# Challenging the Ultra-Light Dark Matter paradigm with Pulsar Timing Arrays

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Cosmology in Miramare, 28 August 2023

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# Outline

- Pulsar Timing Arrays (PTAs)
  - What are PTAs and what do they do?
  - Timing Model (`TEMPO2`)
  - How to look for a signal in PTAs?
- Ultra-Light Dark Matter (ULDM)
  - Brief introduction and motivation
  - ULDM in Galaxies
- PTAs constraints on ULDM (`Enterprise`)
  - Gravitationally coupled ULDM

# Pulsar Timing Arrays

# PTAs: identikit!

PTAs are basically collections of pulsars which are used to search for "special" signatures in their pulse time of arrivals (TOAs). They are composed of milli-second pulsars (MSPs), the most precise celestial clocks. PTAs can be used for a variety of things: characterization of the noise of the pulsars, stochastic gravitational wave background, constraints on primordial black holes etc... Here, we discuss how PTAs can give us some infos about Ultra-Light Dark Matter (ULDM).

# PTAs: Timing Model

Challenge: relate the observed time of arrivals (TOAs) at the observatory to the time of emission at the pulsar. (Edwards et al. 2006)

Convenient to single out three main contributions:

$$t_e^{psr} = t_a^{obs} - \Delta_\odot - \Delta_{IS} - \Delta_B$$

# PTAs: Timing Model - First Piece

$$\boxed{\begin{array}{l} t_a^{SSB} = t_a^{obs} - \Delta_\odot \\ \Delta_\odot = \Delta_A + \Delta_{R\odot} + \Delta_P + \Delta_{D\odot} + \Delta_{E\odot} + \Delta_{S\odot} \end{array}} \quad (1)$$

- $\Delta_A$: *Atmospheric delay*, takes into account *e.g.* refractivity in the troposphere and ionosphere;
- $\Delta_{R\odot}$: *Roemer delay*, vacuum delay between arrival of the pulse at the obervatory and SSB;
- $\Delta_P$: *Parallax delay*, delay between SSB and Earth due to spherical wavefronts;
- $\Delta_{D\odot}$: Solar system dispersion due to Solar wind;
- $\Delta_{E\odot}$: *Einstein delay*, takes into account *e.g.* gravitational redshift by Earth potential;
- $\Delta_{S\odot}$: *Shapiro delay* caused by the planets in the Solar system

# PTAs: Timing Model


So, in conclusion, we have a recipe:

- Take the observed TOAs;
- Fit them accounting for the effects described before;
- Fine! You have your full-fledged timing model and your best-fit parameters!

Unfortunately (or fortunately!) things are not so easy...

# How to look for a signal in PTAs

The main observable in a PTA experiment is the timing residuals, $\vec{\delta t}$, which measure the discrepancy between the observed times of arrival (TOAs) and the ones predicted by the pulsar timing model. In general, each process will affect the timing residuals in a peculiar way. Qualitatively,

$$\vec{\delta t} = \mathbf{M}\vec{\epsilon} + \overrightarrow{W.N.} + \overrightarrow{R.N.} + ? \tag{2}$$

In order to look for a signal in PTAs, we should model how it affects the timing residuals!

# ULDM

# ULDM: where it stands

The CDM paradigm has some well-known issues, for example (Robles et al. 2018):

- *cusp/core problem*
- *Missing satellite problem*
- *Lower-than-expected central densities*

These problems might be alleviated invoking baryonic physics, *e.g.* gravitational stirring by SNe. But in dwarf spheroidal galaxies?

- Need to invoke another mechanism $\rightarrow$ ULDM

# ULDM: which one?

We focus on a scalar field with mass $m \sim 10^{-22}\mathrm{eV}$, whose de Broglie wavelength acts as a *quantum pressure* that suppresses power on very small scales. In formulae:

$$\frac{\lambda_{\mathrm{dB}}}{2\pi} = \frac{\hbar}{mv} \approx 60\mathrm{pc}\left(\frac{10^{-22}\mathrm{eV}}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

# ULDM: some formulae

This behaviour can be easily seen by solving the relevant set of equations, namely the *Schroedinger-Poisson* system:

$$\begin{aligned} i\hbar\frac{\partial\psi}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\psi + mV\psi \\ \nabla^2 V &= 4\pi G(\rho - \bar{\rho}) \end{aligned} \tag{3}$$

Solving this numerically, we have a soliton-like behaviour at the centre and a NFW-like behaviour in the outskirts.

