Fine-Tuning in the 2HDM.

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Recalling the 2HDM

- With respect to the SM Higgs Sector, a new complex $Y = 1/2$, $SU(2)_L$ doublet scalar field is introduced, $\Phi \to (\Phi_1,\, \Phi_2)$. This leads to 8 degrees of freedom: 3 Goldstone bosons ($G^{\pm}$, G) absorbed by the EW bosons, 2 CP-even scalars $(h,\, H)$, 1 CP-odd scalar (A) and one pair of charged Higgses $(H^{\pm})$.
- The most general gauge invariant renormalisable scalar potential is given by:

$$\begin{aligned}\mathcal{V} = {}& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - [m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}] + \\ & \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \left\{\tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \left[\lambda_6(\Phi_1^\dagger\Phi_1) + \lambda_7(\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \text{h.c.}\right\}.\end{aligned} \tag{1}$$

- CP-conserving 2HDM: $\{m_{ij}^2, \lambda_k\}$ real.
- Avoidance of FCNCs: softly-broken discrete symmetry $\mathbb{Z}_2 \implies \lambda_{6,7} = 0$.

Useful equations relating the potential parameters

Both doublets develop vacuum expectation values (v_1, v_2), where $v_1 = v\cos\beta = vc_\beta$, $v_2 = v\sin\beta = vs_\beta$ and $v \approx 246$ GeV. From $\partial_{\Phi_i}\mathcal{V}|_{(v_1,v_2)} = 0$:

$$\mathbf{eq}1 \equiv 2m_{11}^2(1+t_\beta^2) - 2m_{12}^2(t_\beta + t_\beta^3) + v^2\left(\lambda_1 + t_\beta^2\lambda_{345}\right) = 0\,, \tag{2}$$

$$\mathbf{eq}2 \equiv -2m_{12}^2(1+t_\beta^2) + 2m_{22}^2(t_\beta + t_\beta^3) + v^2\left(t_\beta^3\lambda_2 + t_\beta\lambda_{345}\right) = 0\,. \tag{3}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$. Eliminating v^2:

$$\mathbf{eq}\beta \equiv m_{12}^2(-\lambda_1 + t_\beta^4\lambda_2) - m_{11}^2\left(t_\beta^3\lambda_2 + t_\beta\lambda_{345}\right) + m_{22}^2\left(t_\beta\lambda_1 + t_\beta^3\lambda_{345}\right) = 0. \tag{4}$$

On the other hand, the previously mentioned CP-even eigenstates are obtained by a rotation of the basis $(\Phi_1,\, \Phi_2)$ via the angle α, which verifies:

$$\mathbf{eq}\alpha \equiv m_{12}^2(1+t_\beta^2)(t_\alpha - t_\beta)(1+t_\alpha t_\beta) + v^2 t_\beta\left(t_\alpha(-\lambda_1 + t_\beta^2\lambda_2) + t_\beta(1-t_\alpha^2)\lambda_{345}\right) = 0. \tag{5}$$

Alignment limit and the Fine-Tuning in the 2HDM

The **alignment limit** is the situation in which the light eigenstate, h, behaves as the SM Higgs: they both present the same couplings to all the SM fields. By inspection of the normalized couplings to the EW bosons and fermions

$$C_V^h = \frac{g_{hVV}}{g_{h_{SM}VV}} = s_{\beta-\alpha}\,, \quad C_F^h = \begin{cases} s_{\beta-\alpha} + \cot\beta\; c_{\beta-\alpha}\,, \\ s_{\beta-\alpha} - \tan\beta\; c_{\beta-\alpha}\,, \end{cases} \tag{6}$$

we infer a mathematical characterization of the limit: $c_{\beta-\alpha} \to 0$. In this situation, several observables are susceptible of suffering **fine-tuning** (FT), i.e. they depend on a fine adjustment of the fundamental parameters of the theory. From the fact that $c_{\beta-\alpha} = \dfrac{-Z_6 v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}}$ and $Z_{1,6} = \sum\mathcal{O}(\lambda_i)$-terms, $\lambda_i \lesssim 1$:

- Alignment by decoupling: $m_H^2 \gg v^2 \implies m_{ij}^2 \gg v^2$ and we find a FT in $v^2 \backsim \dfrac{\sum\mathcal{O}(m_{ij}^2)\text{-terms}}{\mathcal{O}(\lambda_k)}$.
- Alignment without decoupling: $Z_6 \to 0$ and we find a FT in $c_{\beta-\alpha} \propto Z_6 = \sum\mathcal{O}(\lambda_i)$-terms.

