The 'core-cusp' problem of the dark matter halos and N-body simulations[1]

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Density profiles.

Isothermal profile

$$\rho \sim r^{-2}$$

Navarro-Frenk-White profile

$$\rho_{NFW} = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

Einasto profile

$$\rho_{Ei} = \rho_s \exp\left\{-2n\left[\left(\frac{r}{r_s}\right)^{\frac{1}{n}} - 1\right]\right\}$$

Hernquist profile

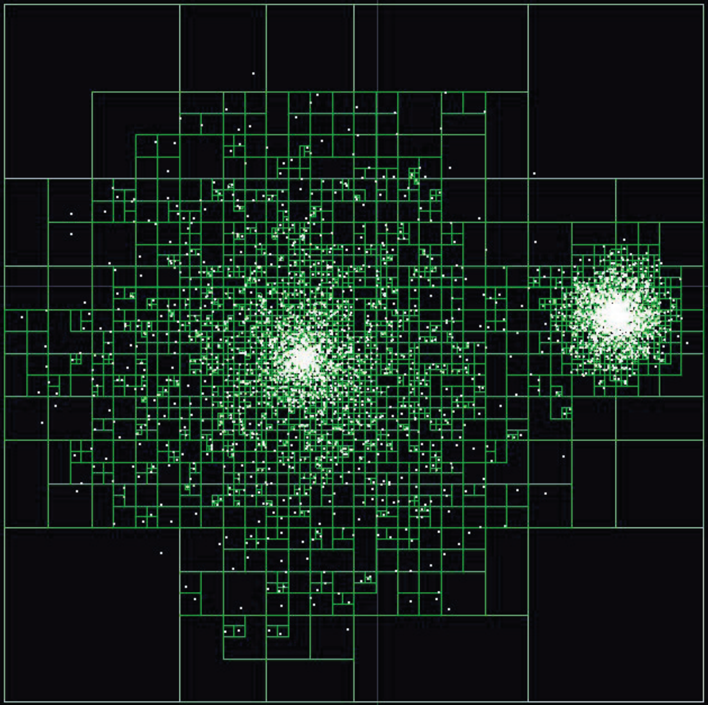
$$\rho_H = \frac{Ma}{2\pi r(r + a)^3}$$

The $\rho_c r_c$ constancy.

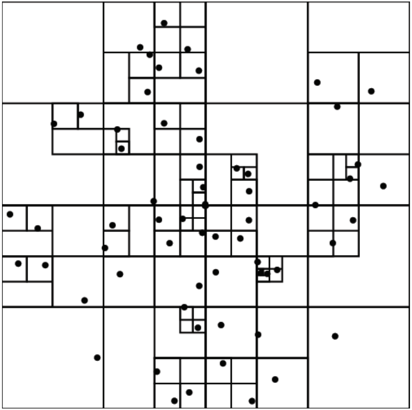
Galaxies: (de Blok et al. 2001), (de Blok & Bosma 2002), (Marchesini et al. 2002), (Gentile et al. 2007), (Chemin et al. 2011), (Oh et al. 2011), (Walker and J. Penarrubia, 2011), (Governato et al. 2012), (Tollerud et al. 2012), (Del Popolo & Pace 2016)

Multiplication $\log(\rho_c r_c) = const \simeq 2.15 \pm 0.2$ $(M_\odot/\mathrm{pc}^2)$ for a wide variety of galaxies (Kormendy & Freeman, 2004), (Donato et.al., 2009)

Galaxy clusters: (Harvey et.al. 2017)



Cell opening criterion $\boxed{\frac{GM_{cell}}{r^2}\left(\frac{l}{r}\right)^2 \leq f_{acc}|\vec{a}|}$



Orbit calculation

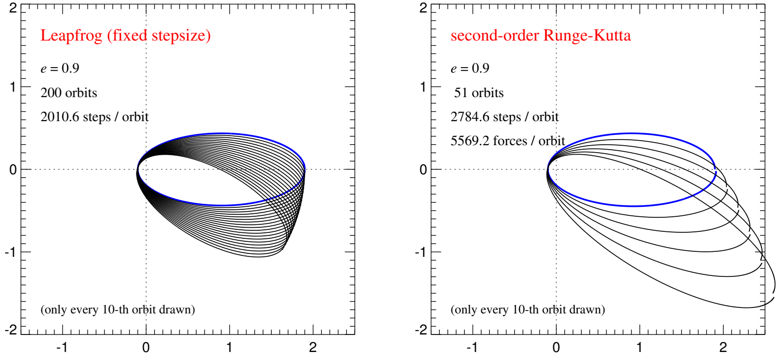


Figure: Leapfrog vs. Runge-Kutta

Numerical effects

Relaxation time

$$\langle \Delta v \rangle \simeq 0 \qquad \langle \Delta v^2 \rangle \simeq \frac{8v^2 \ln \Lambda}{N(r)}$$

$$\tau_r(r) = \frac{N(r)}{8 \ln \Lambda} \cdot \tau_d(r) \qquad \tau_d(r) \sim \frac{r}{v}$$