# PTAs constraints on ULDM

# ULDM: Classical Wave

In the following, we will think of ULDM as a free scalar field. Due to the huge occupation number (Khmelnitsky and Rubakov, 2013), the ULDM field can be thought as a collection of *classical waves*

$$\phi(\mathbf{x}, t) = A(\mathbf{x})\cos(mt + \alpha(\mathbf{x}))$$

Energy momentum tensor:

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\left((\partial\phi)^2 - m^2\phi^2\right)$$

from which

$$\rho_{DM} \equiv T_{00} = \frac{1}{2}m^2A^2$$

# ULDM: Gravitational interaction

To find the gravitational field produced by ULDM, we can write (Newtonian gauge)

$$ds^2 = (1 + 2\Phi(\mathbf{x}, t))dt^2 - (1 - 2\Psi(\mathbf{x}, t))\delta_{ij}dx^i dx^j$$

We can split the potentials in $t$-independent and $t$-dependent part

$$\Psi(\mathbf{x}, t) \simeq \Psi_0(\mathbf{x}) + \Psi_c(\mathbf{x})\cos(\omega t + 2\alpha(\mathbf{x})) + \Psi_s(\mathbf{x})\sin(\omega t + 2\alpha(\mathbf{x}))$$

From the trace of the $ij$ components of Einstein equations

$$-6\ddot{\Psi} + 2\Delta(\Psi - \Phi) = 8\pi G T_{kk}$$

we get:

- $\Psi_0 = \Phi_0$;
- $\Psi_c = \frac{1}{2}\pi G A(\mathbf{x})^2 = \pi \frac{G\rho_{DM}(\mathbf{x})}{m}^2$
- $\Psi_s = 0$

# ULDM: Gravitational delay

Now, remember what we wrote before:

"In order to look for a signal in PTAs, we should model how it affects the timing residuals!"

$$\Delta t(t) = -\int_0^t \frac{\Omega(t') - \Omega_0}{\Omega_0} dt'$$

Skipping a little bit of details (Khmelnitsky and Rubakov, 2013), it turns out that(Porayko, 2018):

$$\Delta t(t) = \frac{\Psi_c(x_e)}{2\pi f} \sin\left[2\pi f t + 2\alpha(x_e)\right] - \frac{\Psi_c(x_p)}{2\pi f} \sin\left[2\pi f\left(t - \frac{d_p}{c}\right) + 2\alpha(x_p)\right] \tag{4}$$

# ULDM: correlated and uncorrelated limit

The stochastic nature of the axion field tells us that:

$$\Delta t(t) = \frac{\Psi_c(x_e)}{2\pi f}\phi_E^2 \sin\left[2\pi f t + \theta_E\right] - \frac{\Psi_c(x_p)}{2\pi f}\phi_P^2 \sin\left[2\pi f t + \theta_P\right] \quad (5)$$

On scales smaller than the de-Broglie wavelength, the ULDM scalar field oscillates coherently, with the same $\phi^2$. Therefore, we have:

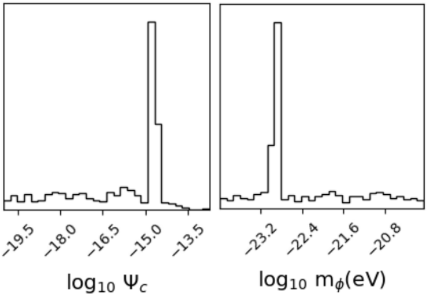
- *correlated limit*: de Broglie wavelength larger than inter-pulsar separation. $\phi_E^2 = \phi_P^2$
- *uncorrelated limit*: de Broglie wavelength smaller than inter-pulsar separation $\phi_E^2 \neq \phi_P^2$

where the de Broglie wavelength is

$$l_c \approx \frac{2\pi}{m_\phi v_\phi} \approx 0.4 \text{ kpc} \left(\frac{10^{-22} \text{ eV}}{m_\phi}\right) . \quad (6)$$

