Parity violating gravitational waves at the end of inflation

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Abstract

Inflaton-vector interactions of the type $\phi F\tilde{F}$ have provided interesting phenomenology to tackle some of current problems in cosmology, namely the vectors could constitute the dark matter component. It could also lead to possible signatures imprinted in a gravitational wave spectrum. Through this coupling, a rolling inflaton induces an exponential production of the transverse polarizations of the vector field, having a maximum at the end of inflation when the inflaton field velocity is at its maximum. These gauge particles, already parity asymmetric, will source the tensor components of the metric perturbations, leading to the production of parity violating gravitational waves. In this work we examine the vector particle production in the weak coupling regime, integrating the gauge mode amplitudes spectrum during the entirety of its production and amplification epochs, until the onset of radiation domination. Finally, we calculate the gravitational wave spectrum combining the vector mode analytical solution, the WKB expansion, valid only during the amplification until horizon crossing, and the numerical solution obtained at the beginning of radiation domination when the modes cease to grow.

Introduction

Cosmological inflation

- Early phase of accelerated expansion [1];
- Currently the preferred solution to address the flatness and horizon problems;
- Inflaton's quantum vacuum fluctuations provide a natural mechanism to generate the observed anisotropies in the Cosmic Microwave Background (CMB).

"Axion-like" inflation models

- Coupling with $U(1)$ vector particles through $\alpha\phi F\tilde{F}/f$;
- As ϕ rolls down it will source a tachyonic amplification of the vector modes from vacuum into a classical state.

Parity violation

- Only one of the vector transverse degrees of freedom is amplified [2, 3];
- Abrupt production of gauge fields sources gravitational waves (GW) [4], also parity asymmetric.

Vector particle production

Relate inflation and reheating with gauge particle production

$$\mathcal{S} = -\int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4}\tanh^4\left[\frac{\phi}{\sqrt{6}\,m_P}\right]m_P^4 + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{4f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

The inflaton will only couple to the transverse modes

$$\left[\frac{\partial^2}{\partial\tau^2} + k^2 \pm 2k\frac{\xi}{\tau}\right]A_\pm(\tau,\tau) = 0,$$

$$\text{where} \quad \xi \equiv \frac{\alpha\dot{\phi}}{2Hf} = \sqrt{\frac{\epsilon}{2}}\frac{\alpha}{f}M_{Pl}, \quad \text{with} \quad \epsilon = -\frac{\dot{H}}{H^2} \simeq \frac{\dot{\phi}^2}{2H^2M_{Pl}^2}$$

where τ represents conformal time defined by $ad\tau = dt$ (during inflation $\tau \simeq -\frac{1}{aH}$) . Taking $\dot{\phi} > 0 \rightarrow \xi > 0$, leads to an instability for one of the modes when

$$k^2 \pm 2k\frac{\xi}{\tau} = k^2 \mp 2k\xi aH < 0$$

- As a result only A_+ is amplified, and A_- will stay in vacuum.

Considering no backreaction on inflaton evolution and treating ξ as constant,

$$A_+(k,\tau)_{\rm WKB} \simeq \frac{1}{\sqrt{2k}}\left(\frac{-k\tau}{2\xi}\right)^{1/4}e^{\pi\xi - 2\sqrt{-2\xi k\tau}}, \qquad \text{for} \qquad \frac{1}{8\xi} < -k\tau < 2\xi,$$

- Good intuition into the behavior of the modes around horizon crossing, [3].

$\rightarrow$ Build a semi analytical function for each mode.

$$A_+(k,\tau) \simeq \begin{cases} A_{BD}(k) & \tau < \tau_{tac} \\ A_{WKB}(k,\tau) & \tau_{tac} < \tau < \tau_h \\ A_+(k,\tau_{end}) & \tau > \tau_h\ . \end{cases}$$