Apart from these two potentially fine-tuned observable, we also take into account the observable t_β, which takes part in the EW breaking since now there are two vacuum expectation values. Therefore, we look over the observables $\{t_\beta,\, v^2,\, c_{\beta-\alpha}\}$

Fine-Tuning computation

By the Barbieri-Giudice Criteria, the FT in an observable Ω w.r.t. θ_i is

$$\Delta_{\theta_i}\Omega = \frac{d\log\Omega}{d\log\theta_i} = \frac{\theta_i\, d\Omega}{\Omega\, d\theta_i}, \qquad \Delta\Omega \equiv \max|\Delta_{\theta_i}\Omega|, \qquad \Omega \text{ is fine-tuned when } \Delta\Omega \gg 1.$$

Instead of obtaining expressions $\Omega(\theta_i)$, which are quite complex (e.g. t_β verifies a quartic equation), we use **eq**1, **eq**2, **eq**α and **eq**β as well as the Implicit Function Theorem:

$$\Delta_{\theta_i}t_\beta = -\frac{\theta_i}{t_\beta}\frac{\partial\mathbf{eq}\beta}{\partial\theta_i}\Big/\frac{\partial\mathbf{eq}\beta}{\partial t_\beta} \tag{7}$$

$$\Delta_{\theta_i}v^2 = -\frac{\theta_i}{v^2}\left(\frac{\partial\mathbf{eq}1(2)}{\partial\theta_i} + \frac{t_\beta}{\theta_i}\frac{\partial\mathbf{eq}1(2)}{\partial t_\beta}\Delta_{\theta_i}t_\beta\right)\Big/\frac{\partial\mathbf{eq}1(2)}{\partial v^2}, \tag{8}$$

$$\Delta_{\theta_i}c_{\beta-\alpha} = \frac{1}{c_{\beta-\alpha}}\left(t_\alpha\frac{\partial c_{\beta-\alpha}}{\partial t_\alpha}\Delta_{\theta_i}t_\alpha + t_\beta\frac{\partial c_{\beta-\alpha}}{\partial t_\beta}\Delta_{\theta_i}t_\beta\right), \tag{9}$$

where:

$$\Delta_{\theta_i}t_\alpha = \frac{\theta_i}{t_\alpha}\left(\frac{\partial\mathbf{eq}\alpha}{\partial\theta_i} + \frac{t_\beta}{\theta_i}\frac{\partial\mathbf{eq}\alpha}{\partial t_\beta}\Delta_{\theta_i}t_\beta + \frac{v^2}{\theta_i}\frac{\partial\mathbf{eq}\alpha}{\partial v^2}\Delta_{\theta_i}v^2\right)\Big/\frac{\partial\mathbf{eq}\alpha}{\partial t_\alpha}.$$

The final analytical formulas are not shown due to their extensions.

Dependence of the FT on the extra-Higgs masses

We consider a nearly realized alignment, $c_{\beta-\alpha} = 0.01$, and that $m_H = m_A = m_{H^\pm} = m_0 \in [200\text{ GeV}, 2000\text{ GeV}]$, where we set $t_\beta = 1.1$. We only display the most relevant results.

