$$\mu = \mu_0 \mu_r$$
$$\mu_0 = 4\pi \cdot 10^{-7}\, Vs/Am$$
$$\varepsilon = \varepsilon_0 \varepsilon_r$$
$$\varepsilon_0 = 8.854 \cdot 10^{-12}\, As/Vm$$
$$c_0 = 2.998 \cdot 10^{8}\, m/s$$

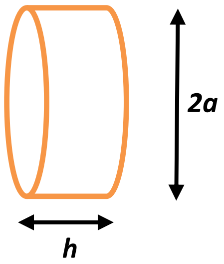
JUAS 2019 – RF Exam

Name: ________________________________ Points: ______ of 20 (25 with bonus points)

Utilities: JUAS RF Course 2019 lecture script, personal notes,
pocket calculator, ruler, compass, and your brain!

(No cell- or smartphone, no iPad, laptop, or wireless devices, no text books or any other tools!!!)

Please compute and write your formulas and results clear and readable, if appropriate on a separate sheet of paper. Any unreadable parts are considered as wrong.

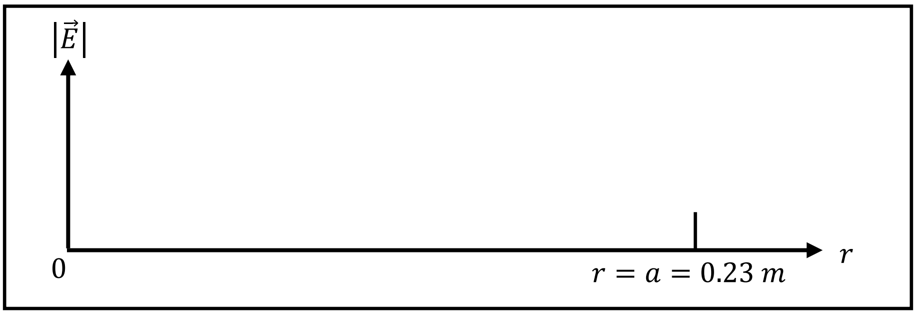


1. Understanding the "Pillbox" Cavity (6+½ points)

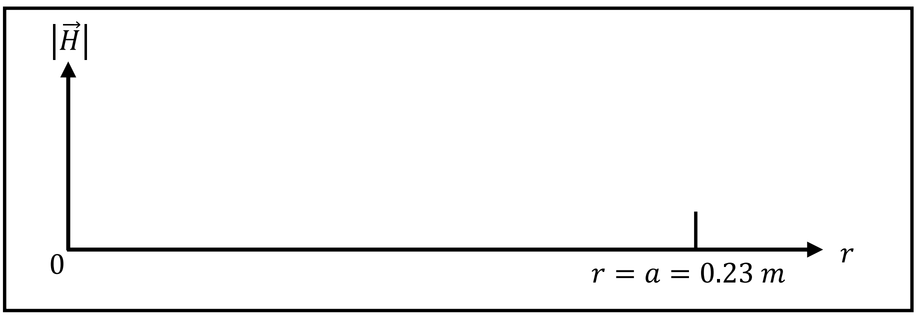
An unloaded, simple cylindrical "pillbox" cavity has a radius of $a = 0.23\ m$, and is of height h. For the prototype tests the cavity is operated in air, the beam-pipe ports are neglected.

a) What is the fundamental mode of the cavity, used to accelerating particles?
Determine the resonance frequency of the fundamental (accelerating) mode? (½ point)

b) Indicate qualitatively (as graph below) the electric field strength as function of the radius $|\vec{E}| = f(r)$ of the fundamental mode. It is proportional to which function? (½ point)



c) Indicate qualitatively (as graph below) the magnetic field strength as function of the radius $|\vec{H}| = f(r)$ of the fundamental mode. It is proportional to which function? (½ point)



d) Which are the dominant electric and magnetic field components of the fundamental (accelerating) mode (in cylindrical coordinates r, φ, z)? (½ point)
Hint: There is only one dominant component for each field type, one for the electric field and one for the magnetic field.