(Power et. al. 2003) $t_0 \leq 1.7\tau_r$
(Hayashi et al. 2003; Klypin et al. 2013) $t_0 \leq 30\tau_r$

Our simulation

$$\rho_H = \frac{Ma}{2\pi r(r+a)^3} \qquad \phi(r) = -\frac{GM}{r+a}$$

$M = 10^9 M_\odot$, $a = 100$ pc. We use $N = 10^5$ test bodies.

$$\tau_d = \frac{r+a}{a}\sqrt{r/a}\times 0.472\cdot 10^6\text{years}; \qquad \tau_r = \frac{2r^2\sqrt{r/a}}{a(r+a)}\times 1.36\cdot 10^9\text{years}.$$

At $r = a$
$\tau_d = 9.45 \cdot 10^6$ years, $\tau_r = 1.36 \cdot 10^9$ years.

The integrals of motion $\epsilon = \phi(r) + v^2/2$, $\vec{K} = [\vec{v} \times \vec{r}]$, r_0:

$$\epsilon = \phi(r_0) + K^2/2r_0$$

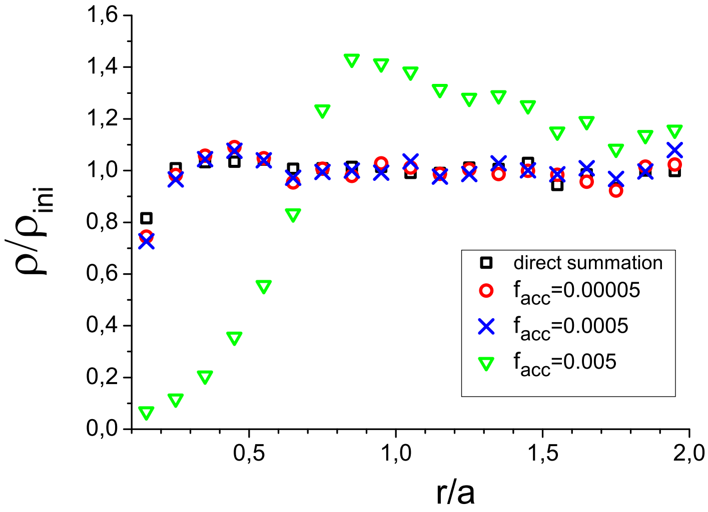


Figure: The density profiles at $t = 0.45 \cdot 10^9$years

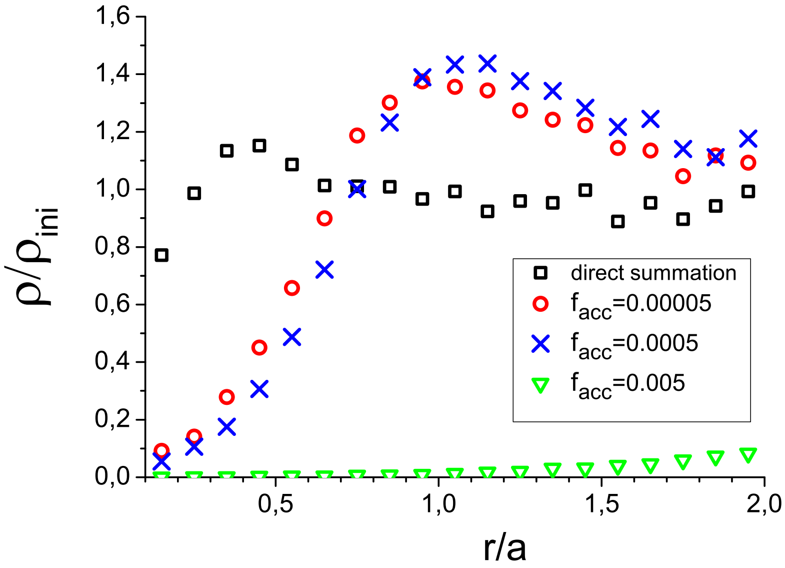


Figure: The density profiles at $t = 2.85 \cdot 10^9$years