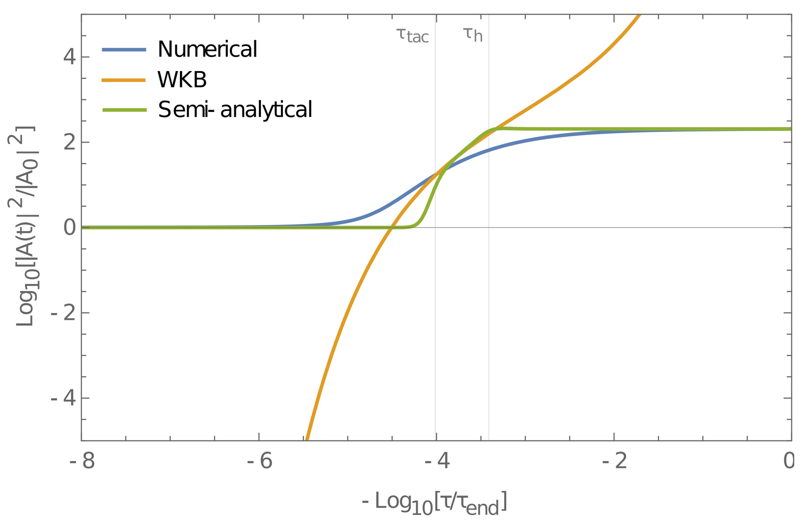

Figure 1: Comparison of the time evolutions of the WKB expansion, the numerical solution for $|A_k|^2$ and the semi-analytical approximation for a fixed mode.

0^{th}- Order backreaction

Backreaction on the inflaton evolution. Taking $\langle F\tilde{F}\rangle = \langle \bar{E}\cdot\bar{B}\rangle$,

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = \dot{\phi}\frac{\alpha}{f}\langle\bar{E}\cdot\bar{B}\rangle, \qquad \dot{\rho}_A + 3H(\rho_A + p_A) = -\dot{\phi}\frac{\alpha}{f}\langle\bar{E}\cdot\bar{B}\rangle,$$

$$\dot{\bar{E}}\cdot\bar{B} + \bar{E}\cdot\dot{\bar{B}} = -4H\,\bar{E}\cdot\bar{B} - \dot{\phi}\frac{\alpha}{f}|\bar{B}|, \text{ with } \langle|\bar{B}|^2\rangle = \frac{1}{8\pi^3a^4}\int d^3k\,k^2\sum_{\lambda=\pm}|A_{+WKB}|^2$$

- Amplification until $\epsilon_H = 2$ for modes that started to be amplified during slow-roll
- The 0^{th}- order backreaction is insufficient to describe non-linear dynamics
- Find an upper bound on $\alpha m_P/f$ for which non-linear effects may be ignored

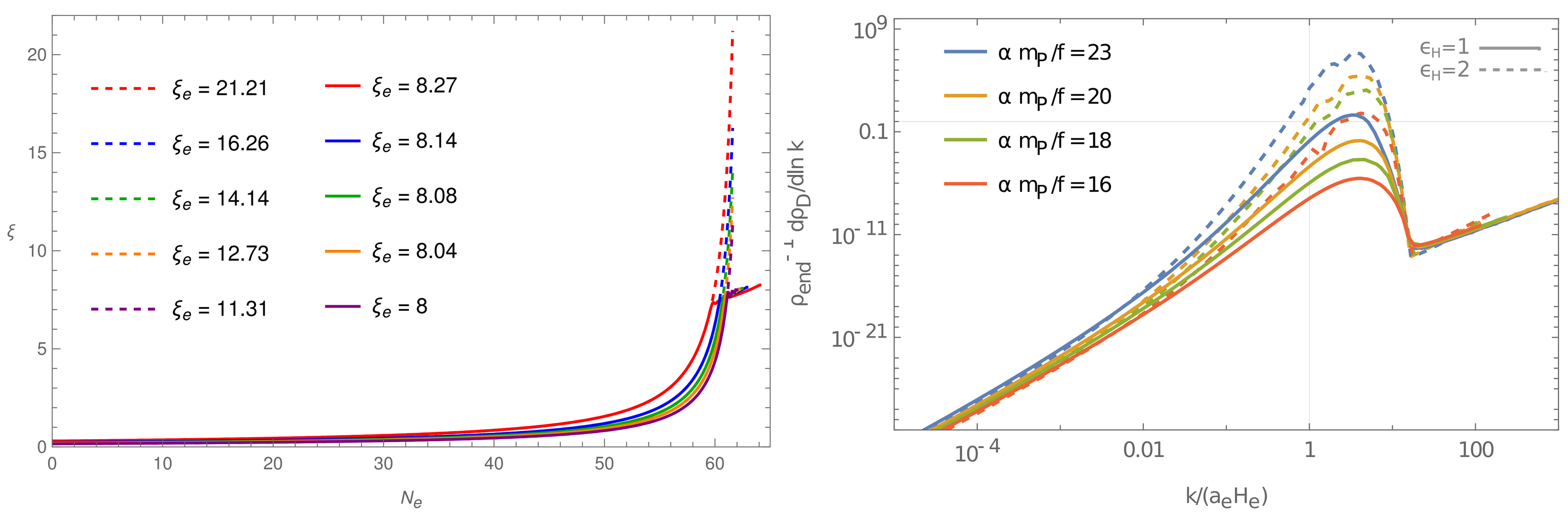

Figure 2: Left panel: comparison of ξ evolution with 0^{th}-order (dashed) and without (solid) backreaction effects. Right panel: Vector energy density spectrum normalized with the total energy density at the end of inflation.

Power spectrum GW's

Solving Einstein's equation for the tensor perturbations

$$h_\pm(\mathbf{k}) = -\frac{2H^2}{M_P^2}\int d\tau' G_k(\tau,\tau')\,\tau'^2\int\frac{d^3\mathbf{q}}{(2\pi)^{3/2}}\Pi_\pm^{lm}(\mathbf{k})\times$$
$$\times\left[A_l'(\mathbf{q},\tau')\,A_m'(\mathbf{k}-\mathbf{q},\tau') - \varepsilon_{lab}q_aA_b(\mathbf{q},\tau')\,\varepsilon_{mcd}(k_c-q_c)\,A_d(\mathbf{k}-\mathbf{q},\tau')\right]$$

The fraction of GW energy density today and the WKB prediction are given by

$$\Omega_{ss'}h^2 = \frac{\Omega_{R0}h^2}{24}\frac{k^3}{2\pi^2}\langle h_sh_{s'}\rangle, \qquad \Omega_{++}^{WKB}h^2 \simeq 1.5\times10^{-13}\frac{H^4}{m_P^4}\frac{e^{4\pi\xi}}{\xi^6}.$$

Inserting the semi analytical function into $\langle h_sh_{s'}\rangle$ we find

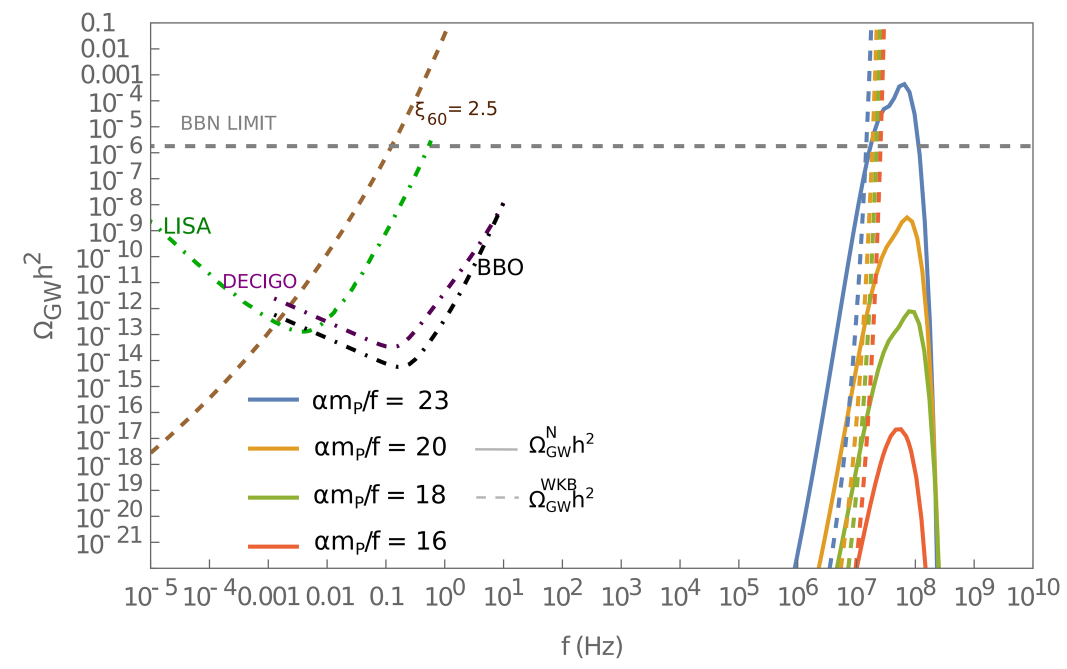

Figure 3: Comparison of the spectral energy density for the GW's produced from the tachyonic amplification estimated by the semi analytical method and the analytical description with the WKB solution for the vector modes

Chaotic quartic vs α-attractors

α-attractor and $\lambda\phi^4$ potentials coincide at the end of inflation and reheating. Comparing the GW spectrum

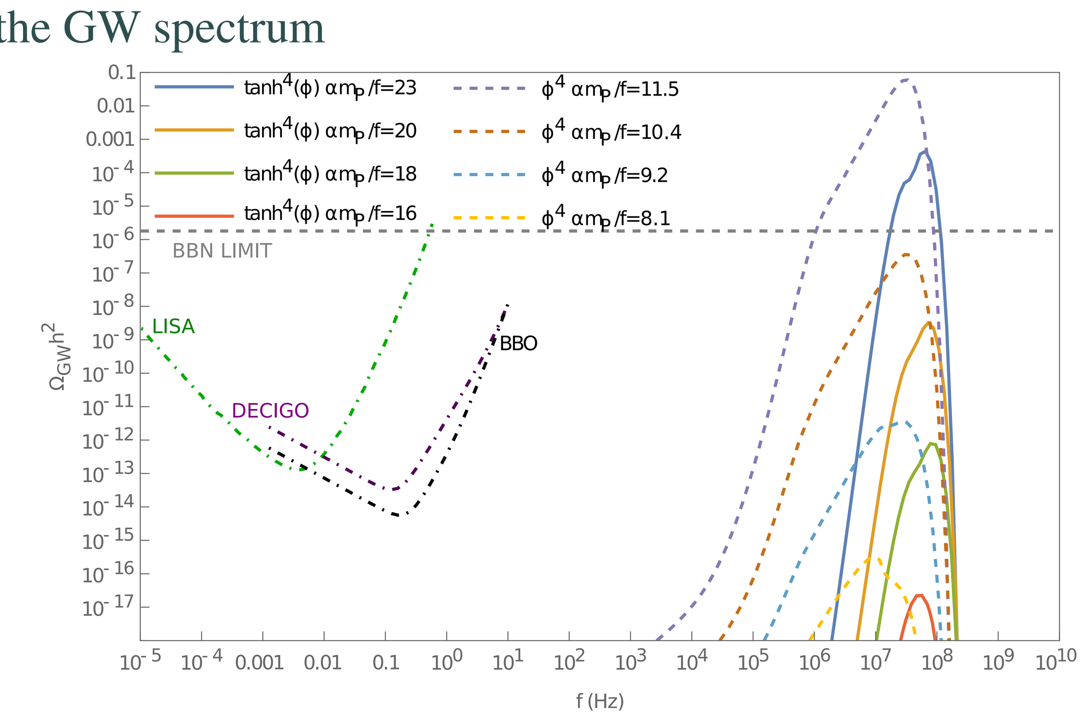

Figure 4: Comparison of the spectral energy density for the GW's produced with an α-attractor (solid) and quartic inflationary potentials (dashed).

- Retain the peak frequencies and a similar shape for the spectrum.
- For $\lambda\phi^4$ we see broader spectrums as the lower k modes are more amplified
- In the α-attractor find a higher peak, a larger amplification at the end of inflation.

Conclusions

- Mechanisms to address early universe puzzles may be tested with GW
- Exponential production of gauge fields during inflation may lead to detectable gravitational wave spectrum
- A_+ to be estimated in the valid regime, full back-reaction needs to be optimized to accomodate upper bound on non-Gaussianities, $\xi_{60} \simeq 2.5$ $(\alpha m_P/f \simeq 249)$,
- Able to correctly reproduce the vector production for $\alpha m_P/f < 16$
- In this linear regime, signals in the GW spectrum are extremely small.
- Peaks in the GW spectrum appear around 10^8 Hz, as expected from the maximal amplification of the vector modes at the end of inflation and reheating

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