e) The cavity should be made out of stainless steel, which has a conductivity of $\sigma_{SS} = 1.32 \cdot 10^6\ S/m$.
Which has to be the value of the cavity height h to achieve a Q-value of approximately $Q \approx 6000$? (1 point)

f) What is the "R-over-Q" of this fundamental mode? (½ point)
Can we use the approximate equation to compute R-over-Q?

g) Determine the lumped elements R, L, and C of the equivalent RLC parallel circuit. (1½ points)

h) Which is the frequency of the closest higher-order mode? (1 point)
Of which type is that mode?
Hint: Make use of the ***Mode chart for a Pillbox cavity – Version 1***

i) **Bonus question:**
What is the unloaded-Q value of the closest higher-order mode? (½ point)

2. Multiple choice (4 points)

Tick ***one*** correct answer like this: ☒.

1. An amplifier is a
   - ○ passive 2-port
   - ○ reciprocal 2-port
   - ○ non-reciprocal 2-port

2. A circulator is
   - ○ non-reciprocal and active
   - ○ reciprocal and active
   - ○ non-reciprocal and passive

3. The measurement of the 3-dB bandwidth of a resonant mode of a cavity in transmission gives
   - ○ the unloaded Q-value Q_0
   - ○ the loaded Q-value Q_L
   - ○ the external Q-value Q_{ext}

4. To reduce the physical length of a microstrip-line, by maintaining the electrical length, you can
   - ○ increase the permittivity of the substrate material
   - ○ decrease the width of the microstrip-line
   - ○ reduce the conductivity of the microstrip-line metal

5. A RF signal of $1\ V_{RMS}$ is divided by using an ideal, lossless 2-way power splitter (T-splitter). What is the signal level measured at each of the two ports which are terminated with loads:
   - ○ $1\ V_{RMS}$
   - ○ $0.707\ V_{RMS}$
   - ○ $0.5\ V_{RMS}$

6. Which is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *without* inner conductor?
   - TE
   - TEM
   - TM

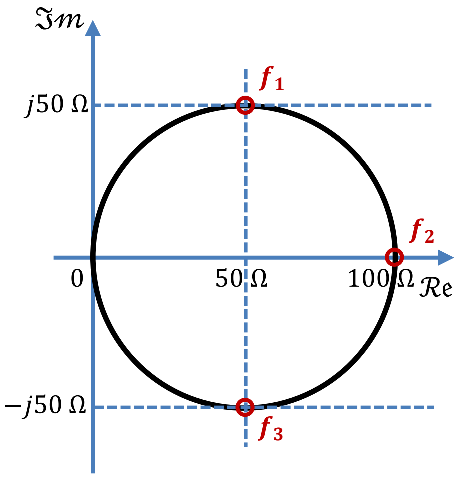
7. Which is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section *with* inner conductor (coaxial line)?
   - TE
   - TEM
   - TM

8. Adding capacitive loading to a cavity
   - increases the resonance frequency
   - decreases the resonance frequency
   - does not change the resonance frequency

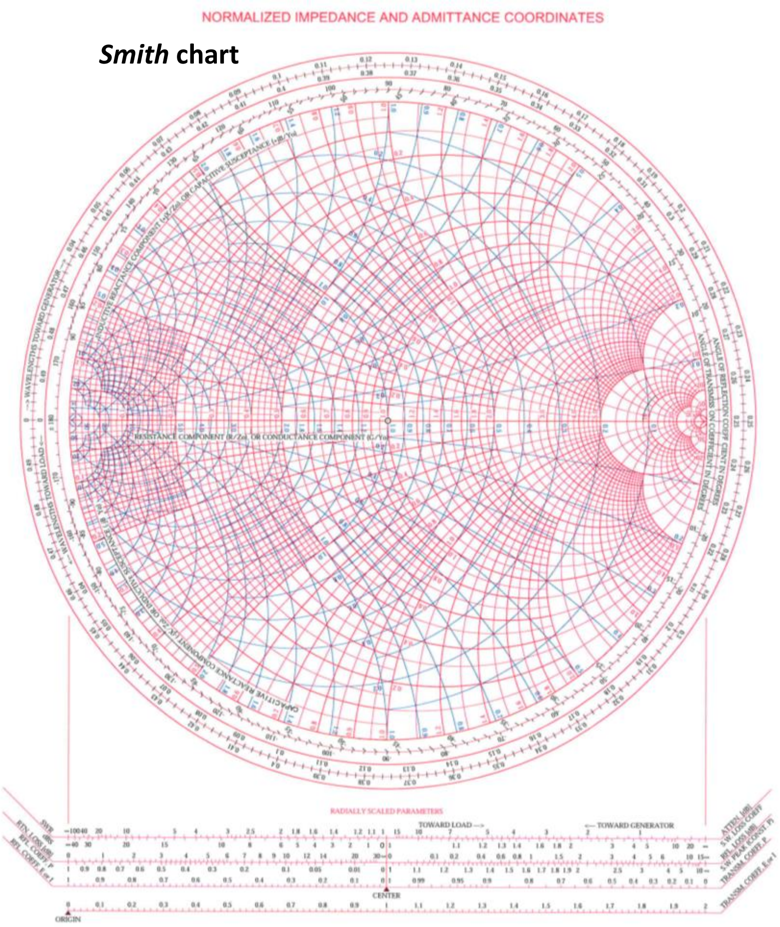
3. Smith chart (3+1½ points)

