Simulating quench dynamics on a digital quantum computer with data-driven error mitigation

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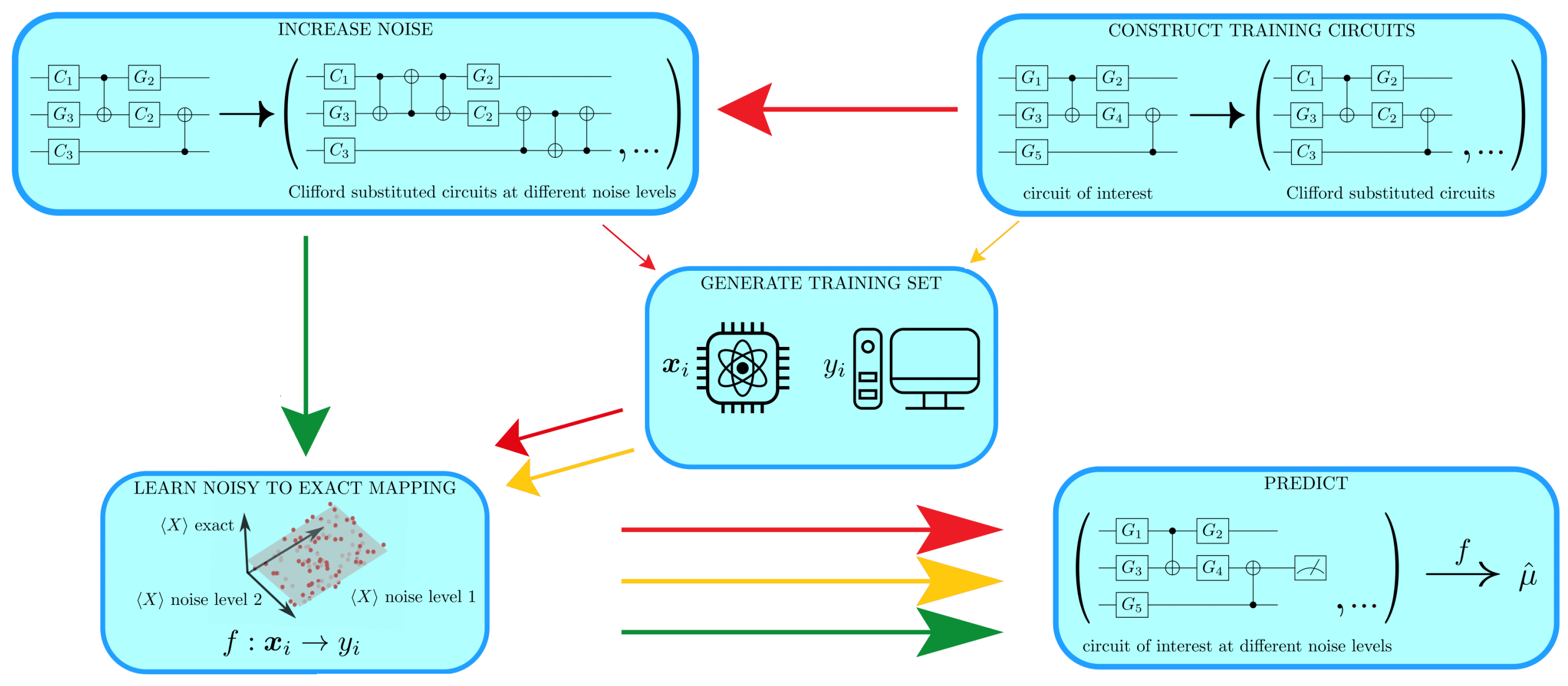
Introduction

Error mitigation is likely to be key in obtaining near term quantum advantage. In this work we present one of the first implementations of several Clifford data regression based methods which are used to mitigate the effect of noise in real quantum data. We explore the dynamics of the 1-D Ising model with transverse and longitudinal magnetic fields, highlighting signatures of confinement. We find in general Clifford data regression based techniques are advantageous in comparison with zero-noise extrapolation and obtain quantitative agreement with exact results for systems of 9 qubits with circuit depths of up to 176, involving hundreds of CNOT gates. This is the largest systems investigated so far in a study of this type. We also investigate the two-point correlation function and find the effect of noise on this more complicated observable can be mitigated using Clifford quantum circuit data highlighting the utility of these methods.