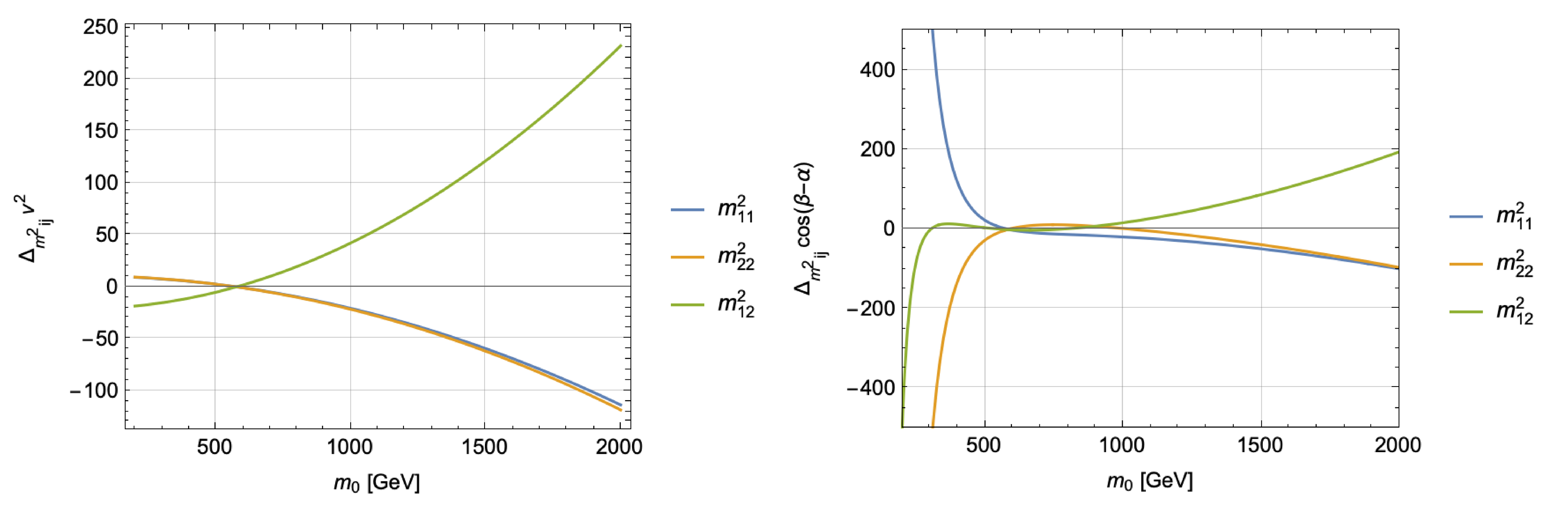

Fig. 1: Dependence on m_0 of the fine-tunings in v^2 and $c_{\beta-\alpha}$ due to m_{ij}^2.

Regarding v^2, $\Delta v^2 \sim \dfrac{m_0^2}{m_h^2}$ for $m_0 \gg m_h$. With respect to $c_{\beta-\alpha}$, using its expression and the definition of the FT:

$$\Delta_{\theta_i}c_{\beta-\alpha} = \frac{d}{d\,\log\theta_i}\left[\log v^2 + \log Z_6 - \frac{1}{2}\log((m_H^2 - m_h^2)(m_H^2 - Z_1 v^2))\right],$$

implying that $\Delta_{m_{ij}}c_{\beta-\alpha}$ has a contribution coming from $\Delta_{m_{ij}}v^2$. As a result, $\Delta_{m_{ij}}c_{\beta-\alpha}$, $\Delta_{m_{ij}}v^2 \gg 1$ for large masses and $\Delta_{m_{ij}}c_{\beta-\alpha} \gg 1$ also for small masses. Both FT are minimum (in absolute value) for $m_0 \approx 550$ GeV. t_β seems not to be fine-tuned.

Dependence of the FT on t_β

As before, $c_{\beta-\alpha} = 0.01$. We consider $t_\beta \in [1, 40]$ and we set $m_H = m_A = m_{H^\pm} = m_0 = 550$ GeV.

