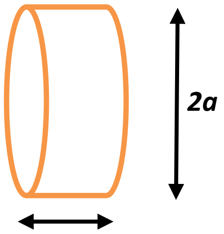
$\mu = \mu_0 \, \mu_r$
$\mu_0 = 4\pi \cdot 10^{-7}$ Vs/(Am)
$\varepsilon = \varepsilon_0 \, \varepsilon_r$
$\varepsilon_0 = 8.854 \cdot 10^{-12}$ As/(Vm)
$c_0 = 3 \cdot 10^8$ m/s

JUAS 2019 – RF Exam

Name: ________________________________ Points: ______ of 20 (25 with bonus points)
Utilities: JUAS RF Course 2019 lecture script, personal notes,
pocket calculator, ruler, compass, and your brain!
(No cell- or smartphone, no iPad, laptop, or wireless devices, no text books or any other tools!!!)
Please compute and write your formulas and results clear and readable, if appropriate on a separate sheet of paper. Any unreadable parts are considered as wrong.



1. Understanding the "Pillbox" Cavity (6+½ points)

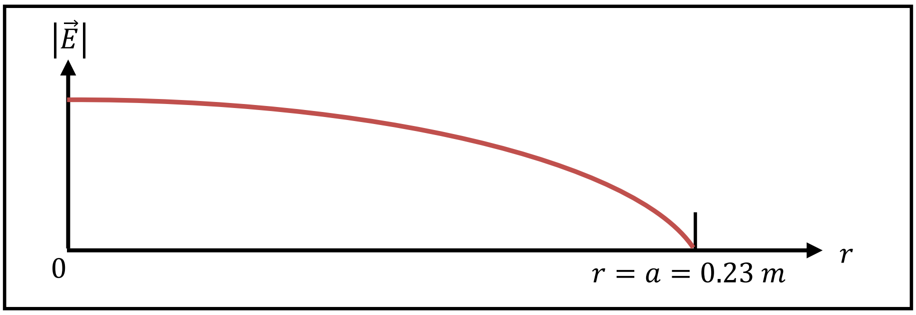
An unloaded, simple cylindrical "pillbox" cavity has a radius of $a = 0.23\ m$, and is of height h. For the prototype tests the cavity is operated in air, the beam-pipe ports are neglected.

a) What is the fundamental mode of the cavity, used to accelerating particles? Determine the resonance frequency of the fundamental (accelerating) mode? (½ point)

Fundamental mode: TM_{010} or E_{010}

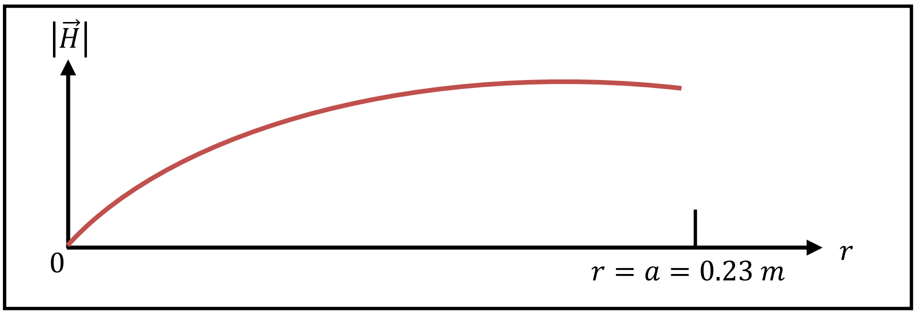
$$a = 0.383\lambda = 0.383\frac{c_0}{f_{TM010}} \Rightarrow f_{TM010} = \frac{0.383}{a} = 500\ MHz$$

b) Indicate qualitatively (as graph below) the electric field strength as function of the radius $|\vec{E}| = f(r)$ of the fundamental mode. It is proportional to which function? (½ point)



Bessel function of 0th order

c) Indicate qualitatively (as graph below) the magnetic field strength as function of the radius $|\vec{H}| = f(r)$ of the fundamental mode. It is proportional to which function? (½ point)



d) Which are the dominant electric and magnetic field components of the fundamental (accelerating) mode (in cylindrical coordinates r, φ, z)? (½ point)
Hint: There is only one dominant component for each field type, one for the electric field and one for the magnetic field.

E_z and H_φ

e) The cavity should be made out of stainless steel, which has a conductivity of $\sigma_{SS} = 1.32 \cdot 10^6\ S/m.$
Which has to be the value of the cavity height h to achieve a Q-value of approximately $Q \approx 6000$? (1 point)

$$\delta_{SS} = \sqrt{\frac{2}{\omega_{TM010}\sigma_{SS}\,\mu}} = \sqrt{\frac{2}{2\,\pi\, f_{TM010}\ \sigma_{SS}\ \mu_0}} = 19.6\ \mu m$$

$$h = \frac{a}{\frac{a}{\delta Q} - 1} = 0.241\ m$$

f) What is the "R-over-Q" of this fundamental mode? (½ point)
Can we use the approximate equation to compute R-over-Q?

$$\frac{h}{a} = 1.05\ > 0.5\ \ \text{the approximation cannot be used!}$$

$$\frac{R}{Q} = 128\frac{\sin^2\left(1.2024\,{}^{h}/_{a}\right)}{{}^{h}/_{a}} = 110.7$$

g) Determine the lumped elements R, L, and C of the equivalent parallel R-L-C circuit. (1½ points)

$$R = \frac{R}{Q}Q = 664.2\ k\Omega$$

$$L = \frac{{}^{R}/_{Q}}{2\,\pi\, f_{TM010}} = 35.3\ nH$$

$$C = \frac{1}{{}^{R}/_{Q}\ \ 2\,\pi\, f_{TM010}} = 2.88\ pF$$

h) Which is the frequency of the closest higher-order mode? (1 point)
Of which type is that mode?
*Hint: Make use of the **Mode chart for a Pillbox cavity – Version 1***

$$\frac{h}{2a} = 0.524$$

from the mode chart follows:

$$\frac{\lambda_{TM010}}{2a} = 1.3 \text{ and } \frac{\lambda_{TE111}}{2a} \approx 0.89$$

$$\Rightarrow \quad f_{TE111} = \frac{c_0}{2a \cdot 0.89} = 730\ MHz$$

The mode is of type TE_{111}

i) **Bonus question:**
What is the unloaded-Q value of the closest higher-order mode? (½ point)

$$Q_{TE111} = 0.206 \frac{\lambda_{TE111}}{\delta_{SS}} \frac{\left[1 + 0.73\left(\frac{2a}{h}\right)^2\right]^{3/2}}{1 + 0.22\left(\frac{2a}{h}\right)^2 + 0.51\left(\frac{2a}{h}\right)^3} = 5428$$

2. Multiple choice (4 points)

Tick ***one*** correct answer like this: ☒.

1. An amplifier is a
   - ○ passive 2-port
   - ○ reciprocal 2-port
   - ☒ non-reciprocal 2-port

2. A circulator is
   - ○ non-reciprocal and active
   - ○ reciprocal and active
   - ☒ non-reciprocal and passive

3. The measurement of the 3-dB bandwidth of a resonant mode of a cavity in transmission gives
   - ○ the unloaded Q-value Q_0
   - ☒ the loaded Q-value Q_L
   - ○ the external Q-value Q_{ext}

4. To reduce the physical length of a microstrip-line, by maintaining the electrical length, you can
   - ☒ increase the permittivity of the substrate material
   - ○ decrease the width of the microstrip-line
   - ○ reduce the conductivity of the microstrip-line metal

5. A RF signal of 1 V_{RMS} is divided by using an ideal, lossless 2-way power splitter (T-splitter). What is the signal level measured at each of the two ports which are terminated with loads:
   - ○ $1\ V_{RMS}$
   - ☒ $0.707\ V_{RMS}$
   - ○ $0.5\ V_{RMS}$

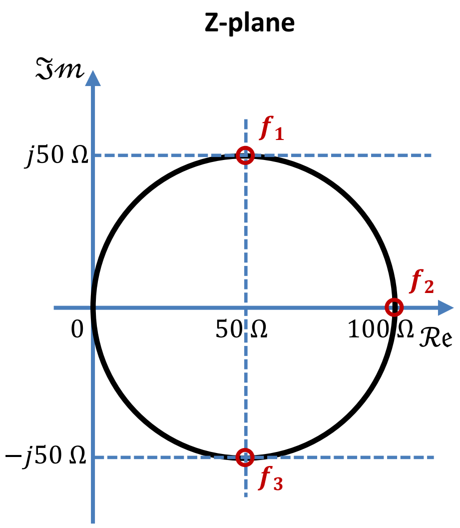
6. Which is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor?
   - ☒ TE
   - ○ TEM
   - ○ TM

