TUTORIAL 2020 solution in ascii format

Exercice 1 :

Uniformly Distributed Random Points Inside a Circle or radius R with the cartesian coordinates :

$$f(x,y) = \begin{cases} \frac{1}{\pi R^2}, \forall x^2 + y^2 \leq R^2 \\ 0 \; elsewhere \end{cases}$$

Other method when changing to r,θ coordinates :

$$x = rcos\theta \; y = rsin\theta$$

We have

$$r = \sqrt{x^2 + y^2}\,, \theta = \arctan(\frac{y}{x})$$

$$I = \iint_{x^2+y^2 \leq R^2} f(x,y)dxdy = \iint_{r=0\; \theta=0}^{r=R,\;\; \theta=2\pi} g(r,\theta)drd\theta = 1$$

$$dx = -rsin\theta d\theta + cos\theta dr, dy = rcos\theta d\theta + sin\theta dr$$

$$at\; first\; order, dxdy = (-rsin^2\theta + rcos^2\theta)drd\theta$$

So $I = \iint_{x^2+y^2 \leq R^2} f(x,y)dxdy = \iint_{\theta=0,r=0}^{\theta=2\pi,r=R} \frac{1}{\pi R^2} r(\cos^2\theta - \sin^2\theta\,)drd\theta$

But $\cos^2\theta - \sin^2\theta = 1 - 2sin^2\theta = 1 - sin2\theta$

And $\int_{\theta=0}^{\theta=2\pi} sin2\theta = 0$

So

$$I = \iint_{r=0\;\theta=0}^{r=R,\;\;\theta=2\pi} \frac{r}{\pi R^2} drd\theta$$

$$Probability\; density\; function : \; P(r,\theta) = \frac{r}{\pi R^2}$$

$$\langle x \rangle = \int_{r=0}^{R}\int_{\theta=0}^{\theta=2\pi} \frac{xr}{\pi R^2} drd\theta = \int_{r=0}^{R}\int_{\theta=0}^{\theta=2\pi} \frac{r^2 cos\theta}{\pi R^2} drd\theta = 0$$

$$\langle x^2 \rangle = \int_{r=0}^{R}\int_{\theta=0}^{\theta=2\pi} \frac{x^2 r}{\pi R^2} drd\theta = \int_{r=0}^{R}\int_{\theta=0}^{\theta=2\pi} \frac{r^3\cos^2\theta}{\pi R^2} drd\theta = \frac{1}{\pi R^2}[\frac{r^4}{4}]_0^R \int_{\theta=0}^{\theta=2\pi} \frac{1}{2}(cos2\theta + 1)d\theta$$

$$\langle x^2 \rangle = \frac{1}{\pi R^2}\left[\frac{r^4}{4}\right]_0^R \frac{1}{2}[sin2\theta + \theta]_{\theta=0}^{\theta=2\pi} = \frac{R^4}{8\pi R^2} 2\pi = \frac{R^2}{4}$$

$$\sigma_x = \sqrt{\langle x^2 \rangle} = \frac{R}{2}$$

1.2 $\epsilon_{\mathrm{N}} = \frac{R}{2}\sqrt{\frac{k_B T}{m_p c^2}}$

$k_B T = 2{,}75 \times 10^{-11} eV$

$m_p c^2 = 938{,}272 \times 10^6 eV$

$\frac{R}{2} = 0{,}0025$ m

$\epsilon_{\mathrm{N}} = 1{,}31 \times 10^{-8}\, m.rad$

$\epsilon_{\mathrm{N}} = 1{,}31 \times 10^{-2}\, mm.mrad$

$\epsilon_{\mathrm{N}} = 4{,}17 \times 10^{-3}\, \pi.mm.mrad$

2. cyclotron frequency

2.1

$$\begin{cases} \dfrac{dv_x}{dt} = \omega v_y \quad (eq.1) \\ \dfrac{dv_y}{dt} = -\omega v_x \; (eq.2) \\ \dfrac{dv_z}{dt} = 0 \qquad (eq.3) \end{cases}$$

$$(eq.3)\; and\; v_z(t=0) = 0 => \; v_z(t) = 0\,; => z(t) = const. = z(0) = 0$$

(eq.2) in $\frac{d}{dt}(eq.1)$ => $\frac{d^2 v_x}{dt^2} = -\omega v_x$

General solution : $v_x(t) = Acos(\omega t) + Bsin(\omega t)$

$v_x(0) = v => A = v$

(eq. 1)=>$v_y = \frac{1}{\omega}\frac{dv_x}{dt} = -vsin(\omega t) + Bcos(\omega t)$

$v_y(0) = 0 => B = 0$

so

$$\begin{cases} v_x(t) = vcos(\omega t) \\ v_y(t) = -vsin(\omega t) \\ v_z(t) = 0 \end{cases}$$

2.2

$$x(t) - x(0) = \int_0^t vcos(\omega t)dt = \frac{v}{\omega}\sin(\omega t) - 0$$

$$\rho = \frac{v}{\omega}\; and\; x(t) = \rho\sin(\omega t)$$

$y(t) - y(0) = \int_0^t -vsin(\omega t)dt = \frac{v}{\omega}\cos(\omega t) - \frac{v}{\omega}$

$y(t) + L = \rho\cos(\omega t) - \rho$

2.3 $v = \rho\omega = \frac{\rho q B}{m}$ => $B\rho = \frac{mv}{q}$ (non relativistic case)

2.4 cyclotronic frequency of an electron :

$\omega = 2\pi f$ and $f = 28\ GHz$

3. Charge breeding

Atom mass is $M = A.amu = 39 \times 941{,}49 \times 10^6 eV/c^2$

Constant of motion : Kinetic energy (T) +potential energy (qV) =constant

$$T + qV = const. = E$$

1) In the 1+ source at potential V_1 , the ion energy is such that $E = qV_1 = 10\ keV$ and $T_1 = 0$
2) After ion extraction, In the beam pipe at ground (so $V_2 = 0$ there), $T_2 + 0 = E$
   => $T_2 = 10\ keV$

$$T_2 = (\gamma - 1)Mc^2$$

$$\gamma = 1 + \frac{T_2}{Mc^2}$$

$\beta = \sqrt{1 - 1/\gamma^2} = 0{,}000742$ NON RELATIVISTIC

$$v = 2{,}23 \times 10^5 m/s$$

3) In the booster at potential $V_3 = 9{,}9\ keV$, the ion energy is such that :

$$T_3 + qV_3 = E = 10\ keV$$

$$T_3 = 100\ eV$$

$v = \sqrt{\frac{2T_3}{M}} = 2{,}23 \times 10^4$ m/s

3.3

$$\sigma\ \sim 4.5 \times 10^{-14}\ g_i\ \frac{ln\left(\frac{T}{P_i}\right)}{TP_i}$$

$T = 90\ eV, P_i = 31{,}63\ eV, g_i = 6$

$$\sigma\ = 4{,}31 \times 10^{-17} cm^2$$

$$n = 10^{12}\ cm^{-3}$$

$$\lambda = \frac{1}{\sigma n} = 231{,}7\ m$$

Coordinate x along the plasma. At x=0 the ions enter in the plasma, the initial number of ions there is $N(0) = N_0$

The number of remaining beam after a thickness x is N(x)

The number of collisions between the plasma of density n and the remaining ions at position x, in a slice of thickness dx is : $Ncollision = N(x).n.\sigma.dx$

N(x) is a decreasing function with x

So $N(x + dx) - N(x) = dN = -N(x)n\sigma dx = -\mathrm{N(x)}\frac{\mathrm{dx}}{\lambda}$

- ⇨ $\frac{dN}{N} = -\frac{dx}{\lambda} => N(x) = N(0)e^{-x/\lambda}$

  At x=L=0,5 m : $\frac{N(L)}{N(0)} = 0{,}9978$

  Conservation of the number of ions : $N(0) = N^{1+}(x) + N^{2+}(x)$

$$\frac{N^{2+}(x)}{N(0)} = 1 - e^{-x/\lambda}$$

$$\frac{N^{2+}(L)}{N(0)} = 1 - e^{-\frac{L}{\lambda}} = 0{,}0022$$