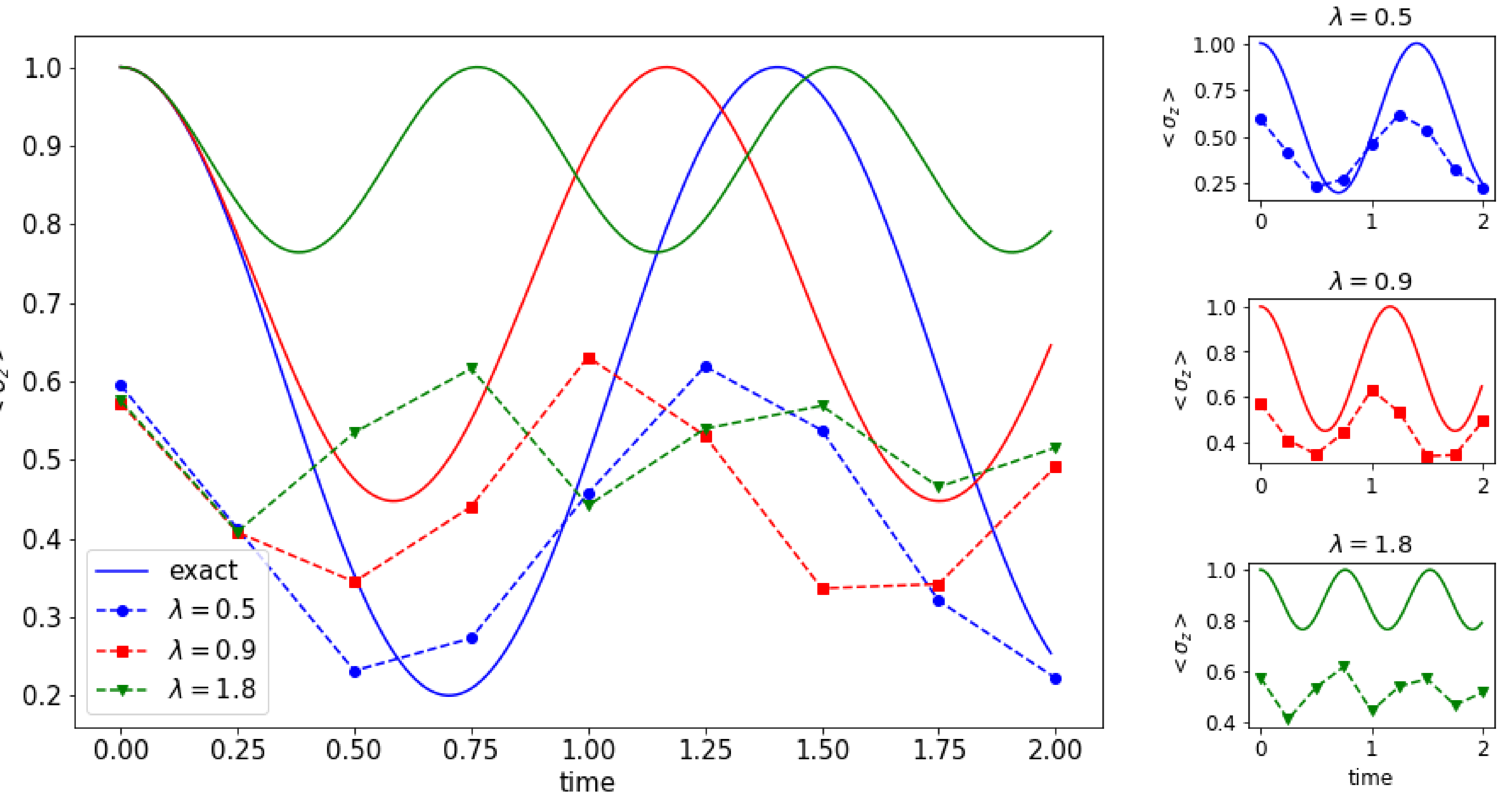
M. Hebenstreit, D. Alsina, JIL, B. Kraus (2017)

Compressed computation scheme



Errors estimated with “validating circuits”





Alba Cervera-Lierta (2018)
IBM Notebook Prize

IBM 16 qubits

Prime State

$$|P(n)\rangle = \frac{1}{\sqrt{\pi(2^n)}} \sum_{p < 2^n \in Primes} |p\rangle$$

G. Sierra, JIL (2013,2014,2018)

Counting primes

Gauss, Legendre
Sieve of Eratosthenes

$$\pi(x) \approx \frac{x}{\ln x}$$

Prime Number Theorem
Gauss, Riemann
Hadamard, de la Vallée Poussin
Density of primes 1/log x

$$\pi(x) \approx Li(x)$$

$$Li(x) = \int_2^x \frac{dt}{\log t} \approx \frac{x}{\ln x} + \frac{x}{\ln^2 x} + \dots$$

$$\pi(10^{24}) = 18\,435\,599\,767\,349\,200\,867\,866$$

Platt (2012)

$$\pi(10^{24}) - \frac{10^{24}}{\ln(10^{24})} = 3.4\;10^{20}$$

$$Li(10^{24}) - \pi(10^{24}) = 1.7\;10^{9}$$

Riemann conjecture and primes

If the **Riemann Conjecture** is correct, fluctuations of primes are bounded

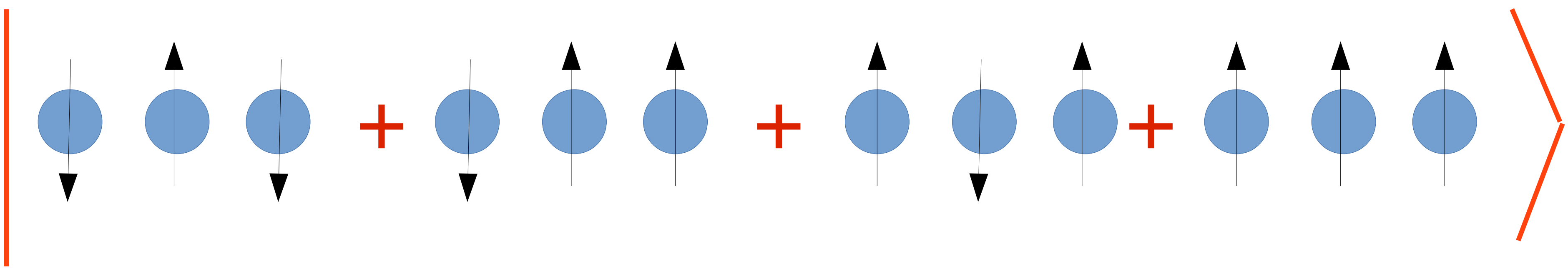
$\zeta(s) = \sum_{n \geq 1} \frac{1}{n^s}$ If $\zeta(s) = 0$ with $0 \leq Real(s) \leq 1$ then $Real(s) = \frac{1}{2}$

$$|Li(x) - \pi(x)| < \frac{1}{8\pi} \sqrt{x} \ln x$$

The prime number function will oscillate around the Log Integral infinitely many times
Littlewood, Skewes

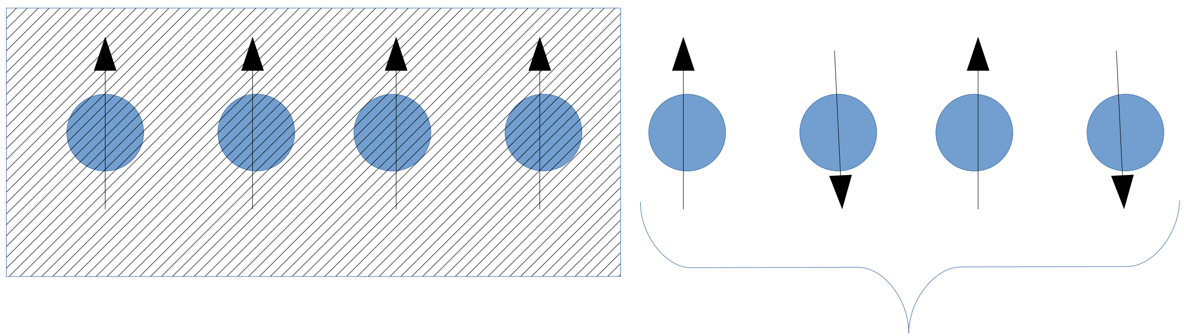
A first change of sign is expected for some $x < e^{727.9513468} \ldots$

$$|P(n)\rangle = \frac{1}{\sqrt{\pi(2^n)}} \sum_{p<2^n \in Primes} |p\rangle$$

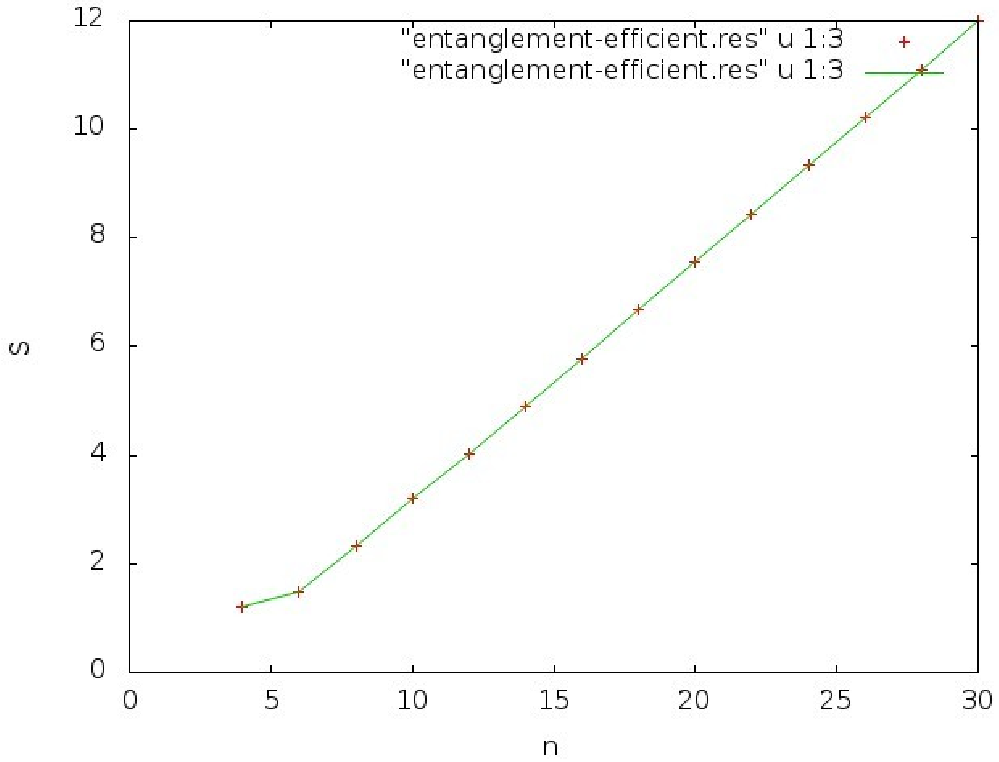


$$|P(3)\rangle = \frac{1}{\sqrt{4}} \left(|2\rangle + |3\rangle + |5\rangle + |7\rangle \right)$$

Entanglement of the Prime state

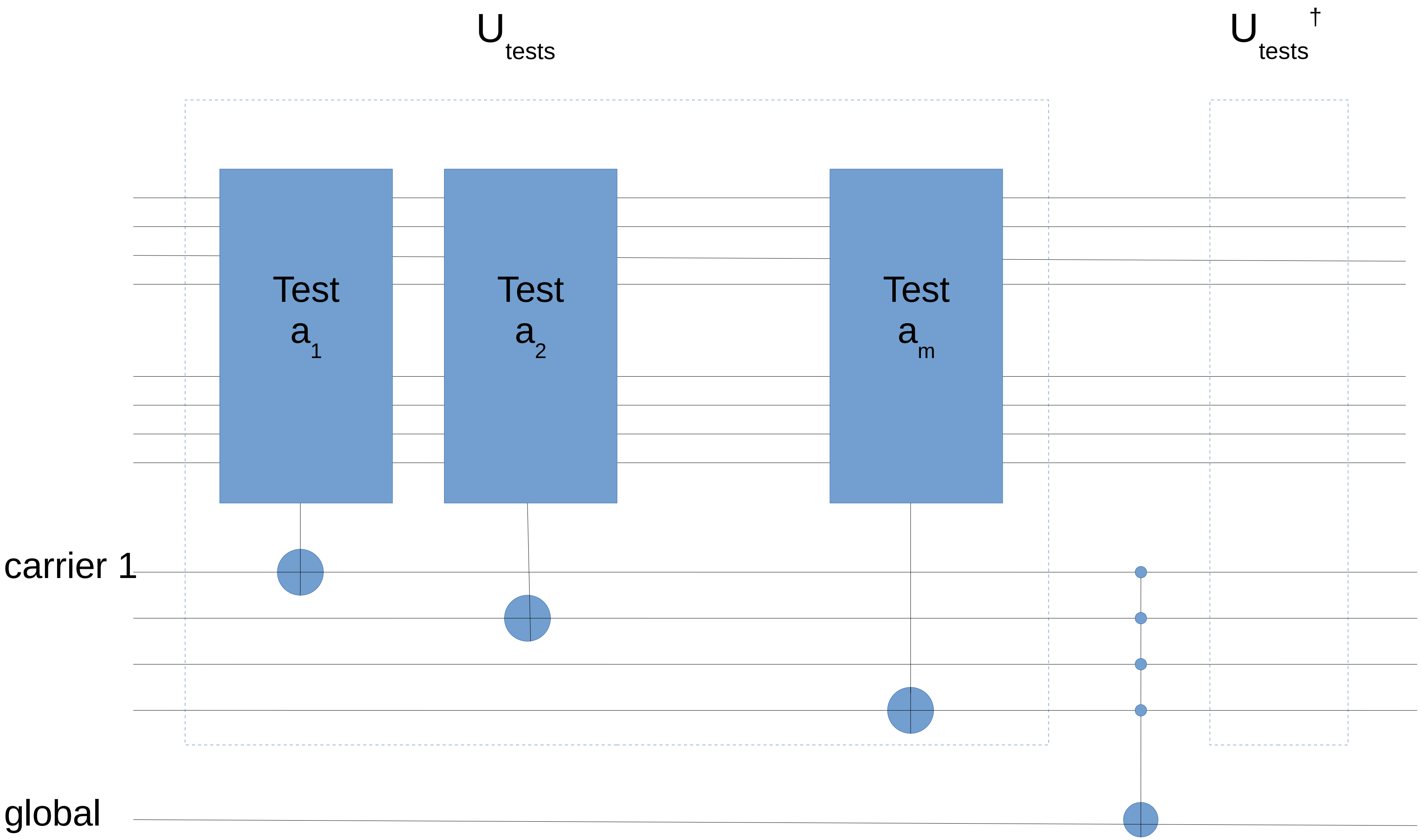


$$\rho_{\frac{n}{2}}$$



"There is entanglement in the Primes"

$$S \sim .8858\, n + const$$



Structure of the quantum primality oracle
Rabin Miller test

Count *M* solutions out of *N* possible states

We know an estimate $\tilde{M}$

$$|\tilde{M}-M|<\frac{2\pi}{c}M^{\frac{1}{2}}+\frac{\pi^2}{c^2}$$

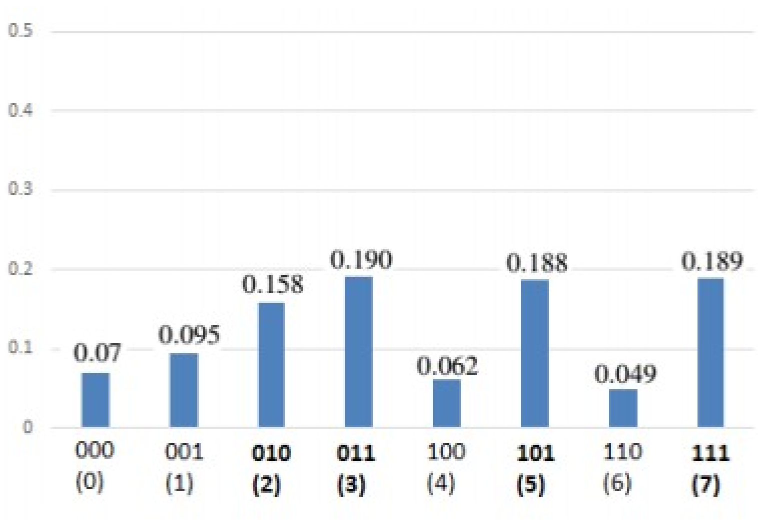
Bounded error in quantum counting using $c\sqrt{N}$ calls

Bounded error in the quantum counting of primes

$$|Li(x)-\pi_{QM}(x)|<\frac{2\pi}{c}\frac{x^{\frac{1}{2}}}{\ln^{\frac{1}{2}}x}$$

Sufficient to disprove a violation of Riemann conjecture

$$|P(3)\rangle$$



Sierra, Diaz (2018) on IBM

Volume law entanglement
Relation of quantum correlations to arithmetic functions
QFT delivers all biases of primes (e.g. Chebichev bias)
Verification of Riemann conjecture
Many other families of numbers

Large entanglement

Rigetti + QUANTIC

caveat

Quantum algorithms that never develop large entanglement can be efficiently simulated with tensor networks

Faithful benchmark of a quantum computer:

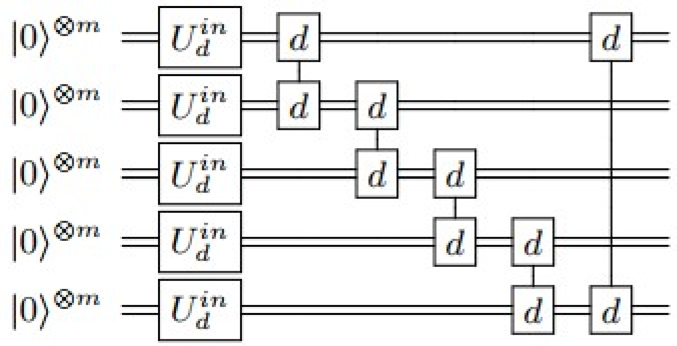
Build states which are largerly entangled in all their partitions

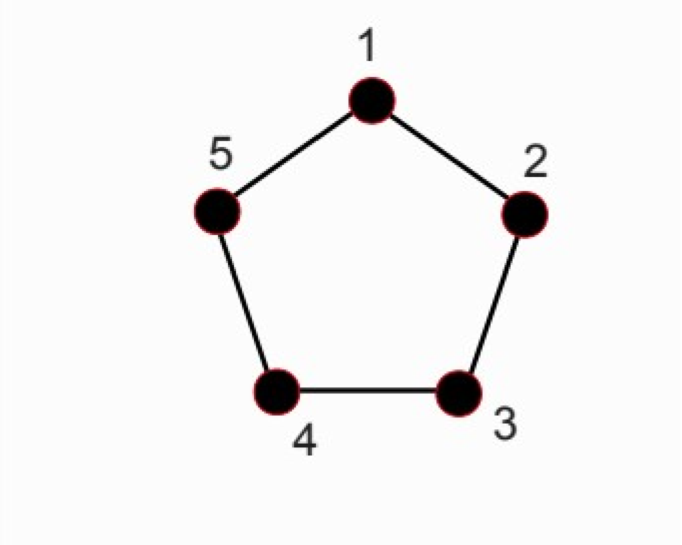
Strategy

- ✓ Identify large entangled states
- ✓ Identify optimal circuits
- Find optimal experimental implementation
- Run circuits
- Explore strategies for tomography
- Use the states for quantum protocoles

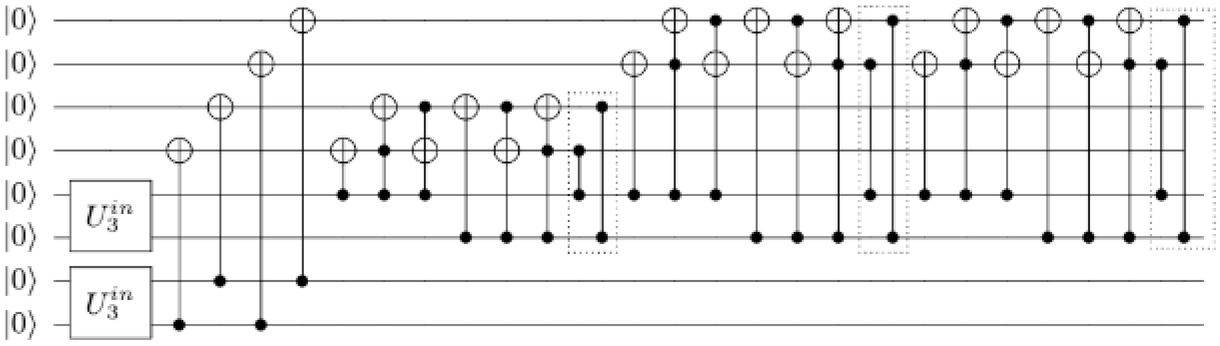
Absolute Maximaly Entangled States

AME(5,2)





AME(4,3)



So far, results are compatible with noise at Rigetti

A. Cervera-Lierta, D. Goyeneche, JIL (2019)