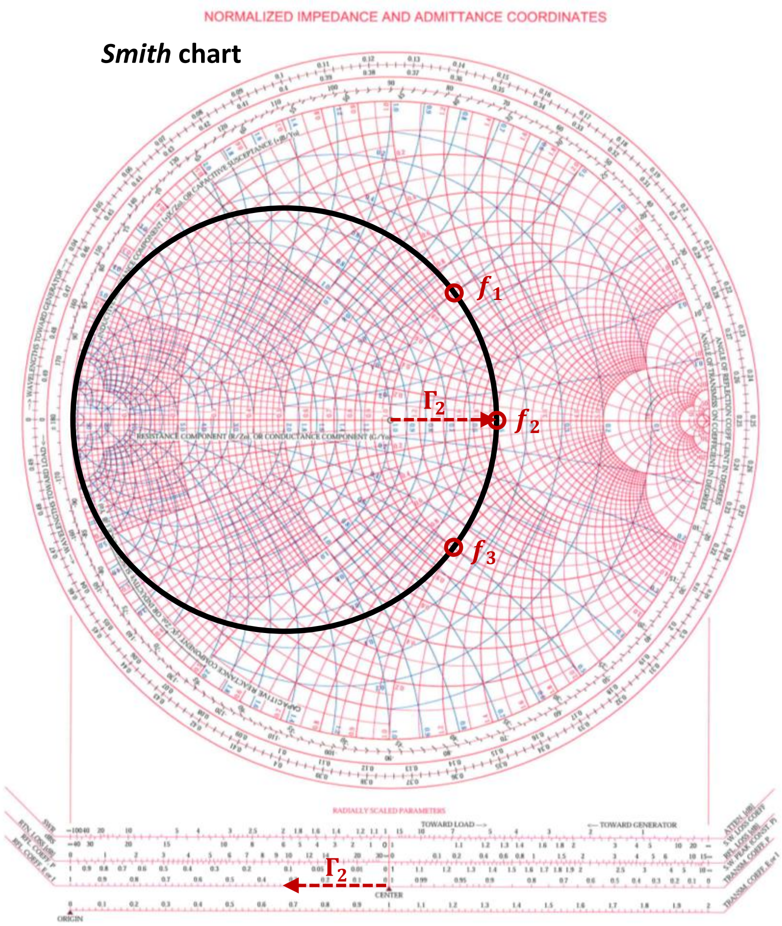
7. Which is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *with* inner conductor (coaxial line)?
   - ○ TE
   - ☒ TEM
   - ○ TM

8. Adding capacitive loading to a cavity
   - ○ increases the resonance frequency
   - ☒ decreases the resonance frequency
   - ○ does not change the resonance frequency

3. Smith chart (3+1½ points)

The locus of the impedance of a resonant mode is given in the complex impedance plane (Z-plane).





a) Transform the impedance points at the frequencies f_1, f_2 and f_3 into the Smith chart. For the impedance normalization, assume a reference impedance of $Z_0 = 50\ \Omega$. (½ point)

b) Indicate the locus of impedance in the Smith chart. (½ point)

c) Indicate the resonance frequency in both, in the Z-plane and in the Smith chart. (½ point)
(select between f_1, f_2 and f_3)

f_2

d) Indicate the 3-dB frequency points in both, in the Z-plane and in the Smith chart. (½ point)
(select between f_1, f_2 and f_3)

f_1, f_3

e) Is the resonant mode critical, under-critical or over-critical coupled? (½ point)
What is the reflection coefficient Γ at the resonance frequency?
(*Hint: Make use of the rulers at the bottom of the Smith chart*)

The resonance mode is over-critical coupled.

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{Z(f_2) - Z_0}{Z(f_2) + Z_0} = \frac{(100 - 50)\ \Omega}{(100 + 50)\ \Omega} = 0.333$$

f) Assume: $f_1 = 399.95\ MHz$, $f_2 = 400.00\ MHz$ and $f_3 = 400.05\ MHz$. (½ point)
Compute the Q-value of the resonance.

$$Q = \frac{f_0}{\Delta f} = \frac{f_2}{f_3 - f_1} = 4000$$

g) **Bonus question:** (1½ points)
The resonator is coupled by a loop which has a transformation ratio $k = 50$.
What are the values of a R-L-C equivalent parallel circuit?

$$R = R(f_2)\, k^2 = 100\ \Omega \cdot 50^2 = 250\ k\Omega$$

$$\frac{R}{Q} = 62.5$$

$$L = \frac{R/Q}{2\pi f_2} = 24.87\ nH$$

$$C = \frac{1}{2\pi f_2\, R/Q} = 6.37\ pF$$

4. S-Parameters (3+1 points)

Match the ideal S-parameters in matrix form to the corresponding components.

$$\boldsymbol{S_A} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \boldsymbol{S_B} = \begin{bmatrix} 0 & \frac{j}{2} & 0.866 & 0 \\ \frac{j}{2} & 0 & 0 & 0.866 \\ 0.866 & 0 & 0 & \frac{j}{2} \\ 0 & 0.866 & \frac{j}{2} & 0 \end{bmatrix} \quad \boldsymbol{S_C} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \boldsymbol{S_D} = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

a) Assign the S-matrices ($\boldsymbol{S_A}$ ... $\boldsymbol{S_D}$) to the components: (2 point)

component	6 dB directional coupler	transmission line, electrical length = λ/4	resistive power divider	circulator
S-matrix	$\boldsymbol{S_B}$	$\boldsymbol{S_D}$	$\boldsymbol{S_A}$	$\boldsymbol{S_C}$

b) Fill the missing dB (coupler) and λ (transmission-line) information (...). (1 point)

c) Bonus question: (1 point)
What has to done to modify a circulator into an isolator?

One of the ports has to be terminated with a load of the reference impedance.

What is the S-matrix of an ideal isolator?

$$S_{isolator} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

5. Coaxial cable (4+2 Points)

A transmission-line in form of a coaxial cable has to be investigated. The dielectric material is air, the characteristic impedance should be $Z_L = 60\ \Omega$. For the inner conductor a metallic rod of diameter $d = 2\ mm$ is available.

a) What is the inner diameter D of the outer conductor? (1 point)

$$Z_l = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\ln\left(\frac{D}{d}\right)}{2\pi\sqrt{\varepsilon_r}} \approx 60\ \Omega \frac{\ln\left(\frac{D}{d}\right)}{\sqrt{\varepsilon_r}} \Rightarrow D = e^1 d = 5.44\ mm$$

b) What is the phase velocity? (1 point)

$$v_{ph} = \frac{c_0}{\sqrt{\varepsilon_r}} = 3 \cdot 10^8\ m/s$$

c) Now the cable is filled with a dielectric material which has a relative permittivity of $\varepsilon_r = 2.25$. For this case, calculate the characteristic impedance and phase velocity. (2 points)

$$Z_l = \sqrt{\frac{\mu_0}{\varepsilon_0}} \frac{\ln\left(\frac{D}{d}\right)}{2\pi\sqrt{\varepsilon_r}} \approx 60\ \Omega \frac{\ln\left(\frac{D}{d}\right)}{\sqrt{\varepsilon_r}} = 40\ \Omega$$

$$v_{ph} = \frac{c_0}{\sqrt{\varepsilon_r}} = 2 \cdot 10^8\ m/s$$

d) **Bonus question:** (2 points)
To which maximum frequency (approximately) this cable with dielectric can be utilized for a

signal transmission free of higher-order modes?

With increasing frequency non-TEM modes can appear in the coaxial cable, the 1st mode is of TE_{11} nature, similar to that of a waveguide with circular cross-section:

$\lambda_{WG\_TE11} = \pi a$

The radius of the waveguide a has to be replace by the average radius of the coaxial conductor arrangement $(D+d)/2$:

$$\lambda_{coax\_TE11} = \pi \left(\frac{D+d}{2}\right)$$

$$\Rightarrow \quad f_{coax\_TE11} = \frac{v_{ph}}{\lambda_{coax\_TE11}} = \frac{c_0}{\sqrt{\varepsilon_r}} \frac{1}{\pi \left(\frac{D+d}{2}\right)} = 17.1\ GHz$$