Data-driven error mitigation

Data-driven error mitigation uses classical post processing of quantum data to improve the zero-noise estimates of some observable of interest. In this work ZNE [1], CDR [2] and vnCDR [3] are used to obtain noise-free estimates of various observables. Furthermore, following the recent work showing the success of a simple mitigation strategy with an assumed noise model [4], we demonstrate the utility of a similar approach where the parameters of an assumed noise model are learned using near-Clifford circuits (pmCDR).



- **Zero-noise extrapolation (ZNE)** uses quantum circuit data collected at various hardware noise levels to estimate the value of a noise free observable. Intuitively, by increasing the noise in a controlled manner and extrapolating to the zero-noise limit one can obtain a more accurate estimate of an observable of interest. Despite widespread success ZNE performance guarantees are limited due to uncertainty in the extrapolation. It is not always obvious how many noise scaled data points to measure and what functional form to use to extrapolate to the zero noise limit. In real devices often the base-level noise is too strong to enable an accurate extrapolation, particularly in circuits with significant depth.
- **Clifford data regression (CDR)** makes use of Clifford circuits to mitigate the effect of noise. Quantum circuits composed of mainly Clifford gates can be evaluated efficiently on a classical computer. In CDR near-Clifford circuits are used to construct a set of noisy and exact expectation values for some observable of interest. This dataset is used to train a simple linear ansatz mapping noisy to exact values,

  $$f(\hat{\mu}_0) = a_1\hat{\mu}_0 + a_2, \tag{1}$$

  where $\hat{\mu}_0$ to be the observable evaluated with hardware noise. This ansatz is motivated by a global depolarising channel which appears to accurately describe the noise in a real device for small system sizes [4].
- **Variable noise Clifford data regression (vnCDR)** conceptually unifies ZNE and CDR into one mitigation strategy where Clifford circuit quantum data is used to inform the functional form of the extrapolation to the zero-noise limit. Intuitively, variable noise Clifford data regression reduces the risk of blind extrapolation and is expected to outperform both ZNE and CDR in deep quantum circuits involving many qubits.
- **Poor man's CDR (pmCDR)** reduces the computational cost of CDR when applied to circuits comprised of several Trotter steps. In the case of a global depolarisation channel, the effective noise of N Trotter steps, ϵ_N, satisfies $1-\epsilon_N = (1-\epsilon)^N$, where ϵ is the noise associated with one Trotter step. The noise of deeper circuits, i.e. with more Trotter steps, can be inferred by applying CDR to circuits with one and two Trotter steps.

Transverse-Longitudinal Ising model

A system which displays interesting many body dynamics is the transverse field Ising model (TFIM) with an additional longitudinal field, providing a clear test bed for these mitigation methods.
The Hamiltonian of the quantum one-dimensional Ising model of length L with transverse and longitudinal fields is given by

$$H = -J\left[\sum_{i}^{L}\hat{\sigma}_i^Z\hat{\sigma}_{i+1}^Z + h_X\sum_{i=1}^{L}\hat{\sigma}_i^X + h_Z\sum_{i=1}^{L}\hat{\sigma}_i^Z\right] \tag{2}$$

where J is an exchange coupling constant, which sets the microscopic energy scale and h_X and h_Z are the transverse and longitudinal relative field strengths, respectively. This model is integrable for $h_Z = 0$ while for $h_Z \neq 0$ it is only integrable in the continuum when $h_X = 1$.