The locus of the impedance of a resonant mode is given in the complex impedance plane (Z-plane).

Z-plane



***Smith* chart**



a) Transform the impedance points at the frequencies f_1, f_2 and f_3 into the Smith chart. For the impedance normalization, assume a reference impedance of $Z_0 = 50\ \Omega$. (½ point)

b) Indicate the locus of impedance in the Smith chart. (½ point)

c) Indicate the resonance frequency in both, in the Z-plane and in the Smith chart. (½ point)
(select between f_1, f_2 and f_3)

d) Indicate the 3-dB frequency points in both, in the Z-plane and in the Smith chart. (½ point)
(select between f_1, f_2 and f_3)

e) Is the resonant mode critical, under-critical or over-critical coupled? (½ point)
What is the reflection coefficient Γ at the resonance frequency?
(*Hint: Make use of the rulers at the bottom of the Smith chart*)

f) Assume: $f_1 = 399.95\ MHz$, $f_2 = 400.00\ MHz$ and $f_3 = 400.05\ MHz$. (½ point)
Compute the Q-value of the resonance.

g) **Bonus question:** (1½ points)
The resonator is coupled by a loop which has a transformation ratio $k = 50$.
What are the values of a R-L-C equivalent parallel circuit?

4. S-Parameters (3+1 points)

Match the ideal S-parameters in matrix form to the corresponding components.

$$\boldsymbol{S_A} = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \boldsymbol{S_B} = \begin{bmatrix} 0 & \frac{j}{2} & 0.866 & 0 \\ \frac{j}{2} & 0 & 0 & 0.866 \\ 0.866 & 0 & 0 & \frac{j}{2} \\ 0 & 0.866 & \frac{j}{2} & 0 \end{bmatrix} \quad \boldsymbol{S_C} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \boldsymbol{S_D} = \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

a) Assign the S-matrices ($\boldsymbol{S_A} \dots \boldsymbol{S_D}$) to the components: (2 point)

component	… directional coupler	transmission line, electrical length = …	resistive power divider	circulator
S-matrix	$\boldsymbol{S_B}$	$\boldsymbol{S_D}$	$\boldsymbol{S_A}$	$\boldsymbol{S_C}$

b) Fill the missing dB (coupler) and λ (transmission-line) information (...). (1 point)

c) **Bonus question:** (1 point)
What has to done to modify a circulator into an isolator?
What is the S-matrix of an ideal isolator?

5. Coaxial cable (4+2 Points)

A transmission-line in form of a coaxial cable has to be investigated. The dielectric material is air, the characteristic impedance should be $Z_L = 60\ \Omega$. For the inner conductor a metallic rod of diameter $d = 2\ mm$ is available.

a) What is the inner diameter D of the outer conductor? (1 point)

b) What is the phase velocity? (1 point)

c) Now the cable is filled with a dielectric material which has a relative permittivity of $\varepsilon_r = 2.25$. For this case, calculate the characteristic impedance and phase velocity. (2 points)

d) **Bonus question:** (2 points)
To which maximum frequency (approximately) this cable with dielectric can be utilized for a signal transmission free of higher-order modes?

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES