

# Particle Source

## Tutorial 2020

Version 2 - corrected

### 1. Thermal emittance from an ion source

A  $H^+$  beam is created from a plasma with a round aperture of  $R=5$  mm radius. Particles are assumed to be extracted uniformly from the disk surface.

1.1 Show that  $\sigma_x = \frac{R}{2}$ .

1.2 The ion temperature is 300K. The  $H^+$  kinetic energy is 100 keV. Calculate the ion beam normalized emittance (see lecture p.15).

### 2. Cyclotron frequency

We propose to demonstrate the formula :  $B\rho = mv/q$ . Consider a particle of mass  $m$  and charge  $q$  located at  $\vec{OM}(0, -L, 0)$  and with velocity  $\vec{v}(v, 0, 0)$  at time  $t=0$  in a cartesian frame  $(0, x, y, z)$ . The magnetic field is uniform with  $\vec{B}(0, 0, B)$ .

2.1 Solve Newton's Law and find that :

$$v_x(t) = v \cdot \cos(\omega t), v_y(t) = -v \cdot \sin(\omega t), v_z(t) = 0 \text{ and } \omega = \frac{qB}{m}$$

2.2 Integrate to find the trajectory:

$$x(t) = \rho \cdot \sin(\omega t), y(t) = -\rho - L + \rho \cdot \cos(\omega t), z(t) = 0, \rho = v/\omega$$

2.3 deduce that :  $B\rho = mv/q$

2.4 what is the electron cyclotron frequency (in Hz) in a field of 1 Tesla

2.5 What magnetic field is needed to curve an electron of 100 keV kinetic energy in a dipole with a 1m bending radius

### 3. Charge breeding

A  $1+$  ion source delivers a  $^{39}K$  beam. The ion source potential is set to 10 kV.

3.1 What is the ion velocity? Is it relativistic?

3.2 The ions are injected into an ECR charge breeder whose plasma is set to the high voltage of 9,90 kV. At what energy do the  $^{39}K$  ion pass into the plasma? What is their velocity?

3.3 The mean electron energy in the plasma is 90 eV. The ionization potential to pass from  $^{39}K^+$  to  $^{39}K^{2+}$  is  $P_i = 31.63$  eV. The number of electrons on the outer ion shell is  $g_i = 6$ . Calculate the electron impact cross section  $\sigma$  of  $^{39}K^+$  to form  $^{39}K^{2+}$ . (see formula in the table below).

3.4 The plasma electron density is  $n=10^{12} \text{ cm}^{-3}$ . Calculate The mean free path  $\lambda = \frac{1}{\sigma n}$  associated with the collision. The plasma length is  $L=50 \text{ cm}$ . Demonstrate that the number of remaining  $^{39}\text{K}^+$  beam after the crossing of the plasma is :

$$N(L) = N(0)e^{-L/\lambda}$$

And deduce the ratio of  $\text{K}^+$  converted to  $\text{K}^{2+}$

Table 1: Electron impact formula

Lotz formula: Ion-Electron impact cross section from charge state q to q+1	$\sigma_{i \rightarrow i+1} \sim 4.5 \times 10^{-14} g_i \frac{\ln\left(\frac{T}{P_i}\right)}{TP_i}$	$\text{cm}^2$
Incoming Electron Kinetic Energy	$T$	eV
Atom Ionization potential to switch from charge state i to i+1	$P_i$	eV
Number of electrons on the outer shell	$g_i$	