- Ferromagnetic phase: $h_X < 1$ (with $J > 0$) → Domain walls between the two ground states of H with $h_X = 0$.
  → Fermions

  $$|i\rangle = |\uparrow\cdots\uparrow_{i-1}\uparrow_i\downarrow_{i+1}\downarrow_{i+2}\downarrow_{i+3}\cdots\downarrow\rangle\,. \tag{3}$$

- $h_Z \neq 0$ → Confining potential between pairs of domain walls which increases linearly with the length of the domain.
  → Mesons

  $$|i,n\rangle = |\uparrow\ldots\uparrow_{i-1}\downarrow_i\ldots\downarrow_{i+n-1}\uparrow_{i+n}\ldots\uparrow\rangle\,, \tag{4}$$

We simulate the induced Hamiltonian dynamics using a first order trotterised evolution of the initial state. We decompose the quantum circuit to execute one Trotter step into the native IBM gate set $\{R_X(\pi/2), R_Z(\theta), X, \mathrm{CNOT}\}$. This decomposition leads to a depth of 11 per Trotter step with $2(Q-1)$ CNOT gates for a system size $Q > 2$, where Q is the number of qubits. For a fixed time step Δt one can evaluate the dynamics up to time t by repeated action of this circuit N_T times, where $N_T = t/\Delta t$.

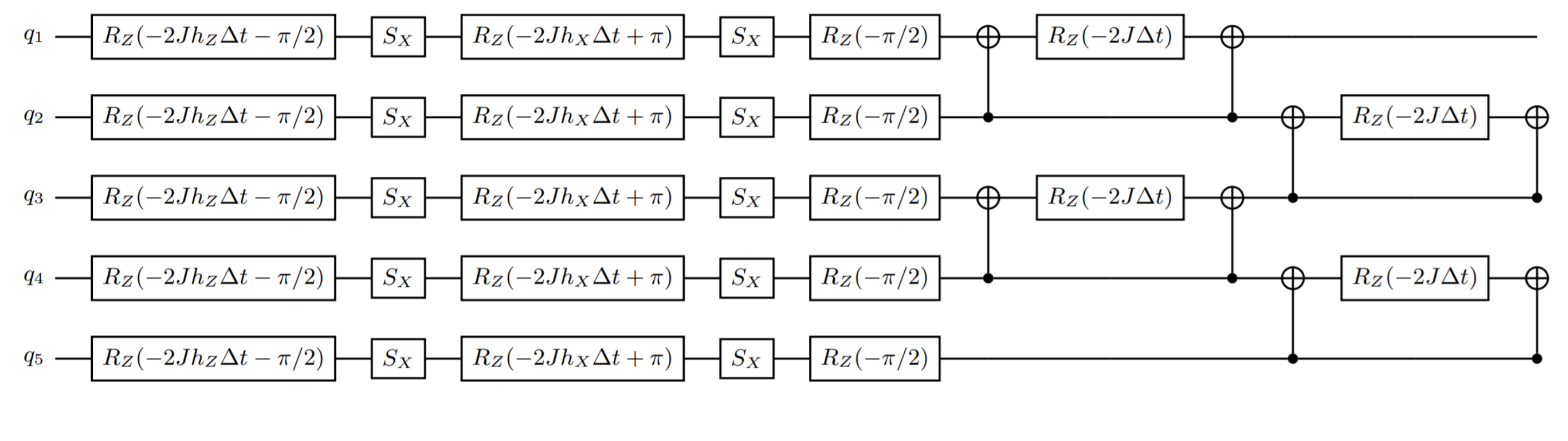


Figure 1: Quantum circuit representation showing one step of a first order Trotter expansion of a 5-qubit encoded spin-system.

Simulated spin chain confinement

We display the results obtained after applying the mitigation methods described above on the trotterised evolution of a system of $Q = 9$ qubits. We explore the signatures of confinement by measuring the probability distribution of kinks $\Delta_i^{ZZ}(t) = \frac{1}{2}(1 - \langle\hat{\sigma}_i^Z\hat{\sigma}_{i+1}^Z\rangle)$, the evolution of the two point correlation function $\sigma_{i,j}^{ZZ}(t) == \langle\hat{\sigma}_i^Z\hat{\sigma}_j^Z\rangle - \langle\hat{\sigma}_i^Z\rangle\langle\hat{\sigma}_j^Z\rangle$ and the meson masses determined by extracting the dominant frequency of the oscillation of the magnetisation $\sigma_i^Z(t) = \langle\hat{\sigma}_i^Z\rangle$.

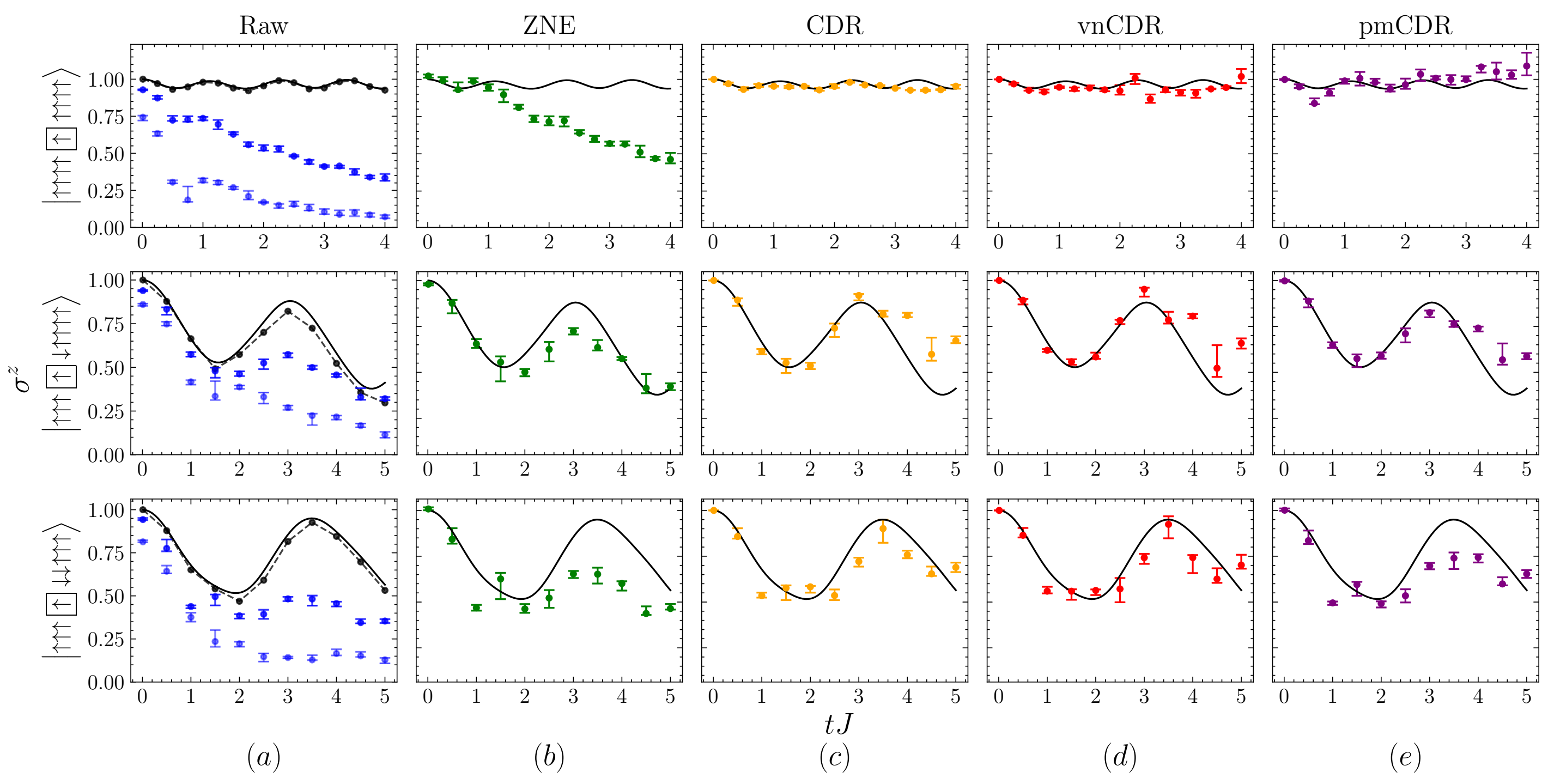


Figure 2: Temporal evolution of Z-axis local magnetisation with $h_X = 0.5$ and $h_Z = 0.9$ for three initial states (in each row) with the observables mitigated using various techniques (columns). In all panels the exact diagonalised dynamics is shown as a black-solid line. Raw observables are shown in the left most column (a) calculated at two noise levels $\mathcal{C} = \{1,3\}$ (blue and light blue points respectively) using the IBMQ Paris quantum computer. Black points and dashed lines show the trotterised dynamics.