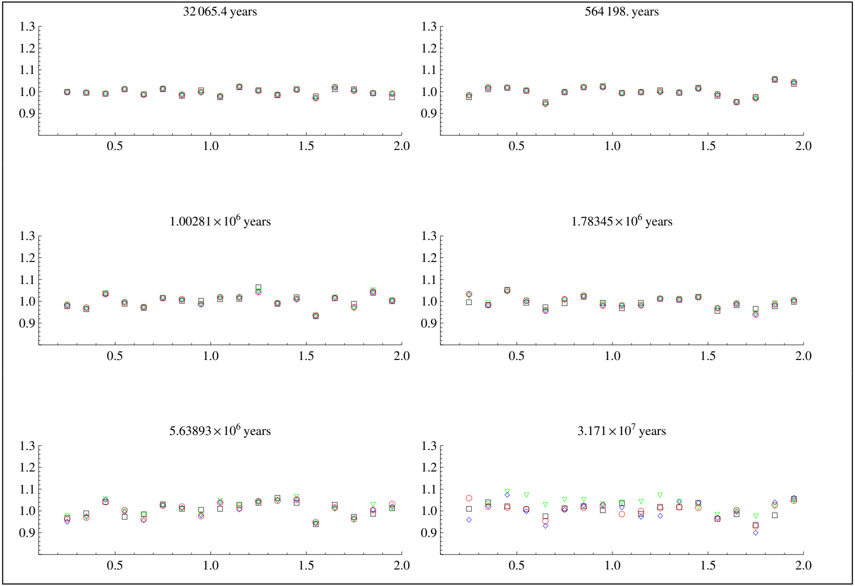


Figure: The density profiles

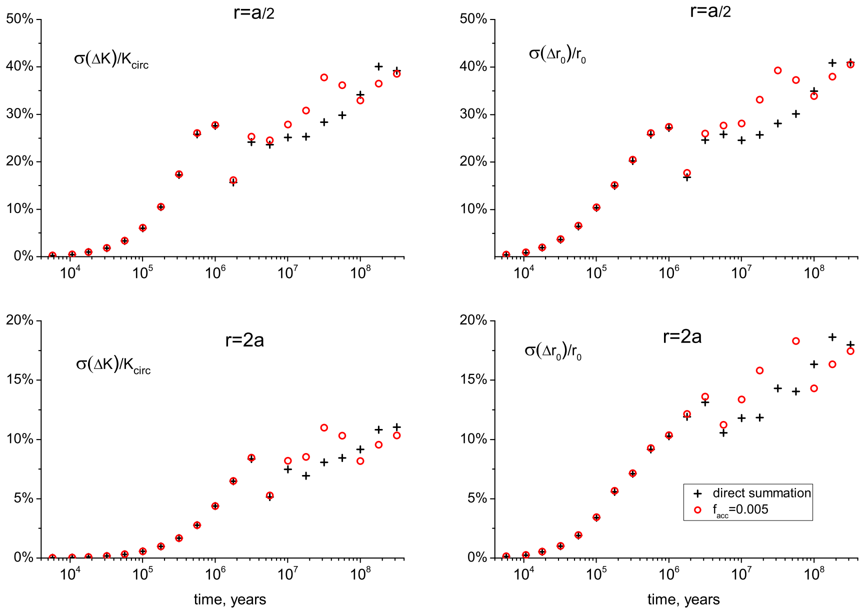


Figure: The temporal behavior of the integrals of motion.

Collisional system

If the halo is spherically symmetric and the velocity distribution is isotropic

$$f(r, \vec{v}) = f(\phi(r) + v^2/2)$$

$$\frac{df}{dt} = \frac{\partial}{\partial p_\alpha} \left\{ \tilde{A}_\alpha f + \frac{\partial}{\partial p_\beta} [B_{\alpha\beta} f] \right\}$$

where $\vec{q}$ is the momentum changing $\vec{p} \to \vec{p} - \vec{q}$ in a unit time.

$$\tilde{A}_\alpha = \frac{\sum q_\alpha}{\delta t} \qquad B_{\alpha\beta} = \frac{\sum q_\alpha q_\beta}{2\delta t}$$

$$\frac{df}{dt} = 0 \qquad \text{vs} \qquad \frac{df}{dt} = \frac{\partial^2 [B_{\alpha\beta} f]}{\partial p_\alpha \partial p_\beta}$$

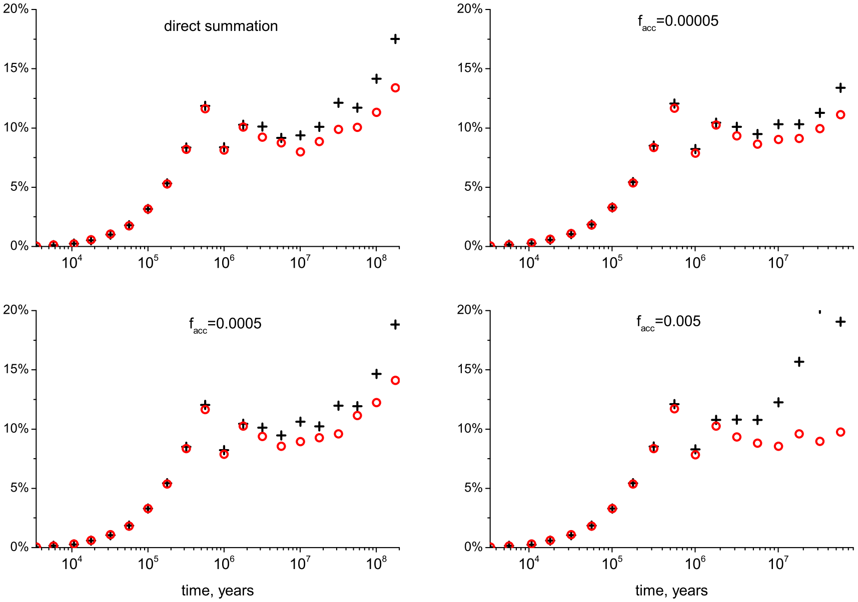


Figure: Radial streams of particles through $r = a/2$.

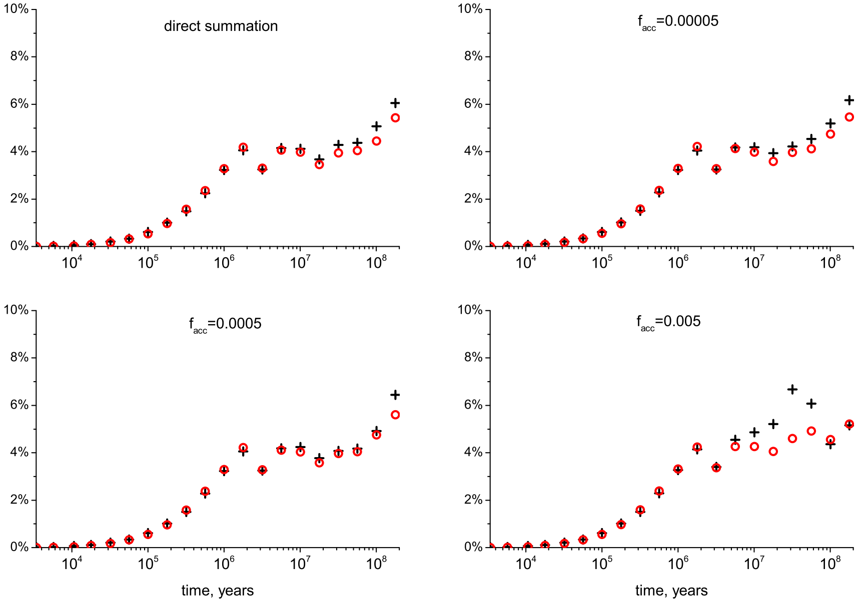


Figure: Radial streams of particles through $r = 2a$.

Conclusion

1) An instability with the characteristic time $\sim \tau_d(r)$ develops immediately after the simulation launch. It leads to a numerical 'violent relaxation': the integrals of motion change (on the average) on 10% from their initial values even at $r \simeq r_s$.

2) Relying on the present-day N-body simulations, one cannot infer that a relaxation (in particular, the collisional one) tends to transform a cusp into a core in the center of DM halos. Theoretical consideration rather suggests the opposite (Baushev, A&A, 2014) .

3) The significant variations of the integrals of motion reveal that the system of test particles in the N-body simulations is essentially collisional, contrary to real DM systems.

4) Much remains to be done in testing of N-body simulation convergence and reliability.