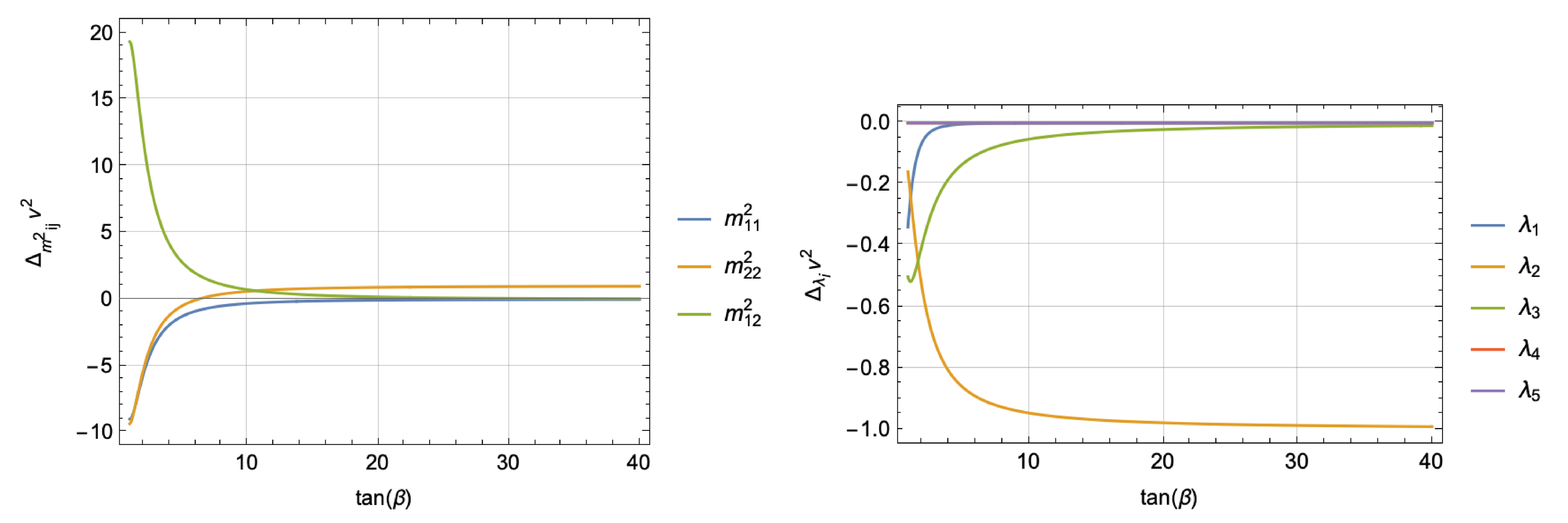

Fig. 2: Dependence of the v^2 fine-tuning on t_β

For $t_\beta \to \infty$ we find that $v^2 \approx v_2^2 \approx \dfrac{-m_{22}^2}{\lambda_2}$ and $m_{22}^2 \backsim m_h^2$. Moreover, $m_{11}^2 \backsim m_A^2$, $m_{12}^2 \backsim m_A^2/t_\beta \implies m_{11}^2 > m_{22}^2 \gg m_{12}^2$. This is, for large values of t_β the observable v^2 is not fine-tuned. Nevertheless, a hierarchy in the mass parameters is required.

The same happens when analysing Δt_β. Once for example, v_2 is fixed, the vacuum expectation value v_1 satisfies $v_1 \sim \frac{m_{12}^2 v_2}{m_{11}^2}$. Again, a hierarchy appears but no FT is found.

Symmetries and Special Regions in the Parameter Space

In the exact alignment limit $t_\alpha = -1/t_\beta$. From the consistency of **eq**α and **eq**β:

$$|m_{12}^2(\lambda_1 - \lambda_2)| - |(m_{11}^2 - m_{22}^2)|\sqrt{(\lambda_1 - \lambda_{345})(\lambda_2 - \lambda_{345})} = 0\,. \tag{10}$$

Alignment is reached when both terms vanish. Possible symmetries in the Higgs potential accommodating this constraint are listed in the following table:

symmetry		transformation law
Π_2	(mirror symmetry)	$\Phi_1 \to \Phi_2, \quad \Phi_2 \to \Phi_1$
$SO(3)$	(maximal Higgs flavour symmetry)	$\Phi_a \to U_{ab}\Phi_b, \quad U \in U(2)/U(1)_Y$
GCP3	(generalized CP symmetry)	$\begin{cases}\Phi_1 \to \Phi_1^*\cos\theta + \Phi_2^*\sin\theta \\ \Phi_2 \to -\Phi_1^*\sin\theta + \Phi_2^*\cos\theta\end{cases}$, for $0 < \theta < \frac{1}{2}\pi$

Table 1: Symmetries in the Higgs potential and their transformations laws.

In practice, these symmetries have to be broken to provide a phenomenologically viable model. Anyway, it is interesting to explore the scalar potential parameter regions where these (approximate) symmetries appear. We are now working on that and we hope to report soon about it.