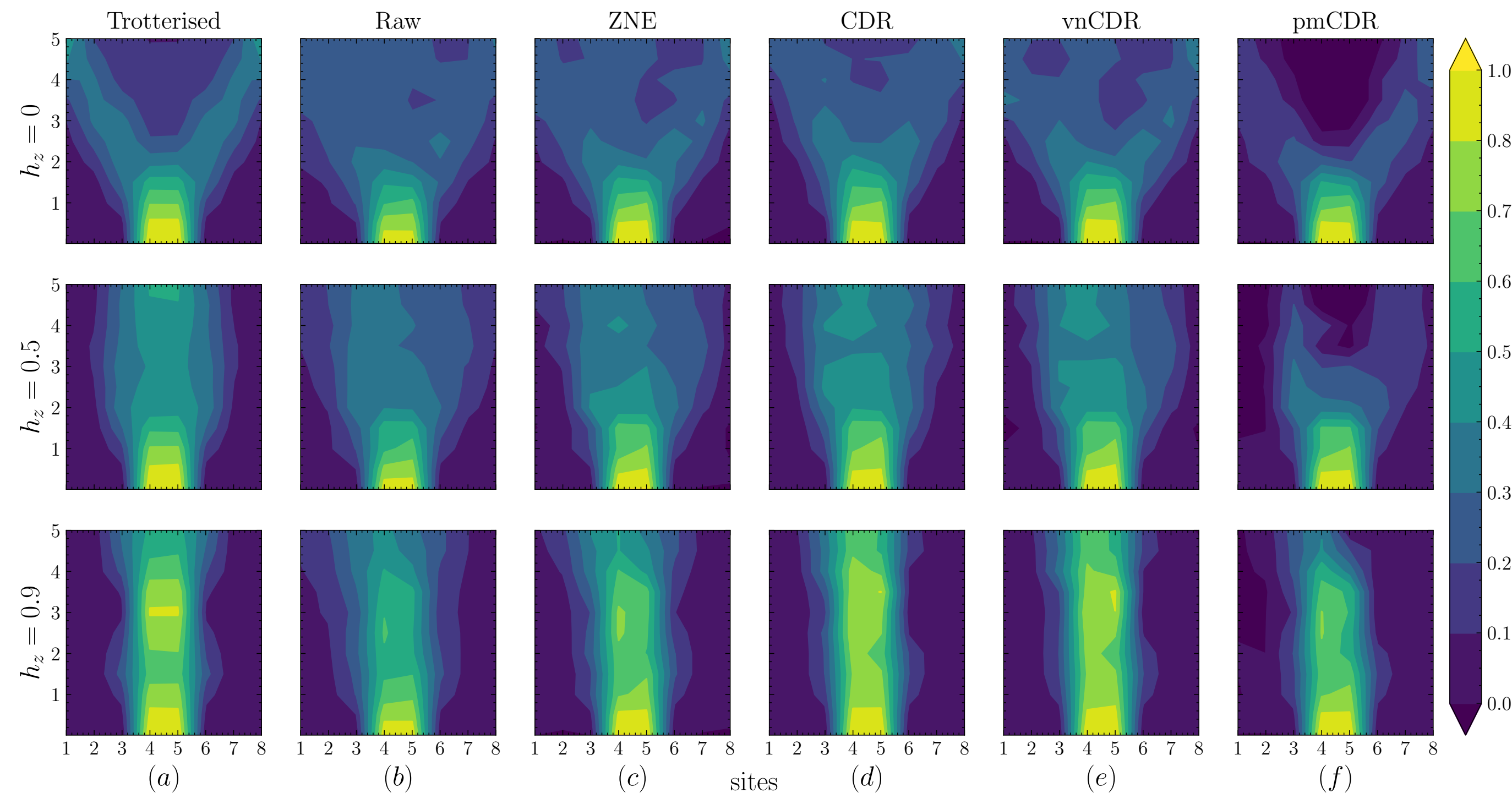


Figure 3: The observable Δ_i^{ZZ} projected into the 2-kink subspace, measured at various sites and Trotter steps when $h_X = 0.5$, $h_Z = 0$ (upper row), $h_X = 0.5$, $h_Z = 0.5$ (middle row) and $h_X = 0.5$, $h_Z = 0.9$ (bottom row) . The initial state of the system is $|\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\rangle$.

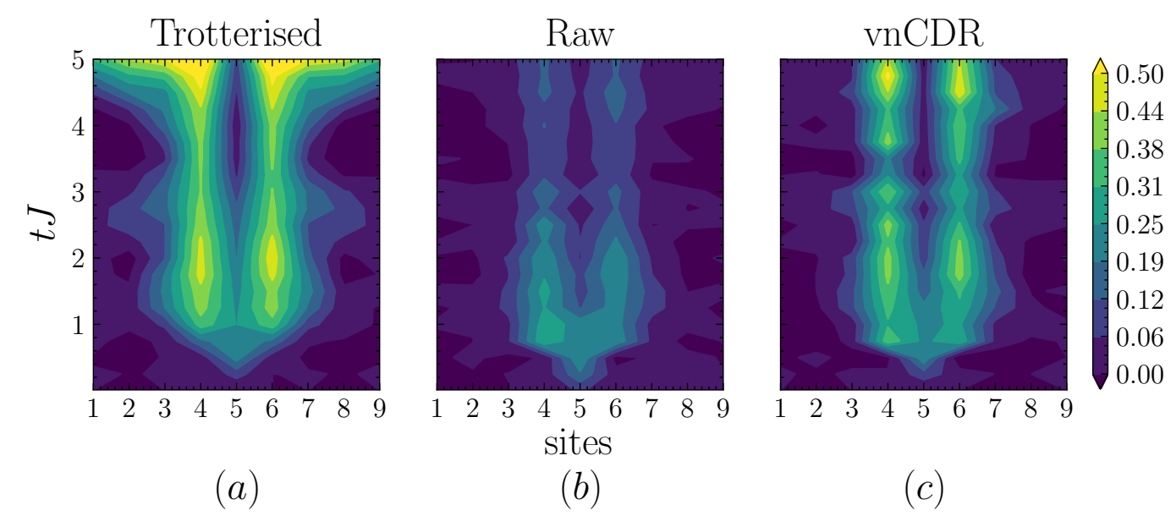


Figure 4: Correlation of qubits at sites along the x axis with the central qubit for the TFIM with $h_Z = 0$ and the system initialised in the $|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$ state.

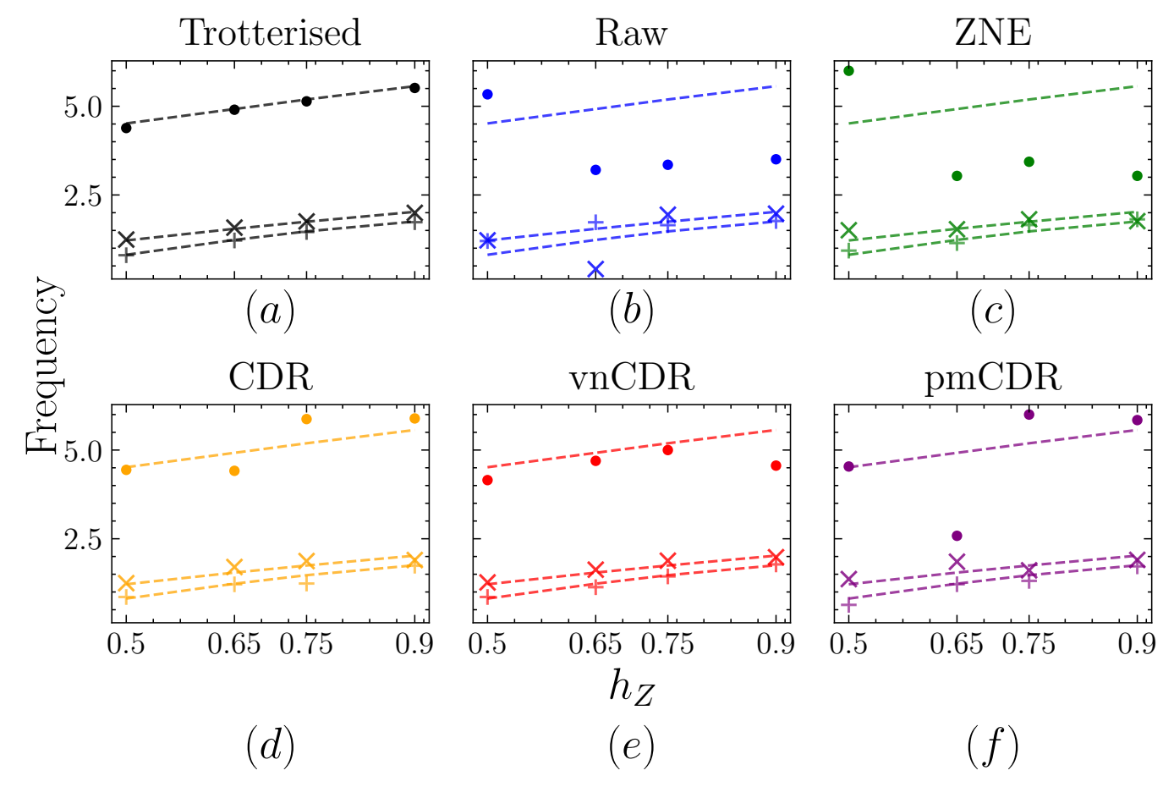


Figure 5: Frequencies obtained at $h_X = 0.5$ and various h_Z values calculated from the exact diagonalised (dashed lines), trotterised (a) and the median raw (b) and median mitigated results (c)-(f). Frequencies obtained for initial states $|\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$ (dots), $|\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow\rangle$ (diagonal cross) and $|\uparrow\uparrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\rangle$ (vertical cross) are plotted.

Conclusions

- We have simulated the dynamics of a quantum quench on the TFIM using a trotterised evolution on a quantum computer.
- We applied several data-driven error mitigation techniques, as well as presenting a simplified implementation of CDR, so-called pmCDR inspired by Ref. [4]. Clifford based mitigation methods show the best performance overall. We have demonstrated quantitative accuracy can be obtained using CDR and vnCDR from observables produced by circuits with depths of up to 176 involving hundreds of CNOT gates.
- We have shown it is possible to calculate the first meson masses with quantitative accuracy for systems of 9 qubits, the largest system explored in a study of this type.

References

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