Scalar Field Dark Matter with a quartic self-interaction potential

Johnatan Osvaldo Román Herrera (1) (jo.romanherrera@ugto.mx)
Luis Arturo Ureña-Lopéz (1), Francisco Xavier Linares Cedeño(2)
(1) División de Ciencias e Ingenierías Campus León, Universidad de Guanajuato, México
(2) Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, México

UNIVERSIDAD DE GUANAJUATO

Introduction

The Cold Dark Matter (CDM) simulations have failed reproducing the universe at small scales, like the one we observe [5]. An alternative for this model is the ultra light Scalar Field Dark Matter (ulSFDM), on which one particle associated with the scalar field is proposed to play the role of dark matter (DM) due to its characteristic ultralight mass about 10^{-22} eV that avoid structure formation below galactic scales [2].

For this model we propose an ulSFDM with quartic potential:

$$V(\phi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{\lambda}{4!}\frac{m_\phi^2}{f_a^2}\phi^4, \qquad (1)$$

where m_ϕ is the boson mass, ϕ is the scalar field, λ is a dimensionless parameter with a natural value of the order of unity and f_a is the characteristic energy scale.

Background

In order to solve the cosmology for a system with homogeneous and isotropic Universe, whose metric is described by the standard Friedmann-Lemaitre-Robertson-Walker (FLRW), the developments described on [1] are proposed for a dynamical systems and change of variables as proposed in [4].
Therefore, the model can be described as a system of differential equations that describes the dynamics of the Klein-Gordon (KG) scalar field equation:

$$\theta' = -3\sin\theta + y_1\sqrt{1+Q^2}, \qquad (2a)$$

$$y_1' = \frac{3}{2}\left[1+\omega_{tot}\right]y_1, \qquad (2b)$$

$$r' = \frac{3}{2}\left[\omega_{tot} - \omega_\phi\right]. \qquad (2c)$$

The new system variables (θ, y_1, r) are related to the equation of state (EoS) for the scalar field ω_ϕ, the mass of boson $y_1 = 2m_\phi/H$ and the scalar field density parameter $r = \Omega_\phi^{1/2}$. Where ω_ϕ is:

$$\omega_\phi = -\cos\theta - \frac{Q^2}{(\sqrt{1+Q^2}+1)^2}(1-\cos\theta), \qquad (3)$$

here Q is a composite parameter defined in [4] as:

$$Q = \frac{\sqrt{2}\lambda^{1/2}}{\kappa f_a}\frac{r}{y_1}(1+\cos\theta), \qquad (4)$$

if $Q \to 0$ the system reduces to the fuzzy cold dark matter model. From (4), we define Q_i as the initial value of Q at early times before the oscillations in ω_ϕ.

The evolution of the system was studied by numerical solutions modifying the CLASS code [3].

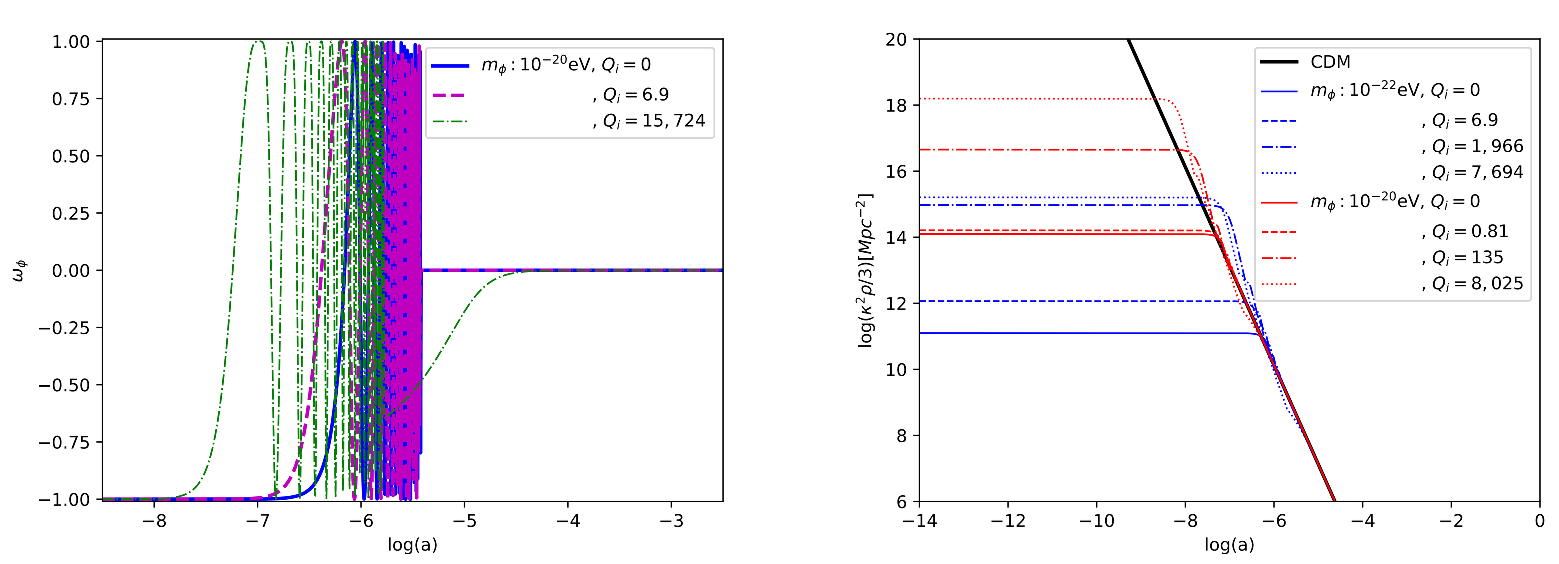

Figure 1: (*left*) EoS for the scalar field as function of the scale factor a, for different values of Q_i and fixed boson mass. (*right*) Evolution of the energy density parameter $\rho_\phi = (1/2)\dot\phi + V(\phi)$, and for the $\rho_{\rm CDM}$ in the ΛCDM model (black line).

Perturbations

Linear perturbations are assumed to be around the background values in the FLRW metric with synchronous gauge, with the general perturbed line element $ds^2 = -dt^2 + a^2(t)(\delta_{ij}+h_{ij})dx^i dx^j$, the perturbation for the scalar field are introduced the same way as in [1].
To calculate the matter power spectrum (MPS), we need the density contrast δ_ϕ, is:

$$\delta_\phi = \sqrt{1+Q^2}\delta_0 - q\frac{r}{y_1}\left(\frac{\sqrt{1+Q^2}-1}{Q}\right)\sin\theta\delta_1, \qquad (5)$$

where $q = \frac{\sqrt{2}\lambda^{1/2}}{\kappa f_a}$, δ_0 and δ_1 are the new variables from KG linearized equations in Fourier space:

$$\delta_0' = -\left[3\sin\theta + \frac{k^2}{2a^2Hy_1}(1-2\cos\theta)qr\cos(\varphi/2)\right]\delta_1 + \left[\left(\frac{k^2}{2a^2Hy_1} + \frac{3}{2}qr\cos(\varphi/2)\right)\sin\theta - 3(1-\cos\theta)\cos^2(\varphi/2)\right]\delta_0 - \frac{1}{2}\bar{h}'(1-\cos\theta)\sin^2(\varphi/2), \qquad (6a)$$

$$\delta_1' = -\left[3\cos\theta + 3(1-\cos\theta)\cos^2(\varphi/2) + \left(\frac{k^2}{2a^2Hy_1} + \frac{3}{2}qr\cos(\varphi/2)\right)\sin\theta\right]\delta_1 + \left[\left(\frac{k^2}{2a^2Hy_1} + qr\cos(\varphi/2)\right)(1+\cos\theta)\right]\delta_0 - \frac{1}{2}\bar{h}'\sin\theta\sin^2(\varphi/2), \qquad (6b)$$

k is the wave number, $\cos(\varphi/2) = \frac{\sqrt{1+Q}-1}{Q}$ and $\bar{h}'$ is the trace of the spatial part of the metric perturbation.

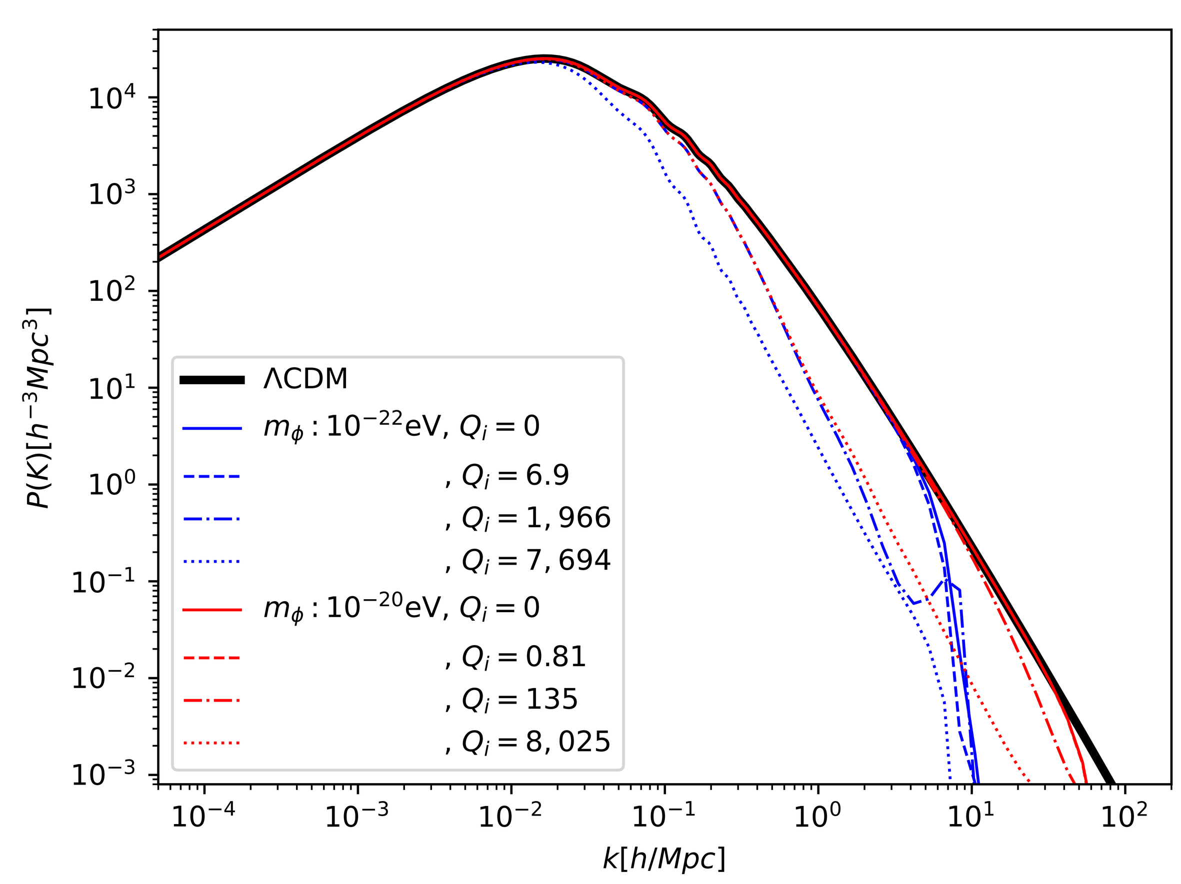

Figure 2: Matter power spectrum for the scalar field model as function of k, and for the ΛCDM model (black line).

Conclusions

- It can be observe that as Q increases in value for fields with the same mass, the oscillations of the field start at earlier times, and thus their behavior is similar to CDM.
- Respect to the magnitude of the MPS, it decreases as Q increases, in some cases, the characteristic cut-off of ulSFDM models is vanishes or is shifted to smaller scales.

References

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