# ULDM: Gravitational interaction - Proof of principle

We might wonder whether the recently observed signal can be explained within the ULDM paradigm. If we do so, then

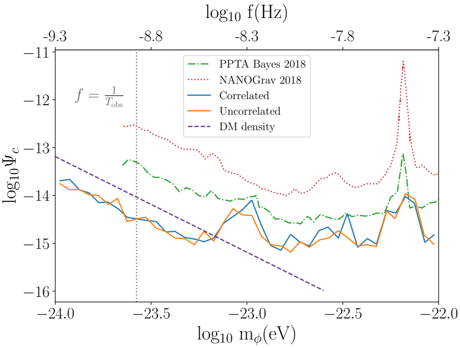


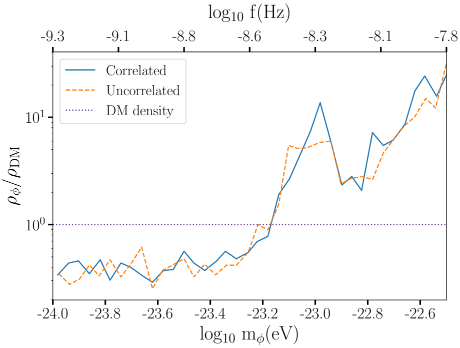

But ULDM gives monopolar, no quadrupolar correlation... so maybe we should *assume* that the signal comes from somewhere else and, *on top of that*, look for ULDM.

# ULDM: Gravitational interaction - parameters table

| Parameter | Description | Prior | Occurrence |
|---|---|---|---|
| | White Noise ($\sigma = E_f^2 \sigma_{TOA}^2 + E_q^2$) | | |
| $E_f$ | EFAC per backend/receiver system | Uniform $[0, 10]$ | 1 per pulsar |
| $E_q$ | EQUAD per backend/receiver system | $\text{Log}_{10}$-Uniform $[-10, -5]$ | 1 per pulsar |
| | Red Noise | | |
| $A_{red}$ | Red noise power-law amplitude | $\text{Log}_{10}$-Uniform $[-20, -6]$ | 1 per pulsar |
| $\gamma_{red}$ | Red noise power-law spectral index | Uniform $[0, 10]$ | 1 per pulsar |
| | ULDM | | |
| $\Psi_c$ | ULDM signal amplitude | $\text{Log}_{10}$-Uniform $[-20, -12]$ | 1 for PTA |
| $m_\phi$ [eV] | ULDM mass | $\text{Log}_{10}$-Uniform $[-24, -22]$ | 1 for PTA |
| $\hat{\phi}_E^2$ | Earth factor | $e^{-x}$ | 1 for PTA |
| $\hat{\phi}_P^2$ | Pulsar factor | $e^{-x}$ | 1 per pulsar |
| $\gamma_E$ | Earth signal phase | Uniform $[0, 2\pi]$ | 1 per PTA |
| $\gamma_P$ | Pulsar signal phase | Uniform $[0, 2\pi]$ | 1 per pulsar |
| | Common spatially Uncorrelated Red Noise (CURN) | | |
| $A_{GWB}$ | Common process strain amplitude | $\text{Log}_{10}$-Uniform $[-20, -6]$ | 1 for PTA |

# ULDM: Gravitational interaction - some plots




Compatible with Lyman-$\alpha$

# Conclusions and Prospects

- PTAs are wonderful laboratories to test signatures in signals coming from pulsars;
- It is possible to constrain ULDM density below the predicted abundance;
- Possibility to extend the bounds to ULDM interacting with matter;
- You want to test some models? Feel free to ask!

