AdS/CFT SUPERCONDUCTORS AND NON-HERMITIAN HOLOGRAPHY

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Ads/CFT: Introduction

AdS/CFT Correspondence (anti-de Sitter/Conformal Field Theory) was proposed by Juan Maldacena in 1999 in the frame of String Theory. It is a duality between a String Theory in d dimensions and a Quantum Field Theory in *d-1* dimensions. In its most fruitful limit, this correspondence relates a **classical theory of gravity** with a **strongly coupled quantum theory**. This correspondence has yield a lot of interesting results in different fields, such as Condensed Matter Physics or Cosmology.

AdS geometry

- Anti-de Sitter geometry is an exact solution of Einstein's field equations for an empty universe with a negative cosmological constant.
- The anti-de Sitter geometry can be expressed as:

$$\boxed{ds^2 = \frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2),}$$

  where t represents the time, $\vec{x}$ represents the vector of spatial coordinates and z is an additional coordinate usually known as *holographic coordinate*.
- When we fix the coordinate z, we get a geometry that is conformally equivalent to Minkowski space.
- In these coordinates, $z \to 0$ defines the conformal boundary of the AdS space.

QFT

An important result in Quantum Field Theory is the LSZ reduction formula, which can be expressed as:

$$\boxed{\langle f|i\rangle = i^{n+n'} \int d^4x_1\, e^{ik_1x_1}(-\partial_1^2 + m_1^2)...d^4x_{1'}\, e^{ik_{1'}x_{1'}}(-\partial_{1'}^2 + m_{1'}^2)\overbrace{\langle 0|T\varphi(x_1)...\varphi(x_{1'})...|0\rangle}^{VEV}\ .}$$

Using this formula, we can calculate any matrix element $\langle f|i\rangle$ by just knowing the VEV (*Vacuum Expectation Value*) $\langle 0|T\varphi(x_1)...\varphi(x_{1'})...|0\rangle$. In QFT, we can calculate this VEV from an object called the **Generating Functional Z(J)**. This object is usually computed using Feynman diagrams, which are a way of classifying terms in a perturbative expansion in the coupling constant. Nevertheless, when the theory presents strong coupling, this procedure is not applicable and other advanced methods, such as Lattice Quantum Field Theory, are required. Then, one can try and describe interesting quantum phenomena via a gravity dual. AdS/CFT proposes thus an alternative way of calculating those objects, and as a consequence, to calculate matrix elements $\langle f|i\rangle$, which allow us to obtain quantities such as cross sections, decay widths, and so on.

The proposal of AdS/CFT: Calculating VEVs from a theory of gravity in AdS

AdS/CFT proposes to identify the source ϕ of a charged operator $\mathcal{O}$ with the asymptotic value of a scalar field ϕ in AdS towards its conformal boundary, $\phi_0(x) = \phi(x, z \to 0)$. One can thus obtain the generating functional of the QFT by computing the generating functional of a gravity theory in the following manner:

$$\boxed{\boldsymbol{Z_{QFT}(\phi_0) = Z_{gravity}(\phi \to \phi_0),}}$$

where $Z_{gravity}(\phi \to \phi_0)$ is the Path Integral in the gravity theory evaluated over all functions which have the value ϕ_0 at the boundary of AdS. In the limit of strong coupling of the QFT, classical gravity dominates and the path integral can be evaluated over the classical gravity solution (*Saddle Point Approximation*), which can be easily calculated.

Holographic Superconductors

In order to test the AdS/CFT Correspondence when studying the strongly coupled regime of a QFT, some quantum phenomena were tried to be described using a gravitational dual. An outstanding example is that of superconductivity. This is a genuinely quantum state that has no classical equivalent. Even so, some of its main characteristics were successfully obtained using only classical formalism due to the AdS/CFT correspondance. One of those features is the spontaneous breaking of the U(1) symmetry that occurs in the phase transition, which is well described by the AdS/CFT Corresponence in [3].

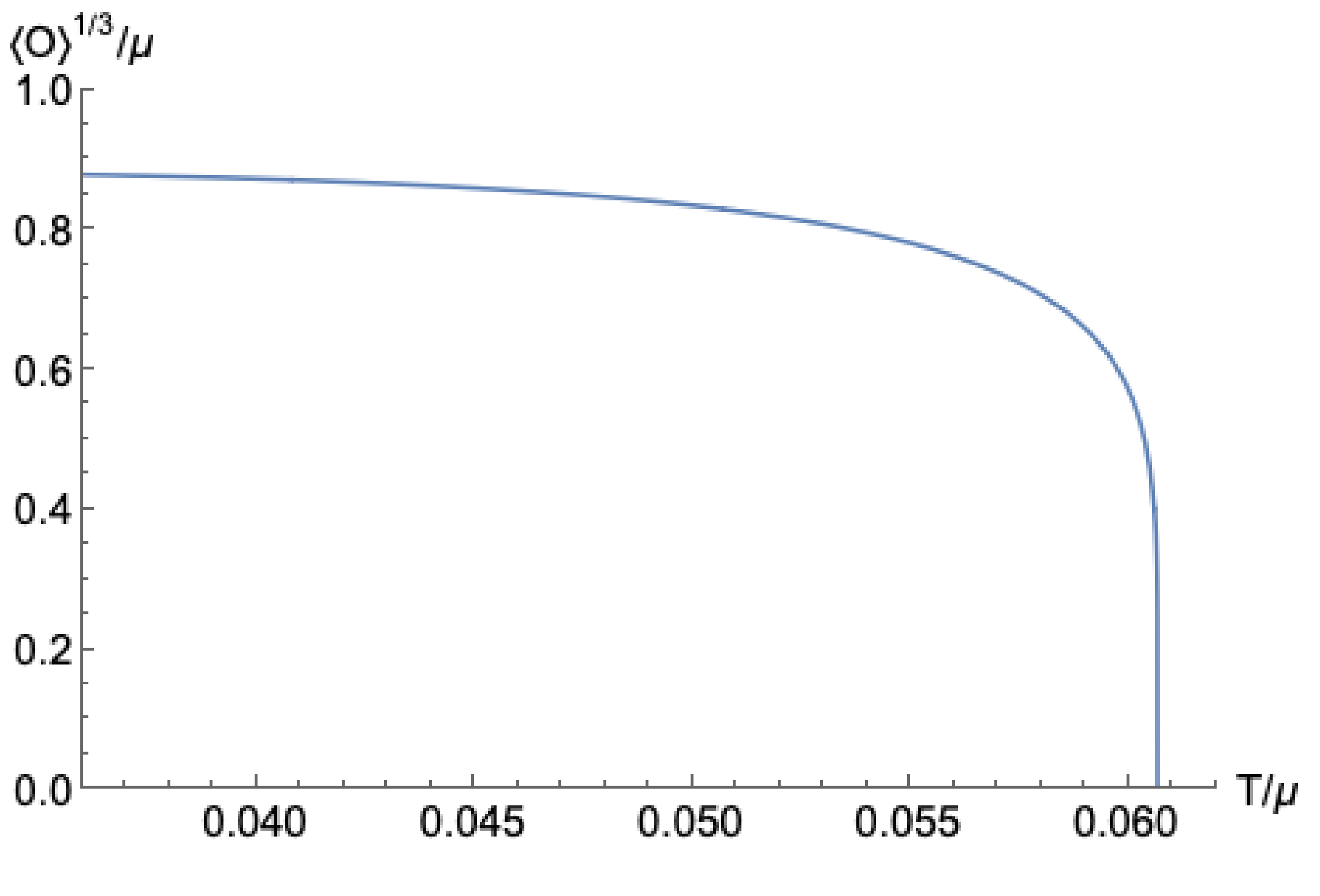

Fig. 1: Spontaneous breaking of U(1) symmetry from AdS/CFT, own authorship

This result is dual to a strongly coupled theory of superconductivity. Some models for high-temperature superconductors include strong coupling, so holography might be a path to better understand the microscopic mechanism of superconductivity at high temperatures.

Non-hermitian holography

It was found in [2] that, contrary to what was thought, Hermiticity is not a necessary condition for hamiltonians to have a real spectra. This condition was then substituted by the less restrictive one of being $\mathcal{PT}$-symmetric. These Hamiltonians have two regimes: $\mathcal{PT}$-symmetric regime, in which real eigenvalues are found, and $\mathcal{PT}$-broken regime, were eigenvalues become complex. We can implement non-Hermiticity in our model by deforming the sources, introducing a parameter X that measures how far away we are from the Hermitian regime (which corresponds to $X = 0$).

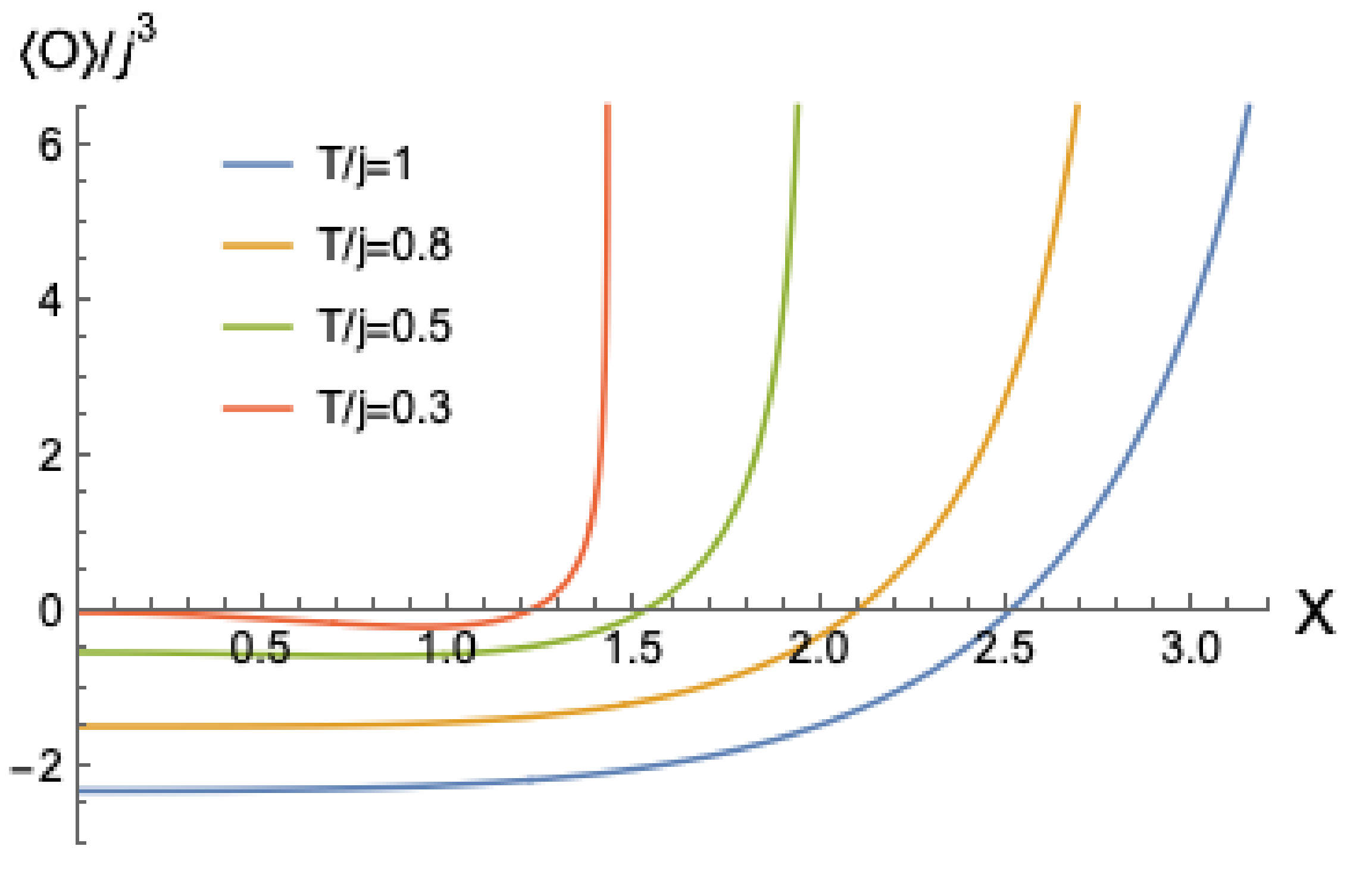

Fig. 2: Real solutions in the non-Hermitian regime.

$X < 1$ is the so called $\mathcal{PT}$-symmetric phase, were real solutions are expected to be found. On the other hand, $X > 1$ is known as the $\mathcal{PT}$-broken regime, and it is surprising to find real solutions there, since this region is not quasi-Hermitian.

References

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