# APPENDIX

# PTAs: Timing Model - Second Piece

$$\boxed{\begin{aligned} t_a^{BB} &= t_a^{SSB} - \Delta_{IS} \\ \Delta_{IS} &= \Delta_{VP} + \Delta_{ISD} + \Delta_{ES} \end{aligned}} \tag{7}$$

- $\Delta_{VP}$: *Vacuum propagation delay*
- $\Delta_{ISD}$: *Interstellar dispersion delay*;
- $\Delta_{ES}$: *Einstein delay*, takes into account special relativistic effects due to relative motion of SSB to BB;

# PTAs: Timing Model - Third Piece

$$\boxed{\begin{array}{l} t_e^{psr} = t_a^{BB} - \Delta_B \\ \Delta_B = \Delta_{RB} + \Delta_{AB} + \Delta_{EB} + \Delta_{SB} \end{array}} \quad (8)$$

- $\Delta_{RB}$: *Roemer delay* related to binary motion;
- $\Delta_{AB}$: *Aberration* related to transverse motion of the pulsar with respect to the observer;
- $\Delta_{EB}$: *Einstein delay*, takes into account relativistic effects from BB to the pulsar;
- $\Delta_{SB}$: *Shapiro delay* due to strong gravitational field of binary companion.

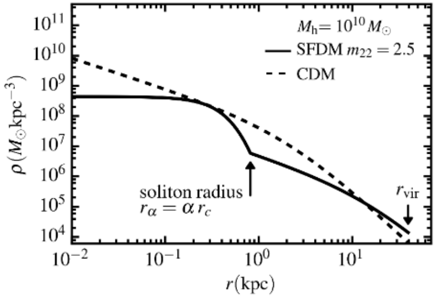
# ULDM: some formulae

Schive et al. (2014) found that the central core region was well fit by the density profile

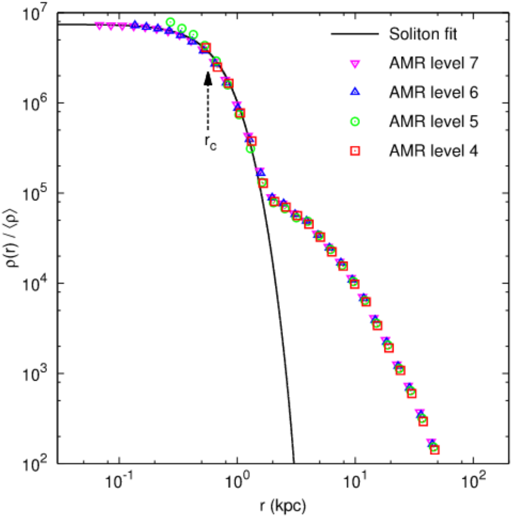
$$\rho_{sol}(r) = \frac{\rho_c}{\left(1 + 0.091\left(\frac{r}{r_c}\right)^2\right)^8}$$

$$\rho_c = 1.93 \times 10^7 m_{22}^{-2} \left(\frac{r_c}{1\mathrm{kpc}}\right)^{-4} M_\odot \mathrm{kpc}^{-3}$$

$$r_c = 1.6\mathrm{kpc} \left(\frac{M_\mathrm{h}}{10^9 M_\odot}\right)^{-1/3} m_{22}^{-1}$$



# ULDM: soliton profile



# Coupling to Matter: Prior Set

| Parameter | Description | Prior | Comments |
|---|---|---|---|
| | | **White Noise** | |
| $E_\mu$ | EFAC per backend/receiver system | Uniform $[0.1, 5]$ | one parameter per pulsar |
| $Q_\mu$ [s] | EQUAD per backend/receiver system | log-Uniform $[-8.5, -5]$ | one parameter per pulsar |
| | | **Red Noise** | |
| $A_{\rm red}$ | red noise power-law amplitude | log-Uniform $[-20, -11]$ | one parameter per pulsar |
| $\gamma_{\rm red}$ | red noise power-law spectral index | Uniform $[0, 7]$ | one parameter per pulsar |
| | | **ULDM** | |
| $A_i$ | ULDM signal amplitude | log-Uniform $[-20, -14]$ | one parameter for PTA |
| $m_\phi$ [eV] | ULDM mass | log-Uniform $[-24, -19]$ | one parameter for PTA |
| $\hat{\phi}^2_E$ | Earth normalized signal amplitude | $e^{-x}$ | one parameter per PTA |
| $\hat{\phi}^2_P$ | pulsar normalized signal amplitude | $e^{-x}$ | one parameter per pulsar* |
| $\gamma_E$ | Earth signal phase | Uniform $[0, 2\pi]$ | one parameter per PTA |
| $\gamma_P$ | pulsar signal phase | Uniform $[0, 2\pi]$ | one parameter for pulsar |